Handbook of the History of Logic:

Mediaeval and Renaissance Logic

Volume 2

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CONTENTS

Preface vii
Dov M. Gabbay and John Woods

List of Contributors ix

Logic before 1100: The Latin Tradition 1
John Marenbon

Logic at the Turn of the Twelfth Century 65
John Marenbon

Peter Abelard and his Contemporaries 83
Ian Wilks

The Development of Supposition Theory in the Later 12th through 14th Centuries 157
Terence Parsons

The Assimilation of Aristotelian and Arabic Logic up to the Later Thirteenth Century 281
Henrik Lagerlund

Logic and Theories of Meaning in the Late 13th and Early 14th Century including the Modistae 347
Ria van der Lecq

The Nominalist Semantics of Ockham and Buridan: A 'rational reconstruction' 389
Gyula Klima

Logic in the 14th Century afer Ockham 433
Catarina Dutilh Novaes

Medieval Modal Theories and Modal Logic 505
Simo Knuuttila

Treatments of the Paradoxes of Self-reference 579
Mikko Yrjönsuuri

Developments in the Fifteenth and Sixteenth Centuries 609
E. Jennifer Ashworth

Relational Logic of Juan Caramuel 645
Petr Dvořák

Port Royal: The Stirrings of Modernity 667
Russell Wahl

Index 701

PREFACE

As modern readers will be aware, one of logic's most fruitful periods filled the comparatively small space from the mid-19^{th} century to the third quarter of the century that followed. While classical mathematical logic remains securely situated in the mainstream of the subject, alternative approaches arising from research programmes in computer science and philosophy have also flourished. It speaks to the intellectual gravitas of mathematical logic that it should have dominated utterly for a century and a quarter, but other developments have had longer runs, none more so than logic in the five centuries between the 11^{th} and 16^{th}. This is staying power of formidable heft, one of the main reasons for which is the rich scope of logical enquiry during this period. The same is true of modern symbolic logic. In the period of its exclusivity, it exhibited an agreeable complexity, with a reach that extends from classical treatments of proof theory, set theory, model theory and recursion theory, to variations that encompass intuitionist, modal, many-valued, relevant and dialogue logic, among others. No doubt this complexity is part of the explanation of the dominance of symbolic logic. If so, a similar explanation attaches to the staying power of mediaeval logic, whose range is easily as robust as its modern cousin. While the mediaeval tradition never abandoned the central fact of the syllogism as its organizing focus, it subjects this focus to the creative tensions of adaptation, extension and creative reinterpretation. All in all, the logicians of the middle ages rang the project of logic through virtually all the changes that characterize modern logic, at times rivaling the new developments in conceptual sophistication and theoretical robustness. The coverage of mediaeval logic is striking, extending from treatments of logical consequence to the intricate rules of logical disputations, from amalgamations of preceding traditions, whether Aristotelian or Arabic, to the development of new insights into quantification, from investigations of meaning to the exploration of modal contexts, from enquiry into the paradoxes to the stirrings of the logic of relations, and much more.

It is agreed that logic lost the vigour and high reputation achieved in the middle ages, as the Renaissance emerged as Europe's dominant intellectual and cultural force, but it is wrong to say that logic was sent into an impotent retirement in this period. It is more accurate to speak of hibernation, a period of quiescence during which logic was renewing itself for the challenge of a looming modernity. Astride the gap between Renaissance and the new world of learning is the work of the Port Royal logicians, included here for its role as a beacon. The Port Royal logicians disliked technical formalisms, and are in that respect the forbears of modern informal logic. But these same logicians did some of the best of the early work in probability theory, thus anticipating the rise of inductive logic.

It remains to be seen whether, on the score of intellectual power, methodological rigour and conceptual inventiveness, the present-day pre-eminence of logic will extend its reign another three hundred and twenty-five years. However we reckon the probabilities, suffice to say that if the longevity of mediaeval logic is indeed bested by our descendants, that will have been an accomplishment of supreme importance in the annals of human intellectual effort.

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THE LATIN TRADITION OF LOGIC TO 1100

John Marenbon

1 INTRODUCTION

The twelfth century, like the fourteenth, was arguably one of the great creative periods in the history of logic. The same is not true for period that precedes it, and forms the subject of this chapter: Latin logic in the early Middle Ages, up to 1100, and the Latin tradition from Cicero to Boethius, Cassiodorus and Isidore, on which, along with some translations of Aristotle and Porphyry, it drew. It had no great logicians, but only influential ones, like Boethius, and outstanding philosophers, like Eriugena and Anselm, who were interested in logic. Yet it is worth studying, for at least two, very different reasons. First, twelfth-century logicians such as Abelard worked in a tradition that continued from the earlier medieval one: the text-books were the same and the methods and questions asked were related. Knowing about the Latin tradition and the early medieval background helps in understanding and assessing their ideas. Second, although the early medieval logicians may not have innovated in the contents of their teaching, they achieved something importantly novel with regard to the position of logic in the curriculum.[1]

In the ancient world, logic had never been a central, or even mainstream, subject in ordinary education. Educated Romans would have studied, above all, rhetoric and the literary classics — Augustine gives a vivid account of such an education in his *Confessions*. But in the Middle Ages logic became a fundamental discipline for every student who went beyond the elementary level of studying Latin grammar. This central placing of logic had its origins in the revival of learning in the Carolingian period: logic was given a position within the standard scheme of education

[1] At the beginning of each section and subsection, I give references to the main secondary works on the area. Parts of the chapter are based on a series of studies I have published on Boethius and on early medieval logic over the last 25 years. I mention these as and where relevant in these initial lists, but I do not then repeat my reference to them, except where I wish to indicate some special discussion. Readers will, therefore, be able to find a fuller exposition of the views here by consulting these more detailed discussions. But in many areas (for example, Eriugena and Anselm), I have reached different conclusions from those I previously argued, and even where my views have remained more constant, I have taken the opportunity to correct errors, add new ideas and take account of the most recent scholarship. There has not been any previous attempt (outside short sections in general histories) to give a complete account of this long period in Latin logic, but the medieval part of was to some extent treated in [Van de Vyver, 1929; 1942; Lewry, 1981].

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which was new; the position was consolidated in the three centuries followed, and logic continued to hold it until the end of the Middle Ages. By studying logic in the early Middle Ages, therefore, we can hope to gain some understanding of how and why the subject had its special role within medieval education and thought. This aim calls for an approach rather different from that of most chapters in this handbook. Consideration of the uses of logic and its connections with other subjects are as important here as the technicalities of logical doctrine. Indeed, there is a sense in which the treatment here, just because it must fit into a handbook of the history of logic, must still remain over-theoretical and insufficiently historical. What is most needed to illuminate the broadly human importance of the subject in this period (and in the twelfth to sixteenth centuries) is a *social history* of medieval logic, a type of study that has never until now been envisaged, let alone attempted. The following pages could be seen as prolegomena to part of that enterprise.

2 THE ANCIENT LATIN TRADITION

When historians, using the medieval terminology, speak of the *logica vetus* — the corpus of ancient logical works used by Latin scholars in the early Middle Ages — they sometimes take it in a narrow sense to mean just the three Greek logical texts, Porphyry's *Isagoge*, and Aristotle's *Categories* and *On Interpretation*, which were available in translation. From this perspective, it seems that the ancient Greek tradition of logic, though in a very curtailed form, is behind the work of the early medieval logicians. But, by the eleventh century, the logical curriculum — the *logica vetus* in a wider and more useful sense — consisted of six works: the three ancient Greek texts, and three textbooks written early in the sixth-century by the Latin thinker, Boethius: treatises on categorical syllogisms, hypothetical syllogisms and on topical argument. Also attached to the curriculum, though more peripherally, was a work on definition attributed to Boethius but in fact by a Latin predecessor of his, Marius Victorinus, and Cicero's *Topics*, along with Boethius's commentary on it. And Boethius's logical commentaries — two on the *Isagoge* and *On Interpretation*, one on the *Categories* — were the tools with which these three texts were studied. Other Latin logical texts had been influential in the eighth to tenth centuries, although they had mostly lost their importance by the eleventh: the *Ten Categories,* a paraphrase of Aristotle's *Categories* from the circle of Themistius, thought at the time to be by Augustine; Apuleius's *Peri Hermeneias*; and the sections on logic in the encyclopaedias of Martianus Capella, Cassiodorus and Isidore of Seville. For all these reasons, though much in early medieval logic goes back ultimately to Aristotle or to the ancient Peripatetics and the Neoplatonists who cultivated Aristotelian logic, they drew more directly on a distinctively Latin tradition. It is here that we must begin. Boethius is the central figure but, as the list of names above suggests, his predecessors and successors should not be neglected.

First, one preliminary, verbal note. Many of the chapters or treatises discussed

here are titled, not 'On Logic' but 'On Dialectic', and the logicians treated here often referred to themselves as 'dialecticians' (*dialectici*) rather than 'logicians'. There certainly was a distinction that could be made, and was in some contexts, between 'logic' and 'dialectic' in which 'logic' stands for the study of demonstrative reasoning (and so the *Analytics*), and 'dialectic' for persuasive reasoning (and so the *Topics*). Usually, however, in the period until 1100, the words were used interchangeably, with 'dialectic' as the more common, to cover the whole of logic. In what follows, I shall speak simply of 'logic', although the word used in the sources will often be *dialectica*. (For a rather different view, see [D'Onofrio, 1986]).

2.1 Before Boethius

The Latin logical works from before Boethius's time divide into two groups: there are Cicero's *Topics,* Apuleius's *Peri Hermeneias,* Marius Victorinus's *On Definition* and Book IV of Martianus Capella's *On the Marriage of Mercury and Philology*, all of which have various links with each other, as will become clear, although they also, except for Martianus, provide independent glimpses of Greek logic. There is also the *Ten Categories*, which unrelated to these other Latin works.

Cicero ([Huby, 1989; Long, 1995])

The earliest Latin logical text is also treatise in legal rhetoric. Cicero wrote his *Topics* ('*Topica*') [Cicero, 1924] in 44 BC, and professes that he will explain there to his friend Trebatius the doctrine of Aristotle's *Topics*. In fact, the work reflects the *Rhetoric* as much as the *Topics,* and probably handbooks of rhetorical and Peripatetic doctrine even more [Long, 1995, 54-7; Stump, 1989, 57-66]. The subject of the *Topics* is said to be the finding of arguments; as his examples show, Cicero has in mind particularly the needs of an orator in the courts. The aim of argument is seen in terms of its persuasive effect: it convinces someone of something that was in doubt (§ 8). The tools for finding arguments are, as in Aristotle, the topics or *loci*. But whereas Aristotle's topics are a proliferation of argumentative strategies, linked to his doctrine of the Categories, Cicero's topics are a short list of types of relationships, consideration of which can help the orator to discover an argument. They include, for instance, genus, species, likeness, cause, effect, comparison. For instance, if the orator needs to show that the heir to a dilapidated house is not bound to repair it, he can argue that the case is similar to that of an heir to a slave, who is not bound to replace him if he has died (§15). All these topics and arguments are intrinsic, but the orator can also argue extrinsically, says, Cicero, by appealing to authority. The importance for logic of this very practically-oriented, rhetorically directed treatise lies in the evidence it gives of how Aristotle's theory of topics had been changed in later centuries, and in the use that Boethius would make of it.

For historians of logic, the most important passage in the book is where (§§53-7) Cicero sets out the *modi dialecticorum* — which consist of an extended and slightly mangled list of the Stoic indemonstrables, expressed, not as the Stoics did,

using numbers as propositional variables ('the first', 'the second'), but rather 'this' and 'that'. In Cicero's list, where there is some confusion because the example sentences do not always match the vaguely given definitions, the first five modes correspond roughly to the five Stoic modes, and Mode 6 is a re-formulation of Mode 3; whilst Mode 7 is (§57) 'Not this and that; but not this; therefore that' — that is to say: $\sim (p\&q); \sim p; q$, an obviously invalid inference. (For discussion, see [Kneale and Kneale, 1962, 179-81; Hadot, 1971, 144-56]). This mistake suggests that, already by Cicero's time, Stoic propositional logic was not properly understood (see below, p. 17).

Apuleius ([Sullivan, 1967; Londey and Johanson, 1987])

By contrast with Cicero's *Topics,* Apuleius's *Peri Hermeneias* ('On Interpretation' — but the Greek title, echoing Aristotle was deliberate and will be retained) written roughly two centuries later, is very straightforwardly a logical treatise, an exposition of Aristotelian syllogistic. Apuleius (c. 125 — c. 171) was a writer in Roman North Africa, most famous for his comic novel, the *Golden Ass*, but responsible too for a number of philosophical works, including an exposition of Plato's thought (*De dogmate Platonis*). Although his authorship of *Peri hermeneias* has been questioned, recent specialists accept it [Sullivan, 1967, 9-14; Londey and Johanson, 1987, 11-15]. The treatise covers much of the material in Chapters 1-8 of Aristotle's *On Interpretation* and in the *Prior Analytics*, excluding modal syllogistics. After treating the nature of propositions and their terms, and the square of opposition, it goes on to explain the principles of the syllogism, giving both the different moods and how the validity of the second and third figure syllogisms can be demonstrated by reducing them to first-figure ones. Apuleius's main differences from Aristotle are that he casts syllogisms as inferences, rather than conditionals; he gives third-figure syllogisms in a different order; and he does not use letters as variables to stand for predicates. He also stands out in the Latin tradition by his terminology, which is sometimes a very literal Latin version of the Greek — for instance, a syllogism is a *collectio*. Not surprisingly, given his period, Apuleius knows something of Stoic logic, but he takes the line about it common among the Peripatetics, condemning it (Chapter 7) without understanding it. For instance, he rejects the inference 'If it is day, it is light; but it is day, therefore it is light', on the grounds that in the conclusion 'it is light' means that it is light now, whereas in the premisses, 'it is light' has a different meaning: it is asserted merely that it follows that if it is day, then it will be light. This criticism shows a clear failure to distinguish between the propositional operator 'if ... then ...' and the propositional contents it links together: a failure, that is, to grasp the propositionality which is central to Stoic logic.

Marius Victorinus ([Hadot, 1971])

Marius Victorinus (c. 280-365) was a philosophically-educated pagan rhetorician who was converted to Christianity about ten years before his death and proceeded to write on the Trinity. He had been one of the rare writers since Cicero to write

on logic in Latin. Two of his logical works survive: a translation of Porphyry's *Isagoge* (used by Boethius for his first commentary) [Hadot, 1971, 371-80] and *On Definition* (*De definitione/definitionibus* [Hadot, 1971, 331-62] that was wrongly attributed to Boethius in the Middle Ages. Victorinus also wrote a now lost commentary on Cicero's *Topics* (Reconstruction in [Hadot, 1971, 118-41]), which is mentioned critically by Boethius in his own commentary on the same work, and from Cassiodorus we know that he wrote, just as Boethius would do, a treatise on hypothetical syllogisms. (See [Hadot, 1971, 323-7] for an assemblage of possible traces of the treatise.) According to Cassiodorus, Victorinus also translated the *Categories* and *On Interpretation* and wrote a long commentary on it. But there is no trace of these works, and it has been argued that Cassiodorus (whose text presents problems: see below, subsection 2.3, pages 21–22) is misleading here [Hadot, 1971, 105-8].

Victorinus placed logic within the framework of rhetoric. Study of the *Isagoge, Categories* and *On Interpretation* led, not as in the Aristotelian organon, primarily to syllogistic (in the *Prior Analytics*) and its use in demonstration (in the *Posterior Analytics*), but to the topics, as expounded by Cicero. And, for Victorinus — to judge from his words as probably reported by Cassiodorus [Cassiodorus, 1937, 127] — it is the very fact that the Ciceronian topics presents arguments that are suited, not just to philosophers, but to orators, lawyers and poets as well, which makes them so precious. *On Definition* and the (lost) treatise on hypothetical syllogisms can be seen as continuations of the commentary on Cicero's *Topics*, since each corresponds to a part of Cicero's text (definition: §§26-37; hypothetical syllogisms 53-7) not treated in the commentary itself, which covers only half the work. *On Definition* draws extensively on Cicero himself in categorizing and explaining the different sorts of definition — rhetorical definition, philosophical definition (which is based on the five predicables: genus, species, *differentia*, *proprium* — i.e. an accident that distinguishes a species, such as ability to laugh in humans — and accident) and other types of definition, such as that from a thing's parts, which does not explain what the things are, but gives some understanding of them. It has been suggested that here a work by Porphyry may have been the source. [Hadot, 1968, I, 482-8]

The 'Ten Categories' ([Kenny, 2005, 128-33])

The *Categorial Decem*(*Ten Categories*), as it came to be called, probably owed some of its early medieval popularity to the fact that it came to be attributed to Augustine. But it also deserved to be used, because it is an excellent work for beginners in logic trying to understand Aristotle. The real author is unknown, but in a couple of comments (Aristotle, 1961, 137:20-1; 175:18-9) he makes clear that he belongs to the circle of Themistius, the fourth-century philosopher and statesman who was perhaps the last member of the ancient Peripatetic school. (Anthony Kenny has suggested that the author of the paraphrase might in fact be Marius Victorinus: [Kenny, 2005, 130-3].)

The paraphraser reveals his approach when (Aristotle, 1961, 148:33 — 149:2),

excusing himself for not going further into the distinguishing features of the first Category, substance, he comments that 'since they are clear in Aristotle himself, it seems superfluous to explain them, especially since this discourse does not aim to put down everything that the Philosopher said, but to recount in a simpler way what seem obscure to the untrained.' In practice, this means that the paraphraser omits some of the more complex ideas and concentrates on providing an elegant, lucid exposition of the *Categories*. On the question that had exercised readers of the text since interest in it revived in the first century BC, whether it is about words or things, the paraphraser manages to maintain a neutral position. Aristotle begins — he says, following Themistius (Aristotle, 1961, 137:20 -138:1) — from the things that are perceived, but in order to discuss them he has to talk about both the things that exist, and the things that are said, because what we perceive is produced by what exists and cannot be demonstrated except with the aid of what is said. What will follow must therefore be a 'mixed disputation'.

Martianus Capella (E.L. Burge in [Stahl, 1971, 104-21], very unsatisfactory)

Martianus Capella was a pagan, living in Roman North Africa either near the beginning of the fifth century or c. 470 [Shanzer, 1986]. His one work is an encyclopaedia, in prose interspersed with verse passages, called *On the Marriage of Mercury and Philology* (*De nuptiis Mercurii et Philologiae*), because Books I and II tell the story of the marriage of Mercury (divine reason) and Philology (the human soul), an allegory of the ascent of the soul to wisdom; Books III — IX [Martianus Capella, 1983] each treat one of the Liberal Arts. Book IV is devoted to logic, and Book V to rhetoric. Both books need mentioning, because in Book IV Martianus treats the material of the *Isagoge*, the *Categories* and *On Interpretation*, followed by a presentation of syllogistic based on Apuleius (cf. [Sullivan, 1967, 170-3]) and, near the end of the book, an account of what he calls the 'conditional syllogism', in which he presents a list of seven modes that are the same as Cicero's, but he is unusual in, at one point, giving these seven modes in the traditional Stoic form, using numbers to stand for propositions ('If the first, the second; the first; so the second') [Martianus, 1983, 144]. Then, in Book V, Martianus goes into a presentation of the Topics, based on Marius Victorinus. It has been argued that Victorinus originally incorporated Apuleius's *Peri hermeneias* into his own work, and so Martianus is putting forward a thoroughly Victorine scheme of logic [Hadot, 1971, 196].

2.2 Boethius

([Chadwick, 1981, 108-73; Marenbon, 2003, 17-65; Cameron, Forthcoming; Ebbesen, Forthcoming; Martin, Forthcoming])

Boethius is the central figure in the ancient Latin tradition of logic. A small part of his importance is due to his continuing the tradition of Cicero and Marius Victorinus; the far larger part to his making — as a translator, a commentator and a writer of textbooks — Greek Aristotelian logic available to the Latin world

in far more breadth and depth than the glimpses which previous Latin texts had allowed. Yet Boethius, though not himself outstanding, or even particularly good, at logic, was not a mere passive transmitter of other, more inventive logicians' ideas: he made what seems to have been a very deliberate choice among the competing theories, approaches and interpretations available to him, and in doing so profoundly influenced the style and contents of medieval logic.

Anicius Severinus Manlius Boethius was born c. 475-7 AD. Although he lived at a time when Italy was ruled by the Ostrogoths, Roman aristocrats like him were free to enjoy their wealth, leisure and a limited local power. Fluent in Greek and in contact with the Greek intellectual world, Boethius chose to devote most of his life to writing or translating works on arithmetic, music and, most of all, logic. His earliest work on logic, his first commentary on Porphyry's *Isagoge*, dates from c. 500 and he continued to write mainly in this area until up to about 523, when he became closely and disastrously involved in the politics of the Gothic ruler's court. At one stage [Boethius, 1880, 79:9-80:9], Boethius announced his intention to translate and comment on *all* of the works of Aristotle he could find, and Plato's dialogues, as well as writing a book to show how the two philosophers are in fundamental agreement with each other. But, in fact, he devoted himself to the logical part of this task and, in the last fifteen years of his life, he was especially occupied in composing text-books as well as commentaries. Although not a priest, Boethius also became involved in the religious controversies of the time, which divided the Catholics of Italy from those of the Eastern Empire. His five short *Theological Treatises* (*opuscula sacra*) [Boethius, 1999] have a place in the history of logic, because his use there of logical techniques would encourage medieval readers similarly to see in logic a tool for theological speculation and debate.

Not long after Boethius's move to Theoderic's court at Ravenna, he was accused of treason and found himself imprisoned, awaiting execution. It was during this time that he wrote *The Consolation of Philosophy*, the work by which he would become best known. Although the *Consolation* is not directly concerned with logical questions, it ends with a discussion of God's foreknowledge and future contingents that returns to some of the issues Boethius had raised in his logical writings.

We shall look at each area of Boethius's contribution to logic in turn: as translator, as commentator and as a writer of text-books.

Boethius the Translator

Boethius put the whole of Aristotle's logical Organon (along with Porphyry's *Isagoge*: see below) into Latin, except for the *Posterior Analytics*. (Ed. in *Aristoteles Latinus* [Aristotle, 1961; 1962; 1965; 1966; 1969; 1975].) He seems to have been led into this task of translation by his work as a commentator. For the first commentary he wrote, on the *Isagoge*, he relied on an existing version by Marius Victorinus. But for his next commentary, on the *Categories*, he made his own

translation, and he made a fresh translation of the *Isagoge* for his second commentary on it. Since Boethius's commentaries, although rarely a word by word exegesis, would often focus on a particular phrase, it was important to have a literal translation, which to a considerable extent preserved rather than ironed out the interpretative difficulties commentators puzzled over. Though elsewhere an elegant stylist, Boethius translates very closely, often following the Greek syntax as far, or further, than Latin idiom permits; and he translated with great care, usually revising his versions at least once [See the prefaces in *Aristoteles Latinus*]. In the case of the *Categories*, the textual history as reconstructed by Minio-Paluello [1962; Aristotle, 1961, ix-lxiii] is very complicated and perhaps open to query: Boethius's final version was combined at some time with another version, also probably by Boethius, to form a composite version; the composite version, itself variously corrupted, was the one most usually used, but the original version is preserved, with corruptions, in some manuscripts and can be reconstructed.

Boethius's translation of the Organon remained the standard version, throughout the Middle Ages. Its accuracy and literalness made it possible for some medieval logicians to step out of the commentary tradition, transmitted by Boethius himself, within which they studied Aristotle's texts, and reach back to grasp Aristotle's own thought. But this return to origins was more a phenomenon of the twelfth century than the early Middle Ages, a period in which the three Boethian translations then available (the *Isagoge*, the *Categories*, *On Interpretation*) had to share their place, as will become clear, with a variety of other material.

The Neoplatonic Aristotelian Tradition ([Ebbesen, 1990; Sorabji, 2004])

As a commentator, Boethius was certainly dependent on the study of Aristotelian logic that had been going on in the Neoplatonic schools of Athens and Alexandria for two centuries. I shall sketch some of the main features of these philosophers' approach to logic, since this important area of logical history is treated no where else in the present Handbook (a mark of how little the 'History of Logic' served up by and for logicians has to do with history?).

Although Plotinus, the founder of Neoplatonism, was hostile to many aspects of Aristotelian logic, this attitude was not shared by Porphyry (c. 232-305), his editor and leading follower. Porphyry believed that the ostensible differences between Plato and Aristotle could be explained by what we can label the 'Different Objects Theory': Plato was discussing the intelligible world, whereas Aristotle focussed his attention on the world as it appears to the senses. He therefore incorporated Aristotelian texts within the Neoplatonic curriculum, and he gave a particular emphasis to Aristotelian logic, a field that seemed especially to interest him. (See [Karamanolis, 2006]; but [De Haas, 2001], suggests by contrast that Plotinus's and Porphyry's views were not very different.) From then on, logic was studied enthusiastically in the Neoplatonic schools. Porphyry himself wrote an introduction (*Eisagoge*; latinized as *Isagoge*) to the *Categories*, which became incorporated into the Aristotelian Organon as the first work to be studied in late antiquity and the Middle Ages, as well as a short commentary on the *Categories*,

and long commentaries, both lost, on this text and on *On Interpretation.* His important follower, Iamblichus, produced a commentary on the *Categories* (also lost) and Boethius's contemporary at the School of Alexandria, Ammonius (435/45 — 517/26) seems to have concentrated on Aristotelian commentary. (See [Sorabji, 1990; 2004], for an introduction to these commentators.)

Porphyry's Different Objects Theory gave Neoplatonists a way to be thoroughly Aristotelian in logic (which was concerned with language, and language with the world as sensibly perceived [Ebbesen, 1981, 133-70]), and he was certainly a keen reader of the greatest of the later ancient Peripatetic, Alexander of Aphrodisias. His *Isagoge* can be seen as fitting neatly into the Aristotelian Organon as an Introduction, although modern assessments of its metaphysical commitments vary (compare [Porphyry, 1998] and [Porphyry, 2003]). Some later Neoplatonists, however, did not follow the Different Objects theory so consistently. Although the Aristotelian texts remained for them the basis of the logical curriculum and much of their discussion concerned the type of issues Porphyry had raised, views of a Platonic kind influenced their exegesis. For example, in his *Categories* commentary, now lost but witnessed through the commentary by Simplicius (early 6th century), Iamblichus seems to have taken advantage of the idea that the originator of the Categories was Archytas, a Pythagorean whom he claimed Aristotle copied, and he attempted to apply the Categories to the supra-sensible realities of Platonic metaphysics; and Ammonius, commenting on *On Interpretation*, tries to insert the teaching of Plato's *Cratylus* that words are not merely conventional signs [Ammonius, 1897, 34:10–41:9].

Boethius and the Commentary Tradition

What was the nature of Boethius's dependence on the Greek Neoplatonic commentators? According to James Shiel, it was total. Boethius simply found a Greek manuscript crammed with marginalia, themselves drawn from the writings of several commentators. 'The translation of these various marginalia and the arrangement of them into a continuous commentary according to the order of Aristotle's words would seem to be Boethius' only title to originality', remarks Shiel [1990, 361], even though he concedes [1990, 370, n. 83] that Boethius himself contributed expansions and references to Latin authors. Shiel founds his position on the fact that Boethius's commentaries are not based on any one known, Greek source. Most specialists now reject his view [Chadwick, 1981, 129-31; Ebbesen, 1990, 375-7], because it assumes the very point it claims to demonstrate, that Boethius was incapable of making his own choice and arrangement of material. Even if his sources included manuscripts with marginal scholia — and it is a moot point whether any such codices existed at that time — there is no reason to think that Boethius did not use them selectively, along with the complete texts of commentaries [De Libera, 1999, 164-8; Magee, 2003, 217].

Boethius, then, was not a mere translating machine. But nor was he an original thinker in his logical commentaries. He was happy in general, just like his Greek

contemporaries, to put down his own version of the views of his chosen authorities. It was mainly in making this choice that he could exercise his independence, and he did so by selecting Porphyry above all as the interpreter to follow. The *Categories* commentary is close to the surviving shorter, question-and-answer commentary by Porphyry [Asztalos, 1993], and from Boethius's own comments about his sources it is clear that his longer commentary on *On Interpretation* is by far the best guide we have now to the contents of Porphyry's lost commentary on this text. By choosing Porphyry, rather than any more recent Greek commentator, as his main guide, Boethius ensured that his presentation of logic would be, by and large, faithfully Aristotelian, in line with the Different Objects Theory.

Boethius on the 'Categories' ([Asztalos, 1993])

Porphyry bore out his general approach to Aristotle with a particular theory about the subject-matter of the *Categories*. Interpreters differed as to whether it is about words or things. Plotinus had taken it to be about things and had been critical of Aristotle's discussion, since it said nothing about the Platonic Ideas. Porphyry's view, as expressed in his question-and-answer commentary on the work [1887, 90-1], was that the *Categories* is about language: it is about words of first imposition, which signify things, as opposed to words of second imposition (such as 'noun' and 'verb') that signify other words and are treated in *On Interpretation*. The things signified are sensible particulars, since it is to these that people first of all attach names; and the secondary substances, on Porphyry's view, are concepts, universals that are abstracted from the particulars, not the Platonic Ideas. The *Categories*, then, and even the assertion in it that (2b5-6) 'if the primary substances' (that is, the sensible particulars) 'did not exist it would be impossible for any of the other things to exist' can be accepted since, in this extension of the Different Objects Theory, Aristotle is not concerned here with Platonic Ideas, but with universals that are mental concepts, gained by abstraction, and dependent for their existence on the primary substances.

Boethius follows Porphyry's question-and-answer commentary closely and reproduces his approach to the subject-matter of the *Categories*. He dramatizes the theory by attaching it to a story about the origins of human language placed at the very beginning of his commentary [Boethius, 1891, 159A-C], in which he tells how, first, names of first imposition, such as 'horse', 'branch' and 'two-foot long' were given their meaning, and then names of second imposition such as 'verb' and 'noun'. Boethius is fairly brief in his comments, because he says that he is writing for beginners. Whether he ever wrote the second, more advanced commentary that he promises [Boethius, 1891, 160A-B] is very uncertain: he may well have been reluctant to produce the type of non-Aristotelian, Iamblichan discussion his description of it suggests [Marenbon, 2003, 23].

Boethius on 'On Interpretation' ([Magee, 1989; Lloyd, 1990, 36-75] a complex discussion of Porphyrian semantics, which may underlie Boethius's theories, [Cameron, Forthcoming; Sharples, Forthcoming])

In commenting *On Interpretation*, Boethius again turned to Porphyry, especially in the second, more advanced commentary [Boethius, 1880] (the first commentary [Boethius, 1877] was designed just as an aid for beginners to a literal understanding of the text). He says so himself [Boethius, 1877, 7:5-9], and it seems very likely that the very influential ideas on semantics and modality he puts forward were developed by Porphyry in his detailed commentary, now lost.

At the beginning of *On Interpretation*, Aristotle says (to translate Boethius's Latin version literally):

> The things therefore which are in the utterance (*in voce*) are the marks (*notae*) of the passions that are in the mind, and the things that are written of those that are in the utterance. And just as the letters are not the same for all, so the utterances are not the same; but what they are primarily signs of, the passions of the mind, are the same for all, and the things too, of which these are likenesses, are the same. [16a1-8; Aristotle, 1965, 5:3-9; cf. Magee, 1989, 21-34]

As the commentary as well as the translation shows, Boethius takes Aristotle to be presenting a semantics which is in many ways a mental language theory. The 'passions of the mind' are the words of this language which, unlike written and spoken languages, is common to all humans, and these passions are in some way likenesses of objects in the world.

To an extent, this is indeed Boethius's approach. He speaks explicitly of mental discourse (*oratio*) [Boethius, 1880, 30], and he believes that the thoughts that are its simple terms are combined into complex thoughts that are true or false. There is, however, an important difference between Boethius's thinking and a straightforward theory of this sort. When I utter 'dog' and you utter 'dog', we are speaking the same word in the sense that utterances are tokens of the same type, in virtue of a physical resemblance between the sounds we produce, which a native speaker would judge to be close enough to take us as uttering the same word. What about my mental concept of dog and your one? For Boethius, they can be called the same just because they both [Boethius, 1880, 21:18-22] originate from the same type of animal. We begin from sensory contact with some object in the world, form a pre-conceptual image in our mind, on which the intellect works in order to have a thought of it. It seems, therefore, that in Boethian mental language, in so far as it is a language, the individual words ('dog', 'cat', 'human') are not imposed on objects, as in a natural language, but derive from them causally.

When he comes to Chapter 9 of *On Interpretation*, Boethius has to give his reading of the much debated passage on future contingents. Earlier (17a1-4) Aristotle had asserted the principal of bivalence: every proposition is either true or false. But it seems that, if propositions about future events are true, or are false, then no future events will be contingent. Suppose p states that a certain event E will happen tomorrow. If p is true, it seems that E *must* happen, and if p is false, it seems that E *cannot* happen, and so, whether p is true or false, E will not be con-

tingent. One way out of the problem is to abandon bivalence for future contingent propositions. Arguably, this was Aristotle's strategy: so the Stoics thought, and some modern scholars agree. Boethius, however, believes that Aristotle retains bivalence, but avoids determinism by holding that future contingent propositions are *indefinitely*, and not *definitely*, true or false.

What does Boethius understand by this position? (See [Kretzmann, 1998] for a different view.) He thinks that, when we assert or deny a proposition about a future event without qualification, we are saying that it is definitely true or false, and we are thereby implying that the event will or will not happen necessarily. In consequence, any proposition of the type 'CE will happen', where CE is a contingent event, is false, because it asserts that a contingent event is not contingent. What the speaker should do is to say 'CE will happen contingently', in which case — so it seems, tying together the different elements of the theory — he is asserting the indefinite truth of 'CE will happen' [Boethius, 1880, 212-3]. In line with this theory, when he discusses in this commentary the question of God's foreknowledge (not raised by Aristotle, but debated in the commentary tradition, as witnessed by Ammonius), Boethius says that God knows that future contingents will take place, but he knows that they will take place contingently, not necessarily [Boethius, 1880, 225:9-226:25]. It may be because even Boethius himself realized that his theory is inadequate as an explanation that, when he returned to the problem of divine prescience at the end of his final work, the *Consolation of Philosophy*, he produced a different and far more ambitious solution (cf. [Huber, 1976; Marenbon, 2003, 125-45; Evans, 2004; Sharples, Forthcoming]).

Boethius on the 'Isagoge': Porphyry's Questions ([De Libera, 1999, 159-280; Marenbon, Forthcoming-B])

Boethius could not, as he usually did, turn to a commentary by Porphyry when he wrote his two commentaries on Porphyry's own *Isagoge* [Boethius, 1906]. The earlier, in dialogue-form, was his first piece of writing on logic, dating from c. 500; the later one (c. 510-5) was by far the most commonly studied of the two at all stages of the medieval tradition. In fact, Boethius continued to show his adherence to Porphyry's approach, since in both commentaries he concentrates largely on straightforward explanation of the ideas in the text, using material some of which is reflected in Ammonius's commentary, the earliest surviving in Greek, written at much the same time as his own. One passage, however, led Boethius and almost every commentator into a long discussion of a subject that Porphyry explicitly refuses to treat here. Close to the beginning of the *Isagoge* [Porphyry, 1887, 1:10-13; Porphyry, 1966, 5:10-14 — Boethius's Latin translation], Porphyry puts a set of questions about universals and how they exist (he uses the word *hufistasthai,* translated by Boethius as *subsistere,* but here it probably means the same as 'exist' [Porphyry, 2003, 40]) or do they consist only in pure thoughts? If they exist, are they bodies or incorporeals, and if they are incorporeals, are they separable or in perceptible things, existing about them? Porphyry himself says he will not try to gives answers, because the subject is too deep for an introductory

work.

Boethius puts forward his fullest and most influential answer in his second commentary. Even here, he is one sense loyal to the Porphyrian project, since he turns to one of Porphyry's preferred sources, the great Peripatetic philosopher Alexander of Aphrodisias, from whom he says he has taken his position (cf. [De Libera, 1999]). He begins with a powerful argument [Boethius, 1906, 161:14–164:2] which claims to show that universals do not exist, and any enquiry into them is pointless. The first premiss resumes Porphyry's first question:

1. Either genera and species exist (*sunt atque subsistunt*) or they are formed by the intellect, in thought alone.

He then argues against the truth of the first disjunct of (1): -

2. Everything that exists is one in number.

3. Nothing that is common to many at the same time can be one in number.

4. Genera and species are common to many at the same time.

5. Genera and species do not exist. (2,3,4)

It follows, therefore, that

6. Genera and species are formed in the intellect, in thought alone. (1,5; disjunctive syllogism)

But to hold (6), Boethius goes on to show, apparently entails a drastic conclusion for the enquirer. Some thoughts — call them 'corresponding thoughts' — correspond to how their object is in reality, some do not.

7. If the thoughts that, by (6), are genera and species are corresponding thoughts, then genera and species exist in reality in the way they are thought.

8. Genera and species do not exist in reality in the way they are thought. (5)

9. Genera and species are not corresponding thoughts. (7, 8; *modus tollens*)

10. Thoughts that are not corresponding are empty and false.

11. All enquiry into universals should cease.

Boethius does not, of course, agree with this argument, but one of the remarkable feature of his answer is that he *accepts* every stage of this argument up to (9). He simply denies (10), and therefore (11); the opponent's argument, in the main, goes through, but without its sting. Boethius denies (10) by appealing to the idea of (mathematical) abstraction. He explains [Boethius, 1966, 164:12–14] that when one thing is abstracted from another 'the thing is indeed not as the thought is, but the thought is not however at all false.' For example, a line cannot

in fact exist apart from a body, but it is thought of in abstraction from any body, and this thought is not false.

Not only does the appeal to abstraction defeat the argument (1)–(11): it also suggests a way of explaining what universals are, which we could label 'Neutral Abstractionism'. According to Neutral Abstractionism, particulars of a given species each have a nature or, as Boethius calls it here, a 'likeness' (*similitudo*), which exactly resembles the likeness of any other member of the species. The species is constituted by the thought which brings together these likenesses, and the genus by the thought that brings together the likenesses of the species. This solution is one of those Boethius puts forward.

At the same time, and without marking the difference, Boethius proposes a rather different solution. He starts by explaining that the likeness 'is sensible to the senses when it is in the singular things, but it is intelligible in universal things, and in the same way when it is sensible it remains in singular things; when it is thought in the intellect (*intelligitur*) it becomes universal' [Boethius, 1906, 166:18-21]. Then he goes on to say — expanding this idea — that singulars and universals are two things in the same subject. Just as the same line is convex or concave depending on how it is regarded, and so it can be considered two things in one subject, so there is the same subject (for instance, this human, John) for singularity and universality. When John is perceived through the senses, among the things in which he has his existence (*esse suum habet*), he is a singular; but when he is grasped in thought (as a human), he is a universal [Boethius, 1966:23-167:7]. This solution differs from Neutral Abstractionism and might be called 'Realist Abstractionism' since it suggests that the process of abstracting does not merely provide a way of regarding a world that is made up only of singular things, some similar, some diverse, but that it allows people to grasp real universals, accessible only to thought. [Marenbon, Forthcoming-B].

Boethius's Logical Treatises([Martin, 1991])

The short *On Division* (*De divisione*; probably 515-20) [Boethius, 1998 — with translation and commentary] stands somewhat apart from Boethius's other logical treatises. Like much else in his logic, it is probably based on a work by Porphyry, prolegomena to a now lost commentary by him on Plato's *Sophist*. Despite the Platonic text, the approach is Aristotelian. Using the language and concepts of the *Isagoge* and *Categories*, Boethius explains the difference between accidental division (dividing, for instance, humans in virtue of accidents of colour into black, white and medium) and intrinsic division. Intrinsic division is of a whole into parts, a genus into species and a word into its different meanings. In the twelfth century, when this treatise was studied most carefully, the section on whole-part division was especially important. Questions about genera and species were treated more fully in the *Isagoge*, and on semantics in *On Interpretation*, but *On Division* was alone in providing a brief introduction to mereology.

All of Boethius's other logical treatises concern types of argument. Two of them, *On the Categorical Syllogism* (*SC*) [Boethius, 2001] and *Introduction to*

Categorical Syllogisms (*ISC*) [Boethius, 1891, 761-94], present Aristotle's doctrine of (non-modal) syllogisms. The relations between the two treatises are complex. *SC* has a first book devoted to preliminaries to syllogistic (as discussed in *On Interpretation*), and a second book which contains a straightforward account of syllogistic itself. *ISC* has one book only, which covers in greater detail the ground treated in *SC*, Book I. It seems to be Boethius's attempt — left unfinished or mutilated in transmission — to rewrite and expand *SC*. Boethius has been accused [Patzig, 1968, 75-6] of making an innovation in presenting Aristotle's theory of syllogisms that made it less clear. According to Aristotle, the validity of first figure syllogisms is evident. In his way of stating them, they read like, for example: 'A belongs to every B, B belongs to every C, therefore A belongs to every C' — a formulation in which, just by looking at the first two premisses, the conclusion is obvious. When, as became usual, the syllogism is constructed using 'is' instead of 'belongs to', the Greek commentators ensured that this transparency was maintained by reversing the order of the premisses: for instance, 'Every C is B, every B is A, therefore every C is A.' But Boethius loses the transparency by using a formulation with 'is' along with the Aristotelian order. In fact, this accusation is not completely fair. It applies only to the way in which Boethius formulated his example syllogisms. When he uses letters for terms, he follows a Latin version of Aristotle's Greek phrasing, such as 'A is predicated of every B.'

Boethius is particularly fascinated by the theory of conversion, and he introduces an explicit discussion of how propositions with infinite names (that is, ones involving negations, like 'not-man' and 'not-animal') convert (*ISC*: [Boethius, 1891, 780-5; Prior, 1953]). Moreover, he extends the theory of conversion to cases where truth is preserved only so long as the terms in the proposition bear a certain relation to each other. For instance, 'Every A is B' does *not* convert to 'Every B is A', but it does if B is a *proprium*, as defined in Porphyry's *Isagoge*, of A: 'Every human is capable of laughing' — 'Every thing capable of laughing is a human being' (*ISC*: [Boethius, 1891, 786B]).

On Hypothetical Syllogisms (*De hypotheticis syllogismis*) [Boethius, 1969] is a far more interesting work than *SC* or *ISC*. It represents a tradition of writing on hypothetical syllogisms that is found, otherwise, only in a brief scholium preserved in a tenth-century Byzantine manuscript [Bobzien, 2002]. Presumably there were some other, now lost accounts of the area (we know that Marius Victorinus gave one), but Boethius says himself [references are to the section divisions in Boethius, 1969: I.1.3-4] that he is giving a fuller and more systematic treatment than anything he can find in the Greek, and there is every reason to accept this claim, certainly about the detailed working out of the theory in Books II and III (cf. [Striker, 1973]), but possibly even about Book I. Moreover, the treatise provides important, though negative, information on Boethius's grasp of Stoic propositional logic.

Hypothetical syllogisms are those in which one or both premisses are compound, rather than simple propositions. Boethius considers two types of compound proposition, those using the connective 'or' (*aut*, understood as exclusive disjunction)

and those using 'if' (*si*/ *cum*). In practice, he concerns himself almost entirely with 'if'. Early in the work [I.3.6-7], he makes a distinction to which he does not return. There are two sorts of conditionals (hypothetical propositions using *si* or *cum*): accidental (*secundum accidens*) and those which have a consequence of nature (*ut habeant aliquam naturae consequentiam*). An accidental conditional is, for instance, 'If (*cum*) fire is hot, the sky is round'. Those which have a consequence of nature are of two types. In one type the consequence is necessary, but does not rest on the positing of the terms (*positio terminorum*), as for example: 'If (*cum*) it's a human, it's an animal'. The other type has a necessary consequence that does rest on the positing of the terms, as for example, 'If (*si*) there is an interposition of the earth, an eclipse of the moon follows'. From the example he gives, the distinction that Boethius seems to have in mind is that, in a true accidental conditional, the antecedent are both necessarily true, but the truth of the one is entirely unrelated to the truth of the other. In a natural conditional, the truth of the consequent is related to that of the antecedent, either by dependency where the conditional rests on the positing of terms — it is *because* there is an interposition of the earth that the moon is eclipsed — or by relevance that is not causal dependency — something's being a human gives us reason to say it is an animal, but Boethius would want to say that it is not because it's a human that it's an animal, but rather, because it's a mortal, rational animal that it's a human. The distinction between accidental and natural conditional was one which would become enormously important in the history of Latin logic from the twelfth century onwards [Martin, 2005]. But Boethius does not dwell on it; his standard example conditional, 'If it is day it is light', seems to be a natural conditional resting on the positing of terms,

Rather, Boethius wants to set out which hypothetical syllogisms are valid. Just as the theory of categorical syllogistic lists the different combinations that are valid, so too hypothetical syllogistic offers a tabulation of valid inference forms. But the table is vastly more complicated. The simplest form of a hypothetical syllogism, where the major premiss involves just two terms, is exemplified by 'If it is day, it is light. It is day. So it is light.' Varying the quality of each of the two propositions making up the major premiss yields four types of syllogism. But the major premiss can involve three terms (for instance, 'If, if it is A, then it is B, it is C) or four terms (for instance, 'If, if it is A, it is B, then if it is C, it is D'). Taking into account all the combinations of quality, and then the different ways of qualifying them modally, the number of combinations reaches the ten thousands (I.8.7), and far more if the propositions are quantified (I.9.2). But Boethius confines himself, wisely, to charting the non-modal, non-quantified combinations — still a long and tedious task, that occupies Books II and III of the treatise.

It is tempting to see Boethius as setting out some variety of propositional logic. 'If it is day, it is light. It is day. So it is light' looks very like an inference of the form '$p \rightarrow q, p$; so q'. Yet it is clear that Boethius conceives himself as doing a sort of term logic (and the same is true for the Byzantine scholium). Each of the

propositions is of the form 'It is (an) $A/B/C\ldots$', where 'A', 'B', 'C' stand for predicates, and the conclusion follows — as in a categorical syllogism — because of the relation between these terms. One moment in the treatise brings out this point especially clearly. Boethius (II.2.3–4.6) has established that, if the major premiss is 'If it is not A, it is B', then, whilst 'It is B' follows when the minor premiss is 'If it is not A', and 'It is A' follows if the minor premiss is 'It is not B', from the minor premisses 'It is A' and 'It is B', no conclusion follows. And yet, Boethius points out, in the nature of things from 'If it is not A, it is B' and 'It is B', it *does* follow that it is not A, and from 'If it is not A, it is B' and 'It is A', it does follow that it is not B. The reason (III.10.4) is that the major premiss will be true only if 'A' and 'B' are immediate contraries — they are such that everything is either A or B. If A and B are immediate contraries, then indeed if something is not A it is B, and if it is not B it is A. This line of thought makes sense only if Boethius has the connections between terms, not propositions, in mind. As Chris Martin has shown in his important study [Martin, 1991], whereas Stoic logic is propositional, Boethius had no grasp of the very notion of propositionality; that is to say, he was unable to see negation or consequence as *propositional* operations.

Boethius is not, though, entirely removed from propositional logic. There are moments, indeed, when he *seems* to come close to propositional logic, as when he gives two of the Stoic indemonstrables using, in Stoic fashion numbers to stand for propositions: 'if the first, then it follows that there is the second', 'if the second is not, it follows necessarily that the first is not' (I.4.6). In his commentary on Cicero's *Topics,* Boethius includes a long passage [Bk. V; Boethius, 1833, 355-9] discussing Cicero's seven modes of inference. This passage may be linked, not just to Cicero's own text, but to the presentation found in Martianus Capella and Cassiodorus, which seems to go back to Marius Victorinus. There is, no doubt through Cicero, genuine Stoic influence at the basis of this list. But, well by Boethius's time, as Anthony Speca has shown [Speca, 2001], the Peripatetic theory of the hypothetical syllogism and Stoic propositional logic had become conflated, to the extent that Boethius (and possibly Marius Victorinus before him), reading Cicero's Stoic-based list of modes of inference, took them for modes of hypothetical syllogisms.

Indeed, there is one point here where Boethius makes clear just how far he was from understanding Stoic logic (pace [Stump, 1989, esp. 19-22]). He knows what Cicero's form of the third mode is, but he deliberately changes it [Boethius, 1833, 362]. Instead of 'Not: A and not B; A; therefore B', Boethius puts: 'Not: if A then not B; A; therefore B' [Boethius, 1833, 356-7]. If this is translated into propositional calculus, it gives $\sim (p \rightarrow\sim q); p; q$, which is clearly invalid. Boethius explains, however, that in this mode it is assumed that A and not-B are incompatible. He understands 'Not if A then not B' as expressing this incompatibility: to negate 'If it is day, is not light' is, for Boethius, to say that it cannot be the case that it is day without its being light. And so it follows that, if it *is* day, as the minor premiss states, it is light (cf. [Martin, 1991, 293-4]). Boethius is therefore, as shown by his very willingness to change the formula he

inherited, trying to think about the meaning of these Stoic-derived formulas, but within the context of his own term-logic.

The strange character of Boethius's hypothetical syllogistic comes out even more clearly when he treats the negation of a conditional in *On Hypothetical Syllogisms*. Not realizing that his discussions should be construed as term logic, earlier commentators [Dürr, 1951; Kneale and Kneale, 1975, 191; Barnes, 1981, 83; cf. Martin, 1991, 295-6] argued that Boethius thought the contradictory negation of 'If p, then q' is 'If p, then not-q', rather than 'Not (If p, then q)' as in classical propositional calculus. Therefore, they concluded, he had a strange propositional system. But, as Martin puts it [1991, 279]: 'The problem is not ... that Boethius' logic is not *classical* propositional calculus but rather it is not propositional at all.' Boethius is in fact both much less strange in his underlying conception than he has been seen to be, and yet at the same time more distant from contemporary logical models. Boethius's account of the state of affairs that must be the case for a conditional to be 'negated' or 'destroyed' is exactly the same as the contemporary logician's account of what obtains when a whole conditional is negated, but Boethius can only offer his explanation, however tortuously, using term logic. For the conditional 'If A is, B is' to be destroyed, we must show 'not that A is not or B is not, but that, when A is posited, it does not immediately follow that B is, but that A can be even if the term B is not' (I.9.7). And even in his extended discussion of the valid forms of hypothetical syllogisms, Boethius seems, whilst always working in term logic, to be mimicking the results which would obtain in a propositional calculus, often employing various extra premisses that allow him to reach the results which, by means of examples, he has identified as the correct ones (cf. [Marenbon, 2003, 53-55, 191]).

Boethius and Topical Argument ([Green-Pedersen, 1984, 37-81])

The remaining logical monograph by Boethius, *On Topical* differentiae (*De topicis differentiis*) (TD) [Boethius, 1990, 1-92; trsl. Boethius, 1978; Boethius, 1891, 1173-1216] was written near the end of his life (c. 522-3), at a time when topical argument became a central concern for him. Shortly before, he had written a commentary on Cicero's *Topics* (TC) [Boethius, 1833; transl. Boethius, 1988] that is closely related to it, translated Aristotle's *Topics* into Latin, and probably written a commentary on it, now lost.

Much as in the case of hypothetical syllogistic, Boethius' work is the only substantial survival from the late ancient tradition of topical argument as it had developed after the time of Cicero. One of its principle exponents had been the fourth-century Peripatetic philosopher, Themistius. He and Cicero are Boethius's special authorities, and one of the main aims of TD is to compare the apparently different lists of topical *differentiae* given by these two philosophers, showing that they really coincide.

In the way it is structured around a list of topical *differentiae*, the topical theory Boethius presents clearly descends from Cicero, though Cicero had just called them 'topics'. Discussion of the topics is no longer, however, the mixture of law and

rhetoric with logic that it had been five centuries before; the proof of this is what might seem to be the exception, the fact that in Book IV of TD, after devoting the rest of the work to the logical topics, Boethius gives a separate treatment of what are identified explicitly as the rhetorical topics. Topical argument has its place within logic as the discipline for *finding* arguments, whereas the *Posterior Analytics* teaches us how to judge them. Boethius thinks of an argument as aiming to decide to between the disjuncts of what he calls a question: 'Either A is B or A is not B'. The argument will be a syllogism with one or other disjunct as its conclusion. Finding the argument is therefore a matter of finding an appropriate middle term, from which a premiss can be formed with A, and a premiss with B, which yield syllogistically the desired conclusion. Topical theory does not, of course, provide a machine, into which the conclusion only need be fed for an argument to emerge. There could be no such machine. Rather, it provides a method for arguers to search their stock of knowledge in the right way to come up with an argument. Since the middle term is predicated of, or has as a predicate, the other two terms, what it designates must be related, even if negatively, to what they designate. The scheme of *differentiae* systematizes the main categories of such relationships — categories such as, for example, part and whole, genus and species, more and less, signified and sign. Suppose [TD II; Boethius, 1847, 1188C — the new edition and translation reproduce the column divisions of the 1891 ed.] I want to show that human affairs are ruled by providence. I mentally scan the list of *differentiae* and my attention is caught by that 'from a whole': it suggests to me that what human affairs stand in relation to as a part to a whole provides me with the middle term I need. I know that the world is ruled is ruled by providence. But human affairs are a part of the world and so they too, I can conclude, are also ruled by providence.

As well as setting out the topical *differentiae*, Boethius lists for each a *maxima propositio* ('maximal proposition') — sometimes giving it in a variety of different forms. The maximal proposition is a statement putting in general terms the pattern of reasoning on which arguments rely which are made using the topical *differentia* in question. So, for the *differentia* 'from a whole' the maximal proposition is: 'What is fitting to the whole, also is fitting to the part.' When I argue,

12. The world is ruled by providence.

13. Human affairs are a part of the world.

so

14. Human affairs are ruled by providence,

I am giving a particular example of how what is fitting to the whole is also fitting to the part. The maxim has another role too, however. (12–14) is not a valid argument, although it may seem to be (just about!) a plausible one. If, however, we add, as an extra premiss

13a. What is fitting to the whole, also is fitting to the part

that is to say, the maximal proposition, the result is (or could be, with a little re-expression) a formally valid argument.

This example illustrates a general rule in Boethius's account of the topics: when the maximal proposition is added as a premiss, the arguments become formally valid deductions. But Boethius does not suggest that this is how maximal propositions should be used. Rather, he says [TD 1185B-D; TC, Boethius, 1833, 280:24-7] that maximal propositions give arguments their force. His idea seems to be that a formally invalid argument such as (12–14) gains its force as an argument from the principle encapsulated in the maximal proposition: we are happy to accept (14) as following, in an informal way, from (12) and (13) because we accept and have in the back of our minds the principle that what is fitting to the whole is also fitting to the part. The strength of a topical argument will therefore depend on the strength of the underlying maximal proposition. Some maximal propositions are analytic truths (for instance: 'the cause of anything brings about what it causes' TD 1189C; 'once the antecedent has been asserted, the consequent follows . . . ' (i.e. *modus ponens*) TD 1198D). Some, like the one in the example here, are much vaguer statements of what is generally the case. Some, like the maximal proposition of the one topic considered to be extrinsic, 'from judgement' — 'What seems so to all or to many or to learned people should not be contradicted' (TD 1190CD) — hardly even amount to useful argumentative rules of thumb.

Boethian Logic and its Survival

Boethius is by far the most important figure in the ancient tradition of Latin logic, but it is important to realize that the Boethian Tradition was not the only ancient Latin one. The logic of the earlier Latin authors, along with, or transmitted by, later encyclopaedic accounts, provided a separate tradition, which would be the one on which, more than Boethius, medieval logic depended in the period up to the late tenth century. It is in the eleventh century that the Boethian Tradition begins to dominate (See §4 below). The twelfth century was the Golden Age of Boethian Logic: the six works that formed the core of the logical curriculum were Boethius's monographs and his translations of the *Isagoge*, *Categories* and *On Interpretation*, which were taught making extensive use of his commentaries. And the *Prior Analytics* and *Sophistical Refutations*, also in his translation, began to be known. As a result of the introduction of the whole range of Aristotle's writing and its adoption, by the mid-thirteenth century, as the Arts course in the universities, and with the development of the *logica modernorum*, branches of logic newly devised by the medieval logicians themselves, Boethian Logic became less important in the thirteenth and fourteenth centuries, although his translations continued to be used by all students of logic, and some outstanding theologians, such as Albert the Great, Aquinas and William of Ockham, made some use of his commentaries. Moreover, *On Division* and TD remained part of the standard university logical collections — and commentaries were even written on TD in the thirteenth century. The monographs on categorical syllogisms were no longer useful now that the *Prior*

Analytics itself was known, and the treatise on hypothetical syllogisms too was forgotten [see Martin, 2007].

2.3 The Encyclopaedists

Between Boethius's death and the late eighth century, no Latin writer worked on logic in a serious and sustained way. But two accounts of logic written within longer encyclopaedic works would be important in the centuries following.

Cassiodorus

Cassiodorus (484/90–590) succeeded Boethius in his post at Theoderic's court, and he probably had a hand in ensuring the preservation and transmission of Boethius's work. In 554, he retired to a monastery at Vivarium in southern Italy and wrote a handbook called the *Institutions* (*Institutiones*) [Cassiodorus, 1937; transl. Cassiodorus, 1946], which was widely read in the early Middle Ages. In Book II he gives short accounts of the seven liberal arts among which is logic (II.3.1-18). There are very summary accounts of the matter of the *Isagoge*, the *Categories* and *On Interpretation* (which is singled out for its difficulty). For all these, he uses Boethius's translations [Courcelle, 1941, 80-3]. He then presents categorical syllogisms, following Apuleius [Sullivan, 1967, 173-7], definition, basing himself on Marius Victorinus, and topical argument, in which it seems he is following Marius Victorinus's now lost work [Hadot, 1971, 115-41]. There is a list (II,13) of what he calls 'the modes of hypothetical syllogisms': the phrase is Boethius's, but the list is closer (though not identical) to that given by Martianus Capella and so, probably, by Marius Victorinus.

At the end of the section on logic (II,18), Cassiodorus includes a list of those 'by whose efforts these things have come into the Latin tongue'. In the standard text of the *Institutions*, Marius Victorinus is credited with the major role — as translator of the *Isagoge*, *Categories* and *On Interpretation*, the writer of monographs on hypothetical syllogisms and definition and the commentator on Aristotle's *Categories* and Cicero's *Topics.* Apuleius is mentioned for his categorical syllogistic, and Cicero, of course, for his *Topics.* Boethius's role is reduced to that of commentator on the *Isagoge* and *On Interpretation* — despite the fact that it was his translations of Aristotle and Porphyry which Cassiodorus had in fact used (and it is uncertain whether Victorinus did in fact translate the *Categories* and *On Interpretation*, or comment on the *Categories*). In one textual tradition, however, the list is changed, and all the writings attributed to Victorinus are attributed to Boethius, even the commentary on Cicero's *Topics,* where Victorinus and not Boethius seems to have been the source. A complex explanation involving an original draft and additions at various stages has been proposed to explain this strange difference in the lists [Courcelle, 1941; Hadot, 1971, 105-9], but perhaps political factors could provide a better explanation. Another feature of the pro-Boethian textual tradition is that these manuscripts contain a series of additions of logical

and other material, including extracts from Boethius's *On Topical* differentiae. [Cassiodorus, 1937, xxv, xxxvii].

Isidore of Seville

Isidore, Bishop of Seville in the early seventh century and relentless collector of facts, was even further from being a logician in his own right than Cassiodorus. His most influential work, surviving in more than a thousand medieval manuscripts, is called the *Etymologies* (*Etymologiae*) [Isidore, 1983], and it is an encyclopaedia which organizes its discussion around the etymologies, usually imaginary and sometimes fantastic, of words. It contains a short account of logic (II, 25-31) mostly borrowed from Cassiodorus, sometimes with mistakes that show a compiler who does not understand his material. Thus the Apuleian account of categorical syllogistic reappears, along with the same brief passage on hypothetical syllogisms, the account of definition and the topics. Isidore also takes some material from Martianus Capella and Marius Victorinus's *On Definition*, which he uses, along with Cassiodorus, in giving summaries of the *Isagoge*, *Categories* and *On Interpretation*. He also seems to be the first author to makes use of the *Ten Categories*, which would become popular two centuries later.

The Character of the Encylopaedic Tradition

At first sight, it would appear that, alongside the Aristotelian logical tradition transmitted through Boethius, the encyclopaedists transmitted a more rhetorically-based tradition, going back to Marius Victorinus, in which Cicero's *Topics* is the central text. The account in Martianus Capella is, arguably, strongly rooted in Victorinus, and Cassiodorus too seems to have looked to Victorinus, and Isidore follows Cassiodorus. But there is also an emphasis on the Categories in these accounts, which would be taken up strongly by their medieval readers, and Martianus Capella's presentation works against a rhetorical conception of logic, by simply placing the account of the topics in his presentation of rhetoric.

3 CAROLINGIAN BEGINNINGS

([Van de Vyver, 1929; Marenbon, 1981])

For a century and a half from the time of Isidore there seems to have been no study of logic in Latin Europe, and no writing about it, even third-hand. Logic was re-discovered at the court of Charlemagne at the end of the eighth century.

3.1 Logic at the Court of Charlemagne [Marenbon, 1997]

There were two different strands of logical work among Charlemagne's court intellectuals, one associated with the Anglo-Saxon Alcuin, the other with Theodulf, who came from Visigothic Spain.

Alcuin as a Logician

Alcuin (d. 806) was educated at the Cathedral school of York and managed, from the 780s until he was made Abbot of Tours in 796, to gain a position and, ultimately, a good deal of influence, in Charlemagne's court at Aachen. Among his compositions are text-books on grammar, rhetoric and logic. The one on logic, *Dialectic* (*Dialectica*) [Alcuin, 1863], written in the form of a dialogue between Alcuin and Charlemagne himself, can claim to be the first medieval work on this discipline. In setting out these three linguistic arts of what came to be called the 'trivium' as basic elements of education, Alcuin was, in a sense, merely following the lead of Cassiodorus, though unlike him neglecting the mathematical arts of the quadrivium. But in a preface to the three works, pointedly called *On True Philosophy* (*De vera philosophia*) [Alcuin, 1863, 951-76], Alcuin makes explicitly the case for the liberal arts as pillars in an education that will finally lead to the temple of Biblical wisdom (cf. [Marenbon, 1984, 172-4]). This was an educational programme that Alcuin ensured was politically underwritten; Charlemagne's role as pupil and interlocutor in *Dialectic* is a flattering endorsement of the direction of royal policy.

Alcuin did not just help to place the arts of the trivium at the centre of medieval education: he helped to make logic the foremost among them. But from *Dialectic,* a text not only wholly derivative, but copied in the main — with one important exception — verbatim from the encyclopaedic accounts of Cassiodorus and Isidore [Prantl, 1885, 16-19; Lehmann, 1917], perhaps with some use of Boethius's first commentary on *On Interpretation* (queried in [Kneepkens, 1998]; but [Bullough, 2004, 404] upholds its use), it is not easy at first to see *how*. Certainly not because of any logical thinking done by Alcuin himself: the one section for which there is no obvious source, Chapter XII on 'Arguments', is simple in the extreme and avoids giving even the summary accounts of types of categorical syllogisms or modes of hypothetical syllogisms available in the encyclopaedias.

Yet Alcuin manages his material with a clear sense of his own particular purpose. In Isidore's *Etymologies*, his main source, the presentation centres on logic as a tool for devising and judging arguments: the section on syllogisms is the longest, and as in Cassiodorus it comes at the end of the work, along with the presentation of topical arguments, punctuated by the discussion of definitions. Alcuin ends with his treatment of material from *On Interpretation*, and, as mentioned, he gives a very simple and brief treatment of syllogistic. The emphasis of *Dialectic* in terms of space and detail falls on the exposition of the Categories. Here alone Alcuin turns to a non-encyclopaedic source, the *Ten Categories,* the intelligent paraphrase produced in the circle of Themistius (see above, 2.1). The excerpts from it occupy two fifths of the whole treatise. To judge from *Dialectic*, logic is above all about the doctrine of the Categories.

There is every reason to believe that Alcuin would have been happy to leave this impression. He believed that the *Ten Categories* was the work of Augustine, no less, and he wrote a prefatory poem for it which would accompany it in the

manuscripts [Aristotle, 1961-1, I, 1–5, , lxxxvii] in which we learn that this 'little book contains ten words which hold everything that we are able to perceive'. The *Categories* are, therefore, a key to understanding the sensible world — and more than that. In a letter dedicating his *On the Faith of the Holy Trinity* (*De Fide Sanctae Trinitatis*; finished 802) to Charlemagne, Alcuin professes the hope that by his treatise he will convince

> . . . those who have been judging as of little use your most noble intention to want to learn the rules of the discipline of dialectic, that in his *On the Trinity,* St Augustine thought them necessary above all, when he showed that the most profound questions about the Holy Trinity cannot be explained except through the subtlety of the Categories. [Dümmler, 1895, 415]

Alcuin has in mind the passage in *On the Trinity* (V.2-5) where Augustine asks whether Aristotle's Categories apply to God, and decides that, properly speaking, only the first, substance, does. In his own work, based in part on Augustine's, this theme is given pride of place in Book I. Just because, as he says in his prefatory poem, the ten Categories provide a comprehensive classification of everything in the sensible world, they are the perfect tool for understanding the distance between God and his creation. Within a programme where studying the liberal arts, especially the trivium, is justified as providing the foundations for Christian wisdom, logic in particular is thus seen to have a special doctrinal role, once the centrality of the Categories is recognized and their part in clarifying some of the central mysteries of the faith.

Theodulf of Orleans and the 'Libri Carolini'

Theodulf of Orleans (d. 821) was another of Charlemagne's court intellectuals. Mainly known as a poet, his part in the history of logic is due entirely to *King Charles's Work against the Synod* (*Opus Caroli Regis contra Synodum* or *Libri Carolini*) [*Opus Caroli,* 1998] — the official response from the Latins to the second Council of Nicaea (787), composed between 790 and 793. Although issued in the Emperor's name, the *Work against the Synod* has now been shown fairly surely to be the work of Theodulf [Freeman, 2003, collecting her articles on this theme; Meyvaert, 1979, for a summary of the discussions]. It is only in one chapter (IV, 23) that logic is employed, but here its use is concentrated. The aim is a simple one: to show that the Greeks are wrong in contending that 'to kiss' and 'to adore' mean the same thing. Theodulf chooses to make his critique of this (strikingly weak) position the excuse for a lesson in, and display of, logic. He borrows from Boethius's first commentary on *On Interpretation*, and takes a long explanation from Apuleius about the truth-values of different propositions. Above all, he sees the chapter as an opportunity to demonstrate his prowess in handling arguments, even though none of this complicated syllogizing is necessary, or even germane, for overturning the Greeks' position.

Theodulf's logical project differs from Alcuin's in the emphasis it places on argument, and its lack of interest in the more metaphysically-inclined logic of the Categories. I once argued [Marenbon, 1997], using the chronology of Alcuin's work established by his biographer, Donald Bullough [Bullough, 1991, 37; Bullough, 1997, 581-2] that the first medieval logician was not Alcuin, but Theodulf. But it turns out to be more plausible to retain the traditional dating of *Dialectic* to 786-90 [Bohn, 2004], and there is evidence, in any case that Theodulf made use of it [Opus Caroli, 1998, 61; English version in Freeman, 2003, 88-9]. Despite the difference in their main aims and interests as logicians, both Alcuin and Theodulf are alike in attaching logic to the official policy of the Carolingian Empire, as part of a scheme of Christian education and the defence of Christian doctrine. Between them, Alcuin and Theodulf map out the three main ways in which logic would be valued throughout the early medieval ages: as one of the Liberal Arts, themselves accepted as the beginnings of an ultimately Christian educational scheme; as a tool in theology; and as a weapon in the fight against heresy.

Theodulf, Alcuin and Alcuin's pupils

A merging of the approaches of Alcuin and Theodulf may be indicated by the contents of the earliest surviving logical manuscript, MS Rome, Biblioteca Padri Maristi, A. II. 1, which belonged to an associate of Alcuin's, Bishop Leidrad, who died c. 814. Along with the *Ten Categories*, Porphyry's *Isagoge* and extracts from Alcuin's *Dialectic*, it includes Boethius's first commentary on *On Interpretation* and Apuleius's *Peri Hermeneias* [Delisle, 1896]. The other piece of evidence that survives about interest in logic at the turn of the ninth century very much reflects Alcuin's interests, perhaps because it is found in a set of passages connected with his pupil, Candidus Wizo [ed., Marenbon, 1981, 151-66]. The logical pieces are mostly linked to the *Ten Categories* (I, XIV, XV), though one (XII) takes an Aristotelian passage from the theologian Claudianus Mamertus, and another (XIII) may be taken from Boethius's *Categories* commentary, though it can perhaps be explained as an elaboration of Cassiodorus.

3.2 Eriugena and the Ninth Century

([Marenbon, 1981, 67-87])

John Scottus Eriugena, who worked in the 850s and 860s at the court of Charlemagne's grandson, Charles the Bald, dominates the logic, as well as the philosophy and theology, of the ninth century. As in all his work, Eriugena's logic shows a strange mixture between ideas that fit closely with the Latin tradition, and ideas that are foreign to it, introduced by the fact that John learned Greek and was deeply influenced by Greek Christian Neoplatonism, combined with an intellectual temperament inclined to take such elements to their extremes. In order to appreciate this combination, it is important to place John in the context both of his Latin predecessors and contemporaries, and the gloss traditions that be-

gin shortly after his lifetime; and to look at John's probably earlier glosses on Martianus Capella's logic, as well as his masterpiece, the *Periphyseon*.

Macarius the Irishman and Ratramnus of Corbie on Universals ([Delhaye, 1950; Erismann, Forthcoming-A])

An interesting discussion about logic from the time of Eriugena survives in an unusual setting. In the early 860s [Bouhot, 1976, 59], Ratramnus of Corbie addressed his *Book on the Soul* (*Liber de anima*) to Bishop Odo of Beauvais [Ratramnus, 1952]. The indexRatramnus of Corbie!*Book on the Soul Book* is directed against an anonymous monk of Saint-Germer de Fly, with whom Ratramnus had been in correspondence, and who claims to be expounding the ideas of his master, an otherwise unknown Irishman named Macarius. Their difference was over the understanding of a passage in Augustine's *On the Quantity of the Soul* (xxxii, 69), where the question is raised, but not resolved, of whether human souls are all one, are many or are one and many. Macarius and his pupil (it is impossible to disentangle their contributions), and even more decidedly Ratramnus, make the issue a logical one: what is the nature of universals?

Macarius's position has been described as Platonic realism [Delhaye, 1950, 19-37] and as 'hyper-Realism' [Marenbon, 1981, 69]. In fact, it seems to be a rather crude and incompletely worked out example of what Christophe Erismann has labelled 'immanent realism' and traced back to Porphyry and the Greek Fathers and through to the twelfth century [Erismann, 2007; Forthcoming-B]. He holds that 'every human being is one human being through substance and every soul is one soul through substance' [Ratramnus of Corbie, 1952, 27:20-2, 29:25-6 and elsewhere]. As Ratramnus reports, he says that 'the human soul is a species, and from this species particular souls descend, which are also contained by it so that they can exist; for which reason if the species does not exist, nor can that into which it is distributed exist' [Ratramnus of Corbie, 1952, 130:20-3]. He looks to a passage from Boethius's *Theological Treatises* [V, 3; Boethius, 2000, 216:213-217:220], where genera and species are said 'just to exist' (*subsistunt tantum*) and individuals 'not only to exist but also to exist as subjects (*substant*); genera and species, Boethius goes on, do not need accidents in order to exist. To Macarius and his pupil, this passage asserts the primacy of genera and species. Although the idea is not explicitly worked out in what Ratramnus's comments preserve, Macarius seems to have held that particular members of a species are individuated by attaching accidents to the species, which is one and the same in them all.

Ratramnus thinks that, so far from particulars deriving their existence from species and genera, the basic constituents of the universe *are* particulars. His opponent has entirely misinterpreted Boethius. Just like genera and species, particulars (which he calls, oddly, *propria*) do not require accidents in order to exist: rather, they make it possible for the accidents themselves to exist, by being subjects for them. [Ratramnus of Corbie, 1952, 86:5-10] Particulars alone exist as subjects, whereas the genera and species just 'subsist'; and Ratramnus, giving an interpretation of Boethius's discussion of universals in the first commentary to the

Isagoge, believes that this means that, in themselves, genera and species do not exist in reality at all, but merely in thought:

> [Boethius] therefore says that species and genera exist, when they are considered in singulars, that is, when they are pronounced (*enunciantur*) of particulars; but when they are universals, they are not then said to exist, but just to be thought, because everything that is universal consists in cogitation, that is, in rational consideration, not in anything subsisting that could appear to the senses of the body. [Ratramnus of Corbie, 1952, 78:3-10].

The controversy between Ratramnus and Macarius's pupil shows that, even outside Eriugena's learned courtly circle, logic was being studied on the basis of the *Ten Categories* and some texts of Boethius, and that it retained the importance in theological controversy it had in the days of Charlemagne. Macarius's position on universals is, arguably, in a basic form that which Eriugena would propose but twist in his own direction, and Anselm would adopt two centuries later. Ratramnus's response anticipates, in a crude form, the line of argument that would be opposed to immanent realism when, in the early twelfth century, the theory of universals became once again a matter for dispute.

Eriugena on Book IV of Martianus ([Marenbon, 1981, 123-4; 2005, 228-30; Von Perger, 2005a, 264-301])

In about 850, John Scottus Eriugena was asked by Archbishop Hincmar of Rheims to write on predestination, against the views of Gottschalk. Although there are no records of Eriugena's life before then, there is reason to think that, at this time and perhaps earlier, he was teaching the Liberal Arts, probably at Charles the Bald's court, and using Martianus Capella's *On the Marriage of Mercury and Philology* as his basic text — his critic, Prudentius of Troyes, remarks [*Patrologia Latina* 115, 1294A] that his 'Capella has led him into a labyrinth.' The continuous commentary on Martianus written by Remigius of Auxerre at the turn of the tenth century draws explicitly on glosses by Eriugena, and from these mentions it is possible to link the marginal glosses in various manuscripts with him. The glosses in MS Paris BN 12960 cover the whole work, including Book IV on logic [John Scottus Eriugena, 1939; for Book IV only, a better edition, giving two different forms of the text, is given. in Von Perger, 2005, 281-301]. Although it is not certain that every gloss on this book is Eriugena's, it is very likely that many of them go back to him; but there are great problems in trying to give an authentically Eriugenian text: two redactions have been distinguished, but further research in the manuscripts might well find more versions, and none of them might correspond exactly to what Eriugena originally wrote [Marenbon, 1981, 117-9; Moran, 1989, 37-9; Von Perger, 2005, 274-7; and see below, 2.3, on the nature of early medieval Gloss Traditions]. Parallels between ideas in the *Periphyseon* and those in the glosses strengthen the case for attributing them to Eriugena, but also raise the

possibility that they were written in the same period as this work and not much earlier in his career.

Martianus divides his exposition roughly equally between a presentation of the material of the *Isagoge* and the *Categories*, on the one hand, and of that of *On Interpretation* and syllogistic (from Apuleius), on the other. Eriugena shows the direction of his interest by writing about twice as much on the first half. Moreover, the glosses on the second half show an intelligent grasp of the material, but not the very individual perspectives he brings to the *Isagoge* and *Categories.* Two moments are of particular interest. When Martianus discusses substance [Martianus Capella, 1983, 117:5], Eriugena writes

> He is speaking about substance which is essence, that is hypostasis, not *usia*, because *usia* is beyond all the Categories and does not receive any accident. Then it descends to substance, that is to hypostasis; then it receives accidents and has the first place in the Categories. [John Scottus Eriugena, 1939, 95:1-4; Von Perger, 2005, no. 82]

The word '*usia*' (i.e. the Greek *ousia*) is not found in Martianus, but is the regular term in the *Ten Categories*. But Eriugena uses it to make a distinction not found in that work, nor indeed anywhere else in the tradition. Eriugena seems to be thinking, just as he does in the *Periphyseon*, that there is some universal being, distinct from the individual beings that receive accidents (such as Socrates and Fido), from which particular things gain their existence. An earlier gloss [John Scottus Eriugena, 1939, 84:10-17; Von Perger, 2005, nos. 68, 80] had already, in line with the theory Eriugena develops in his major work, spoken of 'essence that comprehends all nature, by the participation in which eveything that exists subsists and which is called the most general genus.'

Eriugena also develops a theory about qualities which takes them, not as Aristotelian accidental forms, but as Platonic Ideas. Glossing the lemma 'Qualities can receive <more and less> through the substances' [Martianus Capella, 1983, 121:15-16] he explains:

> that is, in the substances themselves, as if he were to say: when, in the soul, you consider a notion of quality, because it is one and perfect, the quality does not receive in itself more and less. Similarly, if you consider the individual substance, I say that, because it is perfect and individual in itself it does not receive more and less. But if <you consider> the participation of quality through substances, you will find more and less. For there can be a certain substance that participates the same quality more than another. (. . .)[John Scottus Eriugena, 1939, 97:34 — 98:4; Von Perger, 2005, no. 93]

In the case of human beings, Eriugena suggests [John Scottus Eriugena, 1939, 97:25-33; Von Perger, 2005, no. 92], imperfect participation should be seen in terms of 'reaching after': a just or learned person really is perfectly just, and so justice and learning in this case do not receive more or less. But a person can

reach after learning (and, presumably, justice), and this is what is meant when we speak of people being more or less learned or just.

Logic in Eriugena's 'Periphyseon': the project and its sources

It is not in the glosses to Martianus's logical treatise, but in his long and ambitious theological dialogue, the *Periphyseon* (c. 862-6) [John Scottus Eriugena, 1996-2003] where Eriugena develops his views on logic most thoroughly. The *Periphyseon* treats of universal nature in its four different forms: as uncreated and creating (God) — Book I; as created and creating (the Primordial Forms, which are similar to the Platonic world of Ideas) — Book II; created and not-creating (the sensible world) — Book III; uncreated and uncreating (God, as that to which all things return) — Books IV and V). Eriugena's main way of articulating the distinction between God and his creation in Book I is through an extended examination of whether God fits any of the ten Categories. The theme is Augustinian, and Alcuin had highlighted it (above, 2.1). Eriugena, however, will give a sharply different answer from Augustine: even with regard to the first of the Categories — substance or, as Eriugena, following the *Ten Categories* calls it, *ousia* — it is truer to deny it of God than attribute it to him. But the innovativeness of Eriugena's presentation goes far beyond this daring theology: he has unusual ideas about the inter-relation of the Categories, universals, particulars and the nature of *ousia*, and the relationship between bodies and the system of Categories. But, before considering these themes, it is worth considering what Eriugena's sources might have been: although his method differed radically from the paraphrasing, cutting and pasting of Alcuin, his ideas had some definite starting-points.

The number of logical works Eriugena can be shown definitely to have used, from explicit references or definite parallels, is not large. He uses the *Ten Categories* extensively, but there is no sign that he knew Aristotle's *Categories* itself or Boethius's commentary to it. He was also, understandably, influenced by Martianus, which he had taught, and which sometimes leaves its traces on his phrasing. Although scholars, rightly, have seen Porphyry's Tree of genera and species, interpreted in a highly realistic fashion, as a guiding principle of his thought, there is scant evidence that Eriugena knew the text itself or either of Boethius's commentaries, rather than having gathered Porphyry's main ideas through the encyclopaedic accounts. One passage in Book III (702D-3B — references are to the columns of the edition in *Patrologia Latina* 122, since they are preserved in the later editions and translations) on substantial species has some loose parallels with Boethius's second *Isagoge* commentary [Boethius, 1906, 200], but nothing close enough to make borrowing even a probability. (On Eriugena's knowledge, indirect and possibly direct, of the *Isagoge*, see [Erismann, 2004, 405-12].) Eriugena has one, very passing reference to Aristotle's *On Interpretation*, in Book II (597C), where he refers readers to it in order to find out about possibility and impossibility. Although so vague a reference seems second-hand, none of the encyclopaedic accounts indicate that Aristotle discusses modality in this treatise, and

so it does seem likely Eriugena had seen the text or one of Boethius commentaries. There is also strong reason to think that Eriugena knew Boethius's commentary on Cicero's *Topics*. Eriugena thinks (491CD) identifies enthymemes as being arguments of the form $\sim (p \,\&\, \sim q); p$; therefore q. This idea is not found in any of the encyclopaedic accounts, nor in Cicero, but it is found in Boethius's commentary [Boethius, 1833, 364-5]. Other ninth-century scholars may have known more of the logical sources: Porphyry's *Isagoge* is found in Leidrad's manuscript, for instance; the *Categories* and *On Interpretation*, though rare, exist in ninth-century manuscripts, as do both of Boethius's commentaries on the *Isagoge* (the earliest manuscript of the first commentary, though, MS Paris BN 12958, is s. ix/x), the *Categories* commentary and the first commentary to *On Interpretation*. There is little sign that Eriugena had read any of them.

Eriugena's logic is, however, also influenced by some important non-logical sources. Like Ratramnus of Corbie, he found logical material in Boethius's *Theological Treatises*, but it was another theological tradition which most shaped his thought. In the 850s, Eriugena had been given the task of translating the works of (as it was thought) Dionysius the Areopagite into Latin, and he went on to use his skills to translate works by Maximus (the *Difficulties to John* (*Ambigua ad Johannem*) and *To Thalassius*) and Gregory of Nyssa (*On the Making of Man*). The strongly negative theology evident in Eriugena's treatment of the *Categories* comes from this tradition, and Eriugena also weaves other ideas from Maximus and Gregory into his treatment of the concepts he took from the *Ten Categories* and the logical tradition. Especially important is the approach to Porphyry's tree of genera and species mediated to him by Maximus [Erismann, 2004, 411-2; Zachhuber, 2005, 162-7].

Eriugena on the scheme of the Categories ([Marenbon, 2005, 230-1; Von Perger, 2005, 239-64])

In presenting the Categories, Eriugena is not content to list them and explain why none is applicable to God. He also suggests a number of different ways of dividing them up into larger groups, and of relating them to each other. Let us consider two of these divisions (a fuller account of the divisions is found in [Von Perger, 2005]). One of them shows just how idiosyncratic Eriugena is prepared to be in following through his ideas about *ousia,* which will be examined below. The division (471C-2A) is between Categories that are in *ousia,* and so can be considered its accidents — quality, relation, condition (*habitus*), action and being-acted-on — and the others, which are around *ousia* like, Eriugena says, limits set for it, whilst it is like a centre around which they revolve and which they require in order to exist. While the idea of distinguishing between Categories inside and outside *ousia* comes from the *Ten Categories* [Aristotle, 1961, 144:21 — 155:6], and there are echoes of Maximus the Confessor [*Difficulties to John* 30 — in Eriugena's translation: Maximus the Confessor, 1988, 167:4 — 168:25], the actual division that Eriugena makes is his own. Moreover, it turns out that this is not at all intended to be a neat division. Another of Eriugena's novel views is that condition (*habitus*),

ousia itself, quantity and quality can each be found in all the other Categories (466A-8B), because, for instance, there is a condition with regard to relationship between big, small and medium in the Category of quantity, or of father and son (in the Category of relation) to each other; and one can ask, for instance, 'What kind of (*qualis*) relation?', 'What kind of being-acted-on?' There are, therefore, varieties of quality and condition that are *not* in *ousia*, and Eriugena adds that relation too can be outside *ousia*; indeed, it is inside *ousia* only when it is the relation between genera and species.

Another division (469A) is between those Categories that are at rest (*in statu*) — *ousia*, quantity, being-in-a-position (*situs*) and place — and those that are in motion (*in motu*) — quality, relation, condition, time, acting, being-acted-upon. Although these divisions are not altogether intuitively obvious, Eriugena explains them (469B-70D), calling upon various assumptions he expects his readers to share or at least accept: for instance, quantity is at rest because everything is trying to reach its perfect quantity and remain there in it. Von Perger [2005, 246] sees one of the points behind Eriugena's multiplying of different schemes — schemes that agree neither with each other nor with the divisions put forward, in their discussions of God and the Categories by Augustine and Boethius — as an attempt to show that no matter how the Categories are grouped, they give no knowledge of God.

A more straightforward consequence of grouping the Categories, however, is that, contrary to Aristotle, they will no longer be the most universal ways of dividing up things. As Eriugena puts it:

> ... I have said that more diligent research can find some things in nature besides what is comprehended by the ten Categories — for these things have been found by the philosophers — lest someone of limited abilities should think that the careful investigation of things could not go beyond the number of Categories mentioned before. For there is a more general classification (*ratio*) of their genera — that they are in motion and rest. Further, rest and motion are collected by universal essence, which lets itself be divided to infinity. For the substance which has the first place in the Categories, is finite and subject to accidents, but universal essence receives no accidents in itself. Indeed, it is capable of accidents in its subdivisions extending as far as the individuals, but it in itself is simple and not subject to accidents. (597A)

Eriugena goes on to suggest that even the division he has just suggested is not comprehensive, because 'no one of those who correctly philosophize doubts that possible things and impossible things should be counted in the number of things' (597B), and he refers his readers (as noted above) to Aristotle's 'Peri ermenias'. This is an astonishing moment in modal theory, since, at a period when a statistical account of possibility and necessity in temporal terms was normal, Eriugena in a stroke conjures up the idea of innumerable possible and even, more strangely, impossible worlds. But for his broader discussion of logic and the metaphysics he draws from it, what is more important is the division he makes between two sorts

of substance or essence (parallel to the distinction made in the Martianus glosses). On the one hand, there is the first of the Categories, which he calls 'substance' and considers as a concrete individual, much in the way that first substances are envisaged in the Aristotelian logical tradition: it is 'finite and subject to accidents'. On the other hand, there is 'universal essence', which is seen as a super-super-Category, bringing together below it even the super-Categories of motion and rest.

Ousia, universals and bodies in Eriugena ([Eswein, 1930; Marenbon, 1980; Erismann, 2002; 2004; Kavanagh, 2005; Hochschild, 2007; Erismann, 2007; Erismann, Forthcoming-A])

What is the relation between substance and universal essence? It seems to be one of universal to particular, but of a rather special kind. We should say, Eriugena argues (506BC) 'that from the essence which is created one and universal in all things and is common to all things and, because it belongs to all of those which participate in it, is said to be proper to none of the singulars which participate in it, there flows by natural progression a certain proper substance, which belongs to none other except that to which it belongs alone.'

What Eriugena has in mind is clarified by his more general comments on particulars and universals. In his view (471A), a species is just 'the unity of numbers (i.e. numbered things = particulars)' and particulars just 'the plurality of their species', and so there is really no distinction 'in nature' between a subject (for instance, Cicero) and what is 'said of a subject' (*de subiecto*: for instance, human being) — that is to say, between primary and secondary substances — 'except that one is in number and the other in species'. Underlying this assertion is the view that 'the species is whole and one in its particulars (*numeri*) and the particulars are one individual in the species.' These surprising statements are, in fact, just the result of taking to its logical consequences a type of non-Platonic, immanent realism that seems to have been espoused by Ratramnus's opponent (see above, pp. 26–27) and has been traced back through the Greek Neoplatonic tradition [Erismann, 2004; Erismann, Forthcoming-A]. If particulars are thought to be differentiated only by accidents, then, regarded apart from any of their accidents, they are just their species; and the same reasoning applies to species, regarded apart from their *differentiae*, which are then just their genera. Eriugena goes somewhat beyond the ambit of this reasoning, however, by seeing these relationships not merely as an analysis of how the world really is, but as an account of its production. So he glosses Genesis 1, xxiv, 'Let the earth bring forth the living creature after his kind (*genus*)': 'First he puts down the genus, because in it all the species are contained and are one, and it is divided into them, and it is multiplied through general forms and most special species' (748C; see also 529BC). Porphyry's tree is seen — as he presents it explicitly in the preface to his translation of Maximus's *Ambigua* [Maximus the Confessor,1988, 3-4] — as a 'procession' coming from God, a 'multiplication' of God's goodness.

At the top of this tree is *ousia*, and the whole of creation is presented as 'a

division and multiplication' of it into genera and species (869A). Any particular of a natural kind, such as Socrates, is therefore an *ousia* in the Aristotelian sense of being a substance that acts as a subject for accidents, but it is also, when stripped of every addition (accidents and *differentiae*), and so when, as Eriugena would see it, traced back to its origins, *ousia* in the sense of universal essence. Were Eriugena presenting a purely logical analysis of generic and specific relations, designed to clarify predication, then he would certainly be highly confused to suggest that *ousia* could be regarded as a universal in this way. But since he is, rather, using Porphyry's tree to describe a process of creative emanation, his way of thinking makes good sense in its own terms. The existence of things is, for him, a process of specification and particularization. The true reality, God, is, he insists, non-existent because utterly unbounded and so incomprehensible even to himself. God makes himself manifest as the universe by undergoing a whole series of restrictions, into being and then into types of being and ultimately into particulars.

The particulars or *ousiai* are, however, just universal essence seen from the perspective of its greatest multiplicity. They are not, Eriugena insists, at all the same as bodies. The sharp distinction is obvious, he points out (491C-2B), from the way, that *ousia* is not divided up into parts but is the same in all humans as in one, whereas bodies are divisible into parts that are not identical with the whole. Bodies, he explains (492B–4A), are produced from *ousia*, which is incorporeal, and from quantity and quality and other accidents, which are also, themselves, incorporeal. The visible body which results is not a quantity or a quality but a *quantum* and a *quale*. Eriugena's idea seems to be that any body is a particular exemplification of a set of universal accidents (for instance, in the case of Socrates, six-foot-tall, ruddy-complexioned, sitting down and wearing a fetching beret). Those accidents would not, however, produce a natural body on their own, though accidents of quantity can produce a geometrical body — the diagrammatic, designation of a certain shape; they need to be conjoined with *ousia* (493C-4A). Does this mean, then, that bodies are the principle of individuation? Not exactly, since for Eriugena *ousia* is never individuated. Rather, it is a feature of reality, as it comes forth from universal being (which heads the hierarchy not just of substance but of accidents) that, because of the combination of universal, incorporeal accidents and *ousia* that has been specified into most specific species, there come into existence natural bodies, the only particular things in the universe. [Erismann, Forthcoming].

The Character of Eriugena's Logic

There are many other themes related to Eriugena's view of *ousia*: for example, there is the position of *ousia* as one of the primordial causes, and there is the long and fascinating discussion of place and time (esp. 474B-89B) in which, turning to the rhetorical idea of *loci* [cf. Kavanagh, 2003], he develops the idea of these Categories as definitions of things and then elaborates a theory in which, by knowing things, humans create them [Gregory, 1963; Cristiani, 1973]. Although

it is certainly open to wonder whether the many threads in Eriugena's discussion really link together into an entirely coherent theory [for a hyper-critical, and often ill-judged approach, see Marenbon, 1981, 72-87; Marenbon, 1980 (written later)], a charitable interpretation reveals a powerful line of thought, given a certain set of assumptions, especially about the immanence of universals.

In one sense, Eriugena followed an approach to logic that had been pioneered by Alcuin, who looked back to Augustine and Boethius — one which centred around the ten Categories and their use as a way of defining the relations between God and created things. But he developed it to a degree and in a direction that their work would not have led anyone to anticipate. As Abelard and Gilbert of Poitiers would do, nearly three hundred years later, he used ideas stemming from Aristotle's *Categories* to construct a metaphysics. But whereas the twelfth-century writers developed their metaphysical views within Boethius's Aristotelian approach to Aristotelian logic, Eriugena, whose knowledge of Boethius's logic was slight, elaborated his metaphysics under the predominant influence of the Greek Christian Neoplatonic texts he had translated. An intellectual temperament that valued both paradox and systematic coherence, and did not find them incompatible, led him to take this enterprise to an impressive if sometimes bewildering extreme. His may be perhaps the one example of a 'Platonic logic' [Erismann, 2007] in the Middle Ages.

3.3 The Gloss Traditions

([Marenbon, 1981, 116-38; Marenbon, 1997a; Marenbon, 2000b])

As seen above, Eriugena seems to have taught logic using Book IV of Martianus as his text-book, and his teaching is recorded, at least partially, in glosses. In the years after Eriugena, there is a Gloss Tradition to Martianus. The concept of a 'Gloss Tradition' needs explanation. Many manuscripts from the ninth to the eleventh centuries have extensive annotations in the margins and between their lines. Sometimes text and annotations have been copied together; sometimes the annotations have been added — and in some cases apparently at different stages. It is rare that the glosses in any one manuscript are exactly the same as those in another, but it is often possible to distinguish one or more than one tradition of glosses to a particular work, most of which occur in fairly similar form in a whole group of manuscripts, although each individual manuscript will have its peculiar glosses and omissions.

Besides the glosses to Book IV of Martianus, there are two other important early medieval logical gloss traditions — on the most popular of all the logical texts at this period, the *Ten Categories* and also on the *Isagoge.* The other core logical texts available — the *Categories* in Boethius's translation and *On Interpretation* did not generate Gloss Traditions, but isolated glosses in individual manuscripts provide valuable evidence about logical ideas and study.

A certain counter-balance to the anonymity of glossed manuscripts is provided by the fact that they can often be localized to a particular monastery. Careful

study could build up a picture of what logical teaching went on in each of the notable monastic centres. In the case of one monastery, St Gall, there is already some interesting evidence. Not only does an exceptional glossed manuscript of the *Ten Categories* (see below) originate from there; there are also a series of logical passages [Piper, 1882] that can probably be dated to the ninth century [De Rijk, 1963] which, like the glossed manuscript, combine a straightforward interest in mastering logical doctrine with an enthusiasm for Eriugenian ideas.

The Gloss Tradition to Martianus Capella, Book IV

Although the early medieval manuscripts and glosses to Martianus have been scrupulously catalogued [Leonardi, 1959, 1960], the Gloss Tradition has not yet been fully investigated and disentangled. Remigius of Auxerre's continuous commentary [Remigius of Auxerre, 1965] draws both on the set of glosses associated with Eriugena and on another tradition, perhaps linked to Heiric of Auxerre [Pseudo-Dunchad, 1944 — a partial edition from one MS]. There is little evidence of independent logical thinking; some of what Eriugena wrote is transmitted, and some garbled — for example, the gloss on 'What is substance?' reads:

> He defines substance not essence, that is not hypostasis but usia, because hypostasis is beyond all the Categories and receives no accident. Then he descends to usia, that is to substance, which can receive accidents and has primacy in the ten Categories. Substance and the accident of substance are predicated of this, that is of substance, in as many ways. [Remigius of Auxerre, 1965, I, 32:12-16]

Glosses to the 'Ten Categories'

Not surprisingly given its popularity, it is the *Ten Categories* which offers the most important of the early medieval Gloss Traditions [Marenbon, 1981, 185-206 — partial edition], found in nearly twenty manuscripts. Two main strands can be distinguished. There is the Eriugenian strand — glosses which put forward distinctively Eriugenian ideas or use Eriugenian language. And there are the standard glosses, each found in many of the manuscripts. One manuscript, from the later ninth century, MS Milan Ambrosiana B71 sup., contains almost entirely Eriugenian glosses. Two other manuscripts, MS Saint Gall 274, also from the second half of the ninth century and MS Paris BN 12949 (early tenth century) mix some of these Eriugenian glosses with the standard (and some other) material. Among the standard glosses, there are still traces of Eriugenian material.

Many of the Eriugenian glosses seem to be the product of someone whose mind is well stocked with motifs and phrases from the *Periphyseon* who has set about the task of commenting the *Ten Categories* after a pint too many; marginalia, but not perhaps to the history of logic. A more sober example of this style is a gloss entitled 'On Usia' [Marenbon, 1981, 196; no. XIII]. We are immediately told that *usia* is beyond human understanding and 'is better placed in nothing, because

it presents itself stretching beyond our *usia* and makes God himself'. 'Nothing' renders us a truer account of it than any predication. Another of these glosses [Marenbon, 1981, 202; no. XXIII] talks of *usiae* in the plural, as Eriugena also does, to mean particular substances. Whereas *usiae* can be seen and understood without accidents, accidents can be understood apart from a body only in the mind — the glossator seems here to have in mind the type of mathematical abstraction spoken about by Boethius in his second commentary on the *Isagoge*. The gloss then goes on to explain, in line with the ideas of the *Periphyseon*, that accidents are the cause of bodies. The lack of genuinely logical interest and concern with metaphysical or theological questions is typical of this strand of glosses.

Looking at the standard glosses, and at how this Gloss Tradition evolves from the ninth to the eleventh century, the impression is of a gradual turning away from extrinsic theological and metaphysical concerns and an attempt to understand the letter of the text, as witnessed by glosses that put the paraphraser's (already simplified) points in elementary terms. The more sophisticated questions about the doctrine of Categories, raised in the Greek commentary tradition and transmitted by Boethius, do not figure in the standard *Ten Categories* glosses. There is one exception. In MS Saint Gall 274, Eriugenian material and standard glosses are joined by some glosses peculiar to this manuscript, which show that this glossator *did* know Boethius's commentary on the *Categories* and was using it to help understand the pseudo-Augustinian paraphrase. He takes careful note, for instance, of Boethius's description of the subject-matter of the treatise as the utterances that signify things in that they are significant [cf. Marenbon, 1997a, 28-9].

The 'Isagoge'

According to Yukio Iwakuma [his information is presented in Marenbon, 2000b, 99], the early medieval Gloss Tradition on the *Isagoge* has two strands, each found in full form in two manuscripts, while one late-ish (11th-12th-century) manuscript combines the two. The glosses belonging to one strand have been edited in full from MS Paris BN 12949 [Von Waltersharsen and Baeumker, 1924], under attribution to a certain 'Icpa', because a line in the explicit reads 'I, Icpa|||| , wrote this little book, glossing it in some way'. More recent research [Jeauneau, 1985] has shown that the name is that of 'Israel' (written in Greek letters), a tenth-century grammarian of Irish origin, whose interests can be seen more widely in this richly annotated manuscript that includes a set of *Ten Categories* glosses containing the Eriugenian strand. The *Isagoge* glosses themselves are simply a patchwork of quotations, especially from both of Boethius's commentaries, though Macrobius's Commentary on the *Somnium Scipionis*, Isidore of Seville, Alcuin's *Dialectica* and the *Ten Categories* glosses are also used. The glosses are well chosen to illustrate the text, but no individual approach to the material emerges from them.

Glosses to the 'Categories' and 'On Interpretation'

There is no Gloss Tradition for the text of Aristotle's own *Categories*, nor for *On Interpretation*, but individual manuscripts contain some glosses. For example, MS Paris BN 2788 (end of tenth century), which contains Boethius's genuine translation of the *Categories* and MS Cologne Dombibliothek 191 (eleventh century), containing the more widely available composite translation, probably made up from two versions both by Boethius (cf. above, p. 8), both have glosses which draw from Boethius's commentary.

Glosses to *On Interpretation* are exceedingly rare, but one, in Leiden Voss. lat. F 70 (tenth century), to Chapter 9, is very interesting:

> Whatever exists which is impossible not to exist, always exists. Whatever is impossible to exist, always does not exist. And whatever exists that is possible not to exist, does not always exist. Whatever exists that is possible to exist, does not always not exist. Now two are eternal and two temporal. For fire, which is never cold, always heats. Therefore it [heat] is always and eternally in it, but the other [cold] is always absent. Now, of those that are temporal, some are substantially, some accidentally. And substantially not always is whatever is corrupted, and also not always is whatever is generated. What accidentally does not always exist and does not always not exist is whatever is varied by the changeability of some accident in such a way that it is changed from existence into non-existence or from non-existence into existence. These two therefore are contingents, because they happen one way or another (*utrumlibet*). The two above, however, are from simple necessity and can never come about contingently. [Latin text: Marenbon, 1997a, 30, n. 31]

Although the ideas here can be found in Boethius's commentaries, this comment is exceptionally clear-minded in its analysis of modality in terms of time — the statistical view that was one of the dominant models in antiquity and the earlier Middle Ages [Knuuttila, 1993, 1-62].

Gloss Traditions and logic within medieval education

The existence of logical Gloss Traditions (on *De Nuptiis* IV, the *Ten Categories* and the *Isagoge*) is important, not for any logical innovations, since they contain none, but for two other reasons. First, it shows that logic had a full part in the course of studies followed in this period mainly in monastery schools. Second, the fullest tradition, that of the glosses to the *Ten Categories*, shows how scholars turned away from the Eriugenian interests of the ninth century to concentrate on trying to assimilate basic Aristotelian logic. This change of direction is born out by the wider developments, beginning in the year just before 1000, which we shall now consider.

4 THE DEVELOPMENT OF A BOETHIAN TRADITION

([Minio-Paluello, 1972])

The change in Latin logic shortly before the turn of the millennium can be thought of as the founding of a Boethian tradition of logic, which was then dominant until at least the mid-twelfth century. Laurenzio Minio-Paluello (1972) called it, rather 'the second phase in the rediscovery of Aristotle and Boethius', but it is more revealing to see it both as specifically Boethian and, in being such, as a new development rather than merely a 'second phase'. Certainly, the new tradition is Aristotelian, in that two books of the *Organon*, the *Categories* and *On Interpretation*, occupy an important place, and it is Neoplatonic, in that Porphyry's *Isagoge* is read, as in the late ancient schools, as an introduction to the study of logic; but, even more strikingly, it reflects Boethius's particular version of logic as practised by the Neoplatonists, a version heavily indebted to Porphyry and following his tendency to stick to Aristotelian views within logic. Not only were the texts of Aristotle and Porphyry read in Boethius's translations and studied using his commentaries; Boethius's monographs were themselves the authoritative texts for half the curriculum. And, despite the use to a very small extent of his translations and commentaries in the previous two centuries, this Boethian tradition came into being only from the late tenth century: the earliest medieval logic, as the sections above have shown, was *not* strongly influenced by Boethius, but by the *Ten Categories*, the encyclopaedic accounts and even Apuleius.

One of the most telling signs of the new approach is the gradual replacement of the *Ten Categories* by Boethius's translation of Aristotle's text (usually read in the so-called composite version: see above, p. 8). Whereas half of all the surviving manuscripts of the *Ten Categories* date from before c. 1000, only three or four of the 300 odd medieval copies of Aristotle's text in translation date from that period. Copies of the Boethian translation are found at Chartres, Fleury and St Gall around 1000 and, from the eleventh century there remain 25 manuscripts of the translation, around 20 of Boethius' commentary, and just 6 of the *Ten Categories* [Minio-Paluello, 1972, 754-5]. In the case of *On Interpretation*, although there is no dramatic rise in the number of manuscripts of the text, the eight eleventh-century manuscripts of the second commentary, as compared to just three from earlier, indicate that scholars are now studying the work seriously. The letter written by Gunzo of Novara to the monks of Reichenau in 965 — a letter which is designed almost entirely to show off Gunzo's erudition — gives a good indication of the position, in that he comments that his contemporaries 'have hardly tried out or have failed to penetrate' [Gunzo of Novara, 1958, 37:14-15] the obscurity of *On Interpretation*, but he makes sure to cite it, and Boethius's second commentary, frequently.

The most striking change was the reception of Boethius's monographs — on syllogistic, division and topical *differentiae* (along with Victorinus's treatise on definition misattributed to Boethius). A Fleury manuscript from the later tenth century (now split: MS Orleans Bib. mun. 267 and MS Paris BNF n. acq. lat.

1611), containing the whole set of monographs, seems (from a note it contains) to have this material from a collection of Boethius's works put together in Constantinople in the 520s [Van de Vyver, 1935, 131-2]. Whether (as most scholars believe) the exemplar was brought to Europe from Byzantium in the tenth century, or it was simply studied and copied for the first time in this period, this rediscovery would have an important effect on how logic was studied. With regard to syllogistic, both surviving manuscripts and library catalogues show that, after the tenth century, there was a turn from using Apuelian theory (directly from his *Peri Hermeneias*, or indirectly from the encyclopaedias) to using Boethius's treatises [Sullivan, 1967, 193-203]. The theory of the Topics, now much more conveniently available in the *On Topical* differentiae than before through the commentary on Cicero and the encyclopaedic accounts, would become one of logicians' main concerns by the twelfth century.

The three outstanding logicians of the period are also those in whom the new developments can be seen with especial clarity: tentatively in Notker of St Gall, more definitely in Gerbert of Aurillac and most obviously in Abbo of Fleury.

4.1 Notker of St Gall

St Gall had been an important centre for logic since the ninth century (see above, p. 36). Notker Labeo (c. 950–1022) continued this tradition, especially within his project of translating central school-texts into the vernacular, Old High German. Of the four works he chose to translate, two were logical — a mark of the prominence of the subject in the curriculum, especially since the other two translations (Books I and II of Martianus Capella; Boethius's *Consolation*) were general philosophical works rather than texts of a particular discipline. Moreover, the two texts were both Aristotle: the *Categories* and *On Interpretation*, and to the German version some Latin glosses, mainly based on Boethius's commentaries, are added [Notker the German, 1972; 1975]. Notker is therefore, in this way, a pioneer of the new Boethian logic.

Yet the most sophisticated of the writings attributed to him, *On Syllogisms* [Piper, 1882, 597-622], is based closely on works known since the ninth century: Apuleius's *Periermenias* for categorical syllogisms, and, for the hypothetical syllogisms, Boethius's commentary on Cicero's *Topics* and Martianus Capella. By basing himself ultimately on Cicero, Notker produces an account a little nearer the Stoics than he could have found in *On Hypothetical Syllogisms*, had he known it — and he even copies [Notker, 1882, 610-11] from Martianus Capella (cf. p. 6) a presentation of the modes using numbers. But his conception of hypothetical syllogisms is thoroughly Boethian, as his adopting Boethius's deviant form of the third mode indicates. He even elaborates this third mode, because he thinks its major premiss can be formed by preposing a negation to a contradiction of all four forms:- (1) 'If it is day, it is night'; (2) 'If it is not day, it is not night'; (3) 'If it is day, it is not light'; (4) 'If it is not day, it is light.' He has given some thought to how this mode 'arises from the first two':

> For day and light are connected to each other, and for this reason the true propositions of the first and second modes are: 'If it is day, it is light'. Then, if someone wants to destroy the truth by adding a negation, saying, 'If it is day, it is not light', the person who adds a second negation, saying 'Not if it is day, it is not light' returns it again to truth. The first two <modes>, therefore, are through the link of what naturally cohere together (e.g. 'if it is day, it is light'). The third through the negation of opposites: e.g. it cannot be that it is day and it is not light. [Notker, 1882, 612-3]

And another indication of the place of St Gall in this movement is a commentary probably from the late tenth century on the *Isagoge* and *Categories* [Excerpta Isagogarum, 1995], which is mostly extracted from Boethius's commentaries (for the *Isagoge,* both the first and second are used), but takes the form of a dialogue. Despite the fact that one of the manuscripts links this commentary with Fleury, in its earliest form it seems most probably to come from St Gall [Marenbon, 1997a, 26, n. 22]. The commentary also has links with the earlier tradition, though, in the form of material taken from Alcuin, Boethius's commentary on Cicero's *Topics* and the *Ten Categories* [Excerpta Isagogarum, 1995].

4.2 Gerbert of Aurillac

The most famous, by far, of the pioneering logicians of the late tenth century was Gerbert of Aurillac, who taught at Rheims from 972 until he became the archbishop there in 991, ending his life as Pope Sylvester (999-1003). There is an account of his teaching at Rheims by his pupil Richer, which gives some very precise details:

> He ran through dialectic following the order of the books and unravelled their meaning in clear words. For first <he took> the 'isagoges', that is, the Introductions, of Porphyry, using the translation by Victorinus, the Orator, and then he also explained them according to Boethius. Following on from this, he expounded Aristotle's book of the 'Categories', that is, of the predicaments. And he very skilfully showed how to work at the *Periermenias*, that is, the book on interpretation. Then he also made his listeners know about the topics, that is, the seats of arguments, translated by Cicero from Greek into Latin, and explained by the Consul Boethius in a commentary of six books. He foresaw what was useful for the progress of teachers of rhetoric, and he also usefully lectured on and extracted the meaning from the four books *De topicis differentiis*, the two on categorical syllogisms, the three on hypothetical <syllogisms>, the one on definitions and again the one on divisions. [III, 46-7; Richer, 1877, 101]

Since Richer wrote his account in the years immediately after Gerbert became archbishop, it should give a faithful account of what Gerbert taught, at least at

the end of his time as a schoolmaster. If so, then his curriculum seems, then, to have been a fully Boethian one, close to what would be the norm in the twelfth century, though with a few peculiarities, such as the use of Victorinus's translation of the *Isagoge* (a sign, perhaps, that it was the first of Boethius's two commentaries on the work Gerbert turned to). There is no earlier mention of a medieval logician teaching using all of Boethius's monographs (and the treatise on definition by Victorinus, usually misattributed to Boethius), though it is striking that Cicero's *Topics* with Boethius's commentary (available since the ninth century) is mentioned in first place, and the *De topicis differentiis*, which would become the main textbook, only afterwards.

No records of Gerbert's lectures on Porphyry, Aristotle and Boethius survive. His one logical work is a short treatise called *De rationale et ratione uti* [Gerbert, 1867, 297-310], that is to say, a discussion of the meaning of the terms 'rational' and 'to use reason' as used by Porphyry when he discusses what features genera and *differentiae* have in common. The point at issue was that, when we say 'What is rational uses reason', an apparently true proposition, we seem to be saying something false, since not everything that is rational is actually its reason. The basis of the problem is an ambiguity of the kind that could lie at the basis of the sort of sophisms which twelfth-century and which later logicians enjoyed unravelling. Gerbert's approach is less direct, but he succeeds in showing up the logical inadequacies of those contemporaries who had raised this difficulty and showing off his own knowledge of Boethius commentaries on the *Isagoge* and *On Interpretation*.

4.3 Abbo of Fleury

([Van de Vyver, 1935; Abbo of Fleury, 1997: Einleitung; Schupp, 2004])

Abbo led a far quieter life than Gerbert, mostly at the monastery of Fleury, which he entered shortly after his birth in c. 945. He studied in Paris and Rheims, and spent a short period in England (985-7); in 988 he became Abbot of Fleury. His logical work was therefore probably done in the 970s and early 980s, at much the same time as Gerbert's.

Although there is no explicit record, as in Gerbert's case, that Abbo taught the whole Boethian logical curriculum, the manuscript evidence suggests that he at least knew the range of Boethius's monographs, since the earliest manuscript which contains the treatises, mentioned above, also contains Abbo's works on categorical and hypothetical syllogisms and was written at Fleury in his lifetime. These two treatises are themselves the very best illustrations of the change in the curriculum, since each looks both backwards to the mainly non-Boethian sources that had been dominant, and forward, by basing itself to a great extent on Boethius and Aristotle. So, in the treatise on categorical syllogisms [Abbo of Fleury, 1966, 1-64], he uses mainly *On Interpretation* and Boethius's commentaries, but he also draws on Apuleius, and he explicitly compares the two sources, putting their squares of opposition and their moods of the syllogism side by side. Abbo is

not, then, just a copier of authoritative logical texts, but someone who regards them inquisitively. And he produces some surprises. One is his view that all syllogisms, both categorical and hypothetical, are 'like a lifeless body' without the topics [Abbo of Fleury, 1966, 64:14-17]. Another is a discussion of necessity and possibility in which he ends by saying that, although Aristotle holds that substances are susceptible to contraries, these contraries are merely what appear to the senses: understood in themselves, substances are no more susceptible to one contrary than another, and 'in its nature fire is no more hot than cold' [Abbo of Fleury, 1966, 62:25-35].

The treatise on hypothetical syllogisms [Abbo of Fleury, 1997] similarly combines with other material a newly discovered Boethian source *On Hypothetical Syllogisms*, which provides the matter for the first eight chapters. In the ninth, he gives, as 'seven general rules' for hypothetical syllogisms, the doctrine of the seven modes which stems, ultimately, from the Stoic indemonstrables. Although Boethius, and perhaps Victorinus, had already seen these as modes of hypothetical syllogisms, Abbo goes further by claiming explicitly that all the different types of hypothetical syllogism discussed, following Boethius's treatise on them, in the previous chapters can be reduced to these seven forms [Abbo of Fleury, 1997,102:1-15]. The source for the list of modes itself is not obvious. Cicero's *Topics,* Martianus Capella, Cassiodore and Isidore all provide the basic form of this list. But Abbo's list shares with Boethius's commentary on the *Topics*, and Notker's *On Syllogisms,* the peculiarity (see above, , pp. 17–18, 39) of giving the third mode with a conditional first premiss — for instance: 'Not if it is a human, it is not an animal; but it is a human; therefore it is an animal' [Abbo of Fleury, 1997, 94:23-5]. And he explains what Boethius had in mind better than Boethius himself had done: 'The meaning of these propositions is that if a human exists, it cannot be that he is not an animal; that is, that if there is a human, it is impossible that there is not an animal.' Yet Abbo's editor, Franz Schupp, who has brought to light the interesting combination of the two types of doctrine on hypothetical syllogism, considers that Boethius's commentary is not the source for this chapter, and that Abbo probably took it whole from some source later than Boethius and, indeed, than Isidore of Seville. The main reason for thinking so is that, near the end [Abbo of Fleury, 1997, 100-1], Abbo puts forward a distinction between the Latin way of regarding the modes — he names Cicero, Victorinus and Isidore — and the Greek way, as espoused by Eudemus and 'other Greek authors': the Latins consider that modes 4–7 can only be constructed if the terms of the major premiss are immediate contraries (they are such that everything within their field of applicability is one or the other and not both); the Greeks disagree. Abbo certainly could not have taken this (very dubious) information from Boethius or any other source we know, but is that enough to show that this very independently-minded author was simply copying?

5 THE ELEVENTH CENTURY: LOGIC AND THEOLOGY

From the evidence of manuscripts copied, and especially from the position that had been reached by c. 1100, it is clear the eleventh century was a time when the developments in logic of the later tenth century became widespread and were taken further. By the end of the century, the thorough study of the core texts of the *logica vetus* — Porphyry's *Isagoge,* Aristotle's *Categories* and *On Interpretation*, and Boethius's text-books on categorical and hypothetical syllogisms, division and the topics — had become established, at least at the leading schools, and logicians were launching themselves into the interpretative problems stimulated by Boethius's commentaries and by the parallels and discordances with the Stoic semantics found in Priscian's *Grammatical Institutions* (*Institutiones grammaticae*), a work much studied in this period. There is, however, a surprising lack of purely logical texts that can be dated to the eleventh century. Garlandus's *Dialectic,* once dated to before 1050, is now recognized as the work of Garlandus of Besançon, and may well have been written later than 1110 or even than 1120. In cataloguing the continuous commentaries to the *Isagoge*, *Categories* and *On Interpretation*, I found none for which there is compelling evidence of its being written before 1100 [Marenbon, 2000b]; Green-Pedersen [1984, 147-53, 418-9] suggests that two or three surviving commentaries on *De topicis differentiis* may be a few years earlier, but without decisive proof. Since, as the next chapter will explain, these commentaries are layered compositions, it is extremely probable that the earliest layer or layers go back to before the time of the earliest manuscripts, but it is impossible to discern exactly what material belonged to these first forms of the commentaries.

For logic in the eleventh century, then, we must look to four thinkers who used it especially in connection with theology: Lanfranc and Berengar, in their dispute over the Eucharist; Peter Damian, in discussing whether God can undo the past; and Anselm, who was also responsible for two ostensibly non-theological logical discussions.[2]

5.1 Logic and the Eucharist: Lanfranc and Berengar

([Holopainen, 1996, 44-118; Marenbon, 2005a, 232-7])

In the sacrament of the Eucharist, Christians maintain that the bread and wine blessed by the priest are the body and blood of Jesus Christ. Berengar of Tours, writing in the middle of the eleventh century, maintained that this identity must be understood just in terms of signification. The priest's words of consecration do indeed bring about a change, but it is to make the bread and wine into signs for Christ's body and blood. But there is no change in the substance of the bread and wine, and if there were — Berengar maintains — then they could no

[2]The greater part of Odo of Tournai's treatise *De peccato originali* [in *Patrologia Latina* 160, 1071-1102; trsl. Odo of Tournai, 1994] is concerned with the problem of universals (see [Erismann, Forthcoming-A, and Forthcoming-B]). Since it was not written until 1095 at the earliest, and perhaps as late as 1110, and its concerns are intimately linked to the discussions of the early 1100s, it is better discussed in the context of the next chapter.

longer perform their signifying function. In his *De corpore et sanguine Domini* [Lanfranc, 1854; written 1061-70], Lanfranc disagreed strongly with this view, provoking Berengar's most substantial surviving work, where he defends his theory, the *Rescriptum contra Lanfrannum* [Berengar of Tours, 1988]. There is, then, a semantic, or rather semiotic side to the dispute [Rosier-Catach, 2004, 36-40, 355-63], but also a more strictly logical side. Contrary to what was once held [Southern, 1948], Lanfranc seems not to have been very fond of the new Aristotelian logic, and himself avoids the terminology of the *Categories* and the *Isagoge* when he is discussing the Eucharist, although he is willing at times to use ideas from the theory of topical argument [Holopainen, 1996, 44-76]. It is, rather, Berengar who uses Porphyry and Aristotle to argue his case, and even translates Lanfranc's position into these terms, so as to be able most effectively to criticize it.

According to Berengar, Lanfranc wants to hold that, although the bread used in the Eucharist is destroyed as a substance and subject for accidents when it becomes flesh, it also remains, because its accidents remain. He accepts that this continuity condition is necessary if the process is to fulfil its doctrinal role, since in the Eucharist it is not a matter of one thing being replaced by another, but of the bread and wine themselves becoming Christ's body and blood. But he argues, putting his opponent's position into rigorously Aristotelian terms, that Lanfranc's theory cannot satisfy it. Lanfranc, he says, has two alternatives. By the first, the substance of the bread is destroyed and in its place there is a different substance, the body of Christ, to which bread-like accidents are attached, ones the same as those in the original bread, but just in the sense that they are like (*qualia*) them; they are not claimed to be numerically the same. This alternative, Berengar believes, is metaphysically possible and so open to God's power:

> ... it would be easy for God to add to the flesh (which you believe God to make on the altar through the generation of the subject) the same accidents as those which were formerly in the bread ...

But the result would merely be that

> ... it could be said that it is not bread in species, but rather flesh, made now by God through generation of the subject, which has been given accidents like those which were in the bread, and that it is bread because it has the colour and accidents of bread. [Berengar of Tours, 1988, 159:2140-6].

There would be something different, the flesh of Christ, replacing the bread and made to resemble it, and so the continuity condition would not be satisfied.

The second alternative is that what replaces the bread as a substance (the body of Christ) is informed with all or some of the very same accidents numerically as were in the original bread. Berengar suggests that this alternative might satisfy the continuity condition, but, as he explains, it is simply impossible metaphysically:

> If ... it happens that the bread of the altar that was taken away through the corruption of the subject is after the consecration no longer

> the individual bread that was placed on the altar, it is completely clear that it is not the bread according to some of its accidents which, because they were in no other place than in it, when it was taken away through the removal of the subject, could in no way not be taken away themselves.
>
> If the bread and wine ceased to exist when they have been removed from their original essences (*a pristinis essentiis*), nothing which had been in them as subjects could have survived. [Berengar of Tours, 1988, 158:1388-1405; 159:2138-40].

Although Berengar does not explicitly give the reason why it is impossible that the accidents should be numerically the same between the bread and the body of Christ, his position suggests that he considers accidents to be individuated by the substances they inform: they can only persist as those particular accidents so long as their subject persists.

If so, then Berengar not only shows that he has thoroughly absorbed the main ideas of the *Categories*, but also that he has arrived at a view on the relation between substances and accidents, quite plausible as an interpretation of Aristotle, but different from what Abelard and other twelfth-century thinkers would propose.

5.2 Peter Damian

([Resnick,1992; Holopainen, 1996, 6-43; Marenbon, 2007, 116-8])

Peter Damian (1007-72) would probably be horrified to find himself commemorated in a History of logic. He was an ascetic, who thought it was wrong for monks to spend much time studying logic or other secular subjects. But, within the theological context of his *On Divine Omnipotence* (*De divina omnipotentia*) [Peter Damian, 1972], he provides a more sophisticated discussion of modality than any previous medieval author and, although he does not make a display of his knowledge, he seems to have read *On Interpretation* and probably Boethius's second commentary, as well as knowing some of Boethius's teaching on hypothetical syllogisms and on topical argument. His discussion is provoked by a comment of Jerome's that might seem to restrict God's power. 'God can do everything', said Jerome, 'But he cannot restore the virginity of a woman who has lost it.' One way Jerome's comment might be taken is that it denies God's capability to perform a miracle by physically restoring the virginity of a woman who has lost it by making the ruptured hymeneal tissue whole again. Not surprisingly, Damian rejects it if it is understood in this way: he has no doubt that God can produce such miracles [Peter Damian, 1972, 402-6].

Jerome can also be understood, however, as denying, not that God can perform a physical miracle, but that he can change the past: he cannot make it so that the woman in question never lost her virginity. Damian wants in some way to reject Jerome's comment even on this interpretation and so to claim that, in some sense, God *can* change the past. He does not, however, just want to assert this extreme

position, but to argue for it as logically coherent. He begins [Peter Damian, 1972, 412-4] by distinguishing between how the future can be open (*ad utrumlibet*) so far as the 'variable nature of things' is concerned (I may or may not see my friend today, for example), and yet according to the consequence of discourse (*consequentiam disserendi*) they may be fixed. He means by the consequence of discourse, for example, the fact that 'if it will be that it rains, then it is entirely necessary that it will rain, and for this reason it is completely impossible that it will not rain.' Since he insists that such necessity is not genuine, Damian is not wrongly concluding from

15. Necessarily, if it will rain it will rain

to

16. If it will rain, necessarily it will rain.

On the contrary, he is recognizing, that (15) does *not* imply (16). But he does not have the tools to distinguish the scope of modal operators, and so he concludes that the sort of necessity in (15) is purely verbal and does not lead to the conclusion that the rain is inevitable. Although this line of thought is in itself simply an argument against a mistaken logical argument for determinism, it brings out the idea that, though the future cannot be changed, in that the future is simply what will happen, this does not mean that events in the future are not open. Damian goes on explicitly to draw the parallel with the past: 'according to the order of discourse, it is impossible that whatever was, was not' [Peter Damian, 1972, 414:34-6]. The implicit suggestion, then, is that, just as there is a sense 'according to the order of discourse' in which the future is unchangeable, so too with the past. And just as being determined 'according to the order of discourse' still leaves it open to change what that future will be, so, although it is impossible that what was, was not, the past too is open to change.

This implicit argument, however, ignores the fact that the future and the past are not parallel in respect of determinism. Past events have more than a necessity 'according to the order of discourse'. To say about something in the future 'it may happen or it may not' involves no incoherence; but to say of an event in the past 'it may have happened or it may not have happened' makes sense only if taken epistemically, as a profession of ignorance or uncertainty: there is a fact of the matter, and if the event did take place, it cannot now, we feel, not have taken place. Damian does not air such worries explicitly, but an awareness of them may explain why he goes on to give a new turn to his discussion [Peter Damian, 1972, 414:34-6]. He explains, in terms that echo Book 5 of Boethius's *Consolation of Philosophy*, that God has a different relation to time than ours:

> It is clear that omnipotent God holds all the centuries in the treasury of his eternal wisdom so that nothing can come to him nor can anything pass from him through the moments of time. Therefore, remaining constant in that ineffable citadel of his majesty, he contemplates all

> things set up in the sight of his presentness with a single, simple gaze, so that past things never pass from him, nor do future things follow on. [Peter Damian, 1972, 418:3-10].

Damian does not spell out immediately how this point, which was made originally in the context of showing that God's prescience does not determine future events, relates to the question at issue. But near the end of the treatise [Peter Damian, 1972, 478:102-3] he observes that God's 'ability (*posse*) is fixed in eternity, so that whatever he could do at some time he can always do' and he then states that the debate he had entered into can be brought to a conclusion, because 'if the ability to do all things is co-eternal with God, God could makes it that what has been made did not exist.' The most coherent way to put together these remarks is to say that Damian's God has power over all events, but this does not involve his changing anything that is past *to him*, since nothing is past to him: all events are presented to him in his eternal present. But if God were to change a past event — to make it that that there was some day last August when it didn't rain, or that I studied at Oxford — then it is that changed event which would in fact, at the time, have been the one that happened, as brought about by God in his eternal present. God can 'undo' the past, because, since it is present to him, there is in fact no undoing to be done, but just the choice of this over that possibility to actualize. But, living in a temporal frame, we humans cannot change what has happened. [See Knuuttila, 1993, 63-7 for a discussion of how Damian may have somewhat unconsciously innovated here in modal theory.]

5.3 Anselm

Anselm (1033–1109), whose surviving writings all probably date from the 1075 and later, when he was a monk and then abbot of Bec, and from 1093, Archbishop of Canterbury, is unquestionably the outstanding philosopher and theologian of the eleventh century. He was also an accomplished logician, but logic is on the margins of his life's work. In his theological treatises and dialogues, there is only one moment when Aristotelian logic is brought into play explicitly, in order to consider a problem about foreknowledge and determinism. But Anselm tended usually not to parade his sources. In his first theological work, the *Monologion,* he includes a short but innovative discussion of God's relation to the Categories; and, elsewhere in his theological writings, logical reading and thinking underlies many positions in which it is not made explicit, including the famous argument at the beginning of the *Proslogion*. He also wrote a dialogue called *De Grammatico*, containing no theological discussion, which is entirely logical and semantic in its concerns.

Logic in Anselm's Theology: God and the 'Categories' ([Marenbon, 2005, 237-9])

In the *Monologion* (1076) [Anselm, 1946, 1-87; translations of this and the other works of Anselm in Anselm, 1998] attempts to prove by reasoning that God exists and is triune. Although Anselm does not give a Category by Category account

of the relationship between God and the Ten Categories, he is clearly following the tradition of Augustine, Boethius [cf. De Libera, 2005], Alcuin (and Eriugena, though he probably did not know his work), when he considers how God relates to the Categories of place and time (Chapters 18-24), and substance and relation (Chapters 25-7). Augustine had argued that God not only falls into the first Category, which he calls essence, but that he is alone truly essence. Boethius follows, unusually for him, the line of Neoplatonic negative theology which Eriugena would take to its limit, and he declares that God does not really fit into the first Category at all (*Theological Treatise* I, 4). Anselm wishes to say, rather, that it is both true, in one sense, that God is not a substance, since he receives no accidents or *differentiae*, and that he is neither an individual nor a universal substance, since to be individual would mean that he shared an essence with others, and to be universal that he is divided into a plurality of substances. Yet, in another sense, God can rightly be called an individual, spiritual substance. God can be considered, Anselm thinks, an essence, distinct from all substances, but, since it is usual to call the essence of a thing its substance, in this sense God is a substance. Since God has and can have no parts, he cannot be divided up, and so in this sense he is an individual. And since the most worthy (*dignus*) substances are body and spirit, and the spirit is worthier than the body, God can be called a spirit. The treatment of time and place is highly complex [Leftow, 1991, 183-216], but it follows the same pattern, declaring that it is true in one way that God is in place and time, because he is everywhere and there is no time at which he is not, and also that he is in neither Category and so in a sense is never and nowhere.

Anselm seems, then, to sound a note of scepticism about the Categories as tools for understanding where God is concerned, by the virtuosity with which he can argue both for God's inclusion in and exclusion from them. This marks an important difference from his late ancient predecessors. Augustine and Boethius (followed even more emphatically by Alcuin, though arguably not by Eriugena) considered Aristotle's scheme supremely useful in marking the distinction between God and his creation, even though it produced in the case of most of the Categories a negative result. But for Anselm the lack of fit between the Categories and God leads, not to a negative proposition, but to a clash of contradictory propositions, a breakdown in meaningfulness. Does this scepticism extend even further, to the very scheme of the Categories? In his discussion of accidents [Anselm, 1946, I, 43] Anselm extends the thinking of Augustine and Boethius about the extrinsic nature of relations, pointing the example of his relation to a person born next year. At the moment he has no relations to him, but when the person is born he will acquire, without any change in himself, relations of being, for example, older and being similar. Such accidents, says Anselm, are only improperly called such, since they do not 'happen to' (*accidunt*) the thing of which they are predicated. Although such an observation may not at have disturbed Aristotle, it does strike at the way in which medieval authors up until the twelfth century tended to use the Categories as an overall metaphysical scheme.

Logic in Anselm's Theology: logic as a tool

The *Proslogion* (1077-8) [Anselm, 1946, I, 92–122] is Anselm's best known work, because of the famous Argument it proposes (especially in Chapters 2 and 3) for the existence of God. The Argument itself, as well as the *Response*) [Anselm, 1946, I, 130-9] written to Gaunilo's *Reply on behalf of the Fool*, brings up difficult issues of modality and logical consequence, which do not show any clear direct relationship to the tradition of logic Anselm had studied but are certainly informed by it. There is a vast range of modern discussion of Anselm's Argument, and here is not the place to enter into the issues raised by it [for a recent discussion, with bibliography, see Kapriev, 2004]. It is worth observing, however, that when, in the *Proslogion* Chapter 2 argument, Anselm concludes from the fact that the Fool understands 'that than which nothing greater can be thought', that it 'is in his intellect', he is drawing on a semantic theory, going back to Augustine and Boethius, and which he discusses more explicitly in the *Monologion* (Chapters 10 and 33). According to this theory, as well as spoken and written words, there are mental words, which are natural and the same for all people. The process of thinking and understanding requires the production of a mental word, which bears a close resemblance to the object of the thought in the external world. Anselm's claim is that the Fool produces a mental word to correspond to the phrase 'that than which nothing greater can be thought' which he hears and understands. [King, 2004, 85-8]

In the Proemium to the *Proslogion*, Anselm contrasts the work with the *Monologion* by saying that there he used a combination of many arguments, whereas now he will establish the existence and main attributes of God by 'one argument' (*unum argumentum*). But what, exactly, is the *one* argument? Toivo Holopainen has proposed recently [Holopainen, 1996, 133-45] that this question, which has divided interpreters, can be understood properly and answered only in the light of the logical background: in particular, Anselm's knowledge of the theory of topical argument, probably through Boethius's Commentary on Cicero's *Topics.* According to Boethius, the middle which is sought in order to give a convincing answer to a question is an *argumentum.* In this light, Anselm might well mean by his 'single argument' not a chain of reasoning (such as the Chapter 2 'argument') but, Holopainen suggests, the complex term 'that than which nothing greater can be thought', since it is certainly in every chapter through this term that Anselm is able to carry through his plan. It is perhaps not so plausible that a *term* could be an *argumentum*, since the aim of an *argumentum* is carry conviction, and it is hard to see how that can be achieved by an expression that lacks some kind of propositional structure. Maybe the *argumentum* is the claim that God is that than which nothing greater can be thought.

The one place in his theological writing where Anselm explicitly refers to a logical writer (Aristotle in *On Interpretation*, although the name of the treatise is not actually mentioned) is in Book 2, Chapter 17 [Anselm, 1946, II, 120-6] of *Cur Deus Homo*, Anselm's explanation, as the title indicates, of why God became incarnate. Anselm has just argued that before Christ was conceived Mary, the

mother of Christ, was purified of sin by his future death (the so-called 'Immaculate Conception'). The objection is then raised that, if so, then Christ must have died of necessity. The problem, then, is a variant of the problem of prophecy and necessity, which is itself one version of the problem of prescience. In the problem of prophecy, A knows at t^1 that a proposition p about what happens at some future time t^2 is true, and so it seems that what happens at t^2 cannot be contingent, because t^2 cannot turn out to be false if A *knows* it to be true. In the case of the Immaculate Conception, the truth at t^1 of the proposition

17. Mary is purified of sin by Christ's death at t^2

is incompatible with its turning out to be false that

18. Christ dies at t^2.

Anselm tries to solve the difficulty by recourse to a line of thought similar to that Peter Damian had used in his *On Devine Omnipotence*. He distinguishes [Anselm, 1946, II, 125] genuine, preceding (*praecedens*) necessity from what he calls 'sequent necessity' (*sequens necessitas*). Genuine necessity involves constraint. It is necessary that the sky revolves because 'the violence of its natural condition forces' it. But in a case of sequent necessity, such as the necessity that you are talking because (*quia*) you are talking, there is no compulsion. Although the matter is not clear (especially since Anselm varies his connectives between 'if' (*si*), 'because' (*quia*) and 'while' (*dum*)), Anselm seems — just like Damian — not to have the logical equipment to distinguish between

17. Necessarily (if x is what took place/is taking place/will take place, then x is what took place/is taking place/will take place)

and

18. If x is what took place/is taking place/will take place, then x took place/is taking place/will take place necessarily.

Since (17) is obviously true, he feels compelled to accept (18), which he does not discriminate from it, but — like Damian again — describes the necessity involved as being limited in a way that makes the force of (18) no more than that of (17). Anselm then says that it is this sort of non-coercive, sequent necessity that Aristotle talks about in *On Interpretation* and that it is by this necessity that it is said that it was necessary for things to be as the 'faith or prophecy' about Christ's death said. Unfortunately, Anselm has not seen the seriousness of the problem which faced Aristotle and which, in an even more serious form, faces him. He is right to try to suggest, although he lacks the equipment to do so clearly, that from the fact that, necessarily, if I am doing x, then I am doing x, it does not follow that I am necessarily doing x (except if 'necessarily' is drained of its ordinary meaning). But he is unaware that the argument from prescience or prophecy gains its force from the temporal element: when gaining knowledge of an event precedes

that event, and is fixed because it is passed, then it presents a serious obstacle to maintaining that the events known could be otherwise [see Marenbon, 2005b, *passim*]

Elsewhere, Anselm does enter into a far complex discussion of modality, which has been seen as an early attempt 'to codify some aspects of the semantics implied in the Christian use of the notions of necessity and possibility' [Knuuttila, 1993, 70, and see 70-4]. His longest exploration of these themes is found in a set of 'Philosophical Fragments' preserved in a single, Lambeth manuscript [Anselm, 1969, 334-51]. A study of this side of Anselm's thought would be out of place, however, in an account of his logic, since he develops it in the course of thinking about the philosophy of action and of will. Anyone interested in Anselm as a logician should, however, examine the 'Philosophical Fragments', since they show Anselm's logical, and indeed analytical cast of mind, as he ranges widely over areas of philosophy suggested by his theological interests, but approached in a thoroughly philosophical spirit.

Anselm's 'De Grammatico': the argument

The only finished work by Anselm not on a theological subject is a dialogue *De grammatico* [Anselm, 1946]. Anselm's biographer, Eadmer, says that he composed the work in the same period (1080-5), just after the *Proslogion,* as he composed his dialogues on truth and free-will. The dating has been challenged by Richard Southern, who argues that Eadmer is simply misinterpreting Anselm's own preface to *On Truth* [Southern, 1990, 65]; Southern suggests the work was written twenty years earlier. Although he is right to point out that Eadmer's dating is in principle open to question, there is no good reason to make this work much earlier than anything else by Anselm. Indeed, as more knowledge is gained of the grammatical and logical debates around the year 1100, it seems that there is a great deal in common between its main themes and some of Anselm's concerns — a point which should emerge in the course of the next chapter.

The dialogue is about the problem of the place of denominatives, such as *grammaticus*, in the scheme of the Categories. The early medieval notion of denominatives (or 'paronyms': Grk. *parônuma*) comes from a short passage in the *Categories* (1a12-5) and Boethius's commentary on it [Boethius, 1891, 167D-8D]. Aristotle's example of a denominative is *grammatikos*: denominatives, he says, 'get their names from something' — the *grammatikos*, for instance, from grammar. The wider theory that emerges from his brief remark and Boethius's amplification is this: - A thing can often be named from one of its accidents, by using the word for the accident with a change of ending. For instance, the male person who is informed by the accident of knowledge of grammar (*grammatica*) can be called a '*grammaticus*' (the gender-specification is because the ending of the word indicates masculine sex); the man who is informed by an accident of whiteness (*albedo*) can be called '*albus*', 'the white man'. '*Grammaticus*' and '*albus*' are, therefore, denominative words. But Aristotle talks of denominative *things*: in his

and Boethius's view, the things named by denominatives, the *grammaticus* and the *albus*, for instance, are not simply to be identified with the substances, the man or men, who are informed by the accidents of knowledge of grammar or whiteness.

There is, then, an immediate problem for the logician. Is a *grammaticus* a substance or an accident? Anselm spells out the problem immediately at the start of his dialogue. There is a simple argument to show that *grammaticus* is a substance:

19. Every *grammaticus* is a man.

20. Every man is a substance.

21. Every *grammaticus* is a substance.

Yet this conclusion, although supported by the leading grammatical authority, Priscian, contradicts Aristotle, who in the *Categories* (2b28) uses *grammaticus* as an example of a quality. The dialogue explores the problem raised by this clash, treating the question as both one about things — is a *grammaticus* a substance or a quality? — and about the words (*voces*) for the things — is '*grammaticus*' a word which signifies a substance or a quality? (The awareness of these two levels of discussion is one of the striking ways in which *De grammatico* fits the way the *Categories* would be discussed up to the 1130s.)

The dialogue goes through many complex twists and turns, but it seems, about three quarters of the way through [Anselm,1946, I, 163:23-5; 164:5-7], to reach a definitive solution to the problem. This solution has three elements:

22. The word '*grammaticus*' signifies a quality.

23. The word *'grammaticus'* appellates a substance.

24. *Grammaticus* is a quality.

Clearly, this solution is based on the distinction between what it is for a word to signify something and what it is for it to appellate something. For Anselm, as for most medieval writers, signifying means causing a certain sort of thought: as Paul Spade [1975, 213-4] formulates it, 'a word "w" signifies a thing x if and only if "w" causes a thought of x in the mind of a competent speaker of the language in question.' Anselm adds a refinement to this basic notion, which he brings out with a thought experiment [Anselm, 1946, I, 160:4-161:4]. Suppose I say, 'There's an *albus* in the house' ('*albus*' is the masculine singular nominative adjective, which can mean any sort of white masculine thing). From what I say, my listener cannot tell whether I am saying that there is a man in the house, or a horse. But if, pointing at two horses, I say, 'Hit the *album*' (where *album* is the accusative form, as required by the syntax, of *albus*), my listener *does* know that a horse is meant. Anselm would say that *per se*, *albus* does not signify a man or a horse, but it can signify a man or a horse *per aliud*, when the context provides the information that it is a white horse (and not a white man or dog) that we are discussing. The *per*

se signification of a word is, then, that of which it causes a thought irrespective of context; its *per aliud* signification is that of which it causes a thought in a particular context.

The reason why Anselm can assert (22) is, therefore, that the thought which the word '*grammaticus*' causes is not that of a man, who is a substance, who knows grammar, since there is nothing in the word to make the listener think of a man, as opposed to some other sort of being, who knows grammar. Yet this contention seems mistaken. Whereas an *albus* might really be a white man, a white horse or a white snail, who can be a *grammaticus* but a man? Anselm answers that, although it cannot be shown that in fact there are any non-human *grammatici*, there might in principle be some sort of non-human rational animal that knows grammar [Anselm, 1946, 1, 157:30-158:5]. A genuinely contextless, and so *per se*, account of the signification of '*grammaticus*' must exclude even that very general context which makes us think that only humans are *grammatici*.

Most modern analysts of *De grammatico* roughly equate appellation with reference. On this reading, (23) makes the obviously true claim that the objects picked out by the word *grammaticus* — that is to say, men who know grammar — are substances. There may, however, be some anachronism in this interpretation of *appellatio*. Anselm himself [Anselm, 1946, 1, 157:5-6] defines *appellatio* in terms of ordinary usage: the appellative name 'is that by which the thing is called according to the ordinary use of speech' (*usus loquendi*). The way in which we say 'dog' is the word in ordinary usage for dogs, 'white thing' for things that happen to be white, does provide a reference-like way of linking words directly to objects in the world. Anselm seems to be working towards an idea of reference, without having a completely secure grasp of it. He sometimes treats signification *per aliud* and appellation as if they were interchangeable [e.g. Anselm, 1946, 1, 157:1-5], a merging that is understandable, since the *per aliud* signification as given by the general context will correspond to the *usus loquendi*. Indeed, according to Adams [2000, 87], the *per aliud* signification *establishes* 'the semantic connections between terms and things of which they are not *per se* significative' — a position that does not seem to correspond to anything that Anselm himself suggests.

Whereas (22) is straightforward, and (23) is certainly true, though a little hard to interpret, (24) is problematic, since — on the assumption that seems to be being made that something can belong to only one Category — it directly contradicts the conclusion to the initial argument, (19-21), that *grammaticus* is a substance. This argument is valid, and neither of the two premisses has been refuted. The minor premiss, (20), that every man is a substance is unquestionable. The major premiss, (19), that every *grammaticus* is a man, is not at all incompatible with Anselm's assertion that there it is possible for there to be a *grammaticus* who is not a man. How, then, can Anselm be entitled to assert (24)? As Adams points out [2000, 88], Anselm immediately adds, after (24), the point that 'Aristotle appellates words by the name of the things of which they are significative, and not of those of which they are merely appellative' [Anselm, 1946, 1, 163:26 -8]. Aristotle, then, he is saying, sometimes uses a word '*w*' that seems on the page to be a word for a *thing*

(for w) as a word for another *word* (as a word for 'w'), but only where 'w' signifies w, not where it merely appellates w. He gives as an example a passage in the *Categories* where Aristotle says 'every substance seems to signify this something', where by 'substance' Aristotle must have meant 'substance word'. The implication is that, for Anselm, on the Aristotelian reading, the question about denominatives should not really be considered to be one about things at all. The claim that '*Grammaticus* is a quality' must be taken as 'The word *'grammaticus'* signifies a quality' (i.e. as 22, which has already been established). Were a similar analysis, taking the word to refer to another word, applicable to the claim '*grammaticus* is a substance', it would come out as 'The word *'grammaticus*' appellates a substance', that is, as 23). But Aristotle does not accept this analysis for propositions where the relation is one of appellation, not signification, and so there is no acceptable meaning that can be given to '*grammaticus* is a substance'.

Anselm is, therefore, presenting what he recognizes as a narrowly Aristotelian solution to his problem, and the student in the dialogue expresses this limitation when he sums up the reply as being 'according to Aristotle's treatise and according to his followers.' It is interesting to recognize that, earlier in the treatise, Anselm had presented a different solution to the initial problem, which seems a little more broadly based:

M. Tell me, when you speak to me about a *grammaticus*, what should I understand you to be talking about — the word '*grammaticus*', or the things it signifies?

P. The things.

M. What things, then, does it signify?

P. A man and knowledge about grammar

M. So when I hear the word '*grammaticus*', I may understand a man or knowledge about grammar, and when I am speaking about a *grammaticus*, I may be speaking about a man or knowledge about grammar.

P. So it must be.

M. Tell me then, is a man a substance, or is it in a subject?

P. It's a substance, not something in a subject.

M. Is knowledge about grammar a quality and in a subject?

P. It's both.

M. Why then is it strange if someone says that *grammaticus* is a substance and not in a subject with respect to a man (*secundum hominem)* and that *grammaticus* is a quality and in a subject with respect to knowledge about grammar (*secundum grammaticam*)? [Anselm, 1946, 1, 154:7-21]

And a little later it is added that there is no contradiction between holding that *grammaticus* is a substance and that it is a quality because when we are talking about or understanding a *grammaticus*, on some occasions we should speak or understand with respect to a man (*secundum hominem*) and on some occasions with respect to knowledge about grammar (*secundum grammaticam).* [Anselm, 1946, 1, 156:1-4]

According to this solution, then, certain things, such as denominatives, can be envisaged under more than one aspect, and they might fall into different Categories, depending on how they are envisaged.

Interpreting the 'De grammatico' ([Henry, 1964; 1967; 1974; Adams, 2000; Marenbon, 2002; Boschung, 2006, King, 2004])

Until less than fifty years ago, *De grammatico* was Anselm's most neglected work, dismissed even by specialists as trifling in its concerns. Then, in a series of studies and commentaries, D.P. Henry sought to show that this dialogue is a piece of sophisticated and intelligent logical analysis. No one now who understands the text would disagree that Henry was right to see great value in it as a work of logic. He arrived, however, at a very particular interpretation of the dialogue, according to which one of Anselm's main aims was to establish the inadequacy of ordinary logic for discussing logic and to set up a special logical language, which would at times shock ordinary readers by its apparent ungrammaticality. He also considered that Lesniewski's logical system provided the formal tools for making Anselm's project fully explicit. Few have fully accepted Henry's reading and recently it has been intelligently criticized by Marilyn Adams [2000, 90–105] (and also in [Boschung, 2006], itself an interesting and careful interpretation).

Adams has her own view of the point of the dialogue [2000, 105-12]. She sees it as an introduction for students to Aristotle's *Categories*, based on 'concentrated and sustained attention to a single issue', which trains them 'to be meticulous, to slice the distinctions as thinly as texts and problems require' [Adams, 2000, 112]. Behind her decision to regard this dialogue as tool for teaching lies a recognition of its inchoate quality: it is technically finished, but it is not a neat statement and resolution of a problem — it seems to say either too little or far too much. Yet the difficulty and tortuousness of the argument tell against her thesis (and especially the idea that it is intended for students less advanced than those who would study the other, far more straightforward theological dialogues).

Another reason for the impression of confusion might be that Anselm is at once outlining an Aristotelian solution to an Aristotelian problem and — as may also be the case in the *Monologion* — suggesting that Aristotle's doctrine of the Categories may not be as clear or even as reliable as most of his contemporaries took it to be. The 'broader' solution mentioned above, that Anselm presents but then drops, breaks with Aristotelian orthodoxy by supposing that one thing could belong to more than one Category, when seen under different aspects. As one of the very few medieval logicians capable, by his own extraordinary mental powers, of exploring areas of metaphysico-logical argumentation that Aristotle

and his followers had never touched on (as the *Proslogion* and the *Philosophical Fragments* testify), is it unexpected that he should hint — gently but insistently — that even after the bravest efforts of interpreters Aristotle's *Categories* leaves unsolved almost as many fundamental problems as it raises?

5.4 Epilogue. Anselm on Universals and the New Era

([Iwakuma, 1996; Erismann, Forthcoming])

Anselm
indexRoscelin of Compiègneopened his *De incarnatione Verbi* ('On the Incarnation of the Word'; first version 1091-2), an attack on the Trinitarian theology of the logician Roscelin of Compiègne, by criticizing his opponent as one of those who

> think that universal substances are merely the breath of an utterance (*flatum vocis*) and who are not able to understand colour as other than the body, or a person's wisdom as other than his soul .... [Anselm, 1946, I, 285 (and cf. 289); for revised version: II, 9-10].

Anselm's own views about universals can be elicited from passing remarks in his various works, especially the passage from the *Monologion*, discussed above (5.3.i), arguing how God is neither an individual nor a universal substance. Like most medieval writers, he has a place for Platonic-like universals in the Word (or mind) of God. He is also, however, an immanent realist, who thinks of there being a common essence for all the particulars of the same genus, and who thinks of species sharing a common *differentia* and individuals being individuated by a collection of accidental properties peculiar to each one [*De processione Spiritus Sancti*: Anselm, 1946, **?**, 217:17-20; cf. Erismann, Forthcoming-A].

Historically, Anselm's not very developed views on this subject are less important than the fact that he does not pause to develop them, and that even when he attacks Roscelin, he states his point without argument, and so briefly that he has left historians puzzling over what exactly his opponent's position could have been. One of the elements which would distinguish the logic of the turn of the twelfth century and its early decades from that of Anselm and his period was the irruption of the dispute over universals. True, it has been given a more prominent place historiographically than it deserves — Abelard himself deprecated those who behaved as if they thought that logic consisted just in this problem — but it is a striking example of a more general tendency for logic to become a matter for disagreement and dispute between the advocates of different positions. Parallel with this development, most evident in the contest between Abelard and William of Champeaux in the period from 1100 to 1117, was the growth of a far more regular, structured teaching of the curriculum of the *logica vetus* (the *Isagoge*, *Categories*, *On Interpretation* and Boethius on categorical and hypothetical syllogisms, division and topical *differentiae*), which gave rise to a profusion of commentaries. Perhaps this teaching stretched back decades earlier, as the activity of Gerbert or Abbo would suggest; but it is only from c. 1100 that evidence of it survives. Unravelling

this complicated evidence will be the difficult task of the writer of the next chapter. At one stage, a theory was accepted by insufficiently cautious scholars [e.g. Marenbon, 2000b] according to which William of Champeaux was responsible for large numbers of anonymous commentaries in a variety of versions. Now, although William's importance is acknowledged, it is not generally thought that any single commentary can reliably be taken as his own direct work [Cameron, 2004]. At the same time, a re-evaluation is taking place of the so-called 'vocalist' logicians, among whom Roscelin is usually numbered, and whom William of Champeaux is taken to have opposed. What were their positions? When did they flourish? And did they form a coherent group at all?

The crowded logical schools of early twelfth-century Paris worked with the Boethian Tradition of logic that had first come into use late in the tenth century. Their teachers were ultimately informed by the experience of logical discussion and exegesis of texts that stretched back even earlier in the Middle Ages. Yet, in the new conditions of teaching and the new contexts of debate, even the old questions were seen in a different perspective. Intellectual life never observes neat chronological boundaries, but the year 1100 is at least a symbolically good date to mark the end, and the beginning, of an era.

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LOGIC AT THE TURN OF THE TWELFTH CENTURY

John Marenbon

This chapter is not, like the others in the book, a survey of the logical doctrines of a period, because I at least (and perhaps other scholars too) am not yet in a position to write such a survey. It is simply a guide to the material, to the technical problems it presents and some of the theories historians have elaborated about it, along with an indication of a few of the philosophical themes which are waiting to be explored. 'The turn of the twelfth century' is a deliberately vague title, designed to indicate the between where the previous chapter finished and Abelard's work, the subject of the following chapter. But plugging this gap — even as here, with a sponge and a nailbrush — is not a straightforwardly chronological matter. Abelard's *Dialectica*, the main subject of the next chapter, may have been being written as early as 1110: a good deal of what is discussed here was probably written no earlier, and some of the texts may in fact date from the 1120s or 1130s. But the writings and writers discussed here have at least traditionally been seen as providing the prelude to Abelard's work. They form a separate group in their methods and concerns from the writings of Abelard's contemporaries and the later twelfth-century schools considered in the final section of the next chapter.[1]

1 THE CURRICULUM

The study of logic in this period was based around the Boethian curriculum that had come into general use about a century before. Writing around the year 1110, Abelard summarizes the situation neatly [Peter Abelard, 1970, 146]:

> The Latin treatment of this art is furnished by seven books, the work of three authors. For so far in the Latin world there are just two of them by Aristotle, the *Categories* and *On Interpretation*, and one by Porphyry ... We generally use four by Boethius: *On Division*, the *Topics* and his *Categorical* and *Hypothetical Syllogisms.*

[1] This chapter was to have been written by Yukio Iwakuma, who more than anyone else has studied the mostly unpublished material from which an idea of the developments in logic in the years c. 1100 can be formed. I have adapted it, at two days' notice, from a 'Synthèse' written for a Paris conference in February 2007 and made available on the web. I am grateful to Yukio Iwakuma for letting me read his forthcoming 'Vocales Revisited', [Iwakuma, forthcoming-B]. A revised version of the original, longer synthèse will be published in the Proceedings of the conference [Rosier-Catach, Forthcoming-C]. I am very grateful to Irène Rosier-Catach for allowing me to use this text in this volume.

Handbook of the History of Logic. Volume 2: Mediaeval and Renaissance Logic
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By the *Topics* (*Liber topicorum*), *On Topical* differentiae is meant. The commentary on Cicero's *Topics* became much less popular once this text was actively studied and commented on, from c. 1100 onwards. Note the absence of *On Definition*, known but not a regular feature of the logical curriculum. Note too that Abelard is here just listing the texts studied; Boethius's commentaries on Aristotle and Porphyry continued to be indispensable aids.

It has recently been pointed out that there was even in the late eleventh century some knowledge of the text of the *Prior Analytics* [See Iwakuma, Forthcoming-A, adding to the discussion given by Minio-Paluello in *Aristoteles Latinus* III,1-4: ix, 433-6 and in Minio-Paluello, 1954]. But this work had no serious bearing on how logic was studied in the period around 1100, and most of the logicians have a rather inaccurate view of the contents of the *Prior Analytics*. It was when Abelard came to know the *Prior Analytics*, at the time he wrote his *Logica Ingredientibus*, that the text seems to have had some important influence [Martin, Forthcoming]. Similarly, the genuine Aristotelian *Topics* in Boethius's translation did not start to be used until the time of John of Salisbury, but there is a surprising citation from it in a logical text from the early twelfth century [Rosier-Catach, 1986].

2 THE FORM OF COMMENTARIES

The greater part of the evidence for teaching and thought about logic in the period is in the form of continuous commentaries on the texts of the logical curriculum. (For the sake of convenience in referring to so many anonymous commentaries, alphanumeric designations have been assigned to them. In the case of the *Isagoge* (I), the *Categories*(C) and *On Interpretation* (H), they refer to Marenbon [2000+]. For *De topicis differentiis* (B), they refer to the catalogue in Green-Pedersen [1984]. Yukio Iwakuma has also assigned numbers to commentaries on *On Division* (D), *On Categorical Syllogisms* (SC) and *On Hypothetical Syllogisms* (SH) in [Iwakuma, 2004]. They all share the form of being passage by passage treatments of the texts: the first few words of a passage (a lemma) are written out, and there follows a discussion. But they fall into two main types [Marenbon, 2000+, Introduction]. The model for the most widespread type, the 'composite' commentary, was provided by Boethius's commentaries, where each section of the text was discussed discursively, and problems were raised, explained and resolved. (The way in which Boethius acted as a model can be seen from how his commentaries were the main source for the early medieval *Isagoge* glosses and for P2 and C4.) Boethius, however, did not go in detail through every word of the text (although the elementary first *editio* on *On Interpretation* does from time to time gloss individual sentences). The twelfth-century logicians added an element of literal, phrase by phrase commentary, quite often put in the first person, so that the commentator is speaking for Aristotle, Porphyry or Boethius, as if these authors were to have paused to explain their texts more explicitly and ponderously. In the other, slightly less common type, 'literal commentary', this very detailed commentary predominates, and discursive discussion is more limited. Literal commentaries

are, then, distant formally from the model of Boethius, and they are usually distant in content too, whereas some composite commentaries contain many passages borrowed from, or closely based on, Boethius. 'Literal' and 'composite' should not, however, be thought of as designating two completely distinct classes: literal commentaries contain some more discursive comments, and composite commentaries can have sections where the exegesis is merely literal. There were also 'problem commentaries' (the best known is Abelard's *Logica Nostrorum petitioni sociorum*), which concentrated on discussing the difficult issues, with very little or no literal commentary. None of these has been dated to before c. 1120, but they should be born in mind, since it will turn out that the chronology of the commentaries is far less fixable than has been believed.

All these commentaries belong to the activity of teaching and learning logic in the cathedral schools, and especially in the schools of Paris, which were beginning to become important in the early twelfth century. But what exactly is their relation? Were they drawn up to be read out by the master, or are they, rather, lecture-notes taken by students? A few commentaries — notably one on *On Interpretation* (H5) — contain passages recording questions, discussion and humorous (sometimes obscene) asides that appear to be a very direct record of what went on during a particular set of lectures [Iwakuma, 1999, 94-7]. Other commentaries give the impression of having been more formally written up. Probably there is a range of different relationships between the various texts that survive and the lectures with which they are connected, and it goes beyond a simple choice between teacher's text or lecture notes, since lecture notes might be presented to a teacher for correction or they might form the basis of a student's own lectures, with passages revised ands his own particular take on controversial issues added.

These are conjectures, but one thing at least is clear: the twelfth-century logical commentaries were not usually conceived of or created as literary works, produced by a given, single author. They are, for the most part at least, records of teaching and learning, in which individual masters' views on issues may well play an important role, but which draw often on many sources. The relations between different versions of the same basic commentary show how freely one master would feel he could borrow from and adapt the teaching of another: an example is provided by the 'C8 complex' (see [Marenbon, 2000+] in the revised web-based version of the *Categories* list). The result is that commentaries have a layered form, with extra material added, perhaps in a number of stages. Where we have manuscripts of different versions, it is easy to see how the later versions are layered, with a stratum that follows the earlier commentary, and one or more strata added. For example, in P3, after a discussion of Porphyry's questions shared by the three manuscripts, one of the manuscripts, now in Paris, adds a passage giving an alternative discussion of the phrase 'only in bare, pure thoughts', in which it is related to non-existents, such as chimaeras. If we had only the Paris manuscript, it would not be so clear that this paragraph was an added layer. We should suspect, therefore, that there are often layers of this sort within commentaries for which only one manuscript survives, or in the earliest version *we have* of a commentary that

went on to be further revised.

A rough modern parallel might make the nature of these twelfth-century manuscripts more vivid. Imagine someone teaching an elementary logic course who has produced a detailed handout, using a standard textbook which she feels free to copy other logicians (she is just using it to teach, as was intended), and free also to change wherever she can improve on the presentation or disagrees on the stance the author has taken on a controversial issue, or where she finds a passage out of date. Suppose, now, a student downloads the handout, but revises it in line with extra comments the teacher makes in her lectures, and in the light of a conversation he has with her about some issues which he found puzzling. Then, three years later, when he is asked to lecture on the same subject, he turns to his revised handout and uses it as the basis for his own lecture handout, but adding some new material, reflecting his own views and some very recent controversies. What results will be a document that, potentially, can tell a good deal about how logic is taught, and about both teachers' views — but it will not be easy, without further information, to extract this information; and the wrong way to go about it would be to try and find who is the document's *author*.

3 THE 'EARLY TWELFTH-CENTURY' COMMENTARIES

A 'Working Catalogue' has been drawn up that aims to list all the commentaries we know on the *Isagoge*, *Categories* and *On Interpretation* up to the end of the twelfth century [Marenbon, 2000+], and there is also a (chronologically broader) catalogue of *On Topical* differentiae commentaries and list of commentaries on the other Boethian textbooks in [Green-Pedersen, 1984]. Although at the end of this chapter I shall call into question the easy distinction that is often made between logic from the beginning of the twelfth century and logic from the period 1115-40 (hence my scare quotes), there is certainly a group of commentaries from these lists which researchers up until now have assigned to the period c. 1100-1115 and distinguished from other, supposedly later pieces:

Literal commentaries

Commentaries on the *Isagoge* (P5), *On Interpretation* (H4), *On Division* (D7) in MS Paris BN 13368 (edited in [Peter Abelard, 1969]);

Commentaries on the *Isagoge* (*'Disputata Porphyrii'* P7) (Edited in [Iwakuma, 1992, 74-100]), *On Interpretation* (H5) and *On Topical* differentiae (B1) in MS Munich clm 14779;

The commentary on *On Interpretation* in MS Oxford, Corpus Christi College 233 (H7) (distantly related to H4 and H5);

Commentaries belonging to a collection of material in MS Pommersfelden Schlossbibliothek 16/2764, including two fragments of, or notes from, *Isagoge* commentaries (P4a, P4b) (edited in [Iwakuma, 1992, 62-5]), a commentary on *On Topical* differentiae (B3) [edited Hansen, 2005] and another fragment of one (B26),

a commentary on *On Cagegorical Syllogisms* (SC6),a partial commentary on *On Hypothetical Syllogisms* (SH6), and some logical notes.

Almost all of this material, where it has not been published, has been transcribed by Yukio Iwakuma.

(Commentaries in Cambridge MS Fitzwilliam Museum, MacClean 165, P6 and C6: these may belong to this group in principle, but they have been hardly studied.)

The commentaries listed here in MS Paris BN 13368 (P5, H4) have been considered to be Abelard's early works, but this attribution has now been strongly questioned [Cameron, Martin and Marenbon, Forthcoming]. They are related to the commentaries in MS Munich clm 14779, and H5 seems to preserve a fuller version of the same lectures as H4, including personal references that have been interpreted, but questionably, as relating to Abelard [Marenbon, 1997a, Appendix; Iwakuma, 1999, 94-7]. Further, and even more doubtful attributions have been made. Iwakuma [1992, 58-62] once attributed P7 to Roscelin. Luscombe [1962, 225-34] argued that it was by a pupil of Abelard's. Iwakuma also [1992, 61] suggested that the other commentaries and logical notes in this section of MS Munich clm 14779 are probably from the same school and suggests that some might be by Roscelin. It is these supposed links with Abelard — usually, it has been presumed, the young Abelard, or his teacher, that have provided the reason for dating the commentaries in the Paris and Munich manuscripts to the very first years on the twelfth century. Their dating now is uncertain, and C5, at least, could well be from the 1120s or later. The Pommersfelden material must, however, be this early, since the manuscript is dated to the end of the eleventh/ beginning of the twelfth century.

Composite commentaries

There is a group of what might be called 'standard' composite commentaries: commentaries that, unusually, are preserved in more than one manuscript, often in different versions (cf. [Iwakuma, 1999, 101-2; 2004a]):

On the *Isagoge*: P3 (3 MSS; consisting of an earliest-preserved version and two independent revisions) [Iwakuma, Forthcoming-C].

On the *Categories*: C8 complex: C8, C7 and C14 (6 MSS; consisting of an earliest-preserved version and four revisions falling into two groups). This material has been transcribed and studied by Yukio Iwakuma.

On *On Interpretation*: H11 and H9 (3 MSS; H9 is a version of H11, but considerably different). (Edition of H9 by Onno Kneepkens in progress; extracts and information on in [Kneepkens 1994; 2003])

On *On Division*: D8 (= MS Paris BN 13368, f. 191rb-4vb; MS Assisi 573, f. 68ra-78vb)

On *On Topical* differentiae: B8 (3 MSS); B10 (transcribed by Yukio Iwakuma)

6. On *On Hypothetical Syllogisms*: SH3 (= MS Munich clm 14458, f. 59-82 ; MS Orleans 266, pp. 78b-118a; MS Munich clm 14779, f. 66r-7r) (see [Iwakuma,

1999, 101; Forthcoming-a])

Although datings and attributions have been proposed for a number of these commentaries (see page 77, below), there is little that can be established solidly, except to place them somewhere in the period c.1090 — c. 1140.

Other composite commentaries that have been considered to belong to the period include:

P14, which has some relation to P3.

P16, which is thought to be early because it is heavily dependent on Boethius.

P15, which is made up of extracts from P3 and P15.

(All three commentaries have been transcribed by Yukio Iwakuma.)

4 THE TREATISES AND THEIR FORM

Two long and important logical treatises survive from the earlier part of the twelfth century: the '*Dialectica*' by Gerlandus (probably of Besançon) [Gerlandus, 1959], and the *Dialectica* of Peter Abelard [Peter Abelard, 1970]. The logical content of Abelard's *Dialectica* is discussed in detail by Ian Wilks in the next chapter. The comments here are merely about its form and chronology in relation to Gerlandus's treatise.

The two *Dialectica*s do not, as might be expected, make a radical break away from the commentary form. addressed to his brother Dagobert and said to be for the education of his nephews [Peter Abelard, 1970,146:23-5], but comparison with the set of commentaries known as the *Logica Ingredientibus* (c. 1119) shows that Abelard is using his lecture material, though perhaps in a considerably earlier form. He deals, usually section by section, with the material of each of the textbooks in the curriculum, allowing himself some occasional rearrangements. In general, he seems to have included more of the discussion that took place in the lectures here than in the overt commentaries which form the *Logica Ingredientibus*, although he sometimes abbreviates it so severely as to make it nearly incomprehensible. Gerlandus states explicitly in his prologue that his object is to introduce beginners to the teachings of Aristotle, who tends to be too concise, and Boethius, who is prolix and difficult to grasp. In the course of his treatise, Gerlandus goes through each of the ancient textbooks, except for *On Division*, writing terse paraphrases followed by sections full of nit-picking questions (what he calls *sophismata*). Formally, his work is closest to the literal commentaries (but with the added *sophismata*), whereas Abelard's *Dialectica* is close formally to composite commentaries.

The dating of the two *Dialectica*s is difficult. Abelard's *Dialectica* used to be dated towards the end of his life, after 1140, but recent opinion has put its completion before 1117, and quite possibly rather earlier than that (see [Mews, 1985, 74-104; De Rijk, 1986,103-8]; Mews is now inclined to accept a date even earlier

than 1117]. If, as has now been argued (see [Cameron, Martin and Marenbon, Forthcoming]), Abelard did not write the commentaries that have been assigned to his earliest years as a teacher (c. 1103), a dating of start of work on the *Dialectica* even to before 1110 becomes plausible. The *Dialectica* of Gerlandus was thought by its editor to have been written by Gerlandus the Elder, who worked in the first part of the eleventh century. But it has been argued convincingly that the author of the *Dialectica* was Gerlandus of Besançon [Iwakuma, 1992, 47-54]. This attribution still leaves room for a wide range of dates. There are parallels between the treatise and P5 (the literal commentary usually attributed to Abelard) and Abelard's *Dialectica*, but, if there is influence, it is not clear in which direction [Iwakuma, 1992, 52-3]. Gerlandus was still alive in 1149, when he travelled to Frankfurt with Thierry of Chartres [Mews, 1998, 72-3], and the single manuscript of his *Dialectica* could be as late as 1130 [Iwakuma, 1992, 48-9]. His knowledge in this work of a passage from the Aristotle's own *Topics* might also point away from an early dating [Rosier-Catach, 1986]. A date between 1100, at the very earliest, and any time in the 1120s is possible. addition 1 here

It may be the case that a different form of short logical treatise, called '*Introductiones*', also existed even before the beginning of the twelfth century. A number of *Introductiones* from the middle or later part of the twelfth century are known [cf. De Rijk, 1967]. But some *Introductiones* must have been written by c. 1117 or earlier, because Abelard refers in his *Dialectica* to his *Introductiones parvulorum* (which there are no reason to identify, as has often been done, with his so-called literal commentaries) [Peter Abelard (1970) 174:1, 232:10-12, 269:1, 329:4; cf. Mews (1985) 74-5]. Two sets of *Introductiones*, rather similar to each other and attributed, one to a Master G., one to (the same person?) a Master William Paganellus, have been published [Iwakuma, 1993] and placed by their editor, who attributes them both to William of Champeaux, slightly before 1080 [De Rijk, 1967, 130-46; Iwakuma, 2003b]; the earlier of the two manuscripts, however, is mid-twelfth-century. These *Introductiones* are short works that are not concerned at all with the matter of the *Isagoge* or the *Categories*, but with how propositions are constructed from words, and how arguments are made up using propositions; there is a very strong emphasis on topical argument. The ancient textbooks (*On Interpretation,* Boethius's treatises on syllogistic and his *De topicis differentiis*) are ultimately behind the teaching, but often distantly.

Two other logical, or quasi-logical, treatises are known, which were written by theologians. One, because it was definitely written before 1100, is treated in the previous chapter: Anselm's *De grammatico*, a dialogue on the problem of denominatives that is raised by the *Categories.* The second treatise does not sound like a logical work at all: it is the *On Original Sin* (*De peccato originali*) by Odo of Tournai (or Cambrai) [*PL* 160, 1071-1102; transl. Odo of Tournai, 1994]. Odo was a logician, turned ascetic monk. As a master of logic at Tournai, he had apparently written a number of logical treatises, none of which survives. But when, later in life, his monks urged him to write about the problem of Original Sin, he produced a treatise that contains whole chapters that could come from a

manual of logic. *On Original Sin* was probably written between 1096 and Odo's death in 1113 [cf. Odo of Tournai, 1994, 26; Resnick, the translator, suggests 1105 as the latest date].

5 LOGIC AND THE TRIVIUM

There were close connections between studying logic, and studying the other two subjects of the trivium: grammar and rhetoric. Both these disciplines have their own '*synthèses*' on this web-site: my purpose here is just to underline the links with logic.

The longest and most advanced of the textbooks used in the grammar curriculum, Priscian's *Institutions* (*Institutiones*), had been commented on since the ninth century. The early twelfth-century commentary on Priscian is known as the *Glosulae*. The writers of the *Glosulae*, both to the main part of the *Institutions* ('Priscian major'), and the concluding books ('Priscian minor'), knew about the logicians' debates and were willing to bring them into their grammatical commentary — not without reason, because Priscian had his own, Stoic philosophical source, Apollonius Dyscolus. A striking example is the long discussion that begins the Glosulae on the definition of 'utterance' (*vox*), which is almost identical, although differently arranged, to passages in the 'standard' *Categories* commentary (C8) [edited in edited in Rosier(-Catach), 1993]. There are philosophical elements, too, in the related *Notae Dunelmenses,* a series of notes and reports of masters' views, written by someone who knew the *Glosulae* [A complete, web-based edition exists, but is not yet publicly available].

Dating this grammatical material is no less problematic than for the logic. As with the logical commentaries, the *Glosulae* is a layered work, and some, at least, of the layers are discernible through looking at the different manuscripts. It used to be thought that one manuscript of the *Glosulae* on Priscian major (MS Cologne, Dombibliothek 201), dated from the late eleventh century, so providing a relatively early *terminus ante quem* for the earliest surviving version of the commentary, but now that manuscript has been re-dated to the twelfth century. The *Notae Dunelmenses* report above all, and as if the writer had heard them in person, the views of a Master G., and they have been found in a number of cases to correspond with other reports of the teaching of William of Champeaux [Rosier-Catach, Forthcoming-A]. Since it is sometimes made clear that Master G. disagrees with what the *Glosulae* says, some version of the commentary must have been in existence during William of Champeaux's teaching career (if the identification of Master G. is correct): therefore in all probability before he became Bishop of Châlons-sur-Marne in 1113. Moreover, Abelard knows the teaching of the *Glosulae* by the time he wrote his *Dialectica* (perhaps even before 1113).

It was neither in his logical nor his grammatical teaching, but in a course of lectures on rhetoric by William that Abelard, on his own account, made his famous attack on William of Champeaux's theory of universals. William was clearly a teacher of rhetoric. Moreover, positions on the theory of universals have been

found by those studying the rhetorical commentaries of the time, especially one associated with William of Champeaux. [Fredborg, 1986, 13, 29, 30] There is also a close connection between logic and rhetoric brought about by the fact that the fourth book of Boethius's *On Topical* differentiae, a central text for the logicians, is devoted to the rhetorical topics. Most logicians did not comment on it, but Abelard includes a long digression on rhetoric in his commentary, which is copied by one commentator to form a commentary on this final book [Fredborg, 2003].

6 TESTIMONIES AND KNOWN MASTERS

Faced with this mass of material that is mostly anonymous, and therefore hard to place or date, it is important to ask what sort of evidence about where and when particular masters taught. The hope might be that the historian of philosophy could act as a matchmaker, happily uniting names with texts. The danger is that, out of eagerness to earn her keep, she will promote arranged marriages, yoking together couples that have never met and should never have been united.

The two most important testimonies about logic at the turn of the twelfth century are the beginning of Anselm's *On the Incarnation of the Word* and some passages in Abelard's *Story of my Disasters* (*Historia calamitatum*). Both have the advantage of coming from the hands of well-known authors, indeed the two greatest philosophers of their time, and appearing in texts that can themselves be securely dated. There are, however, considerations about the authors' intentions that make their evidence less than straightforward.

At the opening of *On the Incarnation of the Word* (first version 1091-2), Anselm addresses Roscelin. He says that logicians like Roscelin

> think that universal substances are merely the breath of an utterance (*flatum vocis*) and ... are not able to understand colour as other than the body, or a person's wisdom as other than his soul. [Anselm, 1946, I, 285 (and cf. 289); for revised version: II, 9-10]

The problem in interpreting this passage is to judge how accurately Anselm is representing his opponent. Given that he believes that Roscelin's position on the Trinity is heretical, might he be, not describing his logical views, but caricaturing them?

In his *Story of my Disasters*, Abelard tells of how he came to Paris, where the discipline of logic flourished especially, with William of Champeaux as the teacher there; and how William turned from favouring, to persecuting him, when he tried to refute his views and sometimes seemed to have the upper hand in disputations [Peter Abelard (1978) 64:31-8]. Abelard does not say a word about the content of his, or William's, arguments. A few paragraphs later, however, he gives some more detail when he describes an incident that took place when he returned, c. 1108, to Paris after a period of illness spent in his native Brittany:

> Then I returned to him [William] in order to hear his lectures on rhetoric. We exerted ourselves in disputing with one another, and in

> the course of these disputations I forced him through the most clearly reasoned arguments to change his old view about universals, indeed to reject it. He held the view about the commonness of universals according to which the same thing as a thing (*essentialiter*) is at one and the same time whole within its single individuals, which do not differ as things (*in essentia*) but only through the variety of their many accidents. He corrected his view by saying from then on that the thing is the same, not as a thing, but through non-difference (*non essentialiter sed indifferenter*). And, since for logicians the chief question about universals has always been in this — so much so that even Porphyry, writing about universals in his *Isagoge*, does not presume to give a conclusion, saying 'To treat of this is extremely profound' — when William had no choice but to correct, or rather abandon, this view, his lectures came to be so badly regarded that they were hardly accepted on the other parts of logic, as if the whole of this art were contained in that one view, on universals.[Peter Abelard, 1967, 65:80 – 66:100]

The first view held by William is usually labelled 'Material Essence Realism'; a fuller account of it, along with his own counter-arguments, can be found in both of Abelard's mature Porphyry commentaries (from c. 1119, and c. 1125). But Abelard may not have developed by 1108 the same arguments that he would later use. Nor is it clear whether Material Essence Realism was William's invention, or merely the theory he happened to have adopted. Moreover, Abelard wrote his *Story of my Disasters* probably c. 1131, nearly a quarter of a century after this dispute with William, and with the aim of preparing for his re-entry into the Parisian schools by casting his controversial career and personal life in a favourable light, under which he was the victim of envy. It would not be surprising if he had magnified the importance of his difference with William or the extent of his intellectual victory.

There are also chronicle sources which provide some names and suggestions about logic in the late eleventh century. In Hermann of Tournai's account (written 1142 or later) of the restoration of the abbey at Tournai by Odo (who would go on to write *On Original Sin*), he describes a certain master Rainbertus of Lille as reading logic 'in the same way as certain contemporaries *in voce*' [Hermann of Tournai, 1883, 275]. He contrasts Rainbertus unfavourably with Odo, who 'read logic for his pupils *in re* in the manner of Boethius and the ancient doctors' (*... eandem dialecticam non iuxta quosdam modernos in voce, sed more Boetii antiquorum doctorum in re discipulis legebat*). Hermann goes on to apply to logicians like Rainbertus a comment that Anselm addresses to Roscelin (in the revised version of his treatise), that they are not 'dialecticians, but heretics in dialectic' [Hermann of Tournai, 1883, 275]. A chronicle from Fleury (c. 1110) records that at the time when Lanfranc died, that is to say, 1087, the eminent logicians were John, who argued that the art of logic is 'concerned with utterances' (*vocalis*), and his followers, Robert of Paris, Roscelin of Compiègne and Arnulf of Laon [Bouquet, 1781, 3]. Of these names, Roscelin is well known through Anselm's testimony, and it may be

possible to connect Robert and Arnulf with some of the anonymous material. It is common for twelfth-century authors to use their own names at times in logical examples [Iwakuma, 1999, 96-7], and to use place names, river names and so on of their own towns in the same way. The commentaries on *On Topical* differentiae and *On Categorical Syllogisms* in the Pommersfelden manuscript from the turn of the twelfth century use the name 'Arnulfus' in this way, and there is a mention of Laon: so there seems reason to think of Arnulf of Laon as the person who gave these lectures (or, possibly, who wrote them down) [Iwakuma, 1999, 96; Hansen, 2005, 46-7].

7 PHILOSOPHICAL THEMES TO BE EXPLORED

The sections above will have given an idea of the problems that must be tackled before the philosophical doctrines in these early twelfth-century texts can be interpreted. This chapter will not, for the reasons explained at the outset, offer an account of these ideas; here, rather, is an indication of what some of the major themes are, and what has been written about them.

7.1 Universals and ...

The theme which has dominated — and to say 'dominated' is an understatement — scholarship on logic at the turn of the twelfth century is the Problem of Universals, with a capital P and a capital U. So much has this theme dominated, that it was clear to scholars that it was what was at issue, even before they knew of any of the texts. The *dramatis personae* — Roscelin, the radical nominalist, who held that universals are merely puffs air; William of Champeaux, the conservative realist; and Abelard, the 'conceptualist', accredited with genius because he chose the middle way — were already gathered at the stage door before anyone was in a position to give them any lines (see e.g. [Michaud, 1867], containing 200 pages on William of Champeaux's logic, without the author's claiming to know the texts of any of it). Once Abelard's lengthy discussion of universals in his *Logica Ingredientibus* (c. 1118) and *Logica Nostrorum Petitioni Sociorum* (c. 1125) were published [see next chapter for detailed discussion], most attention naturally shifted to them and away from conjecture about what was being discussed twenty years before. Nevertheless, some scholars did begin to try to tell a philosophical story, based on the scraps of evidence to be garnered and, recently, on some attributions that, unfortunately, are far from certain.

Nearly a century ago, François Picavet (1911) did an excellent job of demolishing the various views of Roscelin that had been built up without proper evidence. More recently, Jean Jolivet (1992) brought together the various different reports of Roscelin's views, and argued that his nominalism was linked to a *semantic* theory that concentrates on the reference of words to things, by contrast with the usual Boethian semantic triangle of words, *thoughts* and things; the nominalism developed by his pupil, Abelard, would be very different in its semantics. It is

page refs

by no means clear though what sense is made by the views Jolivet attributes to Roscelin, or whether Roscelin really held them. (See also [Tweedale, 1988, **000-000**]). Another opponent of realism was immediately recognized in Gerlandus (though his work was wrongly place in the eleventh century: see §4 above). In the preface to his edition of the *Dialectica,* perhaps still the best analysis of Gerlandus's position, De Rijk notes what he calls his 'problemless nominalism' [Gerlandus,

page refs

1959, liii-lv; Tweedale, 1988, **000-000**]. Gerlandus clearly hopes to interpret the *Isagoge* and *Categories* as discussions about utterances (*voces*), and yet there is no explicit sign of argument for a metaphysical position (such as that there exist only particulars), and his semantics remains unclear, because some of the most difficult and revealing passages in the *Dialectica* have not yet been properly explained.

Recently, an ambitious attempt has been made by Yukio Iwakuma to bring together what is known about Roscelin and Gerlandus, with some anonymous material he has discovered and the chronicle accounts (see above, §6) into an account of 'vocalism' or 'pre-vocalism' (the term derives from the Latin *vocales,* used in the early twelfth century to describe Abelard and those who followed his position on universals) [Iwakuma, 1992]. The chronicles, he believes, name the main protagonists: John (otherwise unknown); his pupils – Roscelin, Arnulf of Laon and Robert of Paris. To these he adds Gerlandus, now recognized as Gerlandus of Besançon. All the literal commentaries listed in §3 above (except for P6, C6 and H7, on which he has not written) are linked by him to this movement. Constant Mews at one stage suggested that the Priscian *Glosulae* might belong to it too, and that their original author was the John mentioned in the chronicles; but his view has not found much support [Mews, 1992, 14, 33; cf. Mews, 1998, 50 — 55, and cf. 68- 73].

I contributed to this discussion [Marenbon, 1997b, 108-113; 2004, 26-34] by linking the supposed early commentaries by Abelard (P5, H4, D7) with the movement identified by Iwakuma. I also insisted that this group of early 'vocalists' identified by Iwakuma (including the young Abelard) needed to be distinguished sharply from Abelard and the vocalists who were his school. The mature Abelard, I said, had a very definite metaphysical position: that only particular substances and accidents exist. The earlier group of logicians, and Abelard c. 1100-5, were, by contrast just following an exegetical strategy — that of reading the texts of Aristotle, Porphyry and Boethius *in voce*, that is to say, taking the words of the text to refer to other *voces*, not to things. That they read the texts in this fashion did not mean that they must have believed that there *are* no things which correspond to the words; for, as I pointed out, even mentions of individual substances and accidents in the texts were read as referring to words. Iwakuma now accepts this qualification of his position, and distinguishes between 'proto-vocalists' (at the turn of the twelfth century) and 'vocalists' (Peter Abelard and h is followers) [Iwakuma, Forthcoming-B].

My view, however, obscures as much as it reveals. Although it is true that explaining the logical texts was the priority for these teachers, and that it is important not to wish on them a metaphysical position, the idea that they were

engaged in 'mere' exegesis is too easy, and it certainly does not explain the complexities of many passages in Gerlandus. If these logicians were making a conscious decision to treat logic as an art of language, neutral with respect to metaphysical issues, then they were engaged in a rather sophisticated philosophical task. But were they?

Yukio Iwakuma has also advanced a theory about the opponents of the prevocalists, the realists. William of Champeaux is known, from the account in the *Story of my Disasters* above all (see §6 above) to have advanced a realist theory of universals which Abelard refuted, and then to have espoused a different variety of realism. Iwakuma believes that he can attribute a corpus of works to William. Early in his life, Iwakuma contends, William wrote the *Introductiones* of Master G. and of Master William Paganellus (see above, § 4). A little later, he went on to write P3, C8 (in its original version), H11 and P14. B8 and B10 are, he believes, related to a lost commentary by William, and the revised versions of C8 are the works of William's students, as it seems in his view is H9. Iwakuma's arguments for these attributions are based on shared prologue-patterns and the fact of being copied in multiple manuscripts, which he considers to show that they were all written by a single, influential master [Iwakuma, 1999, 101-22; cf. Iwakuma, 2003a]. He also finds parallels between doctrines in these commentaries and those in the *Introductiones,* and between doctrines attributed to William of Champeaux in other sources and some passages in C8 and H11; whilst P14 is attributed to him because it has passages identical with P3 and it refers in passing to the 'indifference' theory of universals (particulars belong to the same species in virtue of being non-different from each other; metaphysically speaking, this theory is not in fact a variety of realism at all), which William adopted, according to Abelard, after he had been forced to give up Material Essence Realism [Iwakuma, 1999, 114].

Unfortunately, the position is less easy to make out with reasonable probability than Iwakuma believes. Although William may have had a part to play in the composition of some of these commentaries, Iwakuma's attribution of them to William of Champeaux and his students is highly questionable [Cameron, 2004]. All of the commentaries are anonymous, and the fact that they have some similarities in form (for instance, in their prologues) need not point to a single author. Indeed, to look for *the author* is to misunderstand what sort of compositions these commentaries are (cf. above, §2). Moreover, it is not yet clear to specialists how to date the different versions of this group of commentaries; there are parallels with Abelard's *Logica Ingredientibus* which very probably result from Abelard's ideas — perhaps from lectures — being borrowed. These composite commentaries may, when studied further, prove a valuable source of information for thinking about universals in the first 30 or so years of the twelfth century. They certainly do not provide an instant guide to what the position was c. 1100. [Cameron, Forthcoming should clarify the position better than has been done so far, and explore the philosophical relevance of this material.]

7.2 ... everything else

When Abelard wrote in his *Story of my Disasters* that, after his defeat of him over universals, William's views came 'to be so badly regarded that they were hardly accepted on the other parts of logic, as if the whole of this art were contained in that one view, on universals' (above, §6), he was criticizing those contemporaries of his who let the problem of universals overshadow all the rest of logic. Historians of philosophy have been even more guilty in this respect. There is much more in logic at the turn of the twelfth century than discussion about universals, but scholars are only just beginning to investigate these questions. Here are some indications of the work that is starting to be done on three of these areas:

Utterances as physical entities

In the *Categories* commentaries of the C8 complex, and also in the Priscian *Glosulae*, there is a long, multi-layered discussion of whether utterances (*voces*) should be considered as belonging to the Aristotelian Category of substance or that of quantity (or even perhaps quality). The debate is complicated both because of its exegetical context — it is not just a matter of providing an internally consistent and explanatorily satisfying answer, but one which is consistent too with Aristotle's text — and because all parties to it know that utterances are not merely physical entities, but also bearers of meaning; that, indeed, it is only by using utterances meaningfully that we can talk about what physically they are. [Rosier-Catach, Forthcoming-C; Cameron, Forthcoming].

Problems in Semantics

Both the Priscian *Glosulae* (studied especially in [Rosier(-Catach), 1993, 2003a-c, 2004a; b, Forthcoming-A; -B]) and the various commentaries on *On Interpretation* (cf. [Kneepkens 1978, 1994, 2003]) are full of material that is rich in ideas about semantics, though difficult to work with, because of the to-and-fro of debate among masters and the exegetical setting. For instance, writers find themselves having to introduce a distinction between utterances signifying potentially and signifying actually, and then having to struggle to maintain a position in which all and only significative utterances have signification. Empty terms, like 'chimaera' cause especial difficulties, as the commentators try to work out the three-way relation between thoughts, words and things [Cameron, 2004; Forthcoming].

Topical Argument

In his *Dialectica*, Abelard would give more space to the theory of topical argument than to any other part of logic (see the following chapter). Although Abelard's version of this theory was innovative, his strong interest in the area reflects the mood of the period from c. 1090 onwards. And, in addition to theoretical study, it became standard to explain arguments (in, for instance, a text being commented)

by citing the topic on which they rested. The basic material for studying this area of logic at the turn of the twelfth century has been set out by Green-Pedersen [1984], and he has also provided [Green-Pedersen, 1974] an account of what are very likely to be the opinions of William of Champeaux on the topics, since they are explicitly attributed to 'Master W.' and they fit Abelard's account of William's views. And a complete commentary on *On Topical* differentiae from just before 1100 has now been published [Hansen, 2005]. Green-Pedersen and Hansen have made a beginning of understanding the material, but this is an area where most of the important philosophical work is still to be done.

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PETER ABELARD AND HIS CONTEMPORARIES

Ian Wilks

Peter Abelard's life extended from 1079 to 1142, with his productive involvement in the philosophical debates of the time beginning in very early adulthood around the turn of the century.[1] He received training in logic and philosophy from figures as eminent as Roscelin and William of Champeaux. His early career as a teacher reached its apogee around 1115, when he became master of the cathedral school of Notre Dame. Previous to this he had held similar positions in lesser centers (Melun, Corbeil, Mont Ste. Geneviève). In 1117 his life was in crisis following his romance with Heloise and castration at the instigation of her uncle. He subsequently lived in monastic circumstances for about 15 years in a variety of locations. This change of milieu consolidated a shift in interests toward work in biblical exegesis and theology, and after returning to Paris around 1132 his teaching curriculum had expanded accordingly. When this second Parisian teaching phase ended is not clear, but it may have been as early as 1136 or as late as 1141. It was certainly over by the latter date since this is when an assortment of views ascribed to him was publicly condemned at the Council of Sens, leading to a sentence of excommunication. He spent the final months of his life near the monastery of Cluny, receiving there the consolation of having that sentence reversed.

The great recovery of ancient texts which would so shape thirteenth century thought was only beginning to occur toward the end of Abelard's life, so his philosophical horizon was comparatively limited. Of absolutely central importance for his work in logic were Aristotle's *Categories* and *De interpretatione*, Porphyry's *Isagoge*, and a series of works by Boethius: *De topicis differentiis* (which concerns topical theory), *De divisione* (which concerns definition), *De syllogismis categoricis* and *De syllogismis hypotheticis* (treatises on categorical and hypothetical syllogisms respectively) and various commentaries, particularly on the above works of Aristotle and Porphyry [Marenbon, 1997a, p. 37]. Of special importance too was Priscian's grammatical treatise *Institutiones grammaticae*.

Abelard wrote at a time when the commentary was still very much a preferred authorial format, and most of his work in logic is styled accordingly. Two works are especially important, both dating around the period from 1115–1120. First, there is the so-called *Logica ingredientibus*, a set of four commentaries on *Isagoge*, *Categories*, *De interpretatione* and *De topicis differentiis*. Whether Abelard intended these to be taken in sequence as parts of a single work remains an open

[1] The following biographical synopsis is based on [Mews, 1995] and [Marenbon, 1997a, pp. 7–35].

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question.[2] And second, there is the *Dialectica*, a single work which does not take the explicit form of a commentary, but nonetheless is closely organized around the content of several of the above ancient sources.[3] My discussion of Abelard's logical theory will be based chiefly on these two works[4]. Note that Abelard was a reasonably prolific author, and produced a wealth of additional material, notably an early set of commentaries on *Isagoge*, *De interpretatione*, and *De divisione*, an extremely valuable later commentary on the *Isagoge* (which goes under the name *Logica nostrorum petitioni sociorum*), a brief treatise on abstraction and knowledge (*Tractatus de intellectibus*), as well as quite a number of works in biblical exegesis, theology and ethics.[5]

About halfway through the *Dialectica* we find a passage which explains the organizing principle behind its presentation of logical theory: "Just as before defining the categorical syllogisms it was appropriate for their content in categorical propositions to be treated, in the same way it is necessary that before defining hypothetical syllogisms their hypothetical propositions ... be treated" [Abelard, 1970, p. 253 (4–7)]. What is suggested here is the appropriateness of presenting logical theory in two sequences, both culminating with discussion of syllogisms. In the first sequence the structure of categorical propositions is discussed to provide insight into categorical syllogisms. In the second the structure of hypothetical propositions is discussed to provide insight into hypothetical syllogisms. But of course what is ideal for both kinds of syllogisms is to have *true* propositions [Abelard, 1970, p. 253 (7–9)]. So we can take the grounding investigation one step further in each case. What makes categorical and hypothetical propositions true? For the truth of categorical propositions we look to their constituent words. And for the truth of hypotheticals, Abelard says, we look to their corresponding topics.[6]

It is standard practice to develop an account of categorical logic which starts with a discussion of words, and moves through categorical propositions to the categorical syllogisms themselves. Abelard consciously adopts a parallel course for the logic of hypotheticals by beginning with a treatment of topics and hypothetical propositions. In general we can think of Abelard's development of material as falling along these two axes: discussion of words and categorical propositions leading to categorical syllogisms; and then discussion of topics and hypothetical

[2]For the view that he did so intend see [Marenbon, 1997a, pp. 47–8].

[3]The relative dating of these texts has been subject to debate. I accept the conclusions drawn in [Mews, 1985, 126–27], which hold the *Dialectica* to be earlier than the glosses of *Logica ingredientibus*. Brief summaries of other positions on this question are available in [de Rijk, 1970, pp. xxi–xxiii] and [Mews, 1987, p. 16, note 2].

[4]In particular my discussion will draw heavily from the commentary on *De interpretatione* in *Logica ingredientibus*; this commentary goes by the name *Glossae super Periermeneias Aristotelis*. An overview of current scholarship on the contents of Abelard's logical works is found in [Marenbon, 2006, pp. 333–342].

[5]For a list see [Marenbon, 1997a, pp. xvi–xix]. Certain works of Abelard are lost; see [Van den Eynde, 1962] and [Mews, 1985].

[6]What is meant by "topic" in this context is, of course, not self-evident, and will be fully treated in Part 2 below. Technically speaking, the label "hypothetical" includes disjunctions as well as conditionals for Abelard, so it is necessary to specify that, in this paragraph and the next, "hypothetical" is to be taken as meaning "conditional."

propositions leading to hypothetical syllogisms. This way of organizing material is overtly embraced by Abelard in the *Dialectica*, in his attempt to depart from the commentary format. So it can be taken as representing his most basic intuitions on how the subject matter of logic should be organized.

I will structure my discussion below accordingly. Part 1 deals with words, categorical propositions and categorical syllogisms. Part 2 deals with topics, hypothetical propositions and hypothetical syllogisms. These two parts complete the treatment of Abelard, and Part 3 turns to his contemporaries and their schools.

PART 1: ABELARD ON WORDS, CATEGORICAL PROPOSITIONS AND CATEGORICAL SYLLOGISMS

The foundation-laying program for categorical syllogisms undertaken by Aristotle in *De interpretatione* amounts to this: describe the propositional basis for those syllogisms, but first describe the lexical basis for those propositions. More than anything else that lexical basis is provided by subject and predicate terms, which typically are general names. This is indeed the most foundational issue in Abelardian logic: the semantics of general names, which, in Abelard's hands, is inseparable from the metaphysical problem of universals.

It would have been impossible for any logician of this time to do logic without directly encountering this metaphysical problem. The very language in which categorical syllogisms are formulated makes it unavoidable. While it is comfortable and intuitive to an English speaker to pluralize both subject and predicate terms in a proposition like "All men are animals," the practice initiated by Aristotle and transmitted by Boethius is to formulate the proposition with both terms in the singular: "All man is animal" (*Omnis homo est animal*). This way of speaking can be taken quite literally as representing the content of the proposition. Here, for instance, is Boethius' explanation of the example: "Therefore we define to be in the whole, or not to be in the whole, thus: to be in the whole, or to be predicated of all, is said when nothing of the subject is able to be discovered of which what is predicated is not able to be said. For nothing of man is discovered of which animal is not able to be said" [Boethius, 1891, 809D–810B]. This description suggests that a universal proposition expresses a relation between two single and distinct items, in this case man and animal, one of which (in this case man) is in the whole of the other (animal). The proposition is then simply telling us how these items relate.

This language is of course used for the other entries in the Aristotelian square of opposition:

A: All man is animal	E: No man is animal
I: Some man is animal	O: Some man is not animal.

The content of the I-form can then be rendered as "Animal is in part of man," the content of the E-form as "Animal is in none of man," and the content of the O-form as "Animal is not in the whole of man." These formulations are understandable but also questionable. Indeed, the question is unavoidable: what sort of things

are these items corresponding to the subject and predicate terms? Speaking of animal instead of animals is to use a mass noun where we would normally use a count noun. We know what animals are, but what is animal?

Notice how easily a Platonist can give a response. Animal can be taken as the form of animality which all animals share, and in which they all participate. Man can be taken as the shared form of humanity (or manness) common to all men. Animal and man, taken thus, are single, distinct things, and are appropriately denoted by unpluralized nouns. The above propositions just describe in various ways, some true, some false, the relationship between these single, distinct things.

So in this way the conventional language of the categorical syllogism fairly invites some kind of theory of shared forms — forms which exist independently of the things that nonetheless share in them.[7] And this is why Porphyry's *Isagoge*, intended as a propaedutic to the study of logic, begins with these questions about genera and species: "(1) Whether genera or species exist in themselves or reside in mere concepts alone; (2) whether, if they exist, they are corporeal or incorporeal; and (3) whether they exist apart or in sense objects and in dependence on them" [Porphyry, 1975, pp. 27 (12)–28 (14)]. Aristotle himself did not ultimately accept the existence of shared forms, but bequeathed a language which made them a live issue.[8] Abelard could not avoid having some position on this issue.

He in fact did not accept the existence of shared forms either. Some people, he says, "maintain that the same thing is essentially in many, so that the same thing that is in this is essentially in that, although <the latter> is furnished with different forms. For example animal is in Socrates and in Brunellus and in others" [Abelard, 1933, p. 515 (17–21)].[9] So animal is shared by all animals; in other words, they are all essentially the same animal because the form that confers animality upon them is the same form. Accordingly, "Socrates is in no way essentially different from Brunellus" [Abelard, 1933, p. 515 (23–24)]. Abelard has a number of objections against this view [Abelard, 1933, pp. 515 (32)–518 (8)]; these emphasize the counter-intuitiveness of what it implies about individuality – such as its implication that Socrates and Brunellus the ass are essentially the same animal, since they share the same form of animality. Shared forms make individuality impossible. The fact that individuals do exist entails that shared forms do not.

Abelard is perfectly willing to speak of forms/genera/species, but not as accompanied by anything like a Platonic theory of participation. While he does

[7] By contrast, a theory of unshared forms posits distinct forms for distinct things. On this theory, the animality of one animal is distinct from the animality of another animal. See note 10 below.

[8] The relationship between Aristotle's syllogistic and Plato's method of division (for which, see [Kneale and Kneale, 1962, pp. 44, 67–68]) would explain how Aristotle came to bequeath such a language.

[9] Abelard is describing the position now referred to as "Material Essence Realism." This position is associated with William of Champeaux, and articulates in some detail a metaphysics of shared forms. For Material Essence Realism, and Abelard's critique of the position, see [Tweedale, 1976, pp. 95–111], [Gracia, 1984, pp. 198–200], and [King, 2004, pp. 66–69]. For Abelard's complex relationship to Platonism generally, see [Marenbon, 1997b].

not articulate the point in this way, it seems clear that he accepts the existence of individual forms: the animality of this animal, the whiteness of this thing.[10] These would be different from the animality of another animal, the whiteness of another thing. In any case, what Abelard is opposing here is itself a fairly radical metaphysical view, and it is far from clear that he himself is being all that radical in opposing it.

While he does not accept the metaphysical theory which seemingly underlies the mass noun language of syllogistic theory, he still uses the language itself. But he insists that it must be taken in a transferred sense:

> It should be understood that questions of this sort are posed by those who have doubts about the designation of things. They fall into such doubts through philosophers' locutions of this sort: "Animal is a genus," "Animal is in many," "Man is a universal," "Man is a species," taking these names "animal" and "man" in their proper and customary signification, but being unaware that philosophers transfer these names to signify themselves in this sense: "Animal is a genus," that is, this word (*sermo*) "animal," or the name "animal," is a genus [Abelard, 1933, p. 525 (15–22)].

This is the key clarification. When we say that animal is in the whole of man we are actually just talking about the words of "All man is animal," and we only mean that "animal" is predicated universally of "man." This simplifying approach is by no means original with Abelard. It is taken up in the late eleventh century by various philosophical predecessors, most notably his teacher Roscelin. The point is to produce *in voce* readings of the logical cannon, as opposed to *in re* ones — readings, that is, which the doctrine treated in those writings is construed as being about words, not things (and, in particular, not shared forms).[11] This approach avoids metaphysical entanglements, but is not itself without difficulties, and certainly calls for theoretical refinements.

How to account for the semantics of general names emerges as the most prominent difficulty. How do we explain their ability to achieve generality of application to a set of objects which are individual and distinct? What is it about those objects which allows them all to correspond to the same name, in spite of their individuality and distinctness? Note that this problem is readily solved by a theory of shared forms. We can say, if such forms exist, that "animal" is predicable of both Socrates and Brunellus because they both have the *same* form of animality; a word which pertains to the form present in the one will then automatically

[10] John Marenbon [Marenbon, 1997a, p. 114] says that, with respect to properties and relations, "Abelard readily accepted the idea that these are particular *differentiae* and accidents though not universal ones . . . , arriving at an ontology similar to that of modern exponents of 'tropes' or 'abstract particulars.' " On this point see his reference to an expanded, unpublished version of [Martin, 1992] in note 44 of that page. Marenbon sums Abelard's basic metaphysical position thus: "There is no thing which is not a particular" [Marenbon, 1997a, p. 117].

[11] Abelard's attitude toward *in re* readings is in fact complex and evolving. See [Marenbon, 1997a, pp. 108-116]. The essential point here is that he will always prefer an *in voce* reading over an *in re* reading where the *res* in question is a shared form.

pertain to the form present in the other. But if there are no shared forms then another sort of explanation must be found. If the animality of Socrates is *not* the same as the animality of Brunellus then why do we not need *different* names — such as "animal 1" and "animal 2" — to denote them respectively? Answering this question is the key labour of the position Abelard adopts. This is why, after his rejection of a theory of shared forms, Abelard supplies, in way of a positive account to follow upon the refutation, not a replacement metaphysical theory, but a semantic theory — a theory about the signification of general names.

There are two aspects of signification that need to be addressed by this theory. They correspond to the "twofold signification" (*duplex significatio*) of names [Abelard, 1970, p. 154 (20)] which Abelard commonly refers to. First there is the *ad rem* or denotative aspect, whereby the name picks out a thing or things. And second there is the *ad intellectum* aspect, whereby the name generates an understanding (*intellectus*) in the mind of the hearer.[12] The technical label for the denotative aspect of signification is "nomination" or "naming" (*nominatio*), and the technical label for the *ad intellectum* aspect is "signification by generation" (*significatio per generationem*) [Abelard, 1970, p. 112 (4)]. When Abelard uses the word "signification" without qualification he means signification by generation, so, following him in this usage, we can say that the twofold intent of his account of general names is to show how, in the absence of shared forms, they can succeed both in naming and signifying. Failing such explanation, general terms will seem as if they cannot "name things as agreeing in anything" [Abelard, 1919, p. 18 (14–15)], or as if they are not able "to establish a sound (*sanus*) understanding of anything" [Abelard, 1919, p. 18 (8–9)], or both. There are two questions here, the first about naming, the second about generating a understanding — i.e., signifying. Both must be answered.

The question about naming is this: what do the things named have in common that allows them, but not other things, to be picked out by the general name? As noted, a shared form would make the question easy to answer; if there was one, the things would have *it* in common. It would be, in Abelard's parlance, the common cause of the name's being imposed on all of those things, where imposition is simply the process of establishing the conventional association of symbol with the thing or things to be named. Having rejected the shared form account of the common cause, this is Abelard's substitute: "Single men, which are discrete from one another because they differ just as much in their essences as in their forms, agree in this: that they are men" [Abelard, 1919, p. 19 (21–22, 23–24)]. What

[12] *Intellectus* is commonly translated "understanding" by Abelard scholars, who therefore speak of a word as "generating an understanding" in the mind of the hearer. John Marenbon prefers "thought" as a translation [Marenbon, 2004, p. 61, note 8], and therefore speaks of "generating a thought." Marenbon's practice has merit, since I think *intellectus* is used in a rather more general and less technical sense than is conveyed by "understanding." To have an understanding of something suggests that one understands it; this is a stronger suggestion than is involved when Abelard speaks of having an *intellectus* of something. While I will follow the now standard translation practice in Abelardian scholarship and use "understanding," I do so with the caveat that the word is to be taken as meaning something like "thought."

they agree in is expressed not as a name ("man") but as a whole clause ("that they are men"), or, equivalently, as an infinitival construction (*esse hominem*) [Abelard, 1919, p. 19 (25)], best rendered in English by a participial phrase: "being a man." Socrates and Plato are like each other, and unlike horses and asses, in being men [Abelard, 1919, p. 19 (31–33)]. What is expressed clausally and infinitivally here is definitely not a thing (as the shared form was supposed to be). And the non-thing so expressed can plausibly be claimed to serve as a cause of something because causal claims are often true when made of non-things; if we say "He was beaten because he did not want to be at the forum" we imputing causal status to the absence of desire, where that absence is clearly not a thing [Abelard, 1919, p. 20 (11–12)]. So, Abelard claims, we can actually talk about a common cause of imposition, about what things have in common, without having to appeal to a shared form, or to any other thing at all. A non-thing will suffice. Abelard gives a name to this non-thing: the *status*, literally the "state" that something is in.[13] He presumably uses the word because it is free from associations with prior metaphysical theories, and thus has a better chance of serving unambiguously as the name of a non-thing.

The second key question Abelard must address is how general terms signify — that is, how they generate an understanding (*intellectus*) of appropriate generality in the mind of the hearer. If one accepts shared forms the question becomes trivial; the understanding achieves the appropriate generality simply by representing such a form. But if there is no such form then what does the understanding represent?

He starts with the notion of mental imaging.[14] The soul produces "a similitude of a thing ... into which the action of its intelligence is directed" [Abelard, 1919, p. 20 (25, 26–7)]. The image remains in place after the thing is no longer evident, and serves as stand-in for the now discontinued sensory presentation. He speaks of this image as a form (*forma*) [Abelard, 1919, p. 20 (31)], and as "an imaginary and fictive thing which the soul produces of itself when and how it wants" [Abelard, 1919, p. 20 (31–32)]. The understanding arises from this image. The understanding is not the same as the image, but relates to it in a way comparable to the way a sensory experience relates to what is sensed [Abelard, 1919, p. 20 (28–30)]; the understanding is an action of the mind directed to the image, just as the sensory experience is an action of the mind directed to what is sensed. So the content of understandings is very much shaped by images, and Abelard formulates the key distinction accordingly: "Let us distinguish the under-

[13] As Abelard makes clear in [Abelard, 1919, p. 20 (1-6)], agreeing in being a man, and agreeing in the *status* of a man, amount to the same. The latter is an attempt to formulate the former in a way that avoids the suggestion that there is a single thing *man* relative to which the agreement arises. The question of what a *status* is remains one of the major issues of Abelardian scholarship. [Tweedale, 1976] represents an extended attempt to explicate the notion. More recent attempts are found in [Marenbon 1997, pp. 190–95] and [Jacobi, 2004, pp. 135–137]; for further references see [Jacobi, p. 154, note 38]. It is most common in Abelard scholarship now simply to leave the word *status* untranslated; rendering it literally as "state" is not common, but one does find this translation in [Tweedale, 1967].

[14] The content, and limitations, of this account are laid out in [Guilfoy, 2004a].

standings of universal and singular <names>. These are differentiated in that the understanding pertaining to the universal name grasps a common and confused image of many, but the understanding which the singular word generates grasps a form which is proper to one and, as it were, singular — that is, related only to one object (*personam*)" [Abelard, 1919, p. 21 (27–32)]. The image arising from many, to which the understanding of a general term is referenced, is "common and confused," while the image arising from only one, to which the understanding of a singular term is referenced, is not "common and confused." The difference in signification between singular and general terms is therefore a matter of the presence or absence of confusion in imaging.

Confusion in imaging just involves leaving out details of what is imaged. This omission is a cognitive limitation of sorts, but one which serves a purpose in that it allows the image to be variously applicable to differing objects — so long as the details it leaves out are the ones that constitute differences between them. Abelard takes for his example the image of the lion, which may be so specified as to mark out the fact that it is limping, having been wounded by the spear of Hercules; or it may lack such distinguishing marks [Abelard, 1919, p. 22 (18–22)]. When lacking such marks, the image is more confused, but capable of conferring upon a term a more general range of application. When retaining such marks, the image loses the confusion but also the general range. A general term, then, works in this way: the understanding associated with it represents no member of the species with individuating detail, and so achieves representation of all members. Through this achievement it confers upon the term the ability to apply generally.

Here, then, is Abelard's way of obviating reference to shared forms in accounting for the meaningfulness of general terms:

> Hence, when I hear "man" a certain figure (*instar*) arises in my soul which is related to single men in such a way as to be common to all and proper to none. But when I hear "Socrates" a certain form arises in my soul which expresses a likeness of a certain person. Hence, through this word "Socrates," which presses upon the soul the proper form of one <man>, a certain thing is indicated and determined; but through "man," the understanding of which rests on a form common to all <men>, the commonness itself stems from the confusion <of the form>, so that we should not understand any one <thing> out of all of them [Abelard, 1919, pp. 21 (32)–22 (2)].[15]

"Form" and "figure" mean the same as "image" in this context. General terms are thus terms which arise from failure to image completely. This failure, however, is a matter of being partial as opposed to complete in their representation, and not a matter of being misleading as opposed to accurate. Neither "Socrates" nor "man" is a misleading term, but the latter is more partially representative than the

[15] "Thing" in this citation translates *res*. For the meaning of this Latin term — so central for semantic theory at this time — see [Jolivet, 1975, pp. 538–543], [Jolivet, 1982, p. 293, note 248] and [Kretzmann, 1982, p. 497].

former. Clearly, on Abelard's account, traditional intuitions about the inherent prestige of generality are challenged. Instead of generality's being characteristic of a higher order of being (the shared form), it is characteristic of a less complete kind of representation.

Universal names, he says, "signify diverse things by nomination not by constituting an understanding that arises (*surgentem*) from those <things> but by <constituting one which> pertains (*pertinentem*) to those single things" [Abelard, 1919, p. 19 (7–9)]. A general understanding is not read off of things directly, but is partially the product of human cognitive processing. Its ability to be useful lies in its ability to pertain to things even while failing to be a copy of them point for point. This ability to pertain *in spite of* that failing stems from the fact that there is something about the things themselves that allows them to be partially represented in the same way. There is, for example, something about Socrates and Plato which allows them both to be represented by an image or figure or form of humanity. What that is, of course, is their being in the same *status*. Obviously, an account of abstraction like Abelard's could easily evolve into a view about the non-objectivity of general names, but he will have none of that. The *status* is what makes objectivity possible. That is why, when Abelard asks whether a word is called common "according to the common cause in which things agree or according to the common understanding" [Abelard, 1919, p. 19 (17–20)], he gives the nod to the former [Abelard, 1919, p. 24 (35–37)]. The common cause in which things agree is the *status*. This common cause is the prior, factual basis for applicability of general terms; the common understanding — the *intellectus* — is the subsequent cognitive response to that fact which actually triggers the production of that term. The *intellectus* explains how general terms are formed, but the *status* explains how they are possible in the first place.

With this semantic account of general terms, a doctrine of shared forms no longer belongs to the subject matter of logic. The true subject matter emerges: logic is concerned not with relations of shared forms (man, animal) but with relations between linguistic items ("man," "animal"). But this conclusion must itself be refined to avoid trivializing the discipline. It becomes important to understand what exactly a word is. If a word is taken just as spoken, and as nothing but an evanescent sound pattern, then this focus on words means that instead of being about the perduring realities of shared forms, logic deals only with these transitory and vanishing sounds. In the derisive phrasing of Anselm [Anselm, 1946, p. 9], logic devolves into the study of mere *flatus vocis*, that is, into the study of what is merely the breath of the word, its physical properties. The need to avoid this result leads Abelard into one of his subtlest innovations: his distinction between *sermo* and *vox*. Both can be translated as "word," but the former encourages a conception of the word as sound invested with meaning, while the latter does not. The *sermo* compares to the *vox*, Abelard argues, as the finished statue compares to the stone from which it is carved [Abelard, 1933, p. 522 (22–27)]. The two are in one sense the same thing, because the statue is materially indistinguishable from its stone. But in another sense the two are different because there is at least

one characteristic which they fail to have in common: the statue is made by the sculptor, but the stone is not.[16] It is likewise for the *sermo* and *vox*: they are the same thing, the same in essence, but do not have all properties in common and therefore differ in definition. The sound taken as meaningful differs just as much from the sound taken *per se* as the statue differs from its constituent stone; the shapeliness of the statue makes it a different thing than the constituent stone, and likewise the meaningfulness of the *sermo* makes it a different thing than the constituent *vox*. In advancing this distinction Abelard is able to suggest how, even though logic does have words for its subject matter, it is not thereby dedicated to a subject matter which is transitory and vanishing. This is so because it studies words *taken as meaningful*, and attends to their general semantic properties, not their arbitrary physical ones.[17] The former constitute worthy and appropriate objects of theoretical investigation. Note that at one stroke Abelard intuits two significant philosophical insights: insight into something like the distinction between type and token[18], and insight into the notion of supervenience. The relation of supervenience — implicitly appealed to through the statuary example — explains how type emerges from individual token. This emergence then explains how logic as a *scientia sermocinalis*, a science of language, can yield the general and abstract theoretical results which have always been its aim.

With this understanding of general terms in place, Abelard can proceed to the details of his treatment of words, propositions and categorical syllogisms. His handling of the first two of these three subjects develops the material bequeathed to the medieval tradition by Aristotle's *De interpretatione*. This treatise is essentially about propositions: how they are formed from their constituents, and how they stand to each other in relations of mutual inference and opposition. The issue of how they stand to each other in relations of inference and opposition is an obvious preparation for the subsequent treatment of syllogistic inference. In turn, the issue of how they are formed from constituents is a preparation for the treatment of mutual inference and opposition, in that these relations ultimately depend on the predicative relation between subject and predicate.

In the opening sections of this work Aristotle is examining the characteristic contents of a proposition — in particular, of the sort of categorical proposition

[16] This point derives from a classification of various kinds of identity which Abelard provides (with some variation) in his theological writings: [Abelard, 1983a, pp. 142 (745)–150 (959)], [Abelard, 1969a, pp. 247 (1677)–255 (1970)] and [Abelard, 1983b, pp. 454 (1411)–457 (1490)]. The identity between stone and statue is identity of essence. The difference is difference of definition. The key points of this classification of kinds of identity and difference are taken up by various scholars, such as: [Jolivet, 1982, pp. 285–293], [Gracia, 1984, pp. 226–232], [Brower, 2004, pp. 226–230] and [King, 2004, pp. 85–92].

[17] The key text for the *vox/sermo* distinction is [Abelard, 1933, pp. 522 (10)–524 (20)]. Peter King [King, 2004, p. 120, note 90] finds evidence of this distinction in earlier Abelardian work. For discussion of the distinction against the background of Abelard's account of identity, see [Tweedale, 1976, pp. 142–157], [Marenbon, 1997a, pp. 176–180] and [Wilks, 1998b, pp. 367–373].

[18] [Boler, 1963, p. 43] does not accept an exact equating of the two distinctions: "It should be mentioned ... that the 'vox'-'sermo' distinction is not quite the same as what is now referred to as the token-type distinction."

which figures in syllogistic reasoning. This is an exercise in studying the compositionality of language, from individual letters to words, and from words to propositions. Three levels of linguistic complexity are implied. (i) At the most basic level there are simply letters and non-significative combinations of letters into syllables. (ii) At the next level there are words and non-propositional combinations of words. (iii) At the final level there are propositional combinations of words, that is, propositions taken as a whole. The essential challenge of this classification will be to explain why certain items belong at one level and not another. It is difficult, for example, to explain what it is about a preposition's signification that makes it classifiable under (ii) and not (i); or what it is about the signification of an expression like "running man" which makes it classifiable under (ii) and not (iii). Part of the challenge is actually maintaining the levels as such, since such examples might otherwise suggest the existence of intermediate levels between these three. This is why what initially may seem a simple classification in Aristotle's hands is able to generate quite a bit of disagreement among commentators.[19]

In pursuing this material, Abelard continues to employ the simplifying semantic assumption that words have the two basic operations of naming things and generating understandings about them in the mind of the hearer. They nominate *res* (things) and signify understandings (*intellectus*). As it turns out, it is the latter operation that plays the larger role in his work. His guiding principle in this regard is expressed thus: "According to the formation and function of words, the understanding is the principle signification, while the signification of the thing is secondary" [Abelard, 1927, p. 309 (23–24); Abelard, 2006, 00.11].[20]

By way of justifying this preference, Abelard gives three pretexts [Jacobi *et al.*, 1996, p. 17]. (i) Names and verbs often denote the same thing, he claims, using for examples the words "runs" and "running" (*currit* and *cursus*; the former is a verb, the latter a name); one can be pointing to the same thing while correctly uttering either word. But "runs" and "running" have obvious semantic difference. This difference, which is not captured by what they name, *can* be captured by how they generate understandings. "A different mode of conceiving varies the understanding in the two cases " [Abelard, 1927, p. 308 (28–29); Abelard, 2006, 00.7]; in the latter we conceive of running directly and in itself (*in essentia*), and in the former we conceive of it as a feature of something else (*in adiacentia*). So in this sort of case denotation fails to capture a real semantic distinction. (ii) In another sort of case, denotation actually creates a spurious semantic distinction. For various contingent reasons names can cease to denote, as "rose" and "lily" do when, with the change of seasons, there are no such flowers in bloom. But this does not prevent the continued functionality of the words. They retain the same signification as before, the reason being that their signification is based on the

[19] A discussion this Aristotelian semantic program, and Abelard's response, is found in [Lenz, 2005]. A survey of some controversies to which this program gives rise is found in [Thorp, forthcoming].

[20] All page references to the existing editions of Abelard's *Glossae super Periermeneias Aristotelis* are supplemented by references to the corresponding section of the Jacobi and Strub edition [Abelard, 2006]. See the comment with which I preface the bibliography below.

understandings they generate, not on the things they denote [Abelard, 1927, p. 309 (5–8); Abelard, 2006, 00.9]. (iii) Semantic theory typically attempts to show, as far as possible, how the meaning of a compound linguistic unit is the product of the meanings of its components. Understandings are able to do a much better job of explaining this sort of compositionality than denotations. Abelard makes this point by reference to the proposition as a whole. The understanding which the proposition produces is very closely linked to the understandings produced by the parts. But we cannot plausibly say that the proposition has a denotation which is similarly linked to the denotations of its parts, since whole propositions do not, strictly speaking, denote [Abelard, 1927, p. 308 (34–40); Abelard, 2006, 00.8]. So we cannot appeal to denotations in way of showing how the proposition's meaning is the product of the meaning of its parts.

Accordingly, as Abelard proceeds through the levels of letters, words and whole propositions, his key analyses generally turn on how understandings are signified, and the modes of signification involved, rather than on how things are denoted.

Since the *Dialectica* is organized not as a commentary but as an independent, free-standing work, it is able to give us a good sense of way Abelard organizes this analysis in his own mind, even if it does not always give us his final solutions to the problems raised therein. The discussion of individual words [Abelard, 1970, pp. 111 (1)–142 (20)] singles out three kinds of words for separate treatment: what are called "indefinites" (i.e., conjunctions and prepositions), names and verbs.[21] This seems intended to be a complete classification of the basic kinds of words there are, because under the heading of names are included pronouns, adverbs and interjections [Abelard, 1970, p. 121 (8-9)]. The inclusion of pronouns under this heading is unproblematic, because of their obvious naming function. The inclusion of adverbs here is less intuitive, but may follow from regarding them as having some kind of adjectival function — as "quickly" in "he runs quickly" can be taken as qualifying the one who is doing the running as opposed to the running itself.[22] Of course Abelard does not even consider adjectives as distinct kinds of words; they are automatically to be taken as names, and if adverbs are interpreted adjectivally then they will be taken as names too. Interjections are names as well, he argues, because they denote feelings such as admiration or fear on the part of the speaker [Abelard, 1970, p. 121 (14–16)].[23] So what a more recent grammatical tradition has labelled the "seven parts of speech" is thus reduced to a simpler lexical classification: names, verbs and indefinites, where indefinites comprise prepositions and conjunctions.

Of these three classes, the first to be treated in the *Dialectica* is the class of indefinites. The main question is how these words signify. They are problematic

[21] [Abelard, 1970] covers these three topics respectively at pp. 118 (1)–120 (20), 121 (1)–129 (5) and 129 (7)–142 (20).

[22] In [Abelard, 1970, p. 121 (13–14)] we encounter the view that interjections, like adverbs, are atttached (*supponuntur*) to names. Note that in [Abelard, 1927, p. 334 (27-36); Abelard, 2006, 02.3] this view is refined to relegate certain interjections and adverbs to the class of indefinites.

[23] "Denote" translates *designat* [Abelard, 1970, p. 121 (15)]. For this translation of *designare* see [Nuchelmans, 1973, p. 140].

because there is no straightforward basis for construing them as generating understandings and denoting as names and verbs do. And yet they clearly play some kind of semantic role within the proposition.

One approach is to argue, against intuitions, that indefinites signify just as names and verbs do, and preserve this signification even when taken in isolation from other words. So what is the isolated meaning of "of"? Since it brings to mind nothing in particular, some who adopt this approach say that it signifies all things: "'Of' said by itself signifies all things, in that anything is of those things" [Abelard, 1927, p. 337 (14-15); Abelard, 2006, 02.17]. In other words, anything at all can be said to be *of*, because one can always identify a group to which it belongs and then identify it as being of that group. So the word when taken in isolation has a specifiable, albeit very broad, signification. When the word is then taken in context with other words, this universal signification will be limited (*reducitur*) and made definite (*certificatur*) through the presence of a name. In the phrase "of man," for example, the word "of" would presumably signify only men [Abelard, 1927, p. 337 (15–16); Abelard, 2006, 02.17].

Against this approach Abelard proposes another, which he prefers, and which indeed seems more intuitive. He simply denies that indefinites preserve signification when taken in isolation from other words — and so presumably signify nothing when said in isolation, as opposed to everything. These words, rather, are established for the sake of co-signifying alongside other words, and when put alongside them acquire signification, just as consonants by themselves make no sound, but acquire sound when uttered alongside vowels [Abelard, 1927, p. 337 (37–40); Abelard, 2006, 02.19]. What Abelard means by indefinite signification, therefore, seems to be *latent* signification. This latent signification is realized by placement of the word in an appropriate context. Now context does not just render the signification of these words more precise; it does that much already for names and verbs [Abelard, 1927, pp. 337 (41)–338 (3); Abelard, 2006, 02.20; Nuchelmans, 1973, pp. 140-141], and must do more for indefinites. On the other hand context does not simply assign the indefinite the same signification as some adjoining definite, since that would yield two words that signify exactly the same, producing a semantic redundancy [Abelard, 1970, p. 119 (20–23)]. Abelard gives several examples of how the real but latent signification of indefinites emerges by their use in context — as when we say that something is "of iron," and mean that it is *made out of* iron; this is to say more of the thing than simply to say it *is* iron. The presence of the preposition is what makes this difference [Abelard, 1927, p. 338 (24–26); Abelard 2006, 02.24].[24]

By contrast with these indefinite words, names and verbs are both classified as definites. Their signification is more than latent. It is analysable even with the words taken in separation. Of the two, names are the simpler case. Abelard's semantic account of names is essentially given already in his discussion of universals,

[24] Abelard's account of indefinites is commented on elsewhere: [Nuchelmans, 1973, pp. 140–142], [Dąmbska, 1977], [Jolivet, 1982, pp. 53–56], [Mews, 1985, pp. 83–84], [de Rijk, 1986, pp. 86-87], [Pinzani, 2003, 65–67], [Rosier-Catach, 2003a, pp. 74–79] and [Wilks, forthcoming].

and further discussion is limited to using the name as a point of comparison for the other kinds of words: indefinites are understood as falling short of the definite signification of names, whereas verbs are understood as adding something extra to that definite signification.

Aristotle defines the name as "a spoken sound significant by convention, without time, none of whose parts is significant in separation" [Aristotle, 1963, p. 43 (16a19)]. The key work of characterization is done here by the word "significant." The last clause ("none of whose parts is significant in separation") is intended to distinguish names from phrases (*orationes*)[25]; the reference to time is meant to distinguish them from verbs (although, as we shall see, Abelard questions this); and the reference to convention is meant to distinguish them from sounds that are meaningful in a way not established by human convention (such as the angry baying of a dog) [Abelard, 1927, 340 (19–29); Abelard, 2006, 02.34]. Only the reference to signification tells us positively what sort of spoken sound a name is: it is one which signifies, where Abelard glosses "signifies" non-denotatively, as "constituting an understanding" [Abelard, 1927, p. 335 (30); Abelard 2006, 02.10]. It is in fact when discussing universals that he gives us a worked out account of naming, and most of what he says about the above definition is focused on how names differ from verbs, and to a lesser extent how they differ from indefinites [Abelard, 1970, pp. 121 (4)–129 (5)] and [Abelard, 1927, pp. 334 (18)–344 (33); Abelard, 2006, 02.1-65]. Abelard's account is also focused on how names do not differ from their own inflected forms. Some inflectional differences such as case and gender represent no difference in signification from the uninflected form at all. They are simply to be explained in terms of grammatical construction [Abelard, 1970, pp. 124 (36)–125 (1)]. The inflectional difference marking plural from singular forms does represent a difference of signification, but only, as Abelard says, a difference in the mode of signification [Abelard, 1970 p. 124 (14–15)], not the underlying signification itself. We are given no further detail about this difference of mode — which presumably involves a different *way* of generating the relevant understanding; but Abelard's use of this notion is noteworthy here as instancing a strategy he often uses in addressing fine semantic distinctions.

This brings us to verbs. What is it about their signification which makes them different from names? Ultimately, Abelard argues, they differ only in that particular mode of signification which they bear by virtue of being able to *propose*. Names do not of themselves propose, and therefore lack this mode. But besides this account Abelard actually discusses two other accounts, both of wide acceptance in his own time, but not acceptable to himself.

The first, deriving from grammatical theory, holds that that the distinguishing feature of verbs is their capacity to express in active or passive voice; what they

[25] *Oratio* proves difficult to translate. It denotes any grammatically well-formed expression of more than one word, including propositions. Tweedale [Tweedale, 1976, p. 14] uses "word-string." Jacobi, King and Strub [Jacobi, King and Strub, 1997, p. 13, note 2] use "expression," with the qualification that single words do not qualify as expressions. I use "phrase," but with a different qualification: whole propositions count as phrases — and will be referred to as "complete phrases."

signify is an action or its passive counterpart ("passion" or "reception").[26] What names signify falls into other categories, not these, and what verbs signify falls into these and not others [Abelard, 1927, p. 346 (1–4); Abelard, 2006, 03.11]. The idea here is that names and verbs differ in their principle signification, not just in a mode of signification, so that they will signify fundamentally different kinds of things. Abelard's response is to point to a participle like "loving" as an example to the contrary; it is adjectival in nature, and therefore a name, but what it refers to is an action [Abelard, 1927, p. 346 (32–34); Abelard, 2006, 03.15]. Obviously, many such examples are available.

The second account derives from Aristotle. It holds that the distinguishing feature of verbs is their capacity to convey tense. The relevant Aristotelian definition, presented in direct contrast with the above definition of a name, is this: "A verb is what additionally signifies time, no part of it being significant separately" [Aristotle, 1963, p. 44 (16b6)]. So, on this account, a verb differs from a name not in having a different principle signification, but in having extra mode of signification above and beyond the signification a name possesses; whatever a name can signify a verb can signify too, but it also signifies tense. Abelard clearly rejects this view in [Abelard, 1970, p. 123 (15–16)]. A verb, he argues, is not distinguished by signifying tense, because names signify tense as well: the present tense. The presumption underlying the use of the name "white" is that its referent is *presently* white, not something which used to be white but is no longer so, or which will be so but is not presently [Abelard, 1970, p. 122 (22–25)]. It is likewise with names in general. The time at which they apply is assumed to be the present, so there is in fact a temporal reference built right into their signification.[27]

But the account of nominal/verbal distinction which Abelard consistently accepts, and which can be taken therefore as his most characteristic view on the question, derives from the second element of Aristotle's definition of the verb: the verb, he says, is a sign of "something's being said of something else" [Aristotle, 1963, p. 44 (16b6)]. The mode of signification truly distinctive of the verb is its predicative force, its ability to effect the sort of combination with a name that yields a proposition. Combining two names alone will not do this. Combining a name and a verb will. Here is a key text on the matter: "There are three acts in the understanding of a proposition: the understandings of the parts, and the conjunction or disjunction of the understood things. For it is not anomalous if that act which is not an understanding be part of the understanding of the whole proposition" [Abelard, 1927, p. 339 (29–32); Abelard, 2006, 02.29]. The first two acts are the understandings corresponding to subject and predicate. The third is the act of combining them in an affirmative or negative proposition. We have already seen how Abelard bases an account of semantic content on an account of

[26] This approach, and its grammatical background — the Priscian commentary tradition — is discussed in [Rosier-Catach, 2003b, pp. 182–186].

[27] On the temporal co-signification of names, see [Tweedale, 1982, p. 146], and, on the other hand, [Jacobi, 1986, pp. 153-154, and 174, note 44]; for adjudication of these views see [de Rijk, 1986, pp. 88-94].

mental acts; general terms achieve generality through selective attention of the soul to some property of a thing and inattention to other properties. But what is specially associated with the verb in a proposition is another kind of mental act [Rosier-Catach, 2003a, p. 78]. It is not itself an understanding, and so involves a different sort of act from the one that yields understandings. It is an act of putting understandings into complete affirmative or negative propositions. The understandings and the conjoining/disjoining are at root the same kind of item — they are all mental acts — and fit together quite naturally. That is why the act of conjoining/disjoining can be part of the understanding of the whole proposition (another mental act) even though it is not itself an understanding.

Note that the juxtaposition of a name and a participle — which of course is itself a name — can produce something whose semantic content seems very much like that of a proposition. "Running man" (*homo currens*), for instance, seems to involve same two understandings as "the man runs" (*homo currit*), and one can be left wondering why the first is not just as much a proposition as the second. But they are, of course, quite different. "Running man" leaves "the soul of the hearer hanging, and it desires to hear something more" [Abelard, 1970, p. 148 (27–28)].[28] But "the man runs" has a completeness about it that does not leave the hearer hanging. This difference is due to the presence of a verb "runs," which actually predicates the semantic content that "running" only juxtaposes. The participle does not serve as a sign of something's being said of something else. The verb does, and therefore possesses distinctive content not present in a name.

This distinctive content can be rendered explicit by re-writing a finite verb like "runs" with a corresponding copula verb phrase: "is running." To say that the signification of a verb reduces to the signification of a name plus some additional mode of signification seems to be confirmed by the fact that "runs" can be rendered thus. This equivalent form gives us a name ("running") plus an additional sign ("is") not having the signification of a name, but having obvious effect on the overall signification of the phrase in which it appears. The copula seems to express exactly that force of predication which the verb has in addition to its name-like signification. What emerges from Abelard's account of the verb, therefore, is an account of the copula. The suggestion is that the copula expresses only a mode of signification inherent in the whole verb phrase "is running," and must be taken in some way inseparably from "running" in the context of the whole phrase. Indeed if it does only supply mode of signification — if its only semantic contribution is indeed just to indicate predicative force — then it may seem appropriate to deny it the status of an independent word. In that case it would be in effect a syllable-equivalent which happens by lexical convention to be written as a separated word.

This latter possibility — a minimalist account of the copula — is in fact endorsed by Abelard in the *Dialectica*, by his construing the whole verb phrase as

[28] Abelard is, of course, just invoking the psychological criterion employed by Aristotle [Aristotle, 1963, p. 45 (16b19)] to demonstrate the non-propositionhood of individual names: "When uttered just by itself a verb is a name and signifies something — the speaker arrests his thought and the hearer pauses — but it does not yet signify whether it is or not."

semantically equivalent to a single word, and the copula as semantically equivalent to a syllable within that word: "When 'is a man' is said, or 'is conceivable' or 'is white,' let us understand as one word 'to be a man' or 'to be white' or 'to be thinkable"' [Abelard, 1970, p. 138 (15–17)].[29] This move is in keeping with a more general tendency of Abelard in the *Dialectica* to preserve as rigorously as possible the Aristotelian distinction between syllables, words and phrases: words contain only non-significative elements (syllables) and phrases contain only fully and independently significant words. A word like *respublica* (treasury) is compounded from the words *res* (thing) and *publica* (public), whose meanings are obviously relevant to the meaning of the compound, but do not directly determine it; that is, its meaning is not a direct additive product of theirs. So Abelard stipulates that its meaning is not a *product* of theirs at all. The word is taken as a single semantic unit, and its seeming constituent words are accordingly regarded as individual syllables, and not as retaining any distinct meaning therein [Abelard, 1970, p. 115 (20–23)]. He proceeds to make this sort of stipulation not only for individual words but whole phrases. A phrase like "dead man" contains an adjectival opposition, in that men are by definition alive, and so "dead man" is a contradiction in terms.[30] Abelard resolves the contradiction by claiming that the meanings of constituent words are not preserved in the phrase, stipulating rather that the constituents are to be taken as meaningless syllables, with meaning conferred only upon the phrase taken as a whole. A similar analysis is attempted in explaining why "good harpist" describes a harpist good at harp-playing, as opposed to a harpist good in a more general — perhaps moral — sense [Abelard, 1970, p. 116 (31)–117 (1)], and also in explaining the meaning of quantified subject phrases ("all men," "some men," and "no men") in syllogistic propositions [Abelard, 1970, p. 188 (26-32)].[31]

Elsewhere Abelard reverts to a less radical view: that the signification of words is affected by context, and words used in context often have their received signification altered by other words in their vicinity. This view is less radical because it falls well of short of saying that the words actually cease to be word-equivalents, and play the role of syllable-equivalents instead. It is simply an account of the *transference* (*translatio*) of signification, not the outright loss of it.[32] This account proves useful for Abelard outside of his work as a logician. In a theological setting, for example, he can argue that God's being three persons does not imply that God is three (i.e., three things), because "three" in "three persons" has different signification than when used in isolation [Abelard, 1969a, p. 269 (134–38); Wilks,

[29] In the translation of [Kretzmann, 1982, p. 509], the phrases are actually rendered as single words: "to-be-a-man," "to-be-white" and "to-be-thinkable."

[30] An account of the long history of commentary on this example is provided in [Ebbesen, 1979].

[31] Abelard does not account for prepositions and conjunctions in this way, however. Since the approach of the *Dialectica* does not allow for any intermediate stage between meaningless syllables and independently meaningful words, he is faced with the choice of construing prepositions in either one way or the other. So he attempts to construe them as independently meaningful. See [Wilks, 1998a, p. 184] and [Wilks, forthcoming].

[32] This important notion has been variously treated: [de Rijk, 1962, pp. 51–56, 94–98], [Jolivet, 1982, pp. 279–84], [Mews, 1987, pp. 23–25], [Rosier, 1988], [Wilks, 1998a], [Rosier-Catach, 1999], and [Wilks, forthcoming].

1998a, p. 175]; he can argue in this way without actually saying that "three" has no signification at all in that context. Likewise, on this approach, the copula is taken to undergo a change of signification when used in the context of a predicate phrase, but is not taken to lose its received signification entirely. This is a helpful result since it makes possible a more worked out account of the function of a copula verb in a predicative structure. Treating the word as a mere syllable-equivalent is a fairly easy, if implausible, way of dismissing the problem. But discussing what is actually involved in a relation of predication requires Abelard to meet the problem head on. This problem, be it noted, is more complex than the issue of what a verb is. It is the issue of what predication is.

Here is a characteristic text, which analyses the predicative relation involved in "Socrates is white":

> Two things are therefore conjoined with Socrates through "white": whiteness is conjoined in adjacence and a white thing (that is, the thing itself affected by whiteness) is conjoined in essence. But only whiteness is predicated, since it alone is what is meant to be conjoined. For not everything that is conjoined is predicated, but only that which is meant to be conjoined. For whoever utters the statement "Socrates is white" only declares that whiteness is in Socrates [Abelard, 1927, p. 360 (23–28); Abelard, 2006, 03.94].[33]

As a first step toward understanding this passage, we must distinguish between the so-called "inherence" and "identity" interpretations of the predicative relation.[34] According to the inherence interpretation, the predicate term signifies a form and the predication expresses the fact that that form actually inheres in the thing signified by the subject term. So "Socrates is white" tells us that whiteness inheres in Socrates. According to the identity interpretation, the predicate term signifies a thing, as does the subject term, and the predication expresses the fact that the two terms signify the same thing. So "Socrates is white" tells us that the white thing signified by the predicate term is the same as the Socrates-thing referred to by the subject term. The semantic account underlying the identity interpretation is gratifyingly simple. It involves only a relation of naming in both subject and predicate, and what is named is in neither case a form. But on the semantic account underlying the inherence interpretation we have a relation of naming only in the subject and a more complex significative relation in the predicate, one which involves a form. On the assumption that Abelard dislikes talk of forms, it in the past seemed plausible that he would ultimately prefer the identity to the inherence interpretation.[35] This assumption is misleading, of course; Abelard only dislikes

[33] [de Rijk, 1986, pp. 108–113] provides a detailed account of the view inherent in this passage, on which my own commentary depends.

[34] The prominence of these labels derives from [Moody, 1953, pp. 36–38].

[35] The resulting developmental interpretation of Abelard is that he moved from embracing an inherence theory of predication to an identity theory: [Pinborg, 1972, p. 52], [de Rijk, 1967, pp. 105–106] and [de Rijk, 1970, pp. xli–xliv]. Correctives are issued in [Tweedale, 1976, pp. 229–231], [Malcolm, 1979, pp. 386-390] and [de Rijk, 1981a, p. 25].

talk of shared forms. So he has no preference as between the two interpretations, and in fact regularly invokes both. That is what he is doing in the above passage. Predicating "white" of "Socrates" actually invokes both of the semantic roles that "white" is able to play: both its ability to name a white thing, and its ability to signify whiteness. The predication "conjoins" both to Socrates, as Abelard says above; that is, it construes Socrates as being in a relation to whiteness *and* as being in a relation to a white thing. The first relation is inherence, the second identity. The predication expresses both.

On the other hand, it is only the inherence relation that is *explicitly* expressed, thus becoming the relation which is explicitly conveyed by the terms of the predication. The identity relation is only implicitly expressed, and remains in the background. So only one of the items Socrates is conjoined with is meant to be appealed to in the predication. But why is the other unintentionally conveyed? Abelard explains this result as in effect forced on the speaker by the peculiar characteristics of the copula verb "is." This word also serves as our way of predicating existence (as in "Socrates is"), and this alternate, existential meaning is not normally divorced from the word, even in its usage as a copula. In this he diverges from the view of the *Dialectica*, where, as we have seen, he is only too willing to construe words as losing their received signification. But the post-*Dialectica* approach requires some notion of existence to be melded with the signification of the predicate name. Accordingly, "white" cannot be used with "is" without some implication, albeit in the background, that what "white" is being used to signify is not just a white thing, but an existing white thing. In other words, the predicate "is white" attributes whiteness, but at the same time unavoidably identifies what is said to be white with a white thing. Even though one only intends to attribute the form to what is named by the subject, one cannot help do something else as well: identify what is implicitly named by the predicate with what is named by the subject. One act is intended, two completed; Abelard refers to both of the two as "conjoining." His account of the copula verb phrase may therefore be summarized thus: each act of predication involves both kinds of conjunction.[36]

This grasp of the semantics of verb function gives us entry into the next level of semantic theory: the proposition.[37] One can group words in all sorts of ways that

[36] Abelard's treatment of predication and the copula verb remains one of the most extensively studied aspects of his logical theory. In addition to the works already cited on this issue, the following should be noted: [Nuchelmans, 1973, pp. 140–141], [de Rijk, 1981b, pp. 33–40], [Jolivet, 1982, pp. 56–62], [Jacobi, 1986], [Mews, 1987, pp. 21–25], [Marenbon, 1999], [Rosier-Catach, 1999, pp. 141–162], [Rosier-Catach, 2003b] and [Jacobi, 2004, pp. 147–150].

[37] "Proposition" is not an ideal translation for *propositio*. For the medievals the *propositio* always means a propositional sign, not propositional content [Kretzmann, 1970, p. 767]; for Abelard in particular it always denotes a sentence-token [Marenbon, 1997a, p. 203, note 2]. Current usage tends to associate "proposition" more with content and less with tokens. Accordingly, John Marenbon has more recently rendered the term as "propositional sentence" [Marenbon, 2004, p. 59, note 2]. One sometimes finds the phrase "statement-making utterance" [Nuchelmans, 1973, p. 146; de Rijk, 1986, p. 95]. Christopher Martin describes the *propositio* as "propositional token" [Martin, 2004a, p. 193, note 4]. I nonetheless use "proposition" in translation to maintain continuity with the standard translation for *maxima propositio* in topical

fail to achieve propositionality. If they are grouped in accordance with grammatical rules of construction they may constitute an appropriate conjunction of words, a *competens dictionum coniunctio* [Abelard, 1970, p. 147 (28)]. But this is no guarantee that they will constitute a proposition. A proposition is a complete phrase, an *oratio perfecta*[38]; what indicates the incompleteness of other phrases is the disposition of the hearer to pause and "desire to hear something more" [Abelard, 1970, p. 148 (27-28)]. When words grouped according to grammatical rules suffice to meet this desire, we have a proposition. The key element is of course the verb. "Without the verb," says Abelard, "there is no completion of sense" [Abelard, 1970, p. 148 (29–30)].

Mere combinations of names do not yield this completion. What they yield is something that, under conditions of grammatical appropriateness, simply remains at the semantic level of a name. "Rational, mortal animal" functions grammatically just as "man" does, even though the one is a phrase and the other a word. This naming function is brought about in the former case by a more complex internal structure of signification, involving more mental acts (*actus intellectus*) than the latter, but in the end both perform as names, and in fact as names of the same thing [Abelard, 1927, p. 325 (24–29); Abelard, 2006, 01.94; Nuchelmans, 1973, p. 143]. At this level no considerations of truth or falsity arise. Names and incomplete phrases may have application to things or fail to do so, but this just means that they are either sound (*sanus*) or not [Abelard, 1927, p. 326 (30–36); Abelard, 2006, 01.97].[39] "The running man" signifies in such a way that it is able to name, and as such is described as being sound. But it is not appropriately said to be true. It differs from "the man runs" by the fact that in using it we determine "running ... as adjacent to man, but we do not also assert the adjacence, which cannot be done without a verb" [Abelard, 1970, p. 149 (12–14)].

The verb's special ability to confer completeness of sense resides in its ability to convey predicative force. The example of predicative force given above is assertive force, the product of which is a proposition. But Abelard notes that there are other kinds of predicative force as well, which yield other kinds of complete phrase. One can predicate interrogatively or imperatively; in uttering a prayer we have a fourth different sort of predication, and in uttering a wish a fifth [Abelard, 1970, p. 151 (4-14)]. These five kinds of complete phrase represent five different ways of joining predicate to subject, and of demonstrating "the inherence of something in something in accordance with various dispositions (*affectus*) of the soul" [Abelard, 1970, p. 149 (21–22)].[40] What is at issue here is the distinction between sentential

theory: "maximal proposition." Note that Abelard takes the token quality of the *propositio* so seriously that he actually denies what we know as the "Tarski biconditional"; from the fact that a man exists it does not follow that "A man exists" is true, since "A man exists" might never have been uttered [Abelard, 1921, p. 291 (26-28)]. See [Guilfoy, 2002, p. 146; Marenbon, 1997a, p. 205, note 8].

[38] See note 25 above.

[39] *Sanus* is contrasted with *cassus*, "empty" [Nuchelmans, 1973, p. 144].

[40] Nuchelmans translates *dispositiones animi* as "mental attitudes" and notes that Abelard uses the similar phrase *affectus animi* to denote the moods of the verb [Nuchelmans, 1973, p.

content and the sort of force with which it is asserted or otherwise put forward. The content may be contained in an incomplete phrase like "running man" and put forward with no sort of force at all. But when a verb is in place to mark the relevant disposition of soul the corresponding force is brought to bear upon the content. In the case of assertion we have seen that this is a function not of the mental acts of conception which constitute the basic signification of verbs and names alike, but rather a function of the mental act supplementary to those, which marks the predicative relation and operates as a mode of that basic signification. Presumably this holds for the other kinds of complete phrase as well; in each case, it may be supposed, their distinctive force arises from a specific mode of signification through a specific way of inflecting or otherwise presenting the verb. So the sentential content of "Socrates is coming" is put here with assertive force, but could be put otherwise with interrogative, imperative, precative or optative force (respectively: "Is Socrates coming?", "Socrates, come!", "Socrates, please come," and "Would that Socrates would come").[41]

For the logician, of course, it is only the first of these five kinds of force — assertive — that constitutes an appropriate object of interest [Abelard, 1970, 152 (27–8)], the others being relegated to non-dialectical subject matters. Abelard has shed much light on how individual words combine to be productive of the propositions they compose. He must now turn to the special semantic properties of propositions themselves. The point of departure is the Boethian definition of the proposition as that which "signifies a truth or a falsehood" [Abelard, 1970, p. 153 (33); Boethius, 1990, p. 2]. To grasp the special character of the proposition as a semantic unit accordingly requires some insight into what is meant by "true" and "false."

To this end, Abelard suggests a key distinction: "Just as names and verbs have a twofold signification — of things and understandings — we also grant that propositions have a twofold signification: according to the understandings composed of the understandings of their parts, and according to their *dicta*" [Abelard, 1927, p. 367 (9–12), Abelard, 2006, 04.26]. Individual names generate understandings and name things, and in some sense the same can be said of propositions. But what has so far been said about the semantics of stating has focussed on understandings only. The key question has been how the combination of words into phrases is made meaningful by the combination of their associated understandings into larger conceptual structures. But this cannot be the whole picture. To satisfy their full semantic function, names are normally expected to refer. Likewise, to satisfy their full semantic function, propositions are normally expected to be true. Just as there must be something for the name to refer to, there must also

147]. Martin Lenz argues that for Abelard the true criterion of completeness is not the fact that the listener does not expect to hear more (the "suspense-criterion"), or the fact that the phrase contains a finite verb (the "grammatical criterion"), or the fact that it signifies a predication (the "logical criterion"), but the fact that one of the propositional attitudes is expressed [Lenz, 2005, pp. 381, 383].

[41] For Abelard's examples (which do not, as mine do, use the same content throughout) see [Abelard, 1970, p. 151 (6–15)].

be something for the proposition to be true of. What it is true of is the *dictum*. Abelard has discussed how individual understanding-generating words merge to make a understanding-generating proposition. Now he must also discuss how this understanding-generating proposition is able to be true.

The *dictum* of a proposition is, translated literally, what is said by it. This translation is not overly helpful, but unfortunately the notion is one which Abelard is clearly struggling to refine, and providing an exact account of it is as difficult as providing one for the notion of the *status*. Abelard does, however, clearly rule out two hypotheses. (i) What is said by a proposition is not itself an understanding. If it were, true conditionals would in most cases fail to involve a necessary connection between antecedent and consequent [Abelard, 1970, p. 154 (30)–155 (5)]. The conditional "If it is a man it is an animal" is supposed to state some sort of necessary connection. But if the connection it states is between understandings then it cannot be a necessary one. After all, it is perfectly possible to have one of the two understandings without having the other. One can conceive of a man, for instance, without also conceiving of an animal. There is no *necessary* connection between the generation of the one understanding and the generation of the other. But since there *is* a necessary connection between what is said by the antecedent and what is said by the consequent, it follows that what is said by a proposition cannot be an understanding. (ii) But what is said by the proposition cannot just be the thing(s) the proposition is about, either. If it were, Abelard argues, a proposition like "Socrates is Socrates" — a bit odd, admittedly, but perfectly well-formed – would be semantically indistinguishable from the name "Socrates." This is because the subject and predicate terms in this case refer to the same thing, and thus to only one thing — Socrates. On the hypothesis, then, that is all the proposition would say: Socrates. How would it then differ in semantic content from the name "Socrates"? "Socrates is Socrates" obviously *does* differ in semantic content from "Socrates," and so the hypothesis fails. What is said by the proposition is not just the thing(s) the proposition is about [Abelard, 1970, p. 157 (14–25)].[42]

The *dictum* cannot be understood as a generated understanding or the thing(s) the proposition is about. What is it then? The best clue is this: not only is the *dictum* said by the proposition, but it possesses key features of the proposition itself, and indeed causes the proposition to have those features: "It is chiefly according to these *dicta* that propositions are judged to be true or false, or to be opposites, or to be necessary or possible, since, that is to say, the *dicta* of those propositions are true or false or opposite to one another or necessary or possible" [Abelard, 1927, p. 367 (13–16); Abelard, 2006, 04.26]. *Dicta* themselves have the properties of truth, falsity, possibility, necessity, and oppositeness to other *dicta*, and then impart them to the propositions by which they are expressed. The fact

[42]These arguments from the *Dialectica* are actually applied not to the *dictum* — which is not referred to in this work — but the *existentia rei*. See note 45 below for these two technical notions. They cannot automatically be presumed to be the same, but nonetheless both represent *what a proposition says*, and so the above arguments seem applicable to both.

that they can be opposite to one another means that they can be contradictories and contraries to one another, and suggests that they likewise can stand in the other relations of the square of opposition, and so can be sub-alternates and sub-contraries. This in turn suggests that they can stand in inferential relations to each other, and accordingly the above puzzle about how inferential relations hold independently of the understandings of the mind, and the things the propositions are about, is able to be addressed through this theory of *dicta*.

So the *dicta* of propositions — what they say — cause their truth, falsity, etc. They are also themselves able to be described as true, false, etc. Above and beyond these features is another already mentioned. The *dictum*, as what is said by the proposition, is its propositional content, and as such subject to the various propositional attitudes under which it can be expressed: it can be expressed with assertive force, interrogative force, imperative force, and so on.[43]

We have, therefore, a variety of features associated with the *dictum*, and the question is, then, what sort of item actually possesses them. Abelard leaves us with no clear response to this question, and indeed seems to struggle with it himself. On the other hand, it is easy to find modern comparisons which answer to these descriptions — understanding, of course, that the key terms involved are extensively nuanced, and cannot easily be transferred to a prior century.

(i) What we call a *state of affairs* can be construed as possessing these features. We can say that true, possible, necessary states of affairs are ones that do, can and must obtain, and that false states of affairs are those possible ones that do not obtain. Two such states can be taken manifesting oppositeness to one another, and standing in a relation of contrariety or contradictoriness. We can speak of one state of affairs as implying another. So states of affairs can be, like *dicta*, described as true, false, etc. They can also be construed as causing these features in propositions — as making a proposition true, false, possible, necessary, opposite to another proposition or such as to imply it. Since states of affairs are closely associated with how the world is in some respect, it is intuitive to think of them as actually *causing* propositions to be true, false, etc. Finally, a state of affairs can be construed as being what is said by a proposition, that is, its content. As such it may be regarded as subject to expression via the various propositional attitudes. One can assert a state of affairs, question it, order it to be brought about, and so on.

(ii) Parallel construals can be attempted for the notion of a *fact*. Facts can be characterized as true, possible and necessary, and then false ones characterized as those possible ones which do not hold.[44] Facts can be mutually exclusive, and one fact can imply another. Facts can be construed as causing propositions to be true, false, possible, and necessary, mutually exclusive and implicative of other propositions. And they can be construed as propositional contents subject to assertive, interrogative, imperative, precative and optative force.

[43]The fact that the *dictum* must be thought of as susceptible to all of the propositional attitudes, not just the assertive one, is emphasized in [Marenbon, 2004, p. 66].

[44]This notion of a false fact would clearly be a departure from the received notion of a fact.

Whatever philosophical problems may beset the propositional semantics arising from the construals in (i) and (ii) are beside the point. The point is that the notion Abelard is attempting to articulate the *sort* of notion that figures commonly in contemporary semantic theory, and that the *dictum* should not be seen, from our perspective, as an especially exotic notion. It is undoubtedly, however, an unfinished one, and in need of further specification.

There is one point about it, however, that Abelard stresses: it is not a thing (*res*) [Nuchelmans, 1973, p 153].[45] Here his concerns echo those expressed in his discussion of the *status*. The *status* is the cause of the applicability of a general name to a plurality of objects; but even though it causes this applicability, it is not a thing. The *dictum* is likewise a cause — of the truth, falsity, etc. of propositions — but again Abelard wants to deny that being a cause entails being a thing. He reverts to the sort of argument, seen earlier in his discussion of the *status*, that what serves as a cause does not necessarily have to be identifiable as a thing. The thief is hung for an act of thievery committed in the past, an act which, at time of hanging, is not a thing. One can starve to death because of the non-act of failing to eat, or be eternally damned for the non-act of failing to act well, and these non-acts are likewise not things. So the non-thinghood of *dicta* is no bar to their causal abilities [Abelard, 1927, p. 369 (2–6); Abelard, 2006, 04.36], or their ability to explain how propositions can be true, false, etc. In this way a further portion of Abelard's semantic theory is set in place without evident appeal to metaphysically controversial items. Given his initial rejection of shared forms from his semantic theory to avoid metaphysical controversy, he is naturally disinclined to re-introduce such controversy elsewhere.

Of course the supposed non-thinghood of the *dictum* does raise its own puzzles.[46] Recall the passage quoted above to introduce the notion of the *dictum*: "Just as names and verbs have a twofold signification — of things and understandings — we also grant that propositions have a twofold signification: according to the understandings composed of the understandings of their parts, and according to their *dicta*" [Abelard, 1927, p. 367 (9–12); Abelard, 2006, 04.26]. This comparison should be approached carefully. If we press the analogy with names too far,

[45] Nonetheless the *Dialectica* is less explicit on this matter. Abelard's preferred term in that work is not the *dictum* [Marenbon, 1997a, p. 204, note 5] but the *existentia rei*; translated literally this means "the existence of a thing." But it would not have been taken in this way, however, given the generality of the word *res*, and is better rendered as "state of affairs" or "matter of fact." Abelard might have come to prefer the word *dictum* because of its non-use of the word *res*, and thus its greater ability to convey the non-thinghood of the cause of truth, falsity, etc. (For comparison, imagine a modern theorist coming to prefer "fact" over "state of affairs" because of the latter's sounding too much like a thing, or collection of things, in the world.) Kevin Guilfoy in fact argues that the *Dialectica* theory is altogether distinct from the one given in the *Glossae super Periermeneias* [Guilfoy, 2004b]. Intriguingly, John Marenbon argues that having come to employ the word *dictum*, Abelard may subsequently have abandoned it [Marenbon, 2004, pp. 76–78].

[46] These are treated in Tweedale's extensive discussion of the *dictum* in [Tweedale, 1976, pp. 216-278]. For other discussions of the *dictum* not so far mentioned see [de Libera, 1981], [de Rijk, 1975], [Jolivet, 1982, pp 77–85], [Lewis, 1987, pp. 84–88], [Jacobi *et al.*, 1996] and [Rosier-Catach, 2004].

we may be led to the conclusion that there is a thing named by the proposition as a whole — its *dictum* — just as there is a thing named by the individual name. But this conclusion makes exactly the wrong mistake about the *dictum*. The *dictum* is signified by a proposition, but is not actually a thing that is named by it. The signifying that a proposition does is different from naming. A proposition does not name what it says.

This requirement proves troublesome when it comes to the correct interpretation of certain kinds of impersonal construction. Subject and predicate terms have so-called "personal" signification when there is something specifiable that they name.[47] Aristotle's defining characterization of the proposition as "a statement of something about something" [Aristotle, 1963, p. 47 (17a25); Abelard, 1927, p. 389 (32–33); Abelard, 2006, 06.2] seems to require that both subject and predicate terms have personal signification. But in many propositions this prerequisite seems not to be met. "It is raining" appears to have a subject term ("it") without personal signification — and hence it is called an "impersonal construction." One of Abelard's examples is *ventum est ecclesiam* [Abelard, 1927, p. 390 (15); Abelard, 2006, 06.5], meaning "It was come to church," or in slightly better English, "There was an arrival at church." This proposition needs to be differently expressed to fit under the definition just given: "There was an arrival at church by some people" or "by them" [Abelard, 1927, 390 (19-20); Abelard, 2006 06.6]. This re-formulation suggests that the proposition does have an implicit subject term ("people," "them" — i.e., "they") with personal signification; it also suggests that the explicit, original form of the proposition is misleading. There really are, in effect, denotative subject and predicate terms.

The question is whether this sort of solution will work in all cases. Deeper problems arise for propositions involving sentential operators such as "It is possible that." If we say, "It is possible that Socrates runs" we appear to be predicating possibility of what is said by "Socrates runs." In other words, we are treating "Socrates runs" as a subject term, with the accompanying suggestion that it names something. But, as noted, propositions do not name. If they did name, it would be their *dicta* that they named, and Abelard will not accept this result. The *dictum* is not a thing, and cannot coherently be treated as if it were personally signified.[48]

This result applies to a wide range of impersonal forms: "It is possible that ...", "It is true that ...", "It is good that ...", and so on, where the omitted portion is seen as being filled by some verbal formula expressing a *dictum*. These impersonal forms need to be clarified, just as "There was an arrival at church" needs to be clarified, so as to reveal a personally signifying subject term. This is done to "It is possible that Socrates runs" by rendering it as "Socrates possibly runs" — that is, by re-stating it as a predication about Socrates, a personal subject.

[47] For some of the historical background to this semantic use of the word "personal" (*personalis*), see [Mews, 1992, pp. 14-16]. This word in these contexts must always be understood with reference to its opposite: "impersonal."

[48] On the other hand, Abelard does refer to *dicta* by the entirely enigmatic phrase *quasi res* (literally, "quasi-things") [Abelard, 1927, p. 367 (12); Abelard, 2006, 04.26]. [Guilfoy, 2004b, p. 42, note 25] discusses the proper translation of this phrase.

Abelard's preference for personal over impersonal formulations is supported by the difficulties involved in applying conversion to impersonal ones. Ability to convert is, of course, taken as a standard property of subject/predicate propositions. A universal proposition "All S is P" is converted to "Some P is S," and along these lines a singular proposition "Socrates is running" becomes "Something which is running is Socrates." In general, if a statement "S is P" is true then at least one of the Ps can be said to be an S, so we can infer that something which is P is S. But notice that this makes no sense when, as in an impersonal proposition, the subject term corresponds to the *dictum* of a proposition.[49] If we say "It is possible that Socrates runs" with impersonal intent, then the predicate term is "possible," and the conversion is this: "Something which is possible is that Socrates runs." Abelard is not willing to credit a proposition like this with any meaning, and concludes in general for impersonal propositions that, unless interpretable as personal constructions, they are "neither universal nor particular nor indefinite nor singular, and on that account do not permit conversion of the simple forms" [Abelard, 1958, p. 14 (15-17); Abelard, 2006, 12.20]. This would suggest that they are not of the subject/predicate structure at all in that event [Tweedale, 1976, p. 259] — and this conclusion challenges the assumption that the subject/predicate analysis is universally applicable to all propositions.

The above account of propositional contents informs another of Abelard's innovations: an account of sentential negation which fully grasps the semantic difference between wide scope and narrow scope negation. Under narrow scope negation the sign of negation is placed between subject and predicate ("Socrates is not a man"); under wide scope, it is placed at the front of the proposition ("It is not the case that Socrates is a man"). Under the latter placement, the sign operates with the entire proposition-to-be-negated in its scope. This placement, in Abelard's view, permits, as the other does not, a complete semantic negation, leaving the negated form in the expected relation of contradictoriness to the pre-negated form: "It is clear ... that to every affirmation, the negation which destroys the entire sense of that affirmation with preposed negation is correctly opposed as the proper dividing statement in contradiction" [Abelard, 1970, p. 178 (28-30)]. Abelard refers to narrow scope negation as "separative," and wide scope as "extinctive" [Abelard, 1927, p. 406 (29-31); Abelard, 2006, 07.28]. His thesis, then, is that it is extinctive negation, not separative, that fully carries out the logical operation of negation. Note that the extinctive negation is not construed as an impersonal construction. It is rendered by placement of *non* at the front of the proposition, and Abelard classifies this sign of negation as an adverb, not as a personal construction [Abelard, 1927, p. 406 (37); Abelard, 2006, 07.29]. So the use of extinctive negation does not encounter the problems that impersonal constructions in general do.

Abelard's prefers extinctive negation because it better conveys the relation of

[49] In that case the linguistic item expressing the *dictum* can be expressed as an accusative and infinitive construction: "for Socrates to run." [Guilfoy, 2004b, pp. 44–53] discusses the bearing of Abelard's account of the *dictum* on the semantic function of this construction.

contradictoriness. Contradictories are supposed to divide completely the conceptual space that they mutually imply. They can neither be true at the same time nor false at the same time, so that the truth of one properly implies the falsity of the other. This relation is formalized in Aristotle's famous square of opposition, where the diagonally opposed propositions are the ones that are contradictorily opposed:

A: All S is P E: No S is P (= Not some S is P)
I: Some S is P O: Not all S is P.

The E- and O-propositions here are rendered extinctively, so as to contradict, respectively, the I- and A-propositions. Say the O-proposition were rendered separatively: "Some S is not P."[50] This version, Abelard argues, does not contradict the corresponding A-form, because both could be false together. The reason lies in the further issue of whether the subject term can be assumed to name something that really exists — that is, whether it carries existential presupposition. If the negation comes *after* the subject term then the proposition as a whole presupposes that there is something corresponding to the subject term. But if the negation comes *before* the subject term then the presupposition disappears. Here is the key claim:

> But it is one matter to disjoin things from each other, another to show that they do not agree with each other. For one who treats of things as being disjoined from one another takes them as if they exist; but one who treats of things as not agreeing with one another does not suggest their existence any more than their non-existence ... Hence the preposed negation has the one import and the interposed negation the other [Abelard, 1970, p. 177 (12–15, 16–17)].

For P to be disjoined from S suggests *removal of P from S*, and therefore presents S as some existing thing from which the removal is made. Existential presupposition is attached to the subject term (and to the predicate term too, since "Some S is not P" is equivalent to "Some P is not S"). But when the negation is put at the front of the proposition, what is suggested is only the disagreement of subject and predicate terms, not removal of one from the other. Mere disagreement does not carry with it the suggestion of an existing thing from which removal is made.[51]

It is this existential presupposition in an O-proposition with separative negation that makes it fail to contradict its corresponding A-proposition. Say no humans exist. Then the A-proposition "All humans are mortal" is false, because it presupposes that humans do exist. But "Some humans are not mortal" is false too, since it presupposes that humans do exist. Two propositions capable of being false together are not contradictories. So the separative version of the O-proposition fails

[50] Note that separatively negating the A-form to yield "All S is not P" would not express what is needed, and would, in fact, be a clumsy way of expressing the E-form

[51] Note that the existential presupposition issue is distinct from the issue of having a subject term with personal signification. "The chimaera is a mythical beast" is a personal proposition, even though its subject term fails to name an actual thing.

to represent the content of that form correctly.[52] But Abelard's conclusion is *not* that there is anything illegitimate about propositions with interposed negation. His point is simply that if we start with the proposition "All S is P," and then build a square of opposition around it, no proposition in that square will contain a separative negation. The sort of negation which produces the logical oppositions is not separative. But this does not mean that separative negations can never appear in a square of opposition. In fact, to the square beginning with "All S is P" Abelard constructs corresponding square beginning with "All S is not P"; the corresponding E-form is "No S is not P," the I-form is "Some S is not P" and the O-form is "Not all S is not P" [Abelard, 1927, p. 407 (20–31); Abelard, 2006, 07.34–35]. The point is that "All S is not P" is treated not as negated form of "All S is P," but as a distinct predication, itself subject to placement as the A-entry on a square of opposition. Given the two different kinds of negation, therefore, "All S is P" in fact gives rise to *two* squares of opposition — or, we might say, a *double square* (or, as it has been called, a rectangle of opposition [Martin, 2004a, p. 168]).

That contrary and contradictory propositions are opposed "not from the nature of the terms but from the very force of assertion" [Abelard, 1927, p. 410 (3–5); Abelard, 2006, 07.45] is a basic point about propositional structure which makes it possible to understand how propositional connectives, like negation, operate.[53] This was later to be a key discovery of Gottlob Frege, and the basis for many of his innovations. But it is already present in the work of Abelard.[54] It confers upon him not only a viable basis for constructing a propositional logic, but gives him heightened awareness of the importance of *scope* in understanding the syntax of propositions under logical analysis — in this case the scope of negation. This awareness naturally shapes his account of modal theory as well, where issues of scope are so ubiquitous. In modal theory, however, his preferences attach to a narrow scope of logical operation, not a wide one, so that he privileges modal propositions formulated with interposed modal operators (the syntactic counterparts of separative negations).

When interposed, modal operators take the form of adverbs. A mode in effect answers a "how" question; one asks "How did he read?" and the respondent formulates the answer by modifying the verb with a word like "well," "badly" or "quickly" [Abelard, 1958, p. 3 (16–21); Abelard, 2006, 12.3]. All three adverbs

[52]Note that the problem of interposed negation is less likely to arise for the E-proposition, since it is normally expressed with preceding a sign of negation. The standard Latin quantifying term for the E-proposition is *nullus*, which is an etymological product of *non* and *ullus*, and in effect means "not some" (more literally: "not any"). *Nullus* already does, in effect, pre-position the *non* [Abelard, 1970, p. 177 (33–34)].

[53]As we shall see below, Abelard realizes the implications of this analysis for understanding the proper negated form of a conditional [Abelard, 1927, p. 406 (29–31); Abelard, 2006, 07.28].

[54]Peter Geach refers to this insight as the "Frege point" [Geach, 1965, p. 449]. Christopher Martin, who cites Geach on this point [Martin, 1991, p. 281], has stressed the significance of finding the Frege point already fully articulated by Abelard: "If he was the first to achieve this understanding in the Middle Ages, and there is no evidence to the contrary, he must be recognized as one of the greatest of all philosophical logicians" [Martin, 2004a, p. 166].

are modes. The modes of modal logic are adverbs too, just very specialized ones. So "necessarily" just qualifies the assertion that something is thus by the addition that it cannot be otherwise [Abelard, 1970, p. 194 (7)]. But note that what are grammatically adverbs do not always, strictly speaking, modify the verb; the adverb "falsely," for example, does not say how a verb applies, but in fact says that the verb does not apply. "Possibly" invokes a similar point. If we say "S is possibly P" we do not deny the truth of "S is P," but we definitely do not affirm it as a truth either. So "possibly" does not operate in typical adverbial fashion to indicate mode of inherence [Abelard, 1958, p. 4 (7–10); Abelard, 2006, 12.4]. Is it even an adverb? Abelard still wants to say it is. His view is that "possible" functions as an adverb syntactically (*secundum positionem constructionis*) even if not semantically (*secundum sensum*); that is, syntax assigns it a modifying role, even if this role is not forthcoming just from its semantic properties plus those of the accompanying verb.

So claims of possibility are still explained as claims about the nature of the subject (i.e., that S is possibly P). These claims, however, are not about what the nature is, but rather about what it is not repugnant (*repugnet*) to [Abelard, 1970, p. 193 (36)]. Something is possible for Socrates when it is not repugnant to his nature. To use Abelard's example, the possibility that Socrates is a bishop lies in the fact that being a bishop is not repugnant to Socrates' nature [Abelard, 1970, p. 193 (35–36)].[55] This fact alone suffices to establish the possibility of Socrates' being a bishop, even if he has never been one and never will be one. In this way, "Socrates is possibly a bishop" is construed as being of subject/predicate form, where a subject is named and a predicate is said of it — with "possibly" modally qualifying the latter.

Besides this adverbial presentation of modes there is another. Modes can also be nominal, and can be indicated by names, not adverbs. When indicated by names they function as predicates of whole propositions, as in "It is necessary that Socrates runs." Here the subject is not a thing but a *dictum* ("that Socrates runs"), and the point of the predication is to impute necessity to this *dictum*: that Socrates runs is necessary. In other words, what is said by "Socrates runs" is necessary. Here we have an impersonal construction, where the subject term fails to denote a thing; this sort of construction characterizes nominal modes generally. The greater complexity of impersonal propositions, as opposed to personal ones, has already been noted, and Abelard always sees nominal modes as posing special problems that adverbial ones do not. The common syntactic form of modal propositions in Abelard's time involves a nominal mode placed with an infinitival construction, as in "It is necessary for Socrates to run." This literally involves a nominal mode — "necessary," as opposed to "necessarily" — which literally functions as predicate term. Nonetheless, Abelard claims, a non-literal, adver-

[55] While Abelard defines possibility in terms of natures, he elects not to define necessity in terms of natures as well (as he could have done by saying that necessity is the incompatibility of the absence of a form with a nature). Instead he speaks of the necessary as the "inevitable" (*inevitabilis*) [Abelard, 1970, p. 194 (8); Abelard, 1958, p. 21 (2); Abelard, 2006, 12.35].

bial construal of the proposition is available in this context as well, under which Socrates is taken as the subject, and running is predicated under the mode of necessity. Perhaps the idea behind this point is that the adverbial interpretation is the basic one anyway, so it is natural to read it even into contexts where it is not explicitly invoked.

The result of this interpretative leniency is to provide alternate construals for standard modal propositions. This ends up being an immensely important point, since the distinction between these two syntactic construals is suggestive of the pervasive semantic distinction between *de re* readings of modals and *de sensu* readings, the former associated with an adverbial construal of the mode, the latter with a nominal construal.[56] The distinction is introduced by Abelard through a familiar Aristotelian example: "It is possible for the standing man to sit" [Abelard, 1958, p. 13 (15–16); Abelard, 2006, 12.18; Aristotle, 1984b, p. 280 (166b24–26)]. The analysis of the example shows that the *de re* and *de sensu* readings are so different that the same proposition true on one can be false on the other.

(i) The *de sensu* reading takes the syntax of the nominal mode as it stands, and interprets the mode ("possible") as the predicate. The subject is then the *dictum* "for a standing man to sit" or "that a standing man sits" — and this is what is said to be possible. Note that the resulting syntax puts "standing" and "sitting" in combination as referring to the same thing. But the combination is impossible, since nothing can be sitting and standing at once. So on the *de sensu* reading the proposition is false. (ii) The *de re* reading takes the syntax of the mode non-literally and interprets the mode as adverbial. This reading puts "standing man" as the subject. "Sits" is what is predicated, with the predication coming under the adverbial modification of "possibly." The effect of this reading is to partition "standing" from "sits." Instead of putting both in the subject term ("that a standing man sits"), one is put in the subject term ("a standing man"), and the other, under modal qualification, is put in the predicate term ("possibly sits"). Because of this partition the *de re* reading avoids referring to its subject as both standing and sitting; it just refers to it as standing, and says of it that it is possibly sitting. The resulting predication, far from being self-contradicting, is perfectly capable of being true. In fact it is capable of being true in more than one way. It can be true when the mode is interpreted *diachronically* to mean that the standing man can sit at some time after the time at which he is standing. Or it can be true when the mode is interpreted *synchronically* to mean that the standing man is now sitting in some possible world co-relative to ours.[57] However

[56] In Abelard's hands, however, the association is not automatic. Paul Thom notes that "the *de sensu/de rebus* distinction has a different *fundamentum divisionis* from the compound/divided distinction" [Thom, 2003, p. 46] (the latter being equivalent to the nominal mode/adverbial mode distinction). See also [Knuuttila, 1993, pp. 85-87].

[57] Whether Abelard's primary understanding of the modes is diachronic or synchronic has been a matter of some debate. We find an argument for the former position in [Knuuttila, 1981, pp. 178-87], and for the latter in [Weidemann, 1981]. Knuuttila replies and refines his position in [Knuuttila, 1993, pp. 93–94]: "Even though one cannot claim that Abelard's modal thinking was systematically based on a model of counterfactual synchronic alternatives, it probably

the notion of possibility involved here is understood, it is clear that the *de re* reading of the example proposition allows it to be true, and the *de sensu* reading does not.

This is not an argument for the superiority of *de re* readings of modal propositions, since, as Abelard notes, the opposite result holds for "It is necessary for the standing man not to sit." This is true taken *de sensu* (since "The standing man does not sit" is necessarily true) and false taken *de re* (since it is false that the standing man *necessarily* does not sit) [Abelard, 1958, p. 14 (8–13); Abelard, 2006, 12.19]. Both this example and the previous one simply convey how very different the two readings are. Nevertheless, Abelard does prefer the *de re* approach. The problem with the *de sensu* approach is that it makes modal propositions into impersonal propositions; these, as we have seen, lack a referring subject term and as such lack features like universality, particularity, and ability to convert, and therefore fit poorly into the forms and figures of syllogistic logic. We can compare the standing of impersonals in this regard to the standing of relational propositions. These likewise fit poorly into the forms and figures, and need to be re-cast somehow into conventional subject-predicate syntax. As they stand, prior to this change, they are perfectly coherent propositions, capable of being true or false; but from a syllogistic perspective, their limitation is that they are not perspicuous expressions of their content. This would seem to be Abelard's view about impersonals generally and, more specifically, about *de sensu* modals.

But there is a more technical limitation to be observed in *de sensu* modals. The infinitival propositions in which they are expressed offer two different points at which negations may be inserted. "It is possible for Socrates to run" can be negated as a whole: "It is not possible for Socrates to run." But the infinitival portion, since it involves a verb, can be negated as well: "It is possible for Socrates not to run." Abelard notes that this second negation in fact makes sense only on a *de re* reading. On a *de sensu* reading the infinitival phrase "for Socrates to run" is of course the subject of the proposition. Now one does not negate a proposition by negating its subject term; for example, one does not regard "Non-Socrates is running" as being a negated form of "Socrates is running." So one must likewise not regard "It is possible for Socrates not to run," taken *de sensu*, as being a negated form of "It is possible for Socrates to run"; again, the subject term on this *de sensu* reading is "for Socrates to run," and that is what the sign of negation has just been applied to. The conclusion is that the *de re* reading engenders a class of negated forms which vanish when the reading is *de sensu*. In fact there are three forms of *de re* negation: the two just mentioned ("It is not possible for Socrates to run"; "It is possible for Socrates not to run") and their combination: "It is not possible for Socrates not to run." In being able to capture only one of these three, the *de sensu* reading must be taken as comparatively less expressive [Abelard, 1958, p. 21 (22)–22 (15); Abelard, 2006, 12.38–39].

When quantifiers are attached and varied, these four *de re* forms (the origi-

played some role in his thought." Marenbon argues that neither approach captures Abelard's understanding of the modal operators [Marenbon, 1991; Marenbon, 1997a, pp. 221–223].

nal plus the three negations) proliferate. When parallel listings are compiled for impossibility and necessity the forms proliferate further. (Note that there is no separate treatment of contingency, since Abelard erroneously takes this mode as identical with possibility [Abelard, 1958, p. 24 (2); Abelard, 2006, 12.41]). So, starting with the simple *de re* statement "All men can possibly run," varying it by quantification, negation, and substitution of other modes, we end up with a twenty-four-entry list which is essentially an expanded square of opposition for quantified modal statements [Abelard, 1958, pp. 25 (19) — 26 (7); Abelard, 2006, 12.44].[58] Modal statements *de dicto* do not proliferate quite so generously, given the restriction just noted, but likwise generate a series of forms. Abelard is content simply to present these relationships, and has no further theory to build upon them. This absence of theoretical enhancement is also the case — and more surprisingly so — for the further relations he observes between the *de re* and *de sensu* modalities themselves. An affirmative possibility *de sensu* is said to imply the corresponding affirmative possibility *de re*, but not *vice versa* [Abelard, 1958, p. 29 (14–16); Abelard, 2006, 12.55]. By contrast, an affirmative impossibility (= a negative possibility) *de re* is said to imply the corresponding affirmative impossibility *de sensu* [Abelard, 1958, pp. 29 (25)–30 (3); Abelard, 2006, 12.56], presumably again not *vice versa*. But for necessity the implication goes both ways. An affirmative necessity *de sensu* both implies and is implied by the corresponding affirmative necessity *de re* [Abelard, 1958, pp. 32 (30)–33 (2); Abelard, 2006, 12.64]. These suggestive principles are tested against various examples, but are not pursued for the fundamental insights they might yield into the nature of the central modal concepts themselves.

These remarks concern Abelard's account of simple modality. He also has an account of determinate modality, which arises from this much-controverted Aristotelian principle: "What is necessarily is when it is, and what is not necessarily is not when it is not" [Aristotle, 1963, p. 52 (19a23)]. Something which was or is the case can be viewed as an irrevocable historical fact, and in this sense gains a necessity just by virtue of the passage of time. This is determinate necessity. The corresponding notion of possibility will be similarly distinct from simple possibility as defined purely in terms of compatibility with nature.

The phrasing of Aristotle's principle exhibits the standard way of expressing determinate modal statements: apply an adverbial while-clause (i.e., a *dum*-clause) to the modal proposition so as to specify its modal content. The purpose of the specification is to indicate an interval of time, and the modal content of the proposition is then understood with reference to this interval. So Abelard works with examples like "It is possible for Socrates to read while he sits" and "It is possible for Socrates to read while he reads" [Abelard, 1958, p. 36 (14–15); Abelard, 2006, 12.73].[59] The possibility in question (reading) is thus characterized as applying

[58] There is some question as to whether Abelard fully grasped the relationships inherent in a square of opposition for quantified modals. See [Knuuttila, 1998, p. 88], [Lagerlund, 2000, p. 37], and [Thom, 2003, p. 55]. On this point, Martin defends Abelard in [Martin, 2001, pp. 122–123].

[59] Notice the structural difference between the two forms. In the first the determining verb,

in the designated interval (sitting in the former case, reading in the latter). So characterized, it becomes a determinate modality.

Abelard considers the inferential relations between determinate and simple modalities. First, determinate possibility implies simple; "It is possible for Socrates to read while he reads" implies "It is possible for Socrates to read." But in general the converse implication, that simple possibility implies determinate, does not hold. And second, simple necessity implies determinate; "It is necessary for me to read" implies "It is necessary for me to read while I read." But again, the converse does not in general hold [Abelard, 1958, p. 36 (11–21); Abelard, 2006, 12.73]. The failure of both converses is instructive.

To say it is necessary for me to read while I read is simply to invoke Aristotle's principle and note the historical irrevocability of my reading while I am reading; for the time period designated by the clause "while I read" my reading is an irrevocable fact. But this says nothing about me and my nature. It does not say that reading is inevitable for me (or that not reading is incompatible with my nature), which is what it would need to say in order to establish the simple necessity of my reading. From this follows the failure of the converse. Determinate necessity does not imply simple necessity.[60]

More complex is the failure of the other converse. Why does simple possibility fail to imply determinate possibility? This failure is masked in the case of internal determination, where the determining verb is the same as the main one; "It is possible for Socrates to read" may well seem to imply "It is possible for Socrates to read while he reads." But certain external determinations make the failure obvious. "It is possible for Socrates to sit" does not imply "It is possible for Socrates to sit while he stands" because the determining verb ("stands") rules out that the property of sitting, with which the nature of Socrates is admittedly compatible, can be realized by him at the moment when he is standing. The lesson here is that simple possibility is temporally indefinite, and the presence of a simple possibility is no guarantee that it will be realized at a given time. The fact that Socrates is sitting at a *given time* implies the determinate necessity of his doing so then, and also the determinate impossibility of his not doing so then; this determinate impossibility holds then even though the simple possibility of sitting holds for Socrates at all times of his life.

Indeed, as Abelard elsewhere suggests, it is still possible for someone with maimed hands to box, for an amputee to walk and for a blind person to see. In each case the possibility is understood abstractly in terms of the nature of

"sits," differs from the main verb, "reads." In the second the determining and main verb are the same: "reads." The former is called an "external" determination, the latter an "internal" one [Abelard, 1958, p. 34 (9–12); Abelard, 2006, 12.67].

[60]Note that there is an assumption about the scope of the temporal determining clause in this argument, that it is to be read in this way: "Socrates possibly (sits while he stands)," not in this way: "(Socrates possibly sits) while he stands. " The former represents a modal reading of the determining clause, the latter an associated temporal reading [Abelard, 1958, p. 37 (5–23); Abelard, 2006, 12.76].

these persons, and not in terms of what their future will hold.[61] The determinate/indeterminate distinction is therefore at a considerable theoretical remove from the necessary/possible one, and is defined in entirely different terms [Lewis, 1987, pp. 82–83]. Abelard follows Boethius in construing determinacy epistemically, as kind of knowability; where Boethius speaks of what is known to nature, Abelard speaks of what is knowable (*cognoscibilis*) of itself. What is knowable of itself is perhaps best understood *in contrast with* what is knowable to God, who, because of the atemporality of the divine perspective, is able to view future and past events alike as completed happenings. What is knowable of itself is what is knowable only from a temporal perspective. Past and present events are knowable in this way, as are future events which are guaranteed to unfold in a certain way due to the natures of the things involved. "Man will die" is an example [Abelard, 1970, p. 211 (11–12)]. This does not mean that all past- and present-tensed propositions qualify as determinate, of course, because some — like "It is true that Socrates will eat" depend on future events for their truth; Abelard dwells on this example at some length, and considers it a case of indeterminacy, the present tense involved notwithstanding.[62]

So in the end, the properly modal notions are, for Abelard, tied to the natures of things, and the properly modal propositions are those which perform the basic subject/predicate operation of imputing a nature to a thing. With this approach, an orthodox subject/predicate grammar is preserved as far as possible; this serves the characteristic structures of syllogistic logic, which absolutely rely on a subject/predicate grammar. In this Abelard is true to his avowed intention of using the more basic levels of semantic analysis as a ground and preparation for his discussion of the categorical syllogism.

This discussion of syllogistic theory, while well-founded in the preceding, is brief, and innovation is to be found more in the details of doctrine than its core. Abelard's handling of the categorical syllogism essentially rehearses the traditional account of the figures and moods, and the reduction of the moods of the second and third figures to the moods of the first. A few salient points can be made here.

(i) A syllogism is defined, following Aristotle, as speech in which "when certain things are posited, it follows necessarily that something other than what is posited comes to be from them" [Abelard, 1970, p. 232 (5–6)].[63] This definition is meant to exclude various other kinds of argument, such as enthymemes and exemplification, and is meant to exclude conditional propositions. It also excludes arguments like this: "Every man is capable of laughter; but everything capable of laughter is capable of laughter; therefore every man is capable of laughter," where the reasoning is overtly circular, on the basis that a syllogistic conclusion must be

[61] This point is extensively discussed in [Martin, 2001, pp. 99–113]. See also [Knuuttila, 1993, pp. 91–92] and [Thom, 2003, pp. 50–51].

[62] This argument is covered at great length in [Lewis, 1987], which responds to the briefer analysis to be found in [Normore, 1982, pp. 361–363].

[63] This definition derives directly from the *Prior Analytics* [Aristotle, 1984a, p. 40 (24b19–20)]. "Speech" translates *oratio*, which is used here in a more general sense than the technical one already encountered.

"something other than what is posited." Abelard refers to such arguments as "absurd syllogisms" [Abelard, 1970, p. 232 (26)]. This is an ambiguous label since, strictly speaking, he has just shown that the argument is not a syllogism at all, and so, by definition, not an absurd one. The lesson Abelard could have taken from this point is that the definition itself is unsatisfactory. What is provided here is not a fully general account of the syllogism. The interest seems to be to direct attention as quickly as possible to the traditionally identified moods of syllogism.

(ii) Aristotle formulates syllogistic propositions in a manner suggestive of containment relations between shared forms: "Animal is in all of man." This places the predicate first. Abelard naturally follows the medieval practice of putting the subject first: "Every man is an animal," which presents a predicative, not a containment, relation. This has the effect of changing subject/predicate order throughout all of the propositions of all of the moods, which changes the formulations somewhat, but not the underlying theory. Abelard is aware of this shift from containment to predicative formulations, but believes that it makes no difference to the underlying theory itself [Abelard, 1970, p. 239 (21–28)].

(iii) In detailing the syllogistic moods, Abelard regularly renders O-propositions with separative negations, as in this example of a Ferio: "No good thing is evil, some just thing is good, therefore some just thing is not evil" [Abelard, 1970, p. 236 (23–25)]. This practice, of course, deviates from his stated preference for extinctive negation. Given this preference, we would expect the above conclusion to be rendered thus: "Not every just thing is evil." One can only speculate on Abelard's motives for employing the less favoured formulation. But one should recall that the basis for favouring the extinctive form in the first place is the fact that the separative form bears existential presupposition, so that if there is no just thing then "Every just thing is evil" and "Some good thing is not evil" will both be false. This failure of contradictoriness arises, of course, *only if* there is no just thing. So long as there is no failure of existential presupposition in A-propositions, there will be no failure to achieve contradictoriness with separatively-formulated O-propositions. Accordingly, so long as the domain of discourse is not lacking in referents for the subject terms of syllogistic propositions, the omission carries no disadvantage. And so long as syllogisms are employed with this presumption in place, there is no actual need to disrupt the traditional language for formulating the moods to accommodate Abelard's non-standard, extinctive formulation of O-propositions.[64]

(iv) Abelard follows the traditional practice of presenting the categorical syllogism in three figures, not four. The moods of the fourth are, with appropriate conversions, assigned to the first. This gives nine moods for the first — the four assigned "on the authority of Aristotle" [Abelard, 1970, p. 236 (17)], plus five additions. (The second and third figures are numbered at four and six moods respectively, as per tradition.) There is nonetheless some kind of partition of con-

[64]Incidentally, de Rijk observes a certain casualness to Abelard's presentation of the separative/extinctive distinction in the *Dialectica*; see [Abelard, 1970, p. lxi, note 6], on Abelard's use of the phrase *vel potius* in presenting the alternatives.

tents in the first figure, since only the first four moods can qualify as complete (*perfecti*) syllogisms, complete syllogisms being ones whose inferential properties are self-evident [Abelard, 1970, p. 236 (26–29)]. The remaining moods of the first figure, and the ones of the second and third, are reduced to these.

(v) Almost all reductions can be carried out by converting the premises or the conclusion in one of three ways (changing "No A is B" to "No B is A"; changing "Some A is B" to "Some B is A"; or changing "Every A is B" *per accidens* to "Some B is A"). For two moods — Baroco in the second figure, Bocardo in the third — the process is more involved. Abelard resorts to the traditional method of indirect proof for such cases. The Baroco syllogism, for instance, is exemplified thus: "Every just thing is good, some evil thing is not good, therefore some evil thing is not just" [Abelard, 1970, p. 240 (8–10)]. The indirect proof will proceed on the principle that if the conclusion does not follow from the premises, then its contradictory must be consistent with those premises [Abelard, 1970, p. 241 (13–15)]. The point is to show that this claim of consistency is untenable. Can there be a situation where the premises are true and the contradictory of the conclusion is true as well? Can there be a situation in which every just thing is good, some evil thing is not good and (by hypothesis) every evil thing is just? Assuming that at least one evil thing exists, the answer is no. (The assumption is needed, remember, because the conclusion is formulated with a separative negation.) Combining the first premise ("Every just thing is good") and the contradictory of the conclusion ("Every evil thing is just") gives us (by Barbara) "Every evil thing is good"; this is not consistent with second premise, which claims that some evil thing is not good. So it must be the case that the contradictory of the conclusion is false; some evil thing is not just.[65] What is interesting in all of this is that Abelard subsequently rejects this line of indirect argument as a complete justification; it establishes its point with "maximum probability," he claims, but not with necessity [Abelard, 1970, p. 244 (35–36)]. The argument, as noted, depends on the principle that if the conclusion does not follow from the premises, then its contradictory must be consistent with those premises. A reason for thinking the principle not applicable in this instance is that we are not actually dealing with contradictories. We cannot overlook the fact that the conclusion of the Baroco is given as a separative negation, not an extinctive one; the corresponding A-proposition is therefore not a contradictory (given the possibility that no evil thing exists). Similar reasoning presumably attaches to Bocardo, since it also requires an indirect argument in order to be reduced to a first figure mood and thereby justified. The result is to set both Baroco and Bocardo apart from the other moods as providing a lesser degree of inferential support for their conclusions.

(vi) The syllogistic moods are deductive inference schemes, and it is noteworthy

[65] Schematically, the *reductio* proposes these as consistent: "Every A is B," "Some C is not B" and "Every C is A." The first and third of these propositions yield, by Barbara, "Every C is B," which is not consistent with "Some C is not B." Therefore "Every C is A" is false, and "Some C is not A" is true. Since this indirect proof employs Barbara, it is considered a reduction to that syllogistic mood.

that in laying down the rules for these schemes, Abelard chooses to express them as conditionals. Here is the rule for Barbara: "If anything $\langle A \rangle$ is predicated of something $\langle B \rangle$ universally, and if something else $\langle C \rangle$ is subject to that subject $\langle B \rangle$ universally, that subject $\langle C \rangle$ is also subject to the predicate $\langle A \rangle$ in the same way, that is universally" [Abelard, 1970, p. 237 (7–9)].[66] Parallel formulations are provided for the other three standard moods of the first figure, though for none of what Abelard takes to be the next five. Such formulations are likewise provided for the first two (but not the next two) moods of the second figure and the first three (but not the next three) of the third. The omissions may simply be an economy on Abelard's part, since the omitted rules are readily supplied on analogy with the ones provided.[67] What is intriguing about the procedure here is that Abelard formulates, in addition to the three conversion rules for reducing moods, another two which license their use: "Whatever follows from the consequent also follows from the antecedent"; and "Whatever implies the antecedent also implies the consequent" [Abelard, 1970, p. 239 (9, 14)]. The care taken to provide these self-evident principles suggests a desire to display the technical rigour with which the reductions may be performed. The fact that the moods are expressed as propositions, and rules are supplied which formalize the process of reducing less basic mood-expressing rules to more basic ones, suggests an axiomatic procedure. Abelard seems to have some insight into how such a procedure is to be carried out here.

In addition to discussing the standard assertoric syllogisms, Abelard broaches the subject of syllogisms varied by mode and tense.

The comments on modal syllogisms are briefer then one might expect. But here Abelard is not able to rehearse a substantial, traditional account of doctrine, since none such has come down to him, given the unavailability of Aristotle's *Prior Analytics*.[68] His first move is simply to provide examples of modal syllogisms from each of the figures, such as this modal Barbara: "Everything just is possibly good; all virtue is just; therefore all virtue is possibly good"; this mood, Abelard notes, works just as well under necessity ("Everything just is necessarily good") as it does under possibility [Abelard, 1970, p. 245 (26-29)].[69] It will be noted that the minor premise in this example ("All virtue is just") is non-modal. The syllogism will *not* work, in Abelard's view, if the minor premise is modal, because then there will be no functioning middle term. Say we modalize the minor premise: "All virtue is possibly just." An adverbial reading of the mode is required in order to make it fit into the syllogistic format at all (as we have seen — otherwise the proposition is unable to convert). But the adverbial reading adds content to the predicate of the

[66] By interpolating letter symbols I follow the practice of [Lagerlund, 2006, p. 8]. Note that without the interpolations, this rule would be all the more inscrutable; the absence of a special notation is keenly felt here.

[67] De Rijk supplies them in his editorial introduction to the *Dialectica* [de Rijk, 1970, p. lxx].

[68] A discussion of Abelard's Boethian sources is found in [Thom, 2003, pp. 36–40].

[69] For different but similarly intentioned examples see [Abelard, 1958, pp. 10 (22) — 11 (16); Abelard, 2006, 12.15].

minor which then differentiates it from the subject of the major.[70] "Everything just is possibly good, all virtue is possibly just, therefore all virtue is possibly good" does not display the required linkage between premises, because "just" in the major and "possibly just" in the (altered) minor are not the same term. Third figure modal syllogisms must likewise contain an assertoric premise for term-sharing to occur. Second figure modals are subject to the same constraint; however, because the shared term is the predicate in both major and minor premises, it turns out that the non-modal proposition appears in the conclusion: "No evil is possibly good; everything just is possibly good; therefore nothing just is evil" [Abelard, 1970, p. 245 (31–34)].[71]

Abelard's commentary on modal syllogisms is essentially limitative, in that its major work is to present many seeming specimens of these as not truly syllogistic. He repeats this kind of commentary for syllogisms with variation in tense. These require, as do modal syllogisms, proper care to ensure that proper middle terms exist, and just as modal variation is supposed to be confined to just two of the three propositions, so with variation in tense. A syllogism like this is obviously defective: "No boy was a young man; but every old man was a boy; therefore no old man was a young man" [Abelard, 1970, p. 249 (3–5)]. The problem apparently lies with the predicate of the minor premise ("was a boy"), which refers to boys who existed in the past; but, by contrast, "boy" in the major premise follows the default semantic presumption that names refer to things existing in the present. So the name bears a different meaning in its two occurrences, referring to present boys in the major and past ones in the minor. There is accordingly no shared middle term.[72] As before, the remedy is to ensure that first figure syllogisms retain a present-tensed minor premise. Observing the constraint yields a perfectly acceptable syllogism like this: "Every man will die; every harpist is a man; therefore every harpist will die" [Abelard, 1970, p. 248 (10–12)]. This constraint is likewise applicable to third figure syllogisms, while second figure ones, as before, will admit of tense variation

[70] Say we modalize the minor premise, and then alter the subject term of the major premise to duplicate the modal content of the minor's predicate term: "Everything possibly just is possibly good." The minor, "Every virtue is possibly just" would then seem to give rise to an appropriate middle term. But (as Abelard appears to argue through another example [Abelard, 1970, pp. 246 (23)–247 (6)]) the subject term of the major must be construed consistently with the distributive function of the quantifier. "Possibly just" refers to any individual, possibly just thing, and therefore means "that which is possibly just." But if "possibly just" is to be a middle term it must bear the same meaning in the minor premise, which must then be read as "Every virtue is that which is possibly just." This reading, however, has the effect of burying the modal concept in the predicate term, so that it no longer plays the adverbial role of qualifying "is." The result is to leave the minor premise, taken whole, as non-modal again [Abelard, 1970, p. 246 (32–33)]. Thus the attempt to modalize the minor in such a way as to produce a middle term neutralizes itself, and leaves the minor back where it began, in assertoric form. See de Rijk's brief comment on this argument in [Abelard, 1970, p. lxxii].

[71] Abelard's modal syllogistic is treated in [Thom, 2003, pp. 58–63].

[72] To circumvent this problem Abelard suggests that "was a boy" might be taken as the semantic equivalent of a single word, meaning "was a past boy"; this suggestion is, however, of no use in establishing a middle, since the subject term of the major premise still reads "present boy" (i.e., "No present boy was a present young man") [Abelard, 1970, p. 249 (15–21)].

in both premises but must have a present tensed conclusion: "Every mortal will die; every mortal is a living thing; therefore some living thing will die" [Abelard, 1970, p. 248 (26-28)].[73]

While Abelard's handling of the standard syllogisms is axiomatic in orientation, his handling of ones with modal and tense variation is taxonomic rather than axiomatic; he wishes simply to identify a few kinds of non-standard syllogism, not to systematize them. And nowhere in his treatment of categorical syllogisms, standard or non-standard, does Abelard follow the deeper purposes of investigating the very nature of the inferential link itself, and how propositions come to be so linked. This project is reserved for his treatment of the hypothetical syllogism, and can indeed be viewed as his chief objective there.

PART 2: ABELARD ON TOPICS, HYPOTHETICAL PROPOSITIONS AND HYPOTHETICAL SYLLOGISMS

Just as categorical syllogisms are broached in the *Dialectica* only after prior ground-laying discussion, so with hypothetical syllogisms. The ground-laying discussion for these latter, indeed, constitutes the largest single section of that work.

It is important to grasp the procedure Abelard has in mind. The predicative relation of terms within categorical syllogisms is what makes those syllogisms work, and so we gain insight into the working of the syllogisms by means of insight into that predicative relation. But hypothetical syllogisms have terms too; they are the antecedents and consequents of the conditionals contained in the syllogisms (or the disjuncts of the corresponding disjunctions). These clause-length terms are in an *implicative relation* which needs to be studied as a preliminary for hypothetical syllogisms, just as the *predicative relation* needs to be studied as a preliminary for categorical ones. Abelard attempts to meet this need, and in so doing gives an extended account of conditionals (and a corresponding account of disjunctions).

This extended account is meant to establish a process for ascertaining when the antecedents and consequents of conditionals actually do stand in an implicative relation. The key notion in this confirmatory process is the notion of the *topic*. It is here that Abelard's expository program takes what is, from a contemporary perspective, a very unfamiliar turn. Clearly one goes nowhere in this explanatory sequence without a good sense of what a topic *is*, and so the first step is to explain it.[74]

By far the commonest traditional conception of logic is as a tool for assessing inferences. To judge whether an inference succeeds or fails requires criteria of adequacy which the inference may be seen either to meet or fail. Logic is supposed to

[73] Abelard's discussion of syllogistic inference under variation of tense is treated in [Tweedale, 1976, pp. 298–302]. See also [Rosier-Catach, 1999, pp. 150–156], [Jacobi, 2004, pp. 148–49] and [Wilks, forthcoming].

[74] Otto Bird's groundbreaking work in topical theory is an especially good point of entry into this material, even if its basic approach is flawed — as argued in [Stump, 1989, pp. 94–95]. See [Bird, 1959], [Bird, 1960, pp. 141–145] and especially [Bird, 1961].

supply these criteria. As such it serves as an *ars iudicandi*, a technique for judging. But it can also be construed as an *ars inveniendi*, a technique for inventing. Instead of just providing constraints on what counts as a successful argument, it may additionally be expected to provide guidance on how to construct a successful argument in the first place, given the desired conclusion [Boethius, 1990, p. 1 (4–5)].

The historically seminal text for logic taken in this less common, creative role is Aristotle's *Topics*. Here we find, in somewhat loosely gathered form, advice on how to advance from the yes- or the no-position on a debatable yes-or-no question to an argument for that position. This advance happens through analysis of the relation between the subject and predicate terms of the proposition which expresses that position. Propositions involving different kinds of predicative relation and different kinds of subject matter will necessitate different kinds of advice; the basic unit of advice is referred to as a topic, or, in the Latin, a *locus*. There are, of course, many topics, and the challenge of the commentators is to impose some order on this profusion of material.

Topics for Aristotle come in the form of a general instruction for argument formation, with some kind of accompanying reason for the instruction.[75] Or they come in the form of an instruction alone without the reason, or just as a reason expressed in such a way as to suggest the instruction which it justifies [Green-Pedersen, 1984, p. 21]. Boethius tries to standardize this presentation of the topics by using two key organizing conceptions: the *maxima propositio* ("maximal proposition"), and the *differentia* (distinguishing principle). I will consider these in turn.

The maximal proposition is the principle which shows the arguer how to identify a salient fact about one of the terms: a fact which reveals how that term may be put in the desired predicative relation to the other term. Here are some Boethian examples of maximal propositions. (i) Say one wishes to show that trees are not animals. This may be derived by using the following principle, i.e., maximal proposition: *What the definition of the genus does not belong to is not a species of the genus defined* [Boethius, 1990, pp. 29 (18–19)]. With this principle in view one may then note that an animal is an animate substance capable of perceiving, and further that a tree is not an animate substance capable of perceiving. The conclusion follows from the maximal proposition plus the two ensuing facts. (ii) Say one wishes to show that justice is natural. This maximal proposition will serve: *Those things whose efficient causes are natural are themselves natural* [Boethius, 1990, p. 35 (9–10)]. With this principle in view one may then note that the society of men is natural, and further that the society of men is the efficient cause of justice. The conclusion follows from the maximal proposition plus the two ensuing facts. (iii) Say one wishes to argue that the rulers of cities should not be chosen by lot. This maximal proposition will serve: *What occurs in a thing must*

[75] In [Stump, 1978, p. 167] we find a slightly different language for expressing this distinction: she speaks of "a strategy or line of argument or tack to take" as opposed to "a principle or a law that explains or justifies the strategy."

occur in what is proportional to that thing [Boethius, 1990, p. 39 (9-10)]. With this principle in view one may note that a ship's pilot is not appointed by lot, and further that the office of pilot on a ship is proportional to the office of the ruler of a city. The conclusion follows from the maximal proposition plus the two ensuing facts.

The maximal proposition is the first basic conception Boethius uses to organize topical theory; the *differentia* is the second. This latter is simply the feature which distinguishes one maximal proposition from another, just as a *differentia* in general will distinguish one species from another within a given genus. For (i) the *differentia* is "from definition"; for (ii) it is "from efficient cause"; and for (iii) it is "from proportion." Just as the maximal proposition helps in locating a salient point about one of the terms out of which the argument may be developed, the *differentia* helps in locating the right maximal proposition. The maximal proposition is a principle which distills the essence of the argument; the *differentia* is a label which distills the essence of the principle.[76]

With these two notions Boethius purports to impose some order upon the variety of topics. These notions represent the point of departure for Abelard in his own handling of topical theory, but, as ever, he adapts the inheritance to serve his own purposes. Two points in particular inform his approach.

(i) Topical theory is first and foremost for Abelard an *ars iudicandi*, a technique for judging adequacy of reasoning. This is not to say that he rejects the other orientation of topical theory, as an *ars inveniendi*; in his initial definitions, indeed, Abelard covers both conceptions of a topic. He allows that it can be defined as an "argument source" (*argumenti sedes*) and as "that from which a suitable argument is drawn" [Abelard, 1970, p. 253 (22-23)].[77] But he also identifies this definition as a narrower one, and adopts what he considers the broader one as his starting point: a topic is the force of an entailment (*vis inferentiae*) [Abelard, 1970, p. 253 (16)], or, as we might say, an inferential warrant, a consideration which ensures that there is entailment. Of course, these narrower and broader definitions are not in conflict, since what warrants an entailment might very well be helpful for discovering it in the first place. Nonetheless Abelard's program here focuses on the topic taken as an inferential warrant. Topical theory is going to be used chiefly for the purposes of assessment, not discovery.

(ii) An interpretative tendency evident in writings from as early as the ninth century is to regard topical theory as having a special association with hypothetical syllogisms [Green-Pedersen, 1984, pp. 140–145].[78] Abelard is much influenced

[76] As Eleonore Stump notes [Stump, 1978, p. 180, note 5] this use of the term "differentia" must be distinguished from its use in denoting one of the predicables.

[77] Abelard associates this approach to topical material with Cicero and Themistius, and treats it in a later section of the *Dialectica* [Abelard, 1970, pp. 414 (4) - 466 (25)]. This is briefer coverage than is given to the other approach, which runs to 261 pages in the de Rijk edition. From this fact we may infer the greater value he imputes to the broader definition.

[78] Niels Jørgen Green-Pedersen believes that "this idea is both un-Aristotelian and un-Boethian and un-Ciceronian" [Green-Pedersen, 1987, p. 411]. However we find in [Martin, 1991, p. 290] an indication of a Boethian source; this lies in the fact that, for Boethius, maximal propositions

by this tendency. He studies this theory as the key element in his account of conditionals. This account, of course, focuses on the inferential linkage between antecedent and consequent, and that linkage, in Abelard's view, should be explained in terms of the topics. It is thus that we will learn, of a significant class of conditionals, which ones are true and why they are so. We have, then, a marked re-orientation of the original theory of the topics. Instead of supplying arguments it is set to the task of confirming conditionals.

Here is how it does so. Consider a very simple conditional like "If it is a man then it is an animal." To explain how the antecedent entails the consequent we look for a logical relationship which a term in the antecedent bears to a term in the consequent. This is easily done in the example. The term in question from the antecedent is obviously man. The corresponding term from the consequent is animal. So what is the logical relationship that man bears to animal? Again the answer is easy. The relation is that of being a species. With this we now have enough to identify the topical *differentia*. This *differentia* is the term of the antecedent which has been identified as key to the entailment — in this case man. But it is not the term taken *per se*. It is the term taken as standing in the logical relationship which has also been identified as key to the entailment [Green-Pedersen, 1984, pp. 166–7]. So in this case the *differentia* is man taken as standing in the relationship of species to another term.

That is the first step. The second then identifies and invokes a maximal proposition. The identification is very much helped by knowing the topical *differentia* but is not made automatic by that knowledge. For every *differentia* there is a multiplicity of maximal propositions, only one of which will be appropriate to the case at hand. Which one this is must be discerned. In the case of "If it is a man then it is an animal" the proposition is "Whatever the species is predicated of, the genus is predicated of too." With this proposition in hand the entailment is confirmed, because we know that animal is the genus of man, and so we know that the conditional in question instantiates the maximal proposition. "Whatever the species is predicated of, the genus is predicated of too" can be instantiated to "Whatever man is predicated of, animal is predicated of too," which is just another way of saying, "If it is a man it is an animal." A maximal proposition has the same self-evident truth as the rule for the Barbara syllogism; a conditional which instantiates a maximal proposition is likewise demonstrably true, just as an argument which instantiates the rule for Barbara is demonstrably valid. We have found a maximal proposition which "If is it is a man then it is an animal" instantiates, and can therefore consider that conditional to be confirmed.

Thus function all the topics. At this point one might say, "There are two things here, the topical *differentia* and the maximal proposition. Which one is actually the topic?" In Abelard's view, both are, presumably because in the sort of analysis just conducted the two are used in tandem [Abelard, 1970, p. 263 (35–37)]. Nonetheless he will from time to time refer to the *differentia* alone as the topic [Abelard, 1970, p. lxxxii (14.051)].

"appear as principles which may be cited in support of conditionals."

He supplies explicit definitions for both. The topical *differentia* is defined as "that thing (*res*) in whose relation to another thing the strength of the entailment resides" [Abelard, 1970, p. 263 (7–8)]. Since he speaks of a thing, not a word, we must understand the *differentia* in the above example as man, not "man" — or, more precisely, as man taken as a species, not "man" taken as a species. Abelard has presumably resolved to his satisfaction the metaphysical problems lurking in this use of general terms, so these need not detain us. Considering man's relation to animal prompts our taking man as a species, at which point the *differentia* is clear to view, and the maximal proposition on the way. It all starts with the relation of man to animal, and so it is in this relation that "the strength of entailment resides."

The account of the maximal proposition is more involved. Abelard defines it as "that proposition which, containing the sense of many entailments, reveals a common method of proof which the *differentiae* of those entailments possess — owing to their having the force of the same relation" [Abelard, 1970, p. 263 (12–14)].[79] This is a densely packed formulation. The key feature of the above maximal proposition ("Whatever the species is predicated of the genus is predicated too") is its standing as a general expression of a whole series of entailments: "If it is a man it is an animal," "If it is a pearl it is a stone," "If it is a rose it is a flower," "If it is red it is a colour," and so on. These entailments have different terms (man and animal, pearl and stone, and so on), but are alike in that the terms are all related as species and genus. We can see each entailment as arising through the substitution of similarly related terms into the maximal proposition, which in this manner can be seen as "containing" the sense of each. This is why the maximal proposition plays a role in confirming each through a "common method of proof." This, as we have seen, starts by construing terms under the guise of the appropriate relation: man taken as species, pearl taken as species, rose taken as species, red taken as species, and so on. So construed they become the *differentiae*, each of which gives rise to a method of proving the conditional from which they arise — and it is the same method in each case. The maximal proposition reveals what this shared and common method is.[80]

This is the sense in which maximal propositions play for conditionals the role that syllogistic rules play for syllogisms: the role of reliably identifying which forms mark genuine entailments. The rule for Barbara ("If anything $\langle A \rangle$ is predicated of something $\langle B \rangle$ universally, and if something else $\langle C \rangle$ is subject to that subject $\langle B \rangle$ universally, that subject $\langle C \rangle$ is also subject to the predicate $\langle A \rangle$ in the same way, that is universally" [Abelard, 1970, p. 237 (7–9)]) confirms as genuine entailments all of those syllogisms which arise from uniform substitution of terms in its open places (marked by "A", "B", and "C".) The rule "Whatever the species is predicated of the genus is predicated too" works in much the same way to con-

[79] On this passage see [Martin, 2004, pp. 174–175].

[80] Note that this account of the essentials of Abelard's topical theory is based on texts from the *Dialectica*. Certain aspects of the account change in his *Super Topica glossae* [Abelard, 1969b]. For a summary of changes see [Green-Pedersen, 1977, pp. 135–137].

firm as genuine entailments all of those conditionals which arise from substitution in its open places — except that the substituting terms must be further delimited as standing in the topical relation of species to genus.[81] The topical rule is more qualified than the syllogistic, but just as confirmatory. To understand this is to understand why Abelard regards topical theory as such an important area of research. Given that the basic rules of form for syllogistic entailments already appear to be discovered and fully expressed, it is natural to seek next the discovery and full expression of rules for conditional entailments.

In support of this program, Abelard tightens the analogy between syllogistic and conditional entailments in two ways.

(i) He distinguishes these entailments as *perfectae* and *imperfectae* respectively. *Imperfectae* means "imperfect" in the sense of "incomplete," not in the sense of "defective."[82] A Barbara syllogism gives us a complete entailment, while "If it is a man then it is an animal" gives us an incomplete one. The point of the distinction is not to mark the quality of the entailments, because both involve the closest possible inferential links. The point is to mark the quality of guarantee for that link. The explicitness of a syllogism is such that we do not even have to pay particular attention to the content of its propositions in order to grasp the fact that the inference succeeds. We need only note the pattern of the terms. We can, indeed, make arbitrary substitutions for the terms, but still be assured that that the resulting entailment succeeds, as in: "If every man is a stone and every stone is wood then every man is wood" [Abelard, 1970, p. 255 (35)]. "If it is a man then it is an animal" cannot be so simply confirmed as this, even though, as we have seen, it can be confirmed. Hence it is an entailment properly speaking, just an incomplete one. Note that the compete/incomplete distinction is traditionally applied within the domain of syllogisms themselves [Abelard, 1970, pp. 233 (35)–234 ((8)]. First figure syllogisms corresponding to the first four moods are complete, in that they are self-evidently valid; all other moods are incomplete in that, while valid, they are not self-evidently so. They require reduction to first-figure ones in order to be shown to be so. This is the sort of distinction being made between syllogistic and conditional entailments. The latter fail in obviousness, not in strength, and need additional means to have obviousness conferred upon them. These means are supplied by topical theory. Calling conditional entailments "incomplete" presents their difference from complete ones as a difference of presentation, not content, and the label thus suggests the underlying likeness of the two.[83]

(ii) Further to this suggestion is Abelard's account of the inferential link itself, which he applies indifferently to both kinds of entailment. He identifies *necessity* as

[81] As Christopher Martin notes [Martin, 1986, p. 566], topical entailments generally will hold for "uniform substitution of terms standing in an appropriate topical relationship." These contrast with syllogistic (i.e., complete) entailments, which hold for "any uniform substitution of terms." See also [Martin, 1992, p. 116].

[82] I will translate *perfecta* as "complete" consistently with my earlier rendering of *oratio perfecta* as "complete phrase." *Imperfecta* is accordingly rendered as "incomplete."

[83] On this distinction see the discussion in [Martin, 2004a, pp. 169–171], under the heading "Perfect and Imperfect Entailment."

essential to entailment: what follows must do so necessarily, so that what it follows from simply cannot be the case without it. But for his purposes this is not enough, and some pages later he adds a further condition on entailment: "There seem to be two necessities of following (*consecutio*), one broader, when what the antecedent says cannot be without what the consequent says, and the other narrower, when the antecedent not only cannot be true without the consequent but of itself compels (*ex se exigit*) the consequent" [Abelard, 1970, pp. 283 (37)–284 (2)]. The verb *exigit* does not admit of an automatic translation, and has elsewhere been rendered as "compels" or "requires."[84] The intended meaning is strongly influenced by the associated phrase "of itself" (*ex se*); the idea is that the content of the consequent is already internal to the antecedent, and the statement of the whole conditional merely exposes this relationship by drawing — or, taking *exigere* literally, driving or forcing — that content out. Abelard's example of a conditional which has this more exacting kind of necessity is "If it is a man it is an animal" [Abelard, 1970, p. 284 (5)]; the conditional has this necessity, Abelard says, because "animal is contained in (*contineatur in*) the substance of man" [Abelard, 1970, p. 284 (7)]. This relationship of nature (being a man, being an animal) creates a semantic relationship between antecedent and consequent whereby the meaning of the latter can itself be said to be contained in the meaning of the former.[85] This sort of containment is clearly present in syllogistic entailments, where the relationship of terms in the conclusion is already present in the relationships stated by the premises. By locating it in conditional entailments as well, Abelard again suggests the underlying likeness of the two.

In pursuing his program of identifying maximal propositions which confirm conditional entailments, Abelard is as uncompromising as Aristotle is in identifying syllogistic rules. Aristotle ends up with a very reduced classification of acceptable syllogisms, and Abelard likewise ends up with a very reduced classification of acceptable conditionals. Given that, for his purposes, a true conditional must express an entailment as strong as the entailment present in a valid syllogistic mood, it stands to reason that he will identify as false many conditionals which otherwise seem true. This indeed happens. Examples of rejected conditionals are: "If it is a man it is not a stone," and "If there is paternity then there is filiation" [Abelard, p. 284 (22–23)].[86] Being a man and not being a stone are linked in that something that is a man is necessarily not a stone. But they are not linked in the stronger sense that specifying the nature of being a man must involve specifying not being a stone; in general, one does not direct attention at what something is by directing attention at what it is not. So there is no relation of containment

[84]We find the former translation in [Stump, 1989, p. 91] and the latter in [Martin, 1987a, p. 392; Martin 1987b, p. 430; Martin 1992, p. 115; Martin 2004a, p. 181].

[85]Christopher Martin discusses this notion of containment in several places, and consistently emphasizes its importance to Abelard's theory of entailment: [Martin, 1986, p. 567; Martin, 1987a, pp. 387–388, 392–393; Martin, 1987b, pp. 423–424; Martin 1992, pp. 115–116; and especially Martin, 2004a, pp. 176, pp. 180–186].

[86]The second example is somewhat literally translated. It means, "If something is in the relation of being a father, then something is in the relation of being a son."

between the antecedent, "It is a man," and the consequent, "It is not a stone."[87] A similar analysis applies to "If there is paternity then there is filiation." Being a father is a relation of having a child but not a relation of being a child, so we do not direct attention to the state of being a child by considering the state of being a father. The state of being a child is not internal to the state of being a father, and so in that sense the latter does not contain the former.[88] In both of the above conditionals there is enough linkage to meet the necessity condition, but not enough to meet the containment one.

This is certainly an uncompromising result. Normally moving in the direction of relevance implication makes an account of conditionals more intuitive (even if at the cost of making it also formally more intractable); but Abelard moves *so* far in this direction, and is so restrictive in his tolerance for what will count as an acceptable conditional, that he risks loss of intuitiveness through being *too* relevantist. One must, however, remember the program: to find rules for conditional entailments that are the equal for certitude with the rules for the syllogistic entailments. In the context of this program Abelard's restrictions make sense. The visible result of these restrictions is that his examples of successful conditional entailments are typically characterized by a shared subject term in antecedent and consequent, as in "If it is a man it is an animal," or schematically, "If x is A then x is B."[89] Whether a conditional of this form turns out to be an entailment will depend on whether A and B are in the requisite relation of containment. If they are, the fact will be established through appeal to the relevant topic.

We can say generally, then, that topics do for incomplete entailments what logical form does for complete ones. But the fact that maximal propositions are indexed to things, and the relationships in which they stand, makes it evident that they are decidedly not formal argument schemata, but semantic principles which themselves require knowledge of natures in order to be grasped. The logician does not study physics (i.e., the philosophy of nature) in order to verify the stated claims of logic, but needs to have some grasp of it nonetheless to understand what is meant by particular words. For this program to be successfully carried out "the property of things must first be investigated" [Abelard, 1970, p. 286 (37)]. Maximal propositions represent the final fruit of this investigation. They are fundamental principles expressive of how natures are. Our knowledge of them is without further basis, and no other such principles are to be discovered which are better known [Abelard, 1970, p. 309 (30–36)].

The many entailments which a maximal proposition yields are themselves expressive of basic relationships of natures, and in fact hold true even in the absence

[87] See [Martin, 2004a, p. 184].

[88] This analysis is further affected by the way in which Abelard understands the category of relation, which, we may say very generally, tends to follow Aristotelian tradition by analysing it more along the lines of a one-place predicate than we are apt to do in modern times. See [Brower, 1998]. See also [Marenbon, 1997a, pp. 143–146] and [King, 2004, pp. 96–98].

[89] It is possible, however, for antecedent and consequent to share the predicate term instead. See note 92 below for an example.

of things which manifest those natures. Abelard distinguishes between the import of a universal affirmative categorical like "All men are mortal" and the corresponding conditional, "If it is a man then it is mortal." The key linking word in the former is the copula "is" (*est*), which "proposes only an act of inherence of things" [Abelard, 1970, p. 279 (9-10)]; but the key linking word in the latter is the connective "if" (*si*), which "proposes necessity of entailment" [Abelard, 1970, p. 279 (13)]. The categorical asserts inherence, and fails to be true if there is no inherence — which is what happens if the terms fail to denote any existing thing. So it is false that all men are mortal if it turns out that there are no men. But the conditional asserts entailment, not inherence, and this entailment will hold whether the terms succeed in denoting or not. "All true conditionals are true from eternity" [Abelard, 1970, p. 279 (18)], Abelard says. Their truth depends only upon facts about natures, not facts about the things which instantiate them. For understanding Abelard's program in constructing a theory of incomplete entailment, the point is clarifying. Incomplete entailment, as we have seen, is as grounded in necessity as complete entailment; what confers that necessity is not a specific argument form but timeless relationships that hold between natures. In defining the domain of incomplete entailments, and attempting to provide maximal propositions as the foundational principles of this domain, Abelard is therefore undertaking a program of considerable philosophical import.[90]

In this program he naturally proposes to whittle down the received list of topics to ones which yield maximal propositions of this sort — ones which imply families of timelessly true conditionals, marked as such by the fact that in each case the sense of the consequent is contained in the sense of its antecedent. The strategy for proceeding with the program is to identify and reject those maximal propositions which in fact give rise to conditionals lacking the requisite properties. This suffices to demonstrate their inadequacy.

The first set of topics he subjects to this scrutiny are the ones listed as topics "from substance," under which heading are grouped various principles relating definitions to the things they define. Here is the first: "Whatever the definition is predicated of, the thing defined is predicated of as well" [Abelard, 1970, p. 331 (23)]. One of the conditionals contained under this would-be maximal proposition is "If Socrates is a rational, mortal animal then he is a man" [Abelard, 1970, p. 331 (31)]. This conditional seems unobjectionable at first sight, but Abelard argues that it is not necessarily true. More is involved in being a man, he argues, than simply being rational and mortal; what qualifies as a man is also able to walk, is two-footed, and shares in learning (*perceptibile discipline*), and there are, indeed, many other such properties involved in being a man "for which we do not have names" [Abelard, 1970, p. 332 (12–13)]. We do not need more than "rational" and "mortal" for the purposes of definition because these suffice to provide an

[90] This program is the interpretative key for grasping the considerable material on topical theory in the *Dialectica* — material which otherwise can seem disjoint and directionless. Christopher Martin has been chiefly responsible for our knowledge of this interpretative key. See especially [Martin 1987a], [Martin 1987b], [Martin 1992] and [Martin 2004a].

(extensionally) "equal definition" [Abelard, p. 333 (21)]. But this definition has such equality with the thing defined only as a matter of fact (*secundum actum rei*), not according to nature (*secundum naturam*), since although no other rational and mortal animals are actually to be found except men, "nature does not seem to require this" [Abelard, p. 333 (32–33)]. In short, the above conditional ("If Socrates is a rational, mortal animal then he is a man") lacks the timeless necessity needed in order for a conditional to be true. This failing is based on the fact that the requisite containment relation between consequent and antecedent does not exist; in particular, the meaning of "rational, mortal animal" does not contain the meaning of "man." So the above definitional principle fails to have a place in topical theory as a maximal proposition. By parity of reasoning so do a number of others: "From whatever the definition is removed, so is the thing defined"; "Whatever is predicated of the definition is also predicated of the thing defined" and "Whatever is removed from the definition is removed from the thing defined" [Abelard, 1970, p. 331 (23-29)]. On the other hand, the nature of the thing defined will be inclusive of whatever properties are appealed to in its definition; so if we take the first of the above would-be maximal propositions and put "definition" for "thing defined," and vice versa, we will acquire a perfectly good maximal proposition: "Whatever the thing defined is predicated of, so is the definition" [Abelard, 1970, p. 338 (4–5)].[91]

Abelard works through a traditional list of maximal propositions and in similar fashion drops many entries from the list, making decisions in this regard which are often at the outset non-obvious, even counter-intuitive. I append a few additional examples, which must suffice to indicate the character of this very extensive undertaking. (i) Under the topic "from genus," Abelard accepts "Whatever the genus is removed from so is the species" and "Whatever does not agree with the genus does not agree with the species." He rejects "Whatever is predicated of the genus in content (*de contento*) is also predicated of the species" and "Whatever the genus taken universally is predicated of, any of its species is predicated of" [Abelard, 1970, p. 340 (5–19)].[92] (ii) Under the topic "from an integral whole" Abelard rejects inferences from whole to part. The maximal proposition "When the whole exists it is necessary that any part of it exist" endorses as true the conditional "If there is a house there is a wall" [Abelard, 1970, p. 343 (34–36)]. But this conditional is false. It is granted that the house as a whole cannot exist without roof, wall and foundation. But saying that the house as a whole exists is simply not the same as saying that any one of those three components exists. The existence claims for the components are not part of the content of the existence claim for the whole — which means that "If there is a house there is a

[91] The topic "from substance" is discussed in [Martin, 2004a, pp. 186–188].

[92] The reasoning behind these results is given in [Stump, 1989, pp. 98–100]. The first (accepted) maximal proposition is exemplified with this true conditional: "If this rock is not an animal it is not a man" [Abelard, 1970, p. 340 (11)]. The second (accepted) maximal proposition is exemplified thus: "If no animal is a stone, no man is a stone" [Abelard, p. 340 (12)]. For the idea of predicating a genus *de contento*, see de Rijk's comment in [Abelard, 1970, p. lxxxiv, note 3].

wall" fails the containment condition for entailment, even though it passes the necessity test [Abelard, 1970, p. 346 (18–28)]. Abelard also argues that attempts to re-formulate the maximal proposition for this topic in such a way as to make it acceptable fail; they fail because they yield variants of the proposition which now fall under a different topic. One such variant is "Whatever wholly agrees with the whole, that is, according to its individual parts, agrees with each one of those parts"; but this variant licenses universal-particular inferences, not whole-part ones, and hence does not qualify as a topic "from an integral whole." Abelard draws the same conclusion of "If anything is predicated of the whole it is also predicated to all the parts of it taken together" [Abelard, 1970, p. 344 (6–24)].[93] (iii) Under the topic "from an equal" Abelard accepts four maximal propositions for what he calls "equality of predication" (that is, co-extensionality): "Whatever one equal is predicated of, the other is too"; "From whatever one equal is removed, the other is too"; "Whatever is predicated of one equal is predicated of the other"; and "Whatever is removed from one equal is removed from the other" [Abelard, 1970, p. 349 (28–32)]. He also accepts two maximal propositions for what he calls "equality of inference," where in this case what are called "equals" are a given conditional and its contrapositive: "When one equal is posited, the other is too"; "When one equal is denied the other is too" [Abelard, 1970, p. 351 (31-32)]. These latter two maximal propositions simply affirm the equivalence of contrapositives.[94] (iv) Under the topic "from antecedent or consequent" Abelard considers the status of various basic rules of propositional logic, such as "If the antecedent is posited then the consequent is posited" [Abelard, 1970, p. 365 (18)]. While not denying the truth of these, he argues that they are not genuine maximal propositions, and so should be re-expressed in terms of other propositions which are. For example, the rule just cited has no explanatory power, he argues, because it does not actually *prove* the truth of conditional; to apply the rule at all in a given case one already needs to accept one proposition as antecedent and the other as consequent — in which case one has already in effect accepted the truth of the conditional the rule is supposed to be justifying [Abelard, 1970, p. 365 (24–26)]. What actually demonstrates the relation between antecedent and consequent, he says, is consideration of how species relates to genus; he therefore refers the reader to the topic "from species" [Abelard, p. 365 (27–28)]. By this sort of reasoning all of the principles associated with the topic "from antecedent or consequent" are rejected as maximal propositions.[95]

The above gives the barest hint of the intricacy of Abelard's argumentation in this section.[96] He takes up a long series of what we would call logical and semantic principles and accepts or rejects these, sometimes after painstaking re-formulation,

[93] The topic "from the integral whole" is discussed in [Martin, 2004a, pp. 188–189] and [Stump, 1989, pp. 100–101].

[94] The topic "from an equal" is discussed in [Stump, 1989, pp. 103-106].

[95] The topic "from antecedent or consequent" is discussed in [Stump, 1989, pp. 106–107].

[96] A summary of maximal principles is provided by de Rijk in [Abelard, 1970, pp. lxxxiii–lxxxix]. For a comment on the general organization of topical theory into three divisions (intrinsic, extrinsic and mediate) see [Martin, 2004a, p. 198, note 65].

as appropriate for topical use. Rejection, as noted, can happen because the principle, while true, does not play the particular justifying role expected of a given topic. Or it can happen because the principle gives rise to false conditionals (such as "If Socrates is a rational, mortal animal then he is a man"). Or it can happen not by direct reasoning in this way but by indirect reasoning, where Abelard shows that justificatory use of a proposed maximal proposition leads to contradiction. This is the form the argument takes when he rejects all the proposed maximal propositions for the topics "from opposites" and "from immediates." And it is here that we find what has so far emerged as perhaps the most interesting of all the sections in Abelard's discussion of topical theory.[97] Here, in order to derive the perceived contradiction, he articulates insights that stand at the very heart of his relevance logic.

Opposites (white, black) are to contraries as immediates (white, non-white) are to contradictories. If one opposite is present then the other must be absent, and likewise with immediates. But the absence of one opposite does not imply the presence of the other since they may both be absent. By contrast, the absence of one immediate *does* imply the presence of the other. So a conditional licensed by the topic "from opposites" is "If it is a man it is not horse" [Abelard, 1970, p. 393 (19)], since nothing can be both at the same time, but something can be neither.[98] A conditional licensed by the topic "from immediates" is "If it is not healthy it is sick," since, presumably, anything which can meaningfully and truthfully be said not to be the one must be said to be the other. Abelard's rejection of both topics proceeds similarly, so I will focus on the one "from opposites."

The principle common to the maximal propositions falling under this topic is that "when the one is posited, the other is removed" [Abelard, 1970, p. 393 (6)]. Consider the conditional, "If Socrates is a man, he is not a stone" [Abelard, 1970, p. 395 (10)]. Being a man and being a stone are in opposition to each other since nothing can be both, while many things are neither. So the topic "from opposites" tells us that the conditional is true; positing man entails removing stone. But note that the same topic also tells us that this conditional is true: "If Socrates is a stone he is not a man" [Abelard, 1970, p. 395 (15)], for the same reason as before (i.e., nothing can be both a stone and a man). So we have what are presumed to be two true conditionals: "If Socrates is a man he is not a stone" and "If Socrates is a stone he is not a man." Abelard now makes one conditional out of the two by conjoining their antecedents into one antecedent, and conjoining their consequents into one consequent: "If Socrates is a man and a stone, Socrates is not a man and a stone" [Abelard, 1970, p. 395 (17)]. This move is intuitive and logically unobjectionable. Its product may be formalized thus: $(p\&q) \supset \sim (p\&q)$. But this represents a logical form which Abelard regards as self-contradicting, so he

[97] The significance of Abelard's treatment of these topics was first brought to light by Christopher Martin in [Martin, 1986, pp. 569–570], [Martin, 1987a, pp. 386–392], and [Martin, 1987b, pp. 432–433]. Further treatments are found in [Martin, 1992, pp. 117–122] and [Martin, 2004a, pp. 189–191]. My own account here is based on these sources.

[98] The corresponding maximal proposition is: "If one opposite is predicated of something, then the other opposite is removed from that same thing" [Abelard, 1970, p. 393 (24–25)].

concludes that the topic from opposites should be rejected.

Why does he regard that logical form as self-contradicting? $[(p\&q) \supset \sim (p\&q)]$ is a substitution instance of the simpler formula $(p \supset \sim p)$. Abelard takes it as self-evident that conditionals of this form are false, because, he believes, no proposition can imply its own negation. Of course it makes sense that he should think this, given his understanding of what counts as a true conditional. If the meaning of the consequent must be contained in the meaning of the antecedent in order for the conditional to be true, then any case where the consequent simply denies the antecedent will represent the most obvious case of non-containment, and so the most obvious case of a false conditional. The form $(p \supset \sim p)$ represents a direct challenge to Abelard's way of thinking about the conditional, and he naturally regards any move in argument which gives rise to a conditional of that form as fallacious. Rejecting this form (a pre-requisite for giving it the diagnostic role that Abelard does) is in fact typical of the approach to relevance logic now known as "connexivism"; a system of connexivist logic is characterized by its not accepting this formula as true.[99] Needless to say, any such system represents a substantial departure from familiar systems of two-valued propositional logic; these interpret a case where a proposition implies its own negation as simply deriving from the fact that it is false. This connexivist principle is derived from Boethius, who himself extracts it from Aristotle's *Prior Analytics*: "It is impossible that the same thing should be necessitated by the being and by the not-being of the same thing" [Aristotle, 1984a, p. 91 (57b3-4)] — in other words, no proposition can be implied both by another and by the negation of that other.[100] The corollary of this, of course, is that no proposition can be implied by its own negation. It is clear from contemporary testimony that the acceptance of the connexivist principle was regarded as an important issue [Martin, 1987a, pp. 378–379], and indeed, as we shall see in the next section, it was to become a major point of contention between Abelard and others.

The upshot of the discussion of the topics from opposites and immediates is that they yield no maximal propositions and serve to confirm no entailments. The kind of entailments they would confirm if this result were otherwise would be ones where one of the clauses of the conditional posited its predicate and the other clause removed its predicate. In other words, they would confirm conditionals of mixed quality, with an affirmative antecedent and negative consequent, or vice versa. The implication of Abelard's result is that no such conditionals will be confirmed by topical theory, and that the sorts of conditional entailments that will be confirmed will be jointly composed out of affirmative clauses or jointly composed out of negative ones.[101] Presumably these limitations establish the

[99] For the notion of connexivism see [McCall, 1966]; this source is cited in [Martin, 1987a, p. 381, note 10].

[100] The origin of this principle is discussed in [Martin, 1987a, p. 378–379]. See also [McCall, 1966, pp. 415–416].

[101] Martin has recently suggested a constraint on Abelard's rejection of mixed quality conditionals as successful entailments: that this rejection applies only in cases where the negated clause is subject to extinctive negation, not separative [Martin, 2004a, p. 196, note 48].

conditions under which containment is able to happen: something's being B is able to be contained by that same thing's being A, or its not being B is able to be contained by its not being A. As established earlier, whether a conditional of this form turns out to be an entailment will depend on whether A and B are in the requisite relation of containment. If they are, the fact will be established through appeal to the relevant topic.

Note that being an entailment in the sense intended here is an exacting status for a conditional to achieve, and not achieving it does not by itself completely strip a conditional of cogency. Compare the standing of Baroco and Bocardo syllogisms, which, as we have seen, fail to reduce directly to a first figure syllogism, and therefore attain only "maximum probability" [Abelard, 1970, p. 244 (35–36)]. This assessment obviously lowers their inferential cogency relative to the other accepted syllogistic moods, but for all that they still retain the capacity to confer maximum probability upon their conclusions, and still figure in the list of categorical syllogisms. Abelard rejects conditionals of mixed quality in the sense that he refuses to accord them the same inferential cogency as he accords the accepted syllogistic forms — but this is no bar to his continuing to regard propositional forms like $(p \supset \sim q)$ and $(\sim p \supset q)$ as worthy objects of study in a propositional logic. And as we turn to his propositional logic we certainly encounter discussion of these forms and the equivalences they bear to other propositional forms. There should be no thought that Abelard is being inconsistent in this.

At the outset of this section on topical theory I cited Abelard's pretext for discussing topical theory before discussing the hypothetical syllogisms; the point, he suggests, is to present an account of the meaning of conditionals prior to studying how such propositions are employed in hypothetical syllogisms. He certainly achieves this goal (while at the same time saying a great deal about topical theory whose worth exists independently of this purpose). When we turn to the section on the hypothetical syllogism itself we are confronted with a treatment that is not extensively elaborated, and indeed not that much longer than what is provided for the categorical syllogism.

When Abelard speaks of hypotheticals he has a notion in mind more general than conditionals, since it also includes disjunctives. His way of distinguishing between conditionals and disjunctives is revealing. Just as a categorical proposition can be divided in three, so can a hypothetical: in case of the categorical, two terms and the copula that joins them; in the case of the hypothetical, two propositions and the conjunction (*coniunctio*) that joins them [Abelard, 1970, p. 472 (1–6)]. Conditional hypotheticals differ from disjunctive ones by virtue of the nature of the conjunction; in the former case we have a conjunctive conjunction (*coniunctiva coniunctio*) and in the latter case a disjunctive one (*disiunctiva coniunctio*) [Abelard, 1970, p. 472 (8-14)]. Of course it is tempting to translate *coniunctio* as "connective" rather than "conjunction," and doing so would suggest a way of speaking more in line with contemporary practice. But the fact that Abelard speaks of the conditional and disjunctive connectives as two kinds of *conjunction*

suggests a way of understanding them which poses an impediment to grasping the truth of the matter — which is, of course, that implication and disjunction are correlative with conjunction, as opposed to being *kinds* of conjunction. Abelard's work on hypotheticals manifests much nascent insight into propositional logic, but also some inherited confusion. He is able to make some useful observations about the conditional and disjunctive connectives, and how they relate to the negative connective. But for this to be a discussion of propositional logic in the fullest sense there needs to be some notion of a connective for conjunction as well. What there is instead is a way of speaking which can only impede the development of such a notion.

There is to be found here, nonetheless, a rich discussion of the interrelationship of conditionals, disjunctions and negations, in which Abelard's sensitivity to the difference between wide-scope and narrow-scope negation plays an important role. Just as a categorical proposition can be negated as a single unit by application of wide-scope negation, so with a hypothetical, which counts as affirmative even if one or other of its clauses happens to be negated: "Whatever hypotheticals are asserted — whether out of two affirmatives, such as this: 'If it is a man it is an animal'; or out of two negatives, such as this: 'If it is not an animal, it is not a man'; or out of an affirmative and a negative or vice versa, such as this: 'If it is night it is not day' — they all seem to assert an affirmative meaning of entailment" [Abelard, 1970, p. 476 (8–15)]. The narrow-scope negation affecting only the consequent in "If it is a man it is not an animal" does not negate the hypothetical as a whole, which still affirmatively expresses a relation of following between antecedent and consequent. If the narrow-scope negation *did* do this then one would not be able to say that a relation of following is being expressed here.

Besides achieving insight into how a conditional is negated, Abelard also discusses at length how it is related to disjunction. He receives from the Boethian tradition the foundational insight that $(\sim p \vee q)$ is equipollent with $(p \supset q)$: "If the conjunctive <hypothetical> is composed of two affirmatives, the disjunctive one will consist of a negative and an affirmative" [Abelard, 1970, p. 488 (29–31)].[102] This means that what he says about the disjunctive relation must be carefully aligned with what he has already said about the conditional relation. This is a good result to arrive at, of course, but it is complicated by his relevance approach to understanding implication.

He argues against defining a disjunction truth-functionally, either inclusively as meaning that *at least one* of the disjuncts is true, or exclusively as meaning that *only one* of them is true.[103] The problem with these definitions is brought out in the example, "Either I am an animal or I am an ass" [Abelard, 1970, p. 490 (24)]. Note that this proposition is true on both definitions; one of the disjuncts is true ("I am an animal"), and in fact *only one* of them is true (since "I am an ass" is false). So taken either inclusively or exclusively, the disjunction is true. Now by the above equipollence the disjunction can be re-written as this conditional: "If I

[102]Reading *si* for *is* in line 29.

[103]On this point see the brief commentary in [Kneale and Kneale, 1962, p. 222].

am not an animal, I am an ass" [Abelard, 1970, p. 490 (28)]. But my being an ass implies my being an animal, so the conditional intuitively implies this one, whose consequent is changed accordingly: "If I am not an animal then I am an animal" [Abelard, 1970, p. 490 (30)]. This is the very sort of conditional which Abelard's connexivism forbids, so it, and the above definitions of disjunction which permit, it must be rejected. So must another definition, again in terms of truth, which takes disjunction as meaning that one of its disjuncts is *necessarily* true. This analysis may work for "Either I am not an ass or I am a man" [Abelard, 1970, p. 490 (33)], where the disjuncts can be seen as advancing predications necessarily true of the subject term "I." But the analysis will obviously fail for "Either I am sitting or I am not sitting" [Abelard, 1970, p. 490 (35)] since both of the disjuncts are obviously contingent. Considerations of necessity do arise, however, in understanding disjunction. The disjuncts need not themselves be necessary, but they are necessarily connected in a certain way, since the denial of one necessitates the other — as if, Abelard adds, "they were not able to have a middle" [Abelard, 1970, p. 491 (11–12)].[104]

To accommodate these results he develops an account of disjunction which defines it *in terms of* conditionality. In the fragment of propositional logic he produces here, the two are not treated as interdefinable, as they typically are now; rather, conditionality and negation are taken as primitive, and disjunction as derived. So, on his account, the disjunction $(p \vee q)$ simply means that the negation of p implies q: "If anyone, therefore, wishes to explain the sense of the disjunctive <hypothetical>, they can only show it through the sense of equipollent conjunctive hypotheticals" [Abelard, 1970, p. 491 (3-4)].

This account of disjunction, however, creates a problem that other accounts would avoid. How do we explain the meaning of the three-part disjunction in "Either it is hot or it is cold or it is tepid"? [Abelard, 1970, p. 492 (8)] On the other accounts we could simply say it meant that at least one of the three disjuncts was true, or only one was true, or one was necessary. But how do we answer the question in terms of the conditional-based account that Abelard prefers? This account, unlike the others, forces each disjunction to be seen as applying to two and only two disjuncts, so $(p \vee q \vee r)$ must be subject to some internal association. We must take it in one of two ways, either as $(p \vee (q \vee r))$ or as $((p \vee q) \vee r)$. But on either approach there is a problem. Say we adopt the second. (The following line of argument would, if adapted, work just as well if we adopted the first.) "Either it is hot or it is cold or it is tepid" is then expressed thus: "Either this disjunction is true: 'either it is hot or it is cold'; or it is tepid" [Abelard, 1970, p. 492 (33–35)]. To grasp the problem, note first that the analysis yields two main disjuncts, the first of which is: "This disjunction is true: 'either it is hot or it is cold."' In Abelard's view, this first disjunct is clearly false. Even if it is in fact hot, or in fact cold, the disjunct is *still* false. Why? Because what "Either it is hot or it is cold" means is "If it is not hot then it is cold." This conditional is false because of a failure of implication; since the possibility of being tepid is put

[104] See [Kneale and Kneale, 1962, pp. 222–223].

in play by the original three-part disjunction, not being hot does not imply being cold. And since the conditional is false, it is also false to say, "This disjunction is true: 'either it is hot or it is cold."' Now go back to the original rendering of the whole disjunction: "Either this disjunction is true: 'either it is hot or it is cold'; or it is tepid" (where the last "it" still refers to the thing whose temperature is under question, not to the disjunction[105]). Shorten it to: "Either this disjunction is true or it is tepid." Express this as a conditional: "If this disjunction is not true then it is tepid" [Abelard, 1970, p. 493 (3)]. What we have been told so far is that the antecedent of the conditional is true (because the disjunction expressed in it is in fact not true). So the conditional as a whole is false when said in circumstances where the consequent is false — in other words, when said of anything which is in fact not tepid. What this all amounts to is that the original disjunction ("Either it is hot or it is cold or it is tepid") is false when said of something not tepid. This is contrary to our intuitions, which tell us that the disjunction may well be true under those circumstances. So the analysis which produces this result must be incorrect.

What, then, is the correct analysis? It is one, Abelard believes, which recognizes *different kinds* of disjunction as present in this case. He proposes to interpret $((p \vee q) \vee r)$ as involving one kind of disjunctive relation between p and q, and then another kind between $(p \vee q)$ and r. For the nested disjunction Abelard reverts to a conception of disjunction which diverges from the above conditional-based account. It is, in fact, a truth-functional account. The nested disjunction is understood to mean that one of p, q is true; it suffices simply that one of the disjuncts be true, with no additional requirement that the negation of one relevantly imply the other. The main disjunction, on the other hand, continues to be understood in accordance with the above conditional-based account. The whole disunctive proposition then reads "Either it is one of these or it is that"; expressed as a conditional it reads, "If it is neither of these then it is that" [Abelard, 1970, p. 493 (14, 16)]. Abelard marks this weakened form of disjunction by writing it as a relation between predicates [Abelard, 1970, p. 493 (12)], saying "Either this is hot or cold," as opposed to "Either this is hot or this is cold." The nested disjunction, then, is written as predicative and interpreted as a truth functional. The main disjunction is written as sentential and interpreted as above. The initial problem was caused precisely because there was an implicative link at work within the nested disjuncts. Now that the link is gone, the problem appears to be solved. "Either this is hot or cold, or it is tepid" says that either the weaker, truth functional disjunction holds between hot and cold, or the stronger disjunction holds between that weaker disjunction and tepid.

Note that in $(p \vee q \vee r)$ either p and q can be grouped into a nested disjunction, or q and r. Abelard is committed by his account to accepting either pattern of association as an equally correct reading of the original statement. He indeed accepts this result, taking not only "Either this is hot or cold, or it is tepid"

[105]This disambiguation is effortless in the Latin, where *disiuncta* has a feminine inflection, *tepidum* a neuter one.

[Abelard, 1970, p. 493 (12)] but also "Either this is hot, or it is cold or tepid" [Abelard, 1970, p. 493 (19)] as allowable readings of the original. Of course on his account these cannot be viewed as having *exactly* the same meaning, because in the former the merely truth functional relation exists between the first and second disjuncts (hot and cold), while in the latter it exists between the second and third (cold and tepid). However subtle, this must be taken as making a difference in the meaning of the whole.

How we group the disjuncts in a proposition of the form $(p \vee q \vee r)$ is intuitively thought *not* to make a difference to the meaning of the whole compound disjunction, but Abelard's analysis suggests otherwise. So this is itself a counter-intuitive result. His initial account of stronger, non-truth-functional disjunction makes it difficult to incorporate that connective into nested contexts. As we shall now see, he encounters a similarly counter-intuitive result with the conditional, where again he must have recourse to a secondary and weaker version of the connective in order to deal with nested propositional structures.

Consider the conditional "If it is not well then it is sick." This proposition is false, given that anything, including rocks, can fail to be well; but only living things (in Abelard's view) can be sick. So failing to be well does not imply sickness. But failing to be well *among animals* does seem to imply sickness, so it seems plausible that if we write the condition of being an animal into the conditional in question then the result will be true. It is therefore tempting to apply a qualifying "if" clause: "If it is an animal, it follows that if it is not well then it is sick." Abelard argues, however, that the result of such qualification, whether it takes this form or others, will not be true, given the requirements for implication. He considers three cases, none of which he finds satisfactory. (i) "If it is an animal, it follows that if is not well then it is sick." This attempt attaches the qualification to the whole conditional, and makes that conditional the consequent in the larger one that results from the attachment. But we know that the conditional standing as the consequent taken by itself is false ("If it is not well then it is sick"). So if the antecedent is true then we have a true antecedent and a false consequent — rendering false the compound conditional taken as a whole when said of an animal [Abelard, 1970, p. 402 (14–15)]. (ii) "If it is not well it follows that if it is an animal it is sick." Here the qualifying "if" clause is attached to the consequent of the original. But "If it is an animal, it is sick" cannot be true since there are many animals that are not sick; animality and sickness are not in the right relation to make that conditional true. So again here the consequent is false, and so if the antecedent is true then the compound conditional as a whole is false [Abelard, 1970, p. 402 (21–23)]. (iii) "(If it is an animal then it is not well) <implies> that it is sick." This attempt attaches the qualifying "if" clause to the antececent of the original conditional. But what we have here is the claim that a categorical proposition follows from a conditional one. This claim cannot be true, [Abelard, 1970, p. 402 (26–27)], presumably because of the existential presupposition expressed in the categorical, which is absent from

conditional contained in the antecedent.[106]

Abelard's remedy lies distinguishing a "temporal" meaning of the conditional connective from the "natural" one already discussed [Abelard, 1970, p. 472 (16–38)]. The temporal interpretation reads "If it rains, it thunders" as "When it rains, it thunders." On this reading, the truth of antecedent and consequent are not linked conditionally or causally, but only temporally, in that the two are simply being said to be true at the same time [Abelard, 1970, p. 472 (26–29)]. They only have *societas comitationis*, an association of accompaniment [Abelard, 1970, p. 481 (22)], and are rendered in Latin by *cum* ("when") as opposed to *si* ("if"). This kind of relation is, unlike the one involved in relevance implication, completely symmetrical, in that if the antecedent holds concurrently with the consequent then the consequent will hold concurrently with the antecedent. It is this temporal relation that Abelard appeals to in qualifying the above conditional "If it is not well then it is sick." He reads it as undergoing temporal qualification; the extra "if" clause is read as determining the time in which the main conditional is true, so "if it is an animal" is read as "when it is an animal." If the temporal qualification is applied to the whole conditional as in (i) above, or applied only to the consequent as in (ii) above the result will, he argues, still be false [Abelard, 1970, p. 403 (19-24)]. But if it is applied only to the antecedent we get this: "If when it is an animal it is not well then it is sick," that is, "If it is an animal and not well at the same time then it is sick" [Abelard, 1970, p. 403 (26–28)]. Abelard is willing to admit that the original conditional under this qualification "perhaps is true" [Abelard, 1970, p. 403 (29)].

This sort of qualification is referred to as *constantia*. Its effect is to make true what would otherwise be a false conditional by limiting what it claims. While the example proposition may be successfully dealt with thus, the result is, in a broader sense, problematic. It reveals the same underlying difficulty for Abelard's discussion of conditionals as arises for his discussion of disjunctions. Just as his conception of disjunction ultimately compels him to regard the two disjunctions in $((p \vee q) \vee r)$ as operating in a different way, so his conception of the conditional compels him to regard the two conditionals in $((p \supset q) \supset r)$ as operating in a different way. This imposes a definite constraint on what Abelard can possibly accomplish in the area of propositional logic. The fragment of this that he develops, which treats various relationships between the conditional, disjunctive and negative connectives, can treat these relationships only as they arise in propositional forms of lesser complexity. Even though Abelard inherits from Boethius an ability to *identify* key connectives, the account he provides of them will not support their successive combination into the full range — or even a representative small range — of compound propositional forms.

The fact that he sees the viable number of these propositional forms as relatively small shapes the way he studies the deductive reasoning patterns to which they give rise. A limited number of propositional forms will naturally give rise to a limited number of deductive patterns, and so Abelard runs through a fairly conventional

[106]This material is discussed in [Martin, 2004a, p. 178].

catalogue of such patterns, presumed to be comprehensive. The treatment of this material is — like the treatment of argument forms in modal and tense logic — taxonomic rather than axiomatic. The intent is a list, not a reduction to principles.

The discussion of deductive patterns arising from the conditional is, naturally, focused on reasoning "through the positing of the antecedent" and "through the destruction of the consequent": modus ponens and modus tollens respectively. These rules end up being variously specified, according to the character of the conditional under question.

Abelard starts with the four *simple conditional* forms: "If it is A it is B"; "If it is A it is not B"; "If it is not A it is B" and "If it is not A it is not B" [Abelard, 1970, p. 498 (27–30)].[107] From these are generated eight moods of hypothetical syllogism, four arising from modus ponens, four from modus tollens [Abelard, 1970, pp. 501 (30)–505 (7)]. The first mood applies "It is A" to"If it is A it is B." The second applies "It is A" to "If it is A it is not B." The third applies "It is not A" to "If it is not A it is B." The fourth applies "It is not A" to "If it is not A it is not B." The fifth to eighth proceed in parallel fashion to apply the denial of the consequent to each of the four forms. One notices the inefficiency of this presentation, since there are, after all, really only two rules at work. The moods strike one as overly specified.

But some need to make modus ponens and modus tollens case-specific emerges when Abelard considers the *compound conditional* forms [Abelard, 1970, p. 505 (9)–516 (14)]. Compound conditionals involve nesting a conditional, either in the consequent, or in the antecedent, or both. As we have seen, for Abelard nested conditionals themselves involve a second kind of conditionality — that of the temporal conditional. The rules of modus ponens and modus tollens are not quite enough to capture the truth conditions of these compounds, and need to be adapted to accommodate the presence of *two* kinds of conditional relation. So a compound like "If it is A, then when it is B it is C" falls under this version of modus ponens: "When the antecedent is posited, its consequences will be at the same time" [Abelard, 1970, p. 506 (4–5)]. A compound like "If, when it is A, it is B, then it is C" falls under a different version: "When the antecedent exists with whatever is posited, any consequent of it is posited" [Abelard, 1970, p. 509 (8–9)]. Compound conditionals with temporal qualification in both antecedent and consequent will need another version again.

Abelard accordingly tailors modus ponens and modus tollens to handle different kinds of content. This is an impediment to recognizing the merits of a very different approach: keep the rules the same for all cases and develop a more sophisticated account of propositional substitution. For all his innovations at a more foundational level of propositional logic, Abelard is evidently not ready to take this step.

The final tally of moods for the hypothetical syllogism gives us one hundred and twenty-eight. These include two further kinds of case. (i) Besides the simple and

[107] The variables "A" and "B" are present in Abelard, and are derived from Boethius. (Upper case forms are used here, not the lower case forms in the text, for typographical clarity.)

compound conditional forms just considered, Abelard lists the *mediate conditional* forms [Abelard, 1970, pp. 516 (16)–530 (26)]. These involve conjunctions of conditionals: "If it is A then it is B; if it is B then it is C." The inference pattern arising from these is an amalgam of what now goes by the name of "hypothetical syllogism" and modus ponens: "If it is A then it is B; if it is B then it is C. It is A. Therefore it is C." The work done by (what is now called) hypothetical syllogism is rather understated in this arrangement, which seems to present the whole inference as just another version of modus ponens — modus ponens applied to a conjunction of conditionals, instead of a single one. In that case the mediate conditional inferences are brought into neat alignment with the others, as involving their own specialized versions of modus ponens (and, in the other half of the cases, modus tollens). But this is at the expense of not fully articulating the distinct presence of (what is now called) hypothetical syllogism. (ii) The list of hypothetical syllogisms is rounded out by considering what now goes by the name of "disjunctive syllogism." The corresponding inference patterns are systematically generated, as above, by listing a full range of substitutional forms. Start with "Either it is an A or it is a B," negate either or both disjuncts to produce an additional three forms of the disjunction, and thus generate four disjunctive forms. The denial of either the first or the second disjuncts in each of those four disjunctions creates a distinct inference pattern for asserting the truth of the remaining disjunct. The product of the analysis is of course eight such patterns [Abelard, 1970, pp. 531 (33)–532 (21)]. No compounded disjunctive inference patterns are considered.

Summing the variant moods arising from conditionals and disjunctions, we get a substantially sized list, to be sure. But of course very little ground is actually covered in this exercise, which, because of the proliferation of the forms considered, falsely conveys the impression that much ground has been covered. The inference patterns are generated by what are often negligible variations in the ways of applying negation. And they cannot include larger, more intricately compounded structures because Abelard's relevance account of the connectives simply raises too many difficulties for this. Even one level of nesting between hypotheticals requires invoking a secondary, temporal conditional; and even one level between disjunctions requires invoking a secondary, predicate disjunction. The result is a taxinomically-oriented presentation whose taxonomy leaves something to be desired.

If nothing else, the above account of Abelard's work on entailment gives the lie to this pre-scholarly story about medieval logic: that it accomplishes very little in the area of propositional logic because of a lack of interest in the area, a lack of interest due to the period's preoccupation with Aristotle, who had no interest in propositional logic at all. Abelard has a great deal of interest in the area and accomplishes much. If we take him at his word and view the discussion of topical theory as a preparation for the discussion of hypothetical syllogisms, then we can fairly say that about half of the *Dialectica* is given over to discussion of propositional logic. The limitation in Abelard's accomplishment here is neither one of interest nor insight. It lies in a basic heuristic problem for propositional logic

itself. The relationships between connectives can be very perspicuously expressed by starting with a material account of the conditional. But this material account is so counter-intuitive that it is not a natural starting point at all. A natural starting point is some kind of relevance account, but on this the relationships between connectives are much easier to mis-state and mis-construe.

Philosophers sometimes refer to very basic propositional logic, rather archly, as "baby logic." Well, it took philosophers about as long to develop a working account of baby logic as it took physicists to develop a working account of relativity theory. The written product of Abelard's intense labours in the former area are helpful in showing us why this was so.

PART 3: ABELARD'S CONTEMPORARIES

Among Abelard's contemporaries, and in the immediate aftermath of his career, no other figure emerges as having anything like his stature and influence within the field of logic. On the other hand he did have rivals during his life, and they, like he, established schools to continue the rivalry.

Among his rivals four names in particular emerge: Adam of Balsham, Alberic of Paris, Gilbert of Poitiers and Robert of Melun. These were all younger contemporaries, whose teaching careers were all launched before Abelard's was ended. According to current scholarly consensus, their adherents were, respectively, known as the *Parvipontani* (or *Adamitae*), the *Montani* (or *Albricani*), the *Porretani* (or *Gilebertini*) and the *Melidunenses* (or *Robertini*) [Iwakuma and Ebbesen, 1992, p. 174]. The adherents of Abelard were the *Nominales*. Gilbert is of course famous in his own right as a philosopher and theologian, and cuts a substantial figure in the twelfth century independently of his work in logic. Alberic and Robert, by contrast, are best known through the literary products of their schools. The most notable logical text to emerge from any of the schools of Abelard's rivals is the *Ars Meliduna*, a large work which provides a rich and varied conspectus of the views of the *Melidunenses*; it has been described as "the climactic achievement of logic in the second half of the twelfth century" [Jacobi, 1988, p. 245].

The above school classification represents the received scholarly view on the matter. It is the product of extensive efforts over the past decades in assembling the scattered and often fragmentary literary remains of the schools. Some rather basic questions have arisen, such as whether the *Montani* were in fact the same as the *Albricani*, and whether the *Nominales* were in fact the followers of Abelard [Iwakuma and Ebbesen, 1992, p. 174]. That points as fundamental as these have been under relatively recent discussion shows how incipient our knowledge of this era in the history of logic is.

The textual evidence ranges from treatises and fragments of treatises to passing references made by non-philosophical authors. The treatises themselves pose substantial interpretative problems, having been composed both by and for those immersed in the technical controversies of the day. They demand a knowledge of context which they do little to provide. A school theorem is often presented

via responses to objections against it, or via objections to competing theorems, the presupposition being that its content and purpose are already clear. The extreme case of this is the widespread use of counter-exemplification as a means of assessing inferential forms. An example of a form is given, and then juxtaposed with a series of proposed counter-examples (*instantiae*), the entire commentary on the form being essentially this list consisting of example and counter-examples [Iwakuma, 1987, p. 438], with little or no supporting commentary. The late twelfth century logicians drew on basically the same logical *corpus* as Abelard, with the exception that in mid-century Aristotle's *Sophistical Refutations* was brought into currency. The *instantiae* technique was doubtless encouraged by resulting interest in the theory behind fallacy identification; but relentless use of the technique served to make the logical literature of this period all the more inaccessible to the uninitiated.

That said, the richness of the resulting debate is undeniable. Its genesis often lies in the reaction of Abelard's rivals to controversial elements in his own philosophical views, the ensuing discussion sharpened by the involvement of the corresponding schools. An outstanding example is the debate arising from Abelard's handling of the topics "from opposites" and "from immediates." Abelard's argument for rejecting these as legitimate topics was adapted by Alberic to pose a problem for Abelard's whole approach to the conditional.

Alberic's argument starts from any obvious and self-evident conditional, such as "If Socrates is a man, Socrates is an animal." Here are the steps, as suggested in a text originating from Alberic's school [de Rijk, 1967b, pp. 65 (35)–66 (4)], and reconstructed thus:

(i) If Socrates is a man and not an animal, Socrates is not an animal

(ii) If Socrates is not an animal, Socrates is not a man

(iii) If Socrates is not a man, it is not the case that (Socrates is a man and not an animal)

(iv) Therefore, if Socrates is a man and not an animal, it is not the case that (Socrates is a man and not an animal).[108]

(iv) follows from the other three propositions by two applications of hypothetical syllogism. So the logic of this argument is unimpeachable. If the argument is to be shown unsound, one of its three premises must be shown to be false. But (i) is a tautology, self-evidently true by simplification. (ii) follows from the original, self-evident conditional by contraposition. And (iii) just contraposes "If Socrates is a man and an animal, Socrates is a man," itself a tautology, self-evidently true by simplification. So (iv) evidently follows. One can start from any true conditional and derive from it, through parallel steps, a proposition having the same logical form as (iv).

[108]See [Martin, 1986, pp. 570–571], [Martin, 1987a, pp. 394–395], [Martin, 1987b, p. 433], [Martin, 1992, p. 122] and [Martin, 2004a, pp. 191–192].

The problem with this result of course is that, as we have seen, (iv) is on Abelard's account obviously false. (iv) is of the form $[(p\&q) \supset \sim (p\&q)]$; this is a substitution instance of the simpler formula $(p \supset \sim p)$. According to Abelard's theory of conditional entailment, no proposition can entail its own negation (since no proposition can contain its own negation). So it is important for Abelard that (iv) be false. Yet it has just been proven true. We have no clear record of his response to this problem, but the approaches of the above-named rivals are known in outline.

Alberic considers the content of the premises (i)–(iii) not adequate to motivate use of hypothetical syllogism. What motivates this argument form, in Alberic's view, is not the fact that antecedents and consequents are related in a certain way (i.e., related so that the consequent of one conditional is the antecedent of another), but the fact that *predicates* are related in a certain way. Remember that the conditionals studied by these logicians typically have a shared subject term between antecedent and consequent ("If Socrates is a man then Socrates is mortal"); in the above argument, "Socrates" is in all the antecedents and consequents as subject, and so the inferences made are interpreted by Alberic as depending on relationships between the predicates. In his view, hypothetical syllogism demands a sequence of predicates such that the first's being predicated of the subject is the *cause* of the second's being predicated of the subject (as Socrates' being a man is the cause of his being mortal), and such that the second's being predicated is likewise the cause of the third's. In Alberic's view this relationship does not exist here. The required causal relation between predicates does not hold in the sequence "being a man and not being an animal," "not being an animal" and "not being a man." In particular, the conjunction of predicates in "being a man and not being an animal" is self-contradicting, and cannot be represented as a cause of anything. So the linkage does not exist which would allow the hypothetical syllogism to work. Alberic's conclusion, then, is that the argument is invalid because the conditions for legitimate use of hypothetical syllogism do not exist [Anonymous, 1967, pp. 64 (31)–65 (3, 10-12); Martin, 1987a, p. 396; Iwakuma, 2004, pp. 325–326].

From Gilbert of Poitiers and his school, and Robert of Melun and his, we get an approach which simply questions the truth of conditionals like (i), not the validity of the argument as a whole. This is an important distinction, because what Alberic's approach essentially does is cease to view the above argument as an exercise in propositional logic at all, construing it instead as an exercise in term logic. By contrast, the approach of these other two schools accepts hypothetical syllogism as an inferential relation between propositions, not terms. The problem raised by the *Porretani* for conditionals like (i) echos Alberic's concern for causal relevance, but this time the relevance problem is expressed in terms of the relation of antecedent to consequent. Their key principle is that the content of the antecedent must have relevance to the content of the consequent, in that the truth of the predication in the former can be seen as causing the truth of the predication in the latter. "If Socrates is a man and an ass, then Socrates is a man," is seen as

failing this requirement, giving grounds for considering it false [Anonymous, 1983, p. 26; Martin, 1987a, p. 397; Iwakuma, 2004, p. 326]. The *Melidunenses*, by contrast, emphasize the importance of a true antecedent rather than the importance of a relevant one; their principle is that nothing follows from a falsehood [Martin, 1987a, p. 398; Iwakuma, 2004, p. 327]. The antecedent of (i), if taken as a free-standing proposition, asserts that Socrates is a man and not an animal, which cannot be true since all men *are* animals. So what the antecedent asserts is false, and the principle that nothing follows from a falsehood accordingly identifies the whole conditional as false. Both the *Porretani* and *Melidunenses* support their analyses by counter-examples to suggest how in other instances the practice of allowing irrelevant or false antecedents will lead to problematic results. In both cases we have genuine propositional logic being done, the search being for constraints on conditionals — either involving relevance conditions or truth conditions — sufficient to demonstrate the falsity of (i). The constraints naturally have a very limiting effect on what is accepted as a true conditional.

The response of the *Parvipontani* is the most forward-looking. They too deal in genuine propositional logic, but are not concerned to constrain it in these ways — because they consider Alberic's argument acceptable as it stands, and feel no incentive to demonstrate the falsity of (i) or any other premise; nor do they wish to question the argument's soundness. They in fact accept the principle that from a contradiction any statement whatsoever follows, so the self-contradicting conclusion (iv), where a proposition entails its own denial, is simply a routine product of the initial self-contradicting antecedent of (i): "Socrates is a man and is not an animal." One adherent of the school states openly that "If Socrates is a man and Socrates is not a man, then Socrates is a crab, and so on for single things, for example a rose, a lily and the rest" [de Rijk, 1967a, p. 290; Martin, 1987a, p. 398; Iwakuma, 2004, pp. 327–328]. The lesson, in other words, is that any conclusion follows from a contradiction.[109] While the *Parvipontani* never advance from this point to constructing a full propositional logic in the modern sense, it is clear that they prefigure in their response to Alberic one of modern propositional logic's most suggestive doctrines.

[109]The result is motivated by this sort of argument [de Rijk, 1967a, p. 290; Martin, 1987a, p. 398; Iwakuma, 2004, pp. 327–328]:

(i) If Socrates is a man and Socrates is not a man, then Socrates is a man.

(ii) If Socrates is a man, Socrates is a man or a stone.

(iii) Therefore, if Socrates is a man and Socrates is not a man, Socrates is a man or a stone.

(iv) If Socrates is a man and Socrates is not a man, then Socrates is not a man.

(v) Therefore, if Socrates is a man and Socrates is not a man, then Socrates is a stone.

(iii) follows from (i) and (ii) by hypothetical syllogism. The inference from (iii) and (iv) to (v) is more abbreviated. "Socrates is a man and Socrates is not a man" implies both "Socrates is a man or a stone" and "Socrates is not a man"; these two latter propositions together imply "Socrates is a stone" (by disjunctive syllogism). Therefore "Socrates is a man and Socrates is not a man" implies "Socrates is a stone" — which is what (v) asserts. With the relevant substitutions, the above argument can be made to derive any conclusion from any contradiction.

Another Abelardian theme which is subjected to much scrutiny and debate by the schools is the doctrine of the propositional *dictum*. The *dictum* comes to be referred to subsequently as the *enuntiabile*, apparently first in the work of Adam of Balsham [Nuchelmans, 1973, p. 169; Iwakuma 1997, p. 19]; whatever differing shades of meaning may initially be imputed to the two terms, they can safely be used interchangeably. The question arising from Abelard's work is what sort of item they are. They are not things, and yet they make propositions true or false, and therefore seem to have a mind-independent standing very comparable to things. The notion of the *dictum* is so important, and yet so enigmatic, that controversy is inevitable. A passage from *Ars Meliduna* describes four basic positions on the issue [Iwakuma, 1997, p. 19; Nuchelmans, 1973, pp. 170–172], and Iwakuma Yukio has suggested that these are in fact the positions staked out by the above four non-Abelardian schools [Iwakuma, 1997, pp. 20–21].

(i) On one view the *enuntiable* is purely conceptual in nature. The words "Socrates reads" generate understandings (*intellectus*) which are conceived by the soul; these together constitute an understanding corresponding to the accusative-and-infinitive structure *Socratem legere* ("that Socrates reads"). The understanding thus constituted is the *enuntiabile*. As such it can easily be identified as something — specifically, some sort of property of the soul [Nuchelmans, 1973, p. 171]. This yields a positive characterization of the *enuntiabile*, and avoids the controversial step of claiming it is nothing at all. But the characterization itself encounters the sorts of problems normally attendant upon this sort of psychologism, of which Abelard is well aware. Since the *enuntiabile* is the cause of truth in propositions, it follows that, on this account, understandings are the cause of truth in propositions as well. We have already seen evidence of Abelard's distaste for this kind of position. Iwakuma suggests that this first position is the view of Alberic and the *Montani* [Iwakuma 1997, pp. 21, 23–24].

(ii) A second position takes the *enuntiabile* as composition (or, in the case of negation, as division) not of understandings but of items outside the mind. The *enuntiabile* corresponding to the proposition that Socrates is white is the actual combination of white with Socrates [Nuchelmans, 1973, p. 171]. Again we have a view which easily identifies the *enuntiabile* as something, and does better than the theory of the *Montani* in explaining how the *enuntiabile* can function as a cause of truth. But these results come at the cost of involving a more involved metaphysics; what is needed is an account of what this composite item consisting of Socrates and whiteness actually is. Supplying this is not a trivial task. Iwakuma identifies this position as arising from the *Porretani* [Iwakuma, 1997, pp. 21-22], whose outlook is not at all metaphysically reductionist, and therefore not at all adverse to taking up this task.[110]

(iii) A third view expressed in this passage from *Ars Meliduna* is attributed by Iwakuma to the *Parvipontani* [Iwakuma, 1997, p. 21]. This is a metaphysicially reductionist view, on which *enuntiabilia* are nothing. The same claim is made of the true and the false; they are just manners of speaking, whose purpose is no

[110] See also [Ebbesen, 2004].

more than to assert, with respect to a given proposition, that things really are as it says they are. This view bears obvious similarities to Abelard's own account of the *dictum*. The associated account of the true and the false is comparable to what is now labelled as a "deflationary" approach.[111]

(iv) This brings us to the remaining view, which is likely that of the *Melidunenses* themselves, given that it is presented approvingly [Iwakuma, 1997, p. 21]. It has a great deal more in common with the view of the *Porretani*, in ascribing to the *enuntiabile* a reality outside the mind at the cost of having to articulate a metaphysically complex view. And like the *Porretani*, the *Melidunenses* do not tend to be metaphysically reductionist. In fact on this matter their proposal is more metaphysically complex. The *Porretani* start with familiar items — objects and properties — and envisage compositions of them to get their *enuntiabile*. But *Melidunenses* get theirs by envisaging an item entirely distinct from the world of objects and properties, accessible only to reason, and not to the senses, and accordingly they confer special metaphysical status upon the *enuntiabile*. Indeed, this entity falls right outside of the Aristotelian categories, and tends to be characterizable more in terms of what it is not than what it is. It is, in particular, not tied to particularities of linguistic formulation, so that the propositions *Marcus est albus* and *Tullius est candidus* both give us the same *enuntiabile* (that Cicero is white) [Nuchelmans, 1973, p. 172].[112] In the end, this proposal explains how *enuntiabilia* can serve as the cause of truth, but is as much in need of additional clarification as Abelard's proposal, and probably more in need of it than that of the Montani and the Porretani.

However one explains it, having a notion of the *dictum/enuntiabile* provides a new avenue for explaining, in the most general sense, what an argument is. Abelard is inclined to view the argument as being nothing but the meaningful words of which it is composed — in particular, the words constituting the premise or premises (the conclusion being that which awaits proof, and the premise or premises being that which supplies it). But this seemingly uncontroversial view does actually occasion some controversy, and the *dictum/enuntiabile*, notwithstanding its source in Abelard's own writings, is appealed to on this issue as a means of staking out different positions. All of the schools register dissent from Abelard's view. One of the reasons for this dissent, apparently, lies in the conventional status of human language, every sound or written character of which could have been established in a different signification than it was [Iwakuma, 2004, p. 319]. How then are propositions able to be stand in a relation of binding inferential linkage? It is tempting to say that what is binding in an argument's inferential linkage is to be found in the *dicta/enuntiabilia* corresponding to its constituent propositions.

[111] [Kneepkens, 1997, pp. 85-86] discusses Parvipontanian views about the *enuntiabile* in connection with a later figure from this school (Alexander Nequam). [Iwakuma, 1997, p. 21] associates this passage from the *Ars Meliduna* with the Parvipontanian school based on similarities of content.

[112] See also [Martin, 1997b].

This indeed is the approach of the *Melidunenses*, *Montani*, and *Parvipontani* alike, although they differ in how this approach is carried through. The *Melidunenses* adapt Abelard's view more directly, taking the argument to consist of the *enuntiabilia* corresponding to the premises used in establishing the conclusion; the arbitrariness of the sounds or letter shapes constituting the spoken or written expressions of those *enuntiabilia* therefore in no sense imparts arbitrariness to the argument as a whole. The *Montani* and *Parvipontani* have a more complex account. They view the relation of premises to conclusion as implicational rather than inferential, so that "A therefore B" is construed as being at heart the conditional "If A then B." For both of these groups the argument is the *enuntiabile* expressed by such a conditional. Given an inference like "Socrates is a man; therefore Socrates is an animal," the *Parvipontani* take the argument as being the *enuntiabile* corresponding to this conditional: "If Socrates is a man he is an animal." The *Montani* adopt much the same approach, except that the conditional takes a more general form: "If something is a man it is an animal." The *enuntiabile* corresponding to this conditional form is thus identified as the argument of "Socrates is a man; therefore Socrates is a mortal," of "Plato is a man; therefore Plato is a mortal," and of all other possible substitution instances of this form. Finally, the response of the *Porretani* lies a little apart from these others in appealing not to an *enuntiabile* at all but to a relation of terms, in particular the relation of the middle term to the others. The argument is simply this relation. Since the relation is assuredly an objective one, it thus avoids what appears to be the problem with holding that an argument is nothing but the meaningful words of which it is composed [Iwakuma, 2004, pp. 318–320].

There is controversy not only about the nature of an argument but about the very relationship between syllogisms and topical inferences. Recall that Abelard distinguishes between complete and incomplete entailment, ascribing to syllogisms the status of being complete by virtue of their syntactic form. The strength of topical inferences is understood, as we have seen, in a very different way [Stump, 1989, pp. 96–97]. This is a unique view at the time Abelard initially propounds it [Green-Pederson, 1987, p. 198]. His predecessors typically view syllogisms as deriving their strength in the same way all topical inferences do, and topics are assigned to them, just as to other inferences. The topics for syllogisms simply tend to be descriptions of the relevant syllogistic mood, descriptions set in the form of topical rules. Abelard's insistence that there is something fundamentally distinct about the syllogism that sets it apart from other inferences is therefore a novel view, and is regarded in his own time as a characteristic view, both of himself and of the *Nominales*. The view that syllogisms are at heart inferences, best expressed as conditionals with the conclusion as consequent, naturally encourages the notion that they, like other conditionals, require topical warrant; but the implicative (as opposed to inferential) approach to the syllogism taken by other schools therefore encourages some level of dissent from Abelard's view. The *Montani* and *Porretani* both maintain that the inferential linkage at work in syllogisms is ultimately a matter of the topical relations that exist between their terms; logical form then

emerges only an indicator that this linkage is present, not as an explanation of how it is present. The *Melidunenses* seem to have a view which simply combines the Abelardian and conventional approaches; they claim that syllogisms are able to draw their strength not only from their formal arrangement but also from topical relations that exist between their terms [Green-Pedersen, 1987, p. 199]. The view of the *Parvipontani* on this matter is not presently known [Iwakuma, 2004, p. 324].

These are a few examples of the school divisions which dominate the study of logic in the late twelfth century. Notice that these examples have all issued from the more distinctively non-Aristotelian elements in Abelard's work. On the more conventional Arisotelian elements in logical theory at this time that natural deference to authority might be expected to have produced more nuanced forms of disagreement, and perhaps more consensus. In fact scholarship has not yet uncovered equally compelling examples of four-way (or five-way, if one includes the *Nominales*) division on logical issues distinct from the ones on which Abelard was most innovative. If this remains the trend then it will be testimony indeed to the magnitude of Abelard's influence upon his contemporaries.

The relationship of the schools is not exclusively one of mutual disagreement. The earliest systematic study into their relationship — found in L.M. de Rijk's *Logica modernorum* — in fact suggests certain themes in common in their treatment of basic semantic theory. While most recent research into this era has sought to distinguish the schools by charting opposing positions in debate, de Rijk has sought to discover how the development of twelfth century theory ultimately leads to the emergence of the terminist tradition in the thirteenth. His approach emphasizes themes held in common by the schools, and the resulting analysis identifies two key lines of historical development.

(i) First, and perhaps most importantly, there is the increasing tendency to pay close attention to the role of context in determining meaning. De Rijk argues that this change is found in the grammatical theory of the period. Eleventh century grammatical approaches are characterized by the belief that what the word means is what it was initially imposed to mean, not how it is used in particular contexts [de Rijk, 1967a, pp. 113-114]. Twelfth century approaches, with their growing interest in syntactic theory, are more open to the idea that placement in a particular construction can legitimately influence a word's meaning in that context [de Rijk, 1967a, pp. 115-17]. This approach is certainly to be found in Abelard's work, and in his later glosses on Aristotle and his theological works the notion of *translatio* is regularly employed. By this term is meant some kind of transferrence of meaning brought about by context, as in "the meadows smile," where "smile" is used to mean "are flowering" [Abelard, 1921, p. 121 (29–35)]. This is not an equivocal use, because equivocation involves a word's having more than one imposed meaning, and that is not the case with "smile" here; it still has just one imposed meaning, and simply acquires another temporary meaning when preceded by "the meadows."

Use of this notion of *translatio* as a staple of semantic theory becomes fairly

commonplace in the twelfth century, and is, for example, particularly in evidence among the *Melidunenses* [de Rijk, 1967a, pp. 298-299]. They adopt, following Abelard, an approach to equivocation which limits equivocity to cases where the word receives multiple meanings at the time of inception, and carries that multiplicity with it for as long as it remains in the language. Other sorts of multiplicity arise just because of the conditions of use to which the word is subjected: "We say ... that nothing prevents the same term from signifying many things in different locutions as a result of different associations (*ex diversis adiunctis*). Nor, however, will it signify many things, since what it signifies through association it does not signify" [de Rijk, 1967a, p. 298]. "Dog" is genuinely equivocal because is imposed on a certain kind of quadruped, and also a certain heavenly constellation. "Healthy" is not equivocal, even though it can be variously predicated of animals, food, and urine, since it was not imposed to signify any of these three. It was imposed to signify health, and is put to these other uses through contextual variation. The lesson here is that initial signification is only part of what is needed by a semantic theory to explain the semantic behaviour of the word. What is needed in addition is an account of how context actually works on signification to produce a variety of references. Discussion of this issue ultimately gives rise to the varieties of supposition articulated by the subsequent terminist tradition.

(ii) The notion of *translatio* comes to prominence, de Rijk additionally argues, under the further influence of fallacy theory. This is a developing interest of the later twelfth century, which reflects the introduction of Aristotle's *Sophistical Refutations* into Western Europe around the middle of the twelfth century. "The different types of supposition," de Rijk says, "are not only illustrated by means of fallacies, but ... derived from the determinations of those fallacies" [de Rijk, 1962, p. 22]. One of the key fallacies in this regard is what is called the fallacy of "univocation" [de Rijk, 1967a, pp. 499-504]. Philosophers have typically viewed univocation in a wholly positive light as representing a general term's success in naming things of the same kind. From this perspective univocation is not readily associated with fallacious reasoning. But there is a specialized tradition in the use of this word common in early medieval fallacy theory which takes a word as being used univocally so long as it preserves its original signification [de Rijk, 1967a, p. 492]. (Outright equivocation occurs when the term is used in a way that departs from its original signification.) Since it is possible for the same term to name things of *different kinds* even though it retains its original signification, univocation can end up being less than semantically straightforward, and can bring with it danger of misleading employment. Hence arises the fallacy designation. For a representative twelfth century treatment of this issue de Rijk turns to the *Fallacie Parvipontane*, a fallacy text emanating from the school of the *Parvipontani*. Here we find description of several varieties of univocation that could sow confusion among the inattentive. In one variety, a word stands for itself, as "teacher" does in "'Teacher' is a name." Or it stands for its logical category, as "man" does in "Man is a species." Or it stands for the kind of things it denotes, as "man" does in "Man is the most worthy of creatures." Or it stands for past and future things

of the kind it denotes, as opposed to just present ones [de Rijk, 1967a, p. 495]. In each of these cases *translatio* takes place (or, as the text puts it, *transsumptio*), whereby the reference of the term is deflected by the adjoining words, even though the term itself sees no change of underlying signification. What is described here, of course, is the very semantic behaviour which later supposition theorists attempt to chart in their various classifications. The supposition of a name is what it stands for in a given context, and the above varieties of univocation just correspond to different things a name can stand for in different contexts — i.e., they correspond to different kinds of supposition.

The forward-looking developments in semantic theory described under (i) and (ii) were not limited to the two schools mentioned. These were very general developments in this era, and credit for them must be widely apportioned. While it is useful to grasp the issues on which the schools differentiated themselves, it should also be understood that there were issues on which they exerted a productive consensus. It does remain difficult to chart this mid- and late-twelfth century period in the history of logic, because essential development occurred through a larger number of smaller voices, as opposed to a smaller number of larger ones. The passage from Abelard to the terminists is an inland delta phase of history, where a distinct channel divides into a network of smaller ones, before these then gather back into another distinct one.

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THE DEVELOPMENT OF SUPPOSITION THEORY IN THE LATER 12^{th} THROUGH 14^{th} CENTURIES

Terence Parsons

INTRODUCTION

Soon after the time of Anselm (1033–1109) and Peter Abelard (1079-1142), universities began to be founded. Logic was taught pretty much as it had been before the year 500; it consisted of work from Aristotle's *Categories* and *On Interpretation*, and material from the first few sections of his *Prior Analytics*. There was also material from the Stoics on propositional logic. In the early 1100's, western scholars acquired Latin versions of additional work by Aristotle, including his *Sophistical Refutations*, which dealt extensively with fallacies. This inspired original new work by medieval scholars. Sometime in the 12^{th} century a number of original texts suddenly appeared containing a fairly extensive group of interrelated theories, usually employing the term 'supposition', which means pretty much what we mean by "reference" or "standing for". The texts were similar, expounding much the same doctrines, and often employing the same or closely similar examples. Clearly there was some common source, but the texts themselves contain no indication of the origin of the doctrines, and the early history of the topic is unknown. This essay is an account of some of the main theories of this tradition, often called Supposition Theory. For want of space, the commonest known themes will be treated, without extensive details of how the theories varied from author to author. The picture given here is then a kind of ideal framework of a large tradition.

Supposition is a relation between an expression and what that expression stands for when the expression appears in a proposition. Much of the theory deals in some way with how changes in what a word supposits for affects the truth value of the sentence containing it. It often focuses on how changes in supposition can create divergences in truth conditions, and it is applied in analyzing fallacies of ambiguity; sometimes it is treated purely on its own, as a study of how language works. Although the theory is not formulated as a recursive theory of truth conditions, in many cases such a picture seems to lurk in the background. One central theme is that authors often allude to structural considerations, such as scope, which we now see as a byproduct of a recursive semantics.

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Medieval authors did not develop an artificial symbolism, as is the custom today.[1] Instead, they used Latin. But this was "regimented" Latin, which made the language under consideration serve many of the purposes of modern logical notation. For example, it was assumed (or perhaps stipulated) that the left-to-right ordering of signs corresponds to their semantic scope — so that '*Every donkey not an animal is*' means that no donkey is an animal, and '*Every donkey an animal not is*' means that for every donkey, there is an animal distinct from it. This was not how ordinary users of Latin understood their language, but it was very useful for logical theory. Also, because of the presence of inflections, Latin has a fairly free word order, so that the direct object can easily precede the subject in a sentence, as in '*Every donkey some man owns*', giving it scope over the subject. Since English does not allow this, we need an artificial symbolism to permit various scope structures; medieval logicians already had this in their own Latin (properly, and artificially, understood).

Because this is a work on supposition theory, some important and interesting parts of medieval logic will not be addressed. Chief among these are the Insolubles (paradoxes), Obligations (rules for specialized debate), Consequences (general theory of inferences among arbitrary propositions), and Syncategoremata (individual studies of special words, such as 'both', 'ceases', 'only'). These will be touched upon only as they bear on Supposition theory. Also, important earlier work, such as that by Abelard, will not be covered.

This essay is based entirely on Latin manuscripts that have been edited and published, and it is based mostly on those works among them that have been translated into English. Each reference to a medieval work is to book, volume, chapter, section, subsection, etc, followed by a page reference to the English translation. Unless otherwise stated, I use the English translations provided in the texts cited.

1 CORE ELEMENTS OF MEDIEVAL LOGIC

Medieval Logic is built on a foundation of logical terminology, principles, and methodology that was contained in the traditional liberal arts, in particular in that part of the Trivium called Logic or Dialectic. This material is mostly from the writings of Aristotle, including also the Stoics and Cicero, much of it as interpreted by Boethius.[2] This section is devoted to the fundamental parts of logic that

[1] However, authors were happy to use schematic letters, as Aristotle did in developing his theory of the syllogism, with forms like "Some A belongs to every B". Also, some post-medieval writers experimented with the introduction of special signs whose function was to force special scope readings. For example, 'Every man is b animal' has the truth conditions 'For some animal, every man is it', because the sign 'b' gives the term following it wide scope. See [Ashworth, 1974, IV.II.1] for a discussion of these special signs.

[2] The parts of Aristotle's symbolic logic that were well-known around the year 1000 were his *On Interpretation*, *Categories*, and material (often second-hand) from the first several sections of his *Prior Analytics*. This subject matter was later called the "Old Logic", to distinguish it from the "New Logic" which was based on several additional writings by Aristotle that became

medieval logicians accepted as the basis of their work.

I begin with an account of the forms of propositions that constitute the subject matter of Aristotle's symbolic logic. In keeping with medieval terminology, I use the term 'proposition' to refer to what we today would call a meaningful sentence. It stands for a sentence, not for an abstract meaning expressed by a sentence which is named by a that-clause. So '*Snow is white*' and '*Schnee ist weiss*' are different propositions.[3]

1.1 Categorical Propositions

The simplest form of a proposition is a categorical proposition. In fundamental cases this consists of two nouns (the "subject" and the "predicate") separated by the copula (*is*), with perhaps some other signs (often quantifier words) which "modify" the nouns. Examples are:

Socrates — is — [an] animal.
Every animal — is — [a] donkey.
Some animal — is — [a] donkey.

in which the subjects and predicates are '*Socrates*', '*animal*' and '*donkey*'. The words '*every*' and '*some*' modify the subject and predicate terms, but they are not parts of the subject or predicate. The indefinite article '*a/an*' is enclosed in brackets since both Latin and Greek lack indefinite articles; in the original texts there is nothing before the noun. The words '*Socrates*', '*animal*' and '*donkey*' are "*categorematic*"; these are the words that stand for individual things, '*Socrates*' for the man Socrates, '*animal*' for each and every animal, and '*donkey*' for each and every donkey. The other words — the quantifier signs and the copula — are called "*syncategorematic*" (meaning "with-categorematic"), since they occur with (and affect) the categorematic terms.

The examples given above are affirmative. If a '*not*' (which modifies the copula) or a '*no*' is added, one gets a negative categorical proposition:

Socrates — is not — [an] animal.
No animal — is — [a] donkey.
Some animal — is not — [a] donkey.

'*Not*' and '*no*' are syncategorematic signs.

Peter of Spain lays out the kinds of non-modal categorical proposition and their ingredients [Peter of Spain *T*I.8-9 (4)]:

available later.

[3]Opinions differed on whether there are also mind-independent entities corresponding to propositions. For many medievals, propositions are tokens, not types, and this is important in certain cases, such as addressing semantic paradoxes, where two tokens of the same type might differ in truth value. But for the most part nothing would be changed in the theory if propositions were types.

> "Some categorical propositions are universal, some particular, some indefinite, and some singular.
>
> A **universal** proposition is that in which a common term determined by a universal sign is subject, as '*every man runs*'; or, a universal proposition is that in which something is signified to be in all or none.
>
> A **common term** is what is apt by nature to be predicated of many, '*man*' of Socrates and Plato and of each and every other man. These are **universal signs**: '*every*', '*no*', '*nothing*', '*any*', '*either*', '*neither*', and similars.
>
> A **particular proposition** is that in which a common term determined by a particular sign is subject, as '*some man runs*'. These are **particular signs**: '*some*', '*a certain*', '*the one*', '*the other*', and similars.
>
> An **indefinite proposition** is that in which a common term without a sign is subject, as '*[a] man runs*'.
>
> A **singular proposition** is that in which a singular term or a common term joined with a demonstrative pronoun is subject, as '*Socrates runs*' or '*this man runs*'; . . .
>
> A **singular term** is what is apt by nature to be predicated of one alone.[4]
>
> Some categorical propositions are affirmative, some negative. An **affirmative** is that in which the predicate is affirmed of the subject, as '*A man runs*'. A **negative** is one in which the predicate is removed from the subject, as '*A man does not run*'."

The theory is primarily applied to eight forms of propositions, classified with respect to Quantity (Universal, Particular, Indefinite, Singular) and Quality (Affirmative, Negative). Some examples are:

	AFFIRMATIVE	NEGATIVE
SINGULAR	*Socrates is [an] animal*	*Socrates is not [an] animal*
UNIVERSAL	*Every animal is [a] donkey*	*No animal is [a] donkey*
PARTICULAR	*Some animal is [a] donkey*	*Some animal is not [a] donkey*
INDEFINITE	*[An] animal is [a] donkey*	*[An] animal is not [a] donkey*

I call these eight forms "standard" categorical propositions, to distinguish them from extended forms to be discussed later.

[4] Medieval authors also call singular terms 'discrete' terms.

1.2 Truth conditions for categorical propositions

Singular, Universal, and Particular categorical propositions are interpreted as you expect from their wording, with a refinement for empty terms. The following explanations are meant to be a theory-neutral statement of when certain forms of propositions are true, and when false.

Singular categorical propositions If Socrates exists now, then the proposition '*Socrates is an animal*' is true if Socrates is now an animal and false if he is not, and vice versa for the negative '*Socrates is not an animal*'. Since Socrates does not exist now, the affirmative form is false and the negative is true.

For the following, assume temporarily that the subject term of the proposition is not empty. Then:

Universal categorical propositions The universal affirmative '*Every animal is a donkey*' is true now if all currently existing animals are now donkeys. Paraphrasing Aristotle's words [*Prior Analytics* I.1.], the proposition is true when no animal can be found of which '*donkey*' cannot be asserted. The universal negative has the obvious truth conditions; it is true when there is nothing of which the subject and predicate are both true.

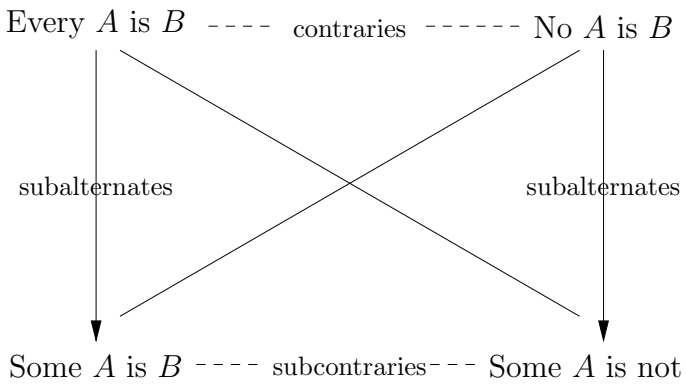
Particular categorical propositions The particular affirmative '*Some animal is a donkey*' is true now if there is now at least one animal which is now a donkey; otherwise false. The particular negative '*Some animal is not a donkey*' is true now if there is now at least one animal which is not now a donkey; otherwise it is false.

Indefinite categorical propositions Indefinite propositions are ones that we would naturally translate using an indefinite article: '*A donkey is running*', '*A donkey is not running*'. These propositions have unclear truth conditions. It is usually said that they should be treated as if they are synonymous with the corresponding particular propositions, and this is the common practice. As a result, '*A donkey is running*' and '*A donkey is not running*' are not contradictories; they are related as '*Some donkey is running*' is to '*Some donkey is not running*'. I'll ignore indefinites in all discussion below.

Except for a few theorists who deal with semantic paradoxes, it is assumed that every well-formed categorical proposition is either true or false (and not both).

1.3 Oppositions

Aristotle devoted considerable study to the four universal and particular forms of proposition just given. These bear certain logical relations to one another, later called relations of "opposition". These relations came to be diagrammed in what is today called the "Square of Opposition". A version of the square used in medieval times is:



The universal affirmative and universal negative proposition at the top are contraries:

> The law of **contraries** is such that if one is true, the other is false, but not conversely; for it is possible for both to be false in contingent matter[5], as '*every man is white — no man is white*'. . . . [Peter of Spain *T*I.14 (7)]

The particular affirmative and particular negative propositions at the bottom are subcontraries:

> The law of **subcontraries** is such that if one is false, the other is true, but not conversely; for it is possible for both to be true in contingent matter. . . . [Ibid]

Diagonally opposite propositions are contradictories:

> The law of **contradictories** is such that if one is true, the other is false, and conversely; for in no matter is it possible for both to be true, or [both to be] false. [Ibid]

Finally, universal and particular affirmative propositions are related by subalternation, as are the universal and particular negatives:

> The law of **subalternations** is such that if the universal is true, the particular is true, but not conversely. . . And if the particular is false, its universal is false, but not conversely. [Ibid]

1.4 Problems with Empty Terms

There are two problems with the relations that Peter gives above. Both have to do with the truth values of propositions with empty subject terms.

[5] A proposition is in contingent matter if the terms in the proposition are not modally related to one another. An example of a proposition in contingent matter is '*Some donkey is grey*'. The proposition '*Some equilateral is equiangular*' is a proposition that is not in contingent matter.

1.4.1 Universal Affirmatives

The first problem concerns universal affirmative propositions when the subject term is empty; for example, '*Every donkey is a mammal*' when there are no donkeys. Suppose that the term '*A*' is empty. Then '*Some A is B*' is clearly false. So according to the principles embodied in the Square, '*No A is B*' is its contradictory, and is thus true. So, by the law of contraries, the universal proposition '*Every A is B*' must be false. This goes against the modern custom whereby a universal affirmative proposition with an empty subject term is trivially true. This is because the canonical translation of '*Every A is B*' into symbolic logic is '$\forall x(Ax \rightarrow Bx)$', which is true when '*A*' is empty.

Modern students often balk at the proposal that universal affirmatives are true when their subject terms are empty. In response they may be told

> This is a convention which is useful in logic — it makes for theoretical simplicity. Ordinary usage is unclear regarding such propositions with empty subjects. If you think that universal affirmatives are false when their subjects are empty, then you may simply represent them by adding a condition: translate them as '$\exists xAx\&\forall x(Ax \rightarrow Bx)$'.

It is apparent that one can also adopt the opposite convention that universal affirmatives are false when their subject term is empty.

> This is a convention that is convenient for doing logic — it makes for theoretical simplicity (see below). If you want to represent ordinary usage differently, then just represent '*Every A is B*' using '*Every A is B or no A is A*'.

1.4.2 Particular Negatives

The other problem with the traditional Square of Opposition concerns particular negatives. Suppose that the term '*A*' is empty. Then '*Some A is B*' is false. So according to the principles embodied in the Square, '*No A is B*' is its contradictory, and is thus true. So, by the principle of subalternation, '*Some A is not B*' is also true. But to modern ears, '*Some A is not B*' should be false if '*A*' is empty. After all, '*some A*', has scope over the rest of the proposition. What is wrong?

This result is built into the diagram in other ways as well. Again, suppose that '*A*' is empty, so that '*Some A is B*' is false. Then its subcontrary, '*Some A is not B*' must be true. Or suppose that '*Some A is B*' is false; then its superalternate '*Every A is B*' is also false; so its contradictory, '*Some A is not B*' is again true.

Some authors did not notice this result, but many did. Almost all of them held that this is the right result:[6] if '*A*' is empty, then '*Some A is not B*' is indeed

[6] Abelard thought that this is wrong; he held that the particular negative form should be read 'Not every *A* is *B*' instead of 'Some *A* is not *B*'. He blamed the latter "misreading" on Boethius, who wrote the latter form instead of the former, which Aristotle had used. But Aristotle [*PA* 27a36] uses both forms interchangeably.

true. This may not accord well with ordinary speakers of Latin, but logicians insisted that this was the right way to read the proposition. This is part of the regimentation that was mentioned earlier. It may be defended in the way that any regimentation is defended, by claiming that it is useful for logical purposes, even if it does not conform well to ordinary usage.

Of course, this proposal will not work unless other parts of logic are formulated with this in mind. For example, we do not want to include the validity of '*Some A is not B ∴ Some A is*'. This inference, of course, is not considered valid.

I said that these proposals make for theoretical simplicity, but I didn't say how. One simplicity is that it makes it possible to maintain these principles:

> Affirmative categorical propositions are false when any of their main terms are empty.
>
> Negative categorical propositions are true when any of their main terms are empty.

These principles hold for singular propositions and for the forms of categorical propositions we have discussed; they will hold without exception, even when categorical propositions are expanded far beyond the forms that Aristotle discussed.

1.5 Conversions

The square of opposition deals with logical relations among propositions which have the same subject and same predicate. There are also other relations. If a proposition is generated from another by interchanging the original subject and predicate, those propositions are candidates for the logical relation of "conversion":[7]

Simple conversion: A proposition is said to *convert simply* if it entails the result of interchanging its subject and predicate terms. Universal Negative and Particular Affirmative propositions convert simply, resulting in equivalent propositions:

> '*No A is B*' converts simply to '*No B is A*'
> '*Some A is B*' converts simply to '*Some B is A*'.

Conversion per Accidens (Accidental conversion): Whereas simple conversion produces a proposition equivalent to the original, conversion *per accidens* is not symmetric. A universal proposition may be converted *per accidens*: you interchange its subject and predicate terms and change the universal sign to a particular sign (adding a negation, if necessary, to preserve quality). This inference is not reversible.

[7] The conversion laws stated below were proved by Aristotle in *Prior Analytics* I.2. They appear (usually without proof) in every major logic text.

'*Every A is B*' converts per accidens to '*Some B is A*'
'*No A is B*' converts per accidens to '*Some B is not A*'.[8]

Since the universal changes to a particular, this form of conversion is sometimes called "conversion by limitation".[9]

1.6 Syllogisms

Syllogisms form the heart of what is commonly called Aristotelian logic. A syllogism is a special form of argument containing two premises and a conclusion, each of which is a standard form categorical proposition. There are three terms; one of them, the "middle" term, occurs once in each premise. Each of the other terms occurs once in a premise and once in the conclusion. An example of a syllogism is any argument having the following pattern:

Every M is P
Some M is S
$\therefore$ *Some S is P*

The first premise is called the *major* premise and the second is called the *minor*. These individual patterns are called "moods", and they are classified into three "figures". (The first figure includes moods in which the middle term occurs as subject in one premise and predicate in the other; in the second figure the middle term is predicate in both, and in the third figure it is the subject in both.) Aristotle discusses some of the first figure moods as well as all of the second and third figure moods in chapters 4-6 of *Prior Analytics I*, and he discusses the remaining first figure moods in chapter 7, for a total of 19 good moods. There are five additional valid moods that neither he nor medieval authors mention; these are moods which conclude with a particular proposition when its super-alternate universal version is also provable from the same premises.[10] An example of such a mood is:

Every A is B
Every C is A
$\therefore$ *Some C is B*

All 24 valid moods are included in the following table.[11]

[8] Cf. Peter of Spain, *T*I.15, (8). Many authors (including Aristotle) did not mention converting the Universal Negative in this way. This conversion is a consequence of other logical relations assumed in the square.

[9] An additional mode of conversion — conversion by contraposition — is discussed in section 2.5.

[10] Arnauld & Nicole [1662, 142]: ". . . people have been satisfied with classifying syllogisms only in terms of the nobler conclusion, which is the general. Accordingly they have not counted as a separate type of syllogism the one in which only a particular conclusion is drawn when a general conclusion is warranted."

[11] Some authors say that Aristotle has only 14 valid moods; these are the ones he discussed explicitly in chapters 4-6 of the *Prior Analytics*. Five more are sketched in chapter 7. Medieval authors included all 19. Aristotle also provides counterexamples for all invalid moods, for a complete case by case coverage of all possible syllogisms.

All Valid Categorical Syllogisms

Figure 1 *(ch 4)*	**Fig 1: indirect** *(ch 7)*	**Figure 2** *(ch 5)*	**Figure 3** *(ch 6)*
Barbara (26a1) Every M is P Every S is M Every S is P	**Baralipton** Every M is S Every P is M Some S is P	**Cesare** (27a5) No P is M Every S is M No S is P	**Darapti** (28a18) Every M is P Every M is S Some S is P
	Celantes (29a26) No M is S Every P is M No S is P		**Felapto** (28a27$*$) No M is P Every M is S Some S is not P
Celarent (26a3) No M is P Every S is M No S is P		**Camestres** (27a8) Every P is M No S is M No S is P	**Disamis** (28b7) Some M is P Every M is S Some S is P
	Dabitis (29a26) Every M is S Some P is M Some S is P		**Datisi** (28b12$*$) Every M is P Some M is S Some S is P
Darii (26a23) Every M is P Some S is M Some S is P	**Fapesmo** (29a22) Every M is S No P is M Some S is not P	**Festino** (27a31) No P is M Some S is M Some S is not P	**Bocardo** (28b18$*$) Some M is not P Every M is S Some S is not P
Ferio (26a25) No M is P Some S is M Some S is not P	**Frisesomorum** (29a22) Some M is S No P is M Some S is not P	**Barocho** (27a36) Every P is M Some S is not M Some S is not P	**Ferison** (28b33) No M is P Some M is S Some S is not P

The unmentioned moods lack names. Chapter numbers indicate the chapter of *Prior Analytics I* in which Aristotle discusses the figure. Specific reference codes indicate where in the chapter he discusses the mood. Transposing the premises of the listed forms also yields good syllogisms; the transposed forms are not treated as independent moods.

The names of the individual moods are from Peter of Spain. [Peter of Spain *T*IV.13 (46)] These names are post-Aristotelian inventions; they are cleverly devised so as to encode useful logical information. Medieval students were expected to memorize a verse consisting of the names:

Barbara Celarent Darii Ferio Baralipton
Celantes Dabitis Fapesmo Frisesomorum.
Cesare Cambestres Festino Barocho Darapti.
Felapto Disamis Datisi Bocardo Ferison.

The names contain codes which show how to mimic Aristotle's reduction of all moods to the first figure direct modes.[12]

[12]Here are Peter's instructions for doing this (translation from [Kretzman and Stump, pp. 224–25 and 222]):

> "In these four verses there are nineteen words that are associated with the nineteen moods of the three figures . . .
>
> "It is important to know that the vowels A, E, I, and O stand for the four types of propositions. The vowel A stands for a universal affirmative; E for a universal negative; I for a particular affirmative; and O for a particular negative.
>
> "Again, there are three syllables in each word (if there is any more, it is superfluous, except for M, as will be clear later). The first of these three syllables stands for the major proposition of the syllogism; the second stands for the minor; the third for the conclusion. For example, the first word — Barbara — has three syllables, in each of which A is used; the three occurrences of A signify that the first mood of the first figure consists of two universal affirmative premises resulting in a universal affirmative conclusion. (The vowels used in the other words should also be understood in this way.)
>
> "Again, it is important to know that the first four words of the first verse and all the other subsequent words begin with these consonants: B, C, D, and F. In this way we are given to understand that all the moods that a word beginning with B stands for should be reduced to the first mood of the first figure; all the moods signified by a word beginning with C, to the second mood of the first figure; D, to the third mood; F, to the fourth.
>
> "Again, where S is used in these words, it signifies that the proposition that the immediately preceding vowel stands for requires simple conversion. And P signifies that the proposition requires conversion *per accidens.* And where M is used, it signifies that the premises require transposition. (Transposition is making the major premise the minor premise, and vice versa.) And where C is used [after a vowel] it signifies that the mood that word stands for should be proved by reduction *per impossibile.*
>
> "The fourth mood [of the second figure] consists of a universal affirmative and a particular negative premise, resulting in a particular negative conclusion. For example [BAROCHO],
>
> Every man is an animal;
> some stone is not an animal;
> therefore, some stone is not a man.
>
> And this is reduced to the first mood of the first figure [BARBARA] by a reduction *per impossibile.*

[Reduction *per impossibile]*

1.7 Quantifying the Predicate Term; Equipollences

The above is mostly Aristotle's logic with some of its consequences spelled out. Some medieval authors in the 1200's expanded the stock of Aristotle's categorical propositions by allowing quantifier signs to combine with predicate terms, and allowing *not*'s to occur more widely, so as to produce propositions like '*No A is not every B*', or '*Not every A some B is not*'. I call Aristotle's forms 'standard Aristotelian categorical propositions', and I call the expanded forms 'categorical propositions'. The forms of categorical propositions will be further extended in section 3.

Along with these expanded forms, writers added a series of equipollences — logically equivalent pairs of propositions that differ only in having one part replaced by an "equivalent" part. These are like contemporary rules of quantifier exchange, such as the equivalence of '$\forall x$' with '$\sim \exists x \sim$'.

"To reduce [a syllogism] *per impossibile* is to infer the opposite of one of the premises from the opposite of the conclusion together with the other premise. For suppose we take the opposite of the conclusion of this fourth mood (namely, 'Every stone is a man') together with the major premise and construct a syllogism in the first mood of the first figure in this way:

> Every man is an animal;
> every stone is a man;
> therefore, every stone is an animal.

"This conclusion is the opposite of the minor premise of the fourth mood. And this is what it is to prove something [by reduction] *per impossibile*."

These instructions work perfectly provided that conversion by limitation is used in the correct order; from particular to universal in premises, and from particular to universal in conclusions.

Sherwood's Equipollences:[13] Replacing a phrase below by another on the same line yields a logically equivalent proposition:[14]

every A	*no A not*	*not some A not*
no A	*not some A*	*every A not*
some A not	*no A*	*not every A not*
some A not	*not no A not*	*not every A*

An example is the equivalence of the old form '*No A is B*' with the new form '*Every A is not B*'. It is also assumed that an unquantified predicate term '*B*' is equivalent to the particular '*some B*', so the old form '*Some A is B*' is equivalent to '*Some A is some B*', which is equipollent to '*Some A is not no B*'. Likewise, '*No A is every B*' is equivalent to '*Every A is not every B*' and also to '*Every A some B is not*'.

Notice that application of any of these equipollences leaves an affirmative proposition affirmative, and a negative proposition negative. (Suppose that '*no*' and '*not*' are (the only) negative signs. A proposition is affirmative if it contains no negative signs or an even number of negative signs; otherwise it is negative.) They thus preserve the principle that affirmatives with empty main terms are false, and negatives with empty main terms are true.

Suppose that we take Aristotle's original list of four (non-singular; non-indefinite) categorical propositions and add the quantifier sign '*every*' to the predicate. This gives us a list of eight forms:

[13]This little chart is from William Sherwood I.19. In applications to categorical propositions the actual form would have the normal Latin word order '*not is*' instead of the English order '*is not*'. It is possible to be precise about what propositions are included in this stock of what I am calling basic categorical propositions. First, the following rules produce categorical propositions whose copula occurs at the end of the proposition. (These are meaningful in Latin.):

1. A denoting phrase consists of a quantifier sign ('*every*', '*some*', '*a*(*n*)' or '*no*') followed by a common noun.
2. '*is*' is a partial categorical proposition
3. If ϕ is a partial categorical proposition with zero or one denoting phrases in it, and if δ is a denoting phrase, then δ followed by ϕ is a partial categorical proposition
4. If ϕ is a partial categorical proposition, so is 'not ϕ'.
5. A categorical proposition is any partial categorical proposition with two denoting phrases.

Examples: '*is*' ⇒ '*no donkey is*' ⇒ '*some animal no donkey is*'.
'*is*' ⇒ '*some donkey is*' ⇒ '*not some donkey is*' ⇒ '*an animal not some donkey is*'
To put the verb in its more natural order in the middle:

> If ϕ is a categorical proposition which ends with '*is*', if there is a denoting phrase immediately to its left, then they may be permuted

Example: 'some animal not no donkey is' ⇒ 'some animal not is no donkey'
(For English readers, change the Latin word order 'not is' to 'isn't': ⇒ 'some animal isn't no donkey.)

[14]In Latin the negation naturally precedes the verb. For applying these equipollences the negation coming before the verb can be treated as if it came after, so that '*No A not is some B*' can be treated as '*No A is-not some B*' ≈ '*No A is no B*'. A similar provision applies to English, where negation follows the copula '*is*'.

Every A is B	Every A is every B
No A is B	No A is every B
Some A is B	Some A is every B
Some A is not B	Some A is not every B

Using William's equipollences together with double negation, every proposition in the expanded notation is equivalent to one of these eight forms.

Notice that throughout this extension of the scope of what counts as a categorical proposition the quantifier signs have not been supplemented; they are '*every*', '*no*', and '*some*', with the indefinite construction (which uses the indefinite article in English, and nothing at all in Latin) treated as if it is equivalent to one with '*some*'.

The semantics of these propositions is straightforward. We can view a categorical proposition in this form as consisting of the copula preceded by some denoting phrases and some negations, each of which has scope over what follows it. This can easily be given a logical form that is familiar to us by flanking the verb with variables, supposing that the preceding denoting phrases bind these variables. Examples:

Every donkey is an animal ⇒
(Every donkey x) (an animal y) x is y

Some donkey is not an animal ⇒
(Some donkey x) not (an animal y) x is y

Some donkey an animal is not ⇒
(Some donkey x) (an animal y) not x is y

No donkey is not an animal ⇒
(No donkey x) not (an animal y) x is y

No animal is not every donkey ⇒
(No animal x) not (every donkey y) x is y

It is not difficult to formulate a modern semantical theory that would apply to the forms that occur on the last lines of these examples, provided that we keep in mind that an affirmative proposition is false when any of its main terms are empty, and a negative one is true.[15]

1.8 Molecular Propositions

Certain combinations of categorical propositions are called *hypothetical* propositions; these include combinations that we call molecular. Their development came

[15]It is often suggested that all the Aristotelian forms can be symbolized in the monadic predicate calculus. The expansions mentioned here require the use of monadic predicate logic with identity, and require using quantifiers within the scopes of other quantifiers.

from the Stoics, not from Aristotle; Boethius wrote about them, and they formed a standard part of the curriculum. They typically included six types:

Conjunctions
Disjunctions
Conditionals
Causals
Temporal Propositions
Locational Propositions

Conjunctions and disjunctions are just what one expects; they are combinations of propositions made with 'and' (*et*) and 'or' (*vel*), or similar words. A conjunction is true iff both of its conjuncts are true, and false if either or both are false; a disjunction is true iff either or both of its disjuncts are true, and false if both disjuncts are false. So they were given modern truth conditions, even including the fact that 'or' was usually treated inclusively.[16]

Conditionals are made with 'if' (*si*), or equivalent words. A conditional was usually taken to be true if it is necessary that the antecedent not be true without the consequent also being true.[17] This makes the truth of a conditional equivalent to the goodness of the related consequence, as some writers observed.[18] Some authors also discuss "ut nunc" (*as of now*) conditionals, which are equivalent to our material implication.[19] But the default meaning of a conditional is that it is true if it is impossible for the antecedent to be true and the consequent false.

Causal propositions are those made by linking two propositions with 'because' (*quia*) or an equivalent word. Such a causal proposition is true if the one proposition causes the other. However, 'cause' must not be read here as efficient causation; it is more like a version of 'because'. An example of such a proposition is 'Because Socrates is a man, Socrates is an animal'. [Ockham *SL* II.34]

A temporal proposition is made from smaller propositions using 'while' or 'when' (*dum*, *quando*), and a locational proposition is one using 'where' (*ubi*). The former is said to be true if both component propositions are true for the same time. The latter is true if the location of the event mentioned in the one proposition is the same as that in the other. These, along with causal propositions, employ notions that are not part of the basic theory discussed here.

[16]Sherwood *SW* XXI.1 (141) holds that ". . . '*or*' is taken sometimes as a disjunctive and at other times as a subdisjunctive. In the first case it indicates that one is true and the other is false; in the second case it indicates solely that one is true while touching on nothing regarding the other part." Later, Paul of Venice *LP* I.14 (132) clearly gives inclusive truth conditions: "For the truth of an affirmative disjunctive it is required and it suffices that one of the parts is true; e.g. 'you are a man or you are a donkey'. For the falsity of an affirmative disjunctive it is required that both parts be false; e.g., 'you are running or no stick is standing in the corner'."

[17]Ockham *SL* II.31; Peter of Spain *T*I.17. Sherwood *IL* I.18 gives truth conditions that are not clearly modal: "whenever the antecedent is [true], the consequent is [true]."

[18]Ockham ibid.

[19]Cf Buridan *TC* I.4.7-7.12. He noted, e.g., that "if the antecedent is false, though not impossible, the consequence is acceptable *ut nunc*".

In modern logic, molecular propositions are thought of as fundamental, since they are used to form general statements, such as

Every A is B $\quad \forall x[Ax \rightarrow Bx]$
Some A is B $\quad \exists x[Ax \& Bx]$

Since connectives are not used to analyze categorical propositions in this way in medieval logic, hypothetical propositions usually receive very little discussion after Abelard.

1.9 Validity and Good Consequences

I have been speaking of "valid" moods of syllogism, whereas Aristotle merely divided potential syllogisms into syllogisms (or "deductions") versus patterns that aren't syllogisms. His criterion for a pattern being a syllogism is:

> a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so. I mean by the last phrase that it follows because of them, and by this, that no further term is required from without in order to make the consequence necessary.[20]

It is unclear what¸ if anything, the second proposition adds to the first.[21] Much medieval logical theory after Abelard seems based on the assumption that the second proposition is merely explanatory, in that the criterion for a "good consequence" is simply that the conclusion follows of necessity from the premises. This is a modal notion of goodness of consequence. There are thus good consequences that are not formal, because of non-formal modality, such as:

Some donkey is grey
∴ Some animal is grey

The conclusion follows of necessity since it is necessary that any donkey be an animal. These examples were taken very seriously.

The word 'consequence' was used to cover two different notions. One is an argument (where a good argument is one in which the conclusion follows necessarily from the premises). Another is a conditional proposition (which is good iff it itself is necessary). There is much debate in the secondary literature over whether Aristotle used arguments or conditionals. I have nothing to say on this. It is clear however that medieval writers tend to use either notion, usually without bothering to distinguish them. For example, the following wordings tend to be used interchangeably:

[20] Aristotle, *Prior Analytics* I.1.

[21] The assumption that something more is needed is crucial to Aristotle's discussion of the fallacies of begging the question and of non-cause as cause in his *Sophistical Refutations* 168b22-26. But in most cases, such as his reduction of all categorical syllogisms to the first figure, he ignores this consideration.

If A and B then C
A, B; therefore C

Since the goodness of an argument and the goodness of the related conditional are equivalent, it doesn't matter which you mean.

The Aristotelian/Medieval modal notion of goodness of argument is purely modal, and it does not appeal to the notion of logical form. However, Aristotle's principles of conversion and his theory of syllogisms look like classifications of good and bad arguments in terms of form. Some later medievals (e.g. [Buridan *TC* 1.4.1]) introduced the notion of a *formally* good consequence; this is a good consequence which remains good no matter how we alter its terms (if we do so uniformly). In practice, the contemporary non-modal notion of formal validity and the medieval notion of formal goodness coincide in how they classify arguments that themselves do not contain modal terms. Because it is cumbersome to speak of formally modally good arguments instead of valid ones, I'll just use the modern term 'valid' as if this is what medieval authors are discussing. I think that this will do no harm, but the reader should be alert for such a possibility.

1.10 Theory of Consequences

The consequence relation itself was well studied in the medieval period. Some very interesting work was done by Abelard on a kind of relevance logic, but the period after that was dominated by what we now call classical logic. Principles are endorsed such as [Buridan *TC* I]:

From a contradiction anything follows.
A necessary proposition follows from anything.
If A is a consequence of B, and B of C, then A is a consequence of C.
Nothing impossible follows from what is possible.
Nothing false follows from what is true.
If 'A, B therefore C' is a good consequence, and if B is necessary, then 'A therefore C' is a good consequence.

2 EXTENDING CATEGORICAL PROPOSITIONS

In contemporary logic, if we wish our theory of validity to apply to sentences not already explicitly treated, we extend the scope of the theory either by showing how new sentences can be symbolized using the resources of the existing theory, or we add new forms (such as modal operators) to the logical notation and explain how their semantics is to work. Since the formal notation of medieval logic is regimented Latin, the task here is to use regimented Latin as an equivalent to non-regimented propositions, or to expand the regimented Latin. Works on logic devote a substantial amount of discussion to this enterprise.

The categorical propositions delimited in section 1 all have the copula 'is' as their verb, and the subject and predicate terms are singular or common nouns.

The purpose of this section is to say how to expand the class of categorical propositions. We need to say how to treat verbs other than the copula, and subjects and predicates that are adjectives, or participles, or that are modified by relative clauses, or that are possessives, and so on.

2.1 Adjectives

Aristotelian logic is awkward when applied to terms that are adjectives. If you apply simple conversion to:

Some donkey is grey

you get

Some grey is [a] donkey

which is ungrammatical; a quantifier word like '*some*' must combine with a noun, not with an adjective. This problem has a solution in Latin, in which it is possible to "substantivate" an adjective — which is to use it as a common noun with a neuter gender. The sentence:

Every woman is tall

contains the adjective '*tall*' which, in Latin, would bear a singular nominative feminine inflection to agree with '*woman*'. But in this sentence:

Every tall is grey (meaning '*Every tall thing is grey*'.)

it is substantivated; it bears a singular nominative neuter inflection of its own, and it is grammatically a noun. So it can be treated as part of the regimented notation as already understood. (In this example '*grey*' is still an adjective; it has an adjectival inflection that agrees with the gender and number of '*tall*'.)

What does such a substantivated adjective supposit for? The obvious option is that substantivated '*tall*' supposits (with respect to a time) for each thing which is tall (at that time). This is the common assumption, which is adopted here.[22]

In ' *Every tall is grey* ' the '*grey*' would not normally be substantivated; it would remain an adjective with adjectival inflections. Presumably, it is here equivalent to its substantivated version; it supposits (with respect to a time) for each thing which is grey (at that time).

2.2 Extending the Theory to Intransitive Verbs

Having explained adjectives, there is a sense in which intransitive verbs have already been accounted for. At least, the contents of these verbe already appear.

[22]Buridan *SD* 8.2.4 (648): "'white [thing]' [*album*], amounts to, 'white thing' [*res alba*], or 'white being' [*ens album*]." In this quote '*album*' is the substantivated version of '*white*' and '*res alba*' means literally '*white thing*', wherein '*white*' is an adjective.

Medieval logicians held that the use of an intransitive verb is equivalent to the use of the copula along with its present participle. For example, they said that '*Brownie runs*' is equivalent to '*Brownie is running*'. This is not the progressive form of the verb '*run*', as it is in English; rather it is the copula followed by the participle '*running*', used adjectivally. It is not clear that this is normal Latin, but it is what they insisted on, and I will go along with this. As a result, if you want to treat a sentence with an intransitive verb, you can take its form to be that of the copula followed by the verb's present participle.[23] Further, in Latin present participles used with '*is*' are adjectives. So in Latin, '*Brownie is running*' has the same grammatical analysis as '*Brownie is grey*'; it's a subject followed by the copula followed by an adjective. And we saw above how ordinary adjectives used with the copula are equivalent to common nouns used with the copula. So we again have a procedure for applying the logical theory to a new class of items. Intransitive verbs are equivalent to a combination of the copula with a participle,[24] which is equivalent to a combination of the copula with an adjective, which is equivalent to a combination of the copula with a substantivated adjective, which is equivalent to a combination of the copula with a common noun. The theory thus needs no emendation.

When the copula '*is*' is used with only one term, as in '*A man is*', it is usually analyzed as an intransitive verb, so that '*A man is*' is equivalent to '*A man is [a] being*'.[25] The word '*being*' here signifies everything that there is; in a present tense non-modal proposition it supposits for every presently existing thing.

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[23]Paul of Venice *LP* I.6. "A categorical proposition is one which has a subject, a predicate and a copula as its principle parts, e.g., 'a man is an animal', 'man' is the subject; 'animal' is the predicate, and the copula is always that verb 'is' because it conjoins the subject with the predicate. And if someone says 'a man runs' (*homo currit*) is a categorical proposition but it does not have a predicate, it can be replied that it has an implicit predicate, viz., 'running'. This is clear by analyzing that verb 'runs' into 'I am', 'you are', 'it is' and its participle."

[24]Buridan 1.3.2 "The third section expounds the above description of categorical proposition, illustrating it by the proposition: 'A man runs', in which the name 'man' is the subject, and the verb 'runs' is the predicate. Concerning this point we should note, as [Peter of Spain] immediately remarks, that the verb is not the predicate, strictly speaking, but it is either the copula joining the predicate to the subject or implies in itself the copula and the predicate together. For the verb 'is' as the third adjacent [*tertium adiacens*] is the copula, and what follows is the predicate."

[The verb is "third adjacent" when it is a third expression combining with two others.] See the continuation of this quote in the next note.

[25]The previous quote continues: "But the verb 'is' occurs as second adjacent [*secundum adiacens*], when I say 'A man is'; but then, like any other verb, it implies in itself the copula with the predicate or with the principal part of the predicate; therefore, to make the subject, predicate and copula explicit, such a verb has to be analyzed into the verb 'is' as third adjacent, provided that the proposition is assertoric and in the present tense, and into the participle of that verb, as for example, 'A man runs' is to be analyzed into 'A man is running', and similarly, 'A man is' into 'A man is a being'."

[The verb is "second adjacent" when it is a second expression combining with one other.] The noun '*being*' must be understood to be a logically special common term, which signifies everything.

2.3 Extending the Theory to Transitive Verbs

Medieval authors often speak as if transitive verbs can be treated exactly as intransitives; just replace the verb with a copula and a participle, treating the participle as a general term that supposits for whatever it is that the verb applies to. For example, Buridan uses this example [*SD* 1.3.3]:

Every horse a man sees ⇒ *Every horse a man is seeing*

If we treat the participles of transitive verbs just like those of intransitives, then, grammatically, things seem to work OK. But something is missing: the logical relationship between a seeing-thing and what it sees. Take the sentence '*Cicero sees every donkey*'. This becomes '*Cicero is every donkey seeing', or 'Cicero is [of] every donkey a seeing-thing*'. But what does the general term '*seeing*' (or '*seeing-thing*') supposit for? The problem is that what this general term supposits for depends on the previous term '*donkey*'. The meaning is: '*Cicero is with-respect-to-every-donkey seeing it*'. But there is no '*it*' in the Latin. Somehow this dependency on the '*donkey*' needs to be built into the analysis. Whenever a transitive verb is analyzed in this way, a dependency on the direct object of the verb needs to be provided for. But turning '*sees*' into '*is seeing*' hides, rather than revealing, the dependency.

As far as I know, medieval authors did not notice this dependency in the supposition of terms from transitive verbs. This is an inadequacy in their theorizing. It may have passed unnoticed because the oversight had little effect on the logic that they developed. Mostly, examples were chosen in which no fallacies resulted. (There is an exception to this; see section 7.6.)

I will call such dependent terms *parasitic*. It would be easy to accommodate parasitic terms in a notation with explicit variables. First, a transitive verb may be moved to the end of the sentence just as we do with the copula. So we first convert '*Every donkey sees no horse*' to '*Every donkey no horse sees*'. The '*sees*' is then converted into '*seeing is*', to get '*Every donkey no horse seeing is* '. The participle of the transitive verb must then be completed with an additional variable, which is to be bound by the denoting phrase that is the direct object. Suppose that for each such participle there is a parasitic term, in which 'v' is a variable:

seeing-v-thing
owning-v-thing
etc

where a '*seeing-v-thing*' is a thing that sees v. These terms contain variables which must be bound. As an illustration, the proposition '*Cicero sees every donkey*' turns into '*Cicero every donkey seeing is*', and corresponds to the formal analogue:

(Cicero x) (every donkey y) (some seeing-y-thing z) x is z

Thus, whenever a parasitic term is used, it carries a variable that is to be bound by some other denoting phrase. (When a parasitic term occurs, a special grammatical

case shows up on the denoting phrase that binds the variable in it; the parasitic term itself is not grammatically affected. For example, in the above example, '*every donkey*' receives accusative case, while '*some seeing-y-thing*' is simply nominative.)

Turning ordinary transitive verbs into participles plus copulas maintains the principle that a categorical proposition consists of terms plus some syncategorematic signs plus a copula. In this sense, one can maintain that the copula is the only true verb. However, it does not maintain the principle that a categorical proposition contains two terms, the subject and the predicate. For analyzing a transitive verb produces a new term. For example, in '*Every horse a man is seeing*' there are three terms: '*horse*', '*man*' and '*seeing*'. This becomes important when truth conditions are stated for categorical propositions, since they are almost always stated for propositions with only a subject and a predicate.[26] One might hope to see truth conditions for a proposition with three terms, or a recursive specification of truth conditions that would extend to more than two terms. So far as I know, nothing like this appears in any currently edited manuscript.[27]

2.4 Using unanalyzed verbs

We have seen how a verb other than the copula can be decomposed into a copula plus a common term used indefinitely. The idea is neat in theory, and it was often used to illustrate the claim that there is only one real verb, the copula. However, in practice, authors tended to treat verbs as if they could be taken to form categorical propositions without being analyzed into a denoting phrase and a copula. I will often follow this practice in quotes and examples.

2.5 Infinitizing Negation and Conversion

In English we have a prefix '*non*' which goes on the front of a noun to indicate things that the noun does not apply to, as in '*Every nonstudent had to pay*'. There is no danger of this prefix being confused with the negation '*not*', because they are spelled differently and do not occur in the same places in sentences. In Latin however (and also in Aristotle's Greek) these two items were spelled the same: '*non*'. Further, if this particle occurs in front of a noun there is no way to tell from the spelling whether it is meant to be prefixed to that noun or whether it is merely a preceding word. So some effort had to be put into distinguishing the readings of a sentence such as

Non homo est animal

which is ambiguous between '*Not: a man is an animal*' and '*A non-man is an animal*'. Aristotle called a term prefixed with '*non*' an "infinite" term. (He did not explain the meaning of 'infinite'.) Medieval authors called this prefix "infinitizing negation" and called the separate word '*non*' "negating negation".

[26]Indeed, they tend to be stated only for the original four Aristotelian forms.
[27]But much work is still undiscovered and/or unedited.

Infinitizing negation does not have the effect that negating negation has in turning affirmative into negative propositions and vice versa; '*Some donkey is grey*' is affirmative, and so is '*Some donkey is non-grey*'. And although this proposition is negative: '*Some donkey is not an animal*', this one is affirmative: '*Some donkey is a non-animal*'. The effect of the '*non*' is confined to the complex term that it occurs in.

Contraposition and Obversion: Recall the theory of conversion inherited from Aristotle: conversion means the interchange of the subject and predicate terms of a categorical proposition. *Simple conversion* was universally recognized as being valid for the universal negative and particular affirmative forms, and *conversion by limitation* valid for universal propositions. Along with simple conversion and conversion by limitation, many authors discussed *conversion by contraposition*; you interchange subject and predicate and prefix each with an infinitizing negation. The conversion by contraposition of the universal affirmative and particular negative forms would be:

Every A is B	$\approx$	*Every non-B is non-A*
Some A is not B	$\approx$	*Some non-B is not non-A*

Several early authors endorse the validity of conversion by contraposition for universal affirmatives and particular negatives.[28] Others point out that this is not correct, since the affirmative form has existential import for the subject and predicate terms, and the contrapositives of such forms have existential import for the negatives of those terms. Buridan [*SD* 1.6.4] points out:

> "We should note that this [type of] conversion is not formal, for although it is valid assuming the constancy of terms, with terms supposing for nothing it need not be valid. For example, 'Every man is a being; therefore, every non-being is a non-man' is not valid, for the first proposition is true, and the second is false, because it is an affirmative the subject of which, namely the infinite term 'non-being', supposits for nothing. Similarly, 'Some chimera is not a man; therefore, some non-man is not a non-chimera' is not valid, for the first proposition is true and the second is false, since the contradictory of the second, namely 'Every non-man is a non-chimera', is true, for every non-man is a being and every being is a non-chimera."

A similar principle is the one that was later called *obversion*. This is the principle that states that you can change a proposition to an equivalent form if you change

[28] Contraposition is discussed in several 12^{th} and early 13^{th} century anonymous texts edited in [De Rijk, 1967], where it is seen as a type of *equipollence*. It is endorsed in several of these: *Excerpta Norimbergenses* (138-39), *Ars Burana* (190), *Tractatus Anagnini* (238), *Introductiones Parisienses* (362), *Logica: Ut dicit* (385), *Logica: Cum sit nostra* (426), *Dialectica Monacensis* (478). Another text, *Ars Emmerana* (157) endorses contraposition, but then says that it does not hold for the particular negative unless understood with "constancy," a term that had to do with assuming instances of the terms in question. Sherwood *IL* 3.3 (59) also endorses it.

it from affirmative to negative (or vice versa) and change the predicate term from finite to infinite (or infinite to finite). Some examples are:

Every S is P	⇒	*No S is non-P*
Every S is non-P	⇒	*No S is P*
Some S is P	⇒	*Some S is not non-P*
Some S is non-P	⇒	*Some S is not P*

This is not a form of conversion because the subject and predicate terms are not interchanged, but the inferences are similar. It is apparent that these inferences are valid when moving from affirmative to negative, but not in the reverse direction, because the terms may be empty (making the negative propositions true and the affirmative ones false). Buridan [*TC* I.8.107 (226)] makes this clear, stating:

> "From any affirmative there follows a negative by changing the predicate according to finite and infinite, keeping the rest the same, but there is no formal consequence from a negative to an affirmative, although there is a consequence under the assumption that all of the terms supposit for something."

The good direction he gives as:

> *Every B is A*; therefore *no B is non-A*.

The fallacious direction is illustrated by

> *A chimera is not a man*; therefore *a chimera is a non-man*.

(Aristotle discussed some instances of obversion. He endorsed many valid forms and he did not endorse the invalid ones, but he did not comment on these.[29] Some medieval writers before Buridan accepted the fallacious versions, and some did not.)

The semantics of 'non-'. There are various ways to give a semantics for '*non-*'. Since it precedes a single word, one might expect it to alter the signification of that term, so that '*non-donkey*' would signify everything that is not a donkey, and it would behave just like a simple term. However, '*non*' can precede any common noun, including nouns that correspond to complex concepts which are themselves made out of mental words. So infinitizing negation may modify complex terms, for which signification may not be defined.

Peter of Spain [*S* 2.8-15 (77-87)] argues that an infinitized term is ambiguous, for '*non*' can either make a term '*non-man*' that is said of absolutely everything that is not a man, including non-beings, or it can make the term be said of every *being*

[29] In *De Interpretatione* 10. The interpretation of Aristotle is not uncontroversial. The Ackrill translation attributes one error to Aristotle: it translates him as asserting the sameness of signification of 'Every non-A is non-B' with 'No non-A is B' (20a.38-39). This, however, is a translator's option; one can just as adequately translate the former as 'Every non-A is not B'; this is a negative form which would lack existential import.

that is not a man. Because of the restrictions by tenses and modalities (section 5) this ambiguity is usually cancelled out in actual use. For example, in the sentence '*Every non-donkey is grey*' the present tense of the sentence (and its lack of a modal word) restricts the supposition of the subject to presently existing things. And when you limit all the things that aren't donkeys, including nonbeings, to the existing things among them, you get the same class as when you limit the beings that aren't donkeys to existing things. It is an interesting question as to whether Peter's ambiguity is always cancelled out in this way.

2.6 Complex Terms with Modifiers

We have been considering terms consisting of a single word — a noun or something behaving like one. Some terms are complex. The simplest examples are nouns modified by adjectives or participles: 'grey donkey' or 'running donkey'. In simple cases their semantics is clear: 'grey donkey' stands with respect to a time for whatever 'grey' and 'donkey' both stand for with respect to that time. (Or perhaps for whatever the substantivated version of 'grey' and 'donkey' both stand for with respect to that time.) The combination of 'running' with 'donkey' is similar. In more complicated cases a more roundabout account needs to be given. Probably the complex term 'donkey seeing every horse' needs to be counted as a "reduced relative clause" — 'donkey which is seeing every horse'. We turn now to relative clauses.

2.7 Relative Clauses and their derivatives

2.7.1 Restrictive Relative Clauses

Relative clauses come in two forms: restrictive and nonrestrictive:

The woman who left early was miffed.	RESTRICTIVE
The woman, who left early, was miffed.	NONRESTRICTIVE

We will only discuss restrictive relative clauses. Restrictive relative clauses make complex terms. In distinguishing restrictive from nonrestrictive relative clauses Buridan says:

> [In 'A man who is white is colored'] there is one predicate here, namely, 'colored', which by the mediation of the copula is predicated of the whole of the rest as of its subject, namely, of the whole phrase: 'man who is white'; for the whole phrase: 'who is white' functions as a determination of the subject 'man'. And the case is not similar to 'A man is colored, who is white', for there are two separate predicates here, which are predicated separately of their two subjects, and there is not a predicate here which would be predicated by the mediation of one copula of the whole of the rest. And although these [propositions] are equivalent, they are not equivalent if we add a universal sign. For

> positing the case that every white man runs and there are many others who do not run, the proposition 'Every man who is white runs' is true, and is equivalent to: 'Every white man runs'; but the proposition 'Every man, who is white, runs' is false, for it is equivalent to: 'Every man runs and he is white'.[30]

In both English and Latin a restrictive relative clause immediately follows a common term; the clause itself has the form of a relative pronoun followed by a full proposition missing a denoting phrase. For example:

> *woman whom some man sees in the corner*

consists of the common term '*woman*' followed by the relative pronoun '*whom*' followed by a partial proposition which is missing a denoting phrase:[31]

> *some man sees ___ in the corner.*

Some examples of propositions containing complex terms with relative clauses are:

> *Some* ***donkey which __ sees every animal*** *kicks Socrates*
> *Some* ***donkey which every animal sees __*** *kicks Socrates*
> *Socrates kicks some* ***donkey which __ sees every animal***
> *Socrates kicks some* ***donkey which every animal sees__***

Although no detailed theory was given, it is apparent from writers' usage (and from inspection of natural language usage itself) that a whole complex phrase stands for a thing x iff the modified term stands for it x and x satisfies the gappy proposition that follows the term (with 'x' itself occurring in the gap). The complex term '*woman whom some man sees in the corner*', stands for a thing x with respect to a time iff the term '*woman*' stands for x with respect to that time and some man sees x in the corner with respect to that time.

2.7.2 Whiz deletion

Relative clauses often appear in reduced form; in addition to 'Some donkey which is seeing every horse is happy' we have 'Some donkey seeing every horse is happy'. This happens whenever the initial relative pronoun is immediately followed by the copula. This possibility is neatly captured by an old idea in transformational grammar called "whiz deletion" (short for 'wh- + is' deletion). Specifically, this

[30] Buridan *SD* 1.3.2 The last example he gives is discussed in Sherwood *SW* I.10, 28-29. Sherwood explains the difference between the two types of relative clauses in terms of Aristotle's notion of "compounded and divided" in which the restrictive reading is the one in which the clause is compounded with the modified noun, and the nonrestrictive reading is the one in which they are divided.

[31] The usual explanation for this is that '*wh-*' originates within the propositional clause, and moves to the front to become a relative pronoun, leaving a blank where it was. This explains why relative pronouns get the grammatical case that a denoting phrase would have in the position from which they move.

rule says that one may optionally delete any relative pronoun followed immediately by 'is' (or any other form of the copula). It is clear that the examples just given are related in this way:

> *Some donkey which is seeing every horse is happy* ⇒
> *Some donkey seeing every horse is happy.*

So we add the principle that:

> If a relative clause begins with '*which is*' or '*which was*' or '*which will be*' then that expression may be deleted. The resulting string of words has the same semantics as the clause without deletion.

There is one additional constraint on reduced relative clauses. A non-reduced relative clause has a main verb which has a tense of its own. So the proposition:

> *Some woman who was seeing a donkey is running*

is true with respect to a time t if and only if a woman who saw a donkey *before* t is running at t. Alternatively,

> *Some woman who is seeing a donkey was running*

is true with respect to a time t if and only if a woman who sees a donkey at t was running *prior to* t. However, this mismatch of times cannot occur with a reduced relative clause; the proposition

> *Some woman seeing a donkey is running*

cannot mean that a woman who previously saw a donkey is running. Briefly, the unpronounced tense in a reduced relative must be identified with the time at which the reference of the whole phrase is evaluated, not with respect to some other time.

2.8 The Genitive

Many examples discussed by medieval logicians contain genitive ("possessive") constructions. The genitive shows up in English in two different forms, as a genitive ending and as a prepositional phrase construction:

> *This is the woman**'s** donkey*
> *This is the donkey **of** the woman.*

In Latin there is only one construction: the genitive noun (the one that in English gets the possessive ending or is the object of the preposition '*of*') is inflected for genitive case. The difference from the English use of "-'s" is that in Latin the possessive noun needn't immediately precede what it modifies. The following word-orders are all OK in Latin:

Every woman's donkey is lame..
A donkey every woman's is lame.
Some woman's some horse sees every donkey

in which the last proposition is to be read something like: Regarding some woman, some horse sees every donkey of hers. These forms may be confusing in English; to increase comprehension I will always use the '*of*' form in the English transliterations. Thus, although the following are not standard English, you should easily be able to figure out what they mean, so long as you keep in mind that the left-to-right ordering of the denoting phrases corresponds to their logical scope:

Of-every woman a donkey is lame
A donkey of-every woman is lame
Of some woman some horse sees every donkey

where the last proposition is ambiguous regarding whether it is the horse or the donkeys that belong to the woman.

2.8.1 What the Genitive means

Some common nouns, such as 'mother', 'owner', . . . are inherently relational; others such as 'woman', 'chair' are not.

Non-relational nouns: Typically the genitive construction used with a non-relational noun indicates some kind of possession ('a horse of Fred's') or something analogous to possession ('Mary's job'), or almost any other kind of relation that can be doped out from context ('Mary's hill' = the hill Mary has been assigned to climb).

Relational nouns: Usually the genitive construction used with a relational noun indicates the relation conventionally associated with the noun; 'Mary's mother' usually means the female person who gave birth to Mary. Relational nouns also have nonrelational uses, as in: 'Four mothers showed up'. I assume that these non-relational uses are best construed as homonyms; 'mother' can be a non-relational noun meaning e.g. "mother of someone". These non-relational nouns then also enter into the first kind of genitive construction, so that 'Mary's mother' can mean the mother (of someone) to whom Mary has been assigned as a case worker.[32]

For simplicity I'll concentrate on examples involving non-relational nouns.

When a genitive is used, the "possessed" term is parasitic, just like a participle of a transitive verb. In '*Cicero's donkey is an animal*' the term '*donkey*' does not stand for donkeys in general; it is restricted to standing for donkeys owned by Cicero. So the semantics has to take this into account. Intuitively, it is as if the denoting phrase containing the possessor term binds a variable in the possessed term. Like this:

(Cicero x)(a donkey-of-x y)(an animal z) y is z

[32]Because of this, constructions with relational nouns are ambiguous. A popular illustration of this was the sophism: "This dog is yours. This dog is a father. So this dog is your father."

The parasitic term occurs in the nominative, with the "binding" term in the genitive. This is like the transitive verb case, where the parasitic term (the participle) is in the nominative case, and the direct object term is in the accusative case.

2.8.2 Additional constructions using the genitive

To generate certain other linguistic forms we need to expand the account. How, for example, are we to analyze this?

Brownie is Socrates's.[33]

I think that this is like the English sentence:

Brownie is mine.

Whereas '*my*' has to combine grammatically with another term, '*mine*' is a term on its own. What does it mean? I think that it means something that is completely unspecific with regard to what sort of thing is possessed. Roughly, '*mine*' means '*my thing*'. In the first person singular there is a difference in spelling between the '*my*' in '*my donkey*' and '*mine*'; but in the third person singular the spelling is the same: '*his donkey*' versus '*his*'. And with a noun the spelling is the same: '*Socrates's donkey*' versus '*Socrates's*'. I suppose that the genitive construction in Latin works pretty much the same (except for the divergence in the spelling of pronouns). If this is right, the meaning of '*Brownie is Socrates's*' is '*Brownie is Socrates's thing*' or '*Brownie is a thing of Socrates*'. I'll indicate this by writing in an invisible '*thing*'; so the sentence above will be equivalent to:

Brownie is Socrates's thing

or

Brownie is of-Socrates a thing

This ellipsis device now allows us to generate another kind of proposition that we need an account of; propositions in which the possessor follows the possessed, as in:

Some donkey of-every farmer is brown

meaning that there is a donkey that is owned by every farmer and is brown. We can generate this from:

Some donkey which is of-every farmer is brown.

using whiz-deletion. The generation route for this sentence is a bit convoluted, but we also need to generate each of the forms we got along the way anyway, so it seems unobjectionable. The meaning is:

[33]Ockham *SL* I.2 (52): "'Hominis est asinus' is [well-formed]." That is, 'A man's is a donkey' is well-formed.

Some donkey which is of-every farmer a thing is brown.

or

Some donkey which is every farmer's thing is brown.

In general then if the possessor term precedes the possessed, the two terms occur in independent denoting phrases. If the possessed precedes the possessor, then together they form a complex term with the possessor term occurring in a phrase that modifies the possessed term.[34]

2.9 Syncategoremata and Exponibles

It was common for writers to discuss categorical propositions which have additional syncategorematic signs in them which affect semantic interpretation. One standard example is '*only*', as in:

Only a man runs

Such a proposition is *exponible,* that is, it can be expounded into some other form of words which display the semantics of the proposition. The usual expounding of the sentence given is:

A man runs and every running-thing is a man[35]

or

A man runs and nothing other than a man runs[36]

Exceptives are usually treated as exponibles:

No man except Socrates is running

is expounded as:

Socrates is running and no man other than Socrates is running

Also popular were categoricals with constructions with certain special verbs, such as '*begins*' and '*ceases*'. Sherwood [*SW* XV.3 (110)] gives a typical analysis:

[34]These options were studied in some detail by later followers of Buridan. See [Karger, 1997].

[35]Buridan *SD* 1.6.3 (54) has "But as far as 'Only a man runs' is concerned, it should be said that it has to be converted into a universal affirmative, namely, 'Every runner is a man'." This omits the '*A man runs*' part, but the analysis is an affirmative proposition with existential import, and so it entails '*A runner is a man*' which converts to '*A man is a runner*'.

[36]Buridan *SD* 4.2.4 gives this analysis. The expression '*other than*' is itself a syncategorematic sign, and so the second exponent itself needs exponing. Peter of Spain *S* III.6 (107) gives a similar analysis: "As to the other problem, what an exclusive word signifies, the answer should be that it signifies the same as the expression 'not with another', or a privation of the connection a whole has with a part; *e.g.* 'Only Socrates is running', that is *Socrates is running and nothing else* or *Socrates is running and no other man is running.*"

> For example, 'he begins to be healthy' — he is [healthy] and he was not [healthy] earlier; 'he ceases to be sick' — he is not sick [and he was sick earlier].

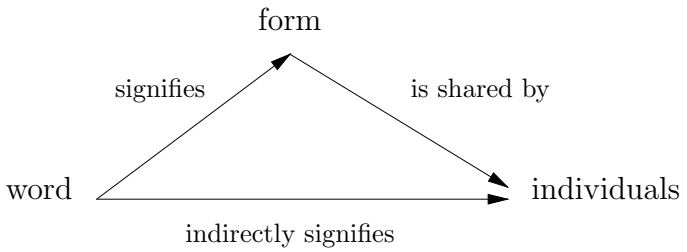
Appeals to exponibles often turn up when some topic is being discussed where a certain sentence seems to be a counterexample to a proposal under discussion, or where it is not clear what a certain proposal would say about the example. Expounding yields a new form that fits with the proposal. In general it seems to be the case that the semantics of an exponible proposition is taken to be whatever is the semantics of its expounded version.[37]

3 SIGNIFICATION AND SUPPOSITION

3.1 Signification

There are different theories about how language relates to the world. I concentrate here on two accounts that were commonly appealed to by semanticists. One is an older realist account that appeals to real universals. The other appeals instead to mental concepts.

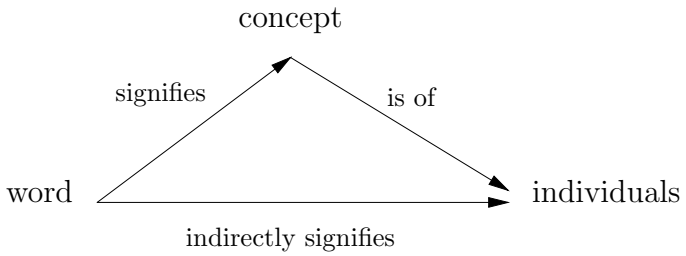
On the realist account, there are universals existing outside of the mind, with individual things falling under them. These universals are usually called forms. A common term gets into the language by being conventionally imposed on a common form. For example, the term '*donkey*' is imposed on a certain form (the species, donkey), which is shared by all donkeys. This imposition is either done by the will of a speaker, or, more usually, is done by convention (with details not given). As a result, the word is a sign of, or *signifies*, that form. Signification is the basic semantic notion; it is a kind of meaning that a word possesses independent of any particular use of the word. A sign is also said to (indirectly) signify the things that share in the form, so that '*donkey*' is also said to (indirectly) signify each and every individual donkey:



On a nominalist account, there are no universals existing outside of the mind. Instead, there are concepts in the mind that naturally are concepts of various individuals. (This view is often backed up by an epistemological theory about

[37] But see section 7.8 for an exception to this principle.

the acquisition of concepts.) A common term gets into the language by being conventionally imposed on such a concept. For example, the term '*donkey*' is imposed on a certain concept, a concept that is of donkeys. This imposition is either done by the will of a speaker, or, more usually, is done by convention (with details not given). As a result, the word is a sign of, or *signifies*, that concept. Signification is the basic semantic notion; it is a kind of meaning that the word then possesses independently of any particular use of the word. A sign is also said to (indirectly)[38] signify the things that the concept is a concept of, so that '*donkey*' is also said to signify each and every individual donkey:



The two accounts clearly have the same structure, and if the concept is called a *form*, then they even have the same wording. For logical purposes, it often doesn't matter which account is being used.

3.2 Supposition

A significant word (one which signifies) may be used in a proposition to stand for something, or for some things. This "standing for" is the medieval relation of *supposition*. Supposition is a relation that an already signifying word has within a proposition. Depending on the proposition, a word can supposit for any one of the three items in the diagrams above. It may be used to supposit for itself, as in '*Donkey is a noun*', or for its related form or concept, as in '*Donkey is a species*', or for the individual things that fall under the form or concept, as in '*[A] donkey is an animal*'. When it is used to stand for itself, the type of standing for is called "material supposition"; when it is used to stand for its related form or concept, it is called "simple supposition", and when it is used to stand for (some of) the items that it indirectly signifies, it is called "personal supposition".

Material and simple supposition are discussed in section 4. Personal supposition is the ordinary kind of reference. In '*Every donkey is an animal*' the word '*donkey*' supposits for each and every presently existing donkey. This is because the word '*donkey*' is imposed on the form or concept "donkey", which itself signifies donkeys. The word thus indirectly signifies individual donkeys. The personal supposition of

[38]For Ockham [*SL* 1.1 (49–50)] a spoken word directly (not indirectly) signifies the things that the associated concept signifies, though it does so "secondarily". The important point is that when spoken words signify things in the world they so do because the concept on which they are imposed does so.

a term in a proposition is determined by its place in the proposition and what it signifies. In the simplest case of a present tense nonmodal proposition containing no special terms (such as '*believe*'), a term personally supposits for all of the presently existing things that it signifies. So in '*Every donkey is an animal*' the term '*donkey*' supposits for all presently existing donkeys, and the term '*animal*' supposits for all presently existing animals.

For early writers such as Peter of Spain, predicates do not supposit personally; they supposit simply, for they directly represent a form.[*T* VI.10 (72)]. Sherwood has an in-between view. For him, predicates do not supposit at all; instead they appellate.[*IL* 5.15 (122)]. However, appellation for a predicate seems to behave exactly like supposition for a subject. Later nominalists hold that predicates supposit just as subjects do.

3.3 Supposition and Truth Conditions

Supposition is important because it, together with the structure of the proposition, determines the truth conditions of the proposition. In contemporary semantics we often characterize the truth conditions for sentences in terms of what their parts stand for. The same idea is pursued by medieval nominalists using the notion of supposition. Paraphrasing somewhat, nominalists give the following account for standard categorical propositions (e.g. in [Ockham *SL* II.3-4 (92, 96-97)], and [Buridan *SD Sophismata* 2 12^{th}-14^{th} conclusions (857-59)]).

> An affirmative proposition is true iff the subject and predicate "supposit for the same". In detail, a particular affirmative proposition is true iff there is something that the subject and predicate both supposit for, and a universal affirmative proposition is true iff (the subject supposits for at least one thing and) the predicate supposits for everything that the subject supposits for.
>
> A negative proposition is true iff the subject and predicate "do not supposit for the same". In detail, a universal negative proposition is true iff there is nothing that the subject and predicate both supposit for, and a particular negative proposition is true iff the predicate does not supposit for everything that the subject supposits for (or the subject supposits for nothing).

These accounts do not seem to be given because the authors think that it is a goal of semantics to provide truth conditions for sentences, but rather they are given in order to show how to account for the truth of categorical propositions without appeal to universals. Of course, this will not succeed if supposition is defined in terms of signifying universals, but the previous section shows how universals can be avoided by an appeal to naturally signifying concepts.

If we were interested in extending this account to propositions in general, some impediments would have to be overcome. One is the extension of the above ac-

counts to nonstandard propositions containing universal predicate terms. One would need something like:

> A proposition of the form '*No A is every B*' is true iff there is nothing for which the subject supposits which is everything for which the predicate supposits.

This looks a bit ad hoc, but as shown in section 1.7 we could account for a wide class of propositions simply by covering eight basic forms. However, things get more complicated with the extensions of section 2. Consider for example the sentence:

> *Some A sees every B*

There are two options here: to leave the verb unanalyzed, or to replace it with the copula and a participle. If the verb is left unanalyzed then the truth conditions using supposition would be something like:

> Something for which the subject supposits sees everything for which the predicate supposits.

This is correct, but not part of a general account of truth conditions, because it contains the special word '*sees*'. The other option produces a proposition like this:

> *Some A is of every B a seeing-thing.*

That is, the form is:

> *Some A is of every B a C.*

This can be given an analysis using the copula 'is', but there are three quantified terms, and so we need an additional analysis that is not included in the original eight forms mentioned above. It also needs something conceptually new, since one of the terms is parasitic, and so its supposition is characterized relative to another term.

These issues were not pursued in detail.

3.4 Mental Language

The relation of a form to the individuals falling under it, or of a concept to the individuals that it is a concept of, is a natural relation, independent of language. The relation of a word to its form or concept (namely, signification) is a conventional relation. Setting aside material supposition, which may be possessed by a nonsense word such as '*bu*', supposition by a written or spoken word is always partly conventional because it is characterized in terms of direct or indirect signification.

Ockham added an innovation to this framework. He held that the mental concepts discussed above are themselves words of a mental language. These words signify naturally, not by convention. All humans have such concepts, and they

signify the same from person to person — unlike spoken and written words, which change signification from one spoken language to another. So there is a single mental language common to everyone. The words and propositions of written and spoken language get their signification by being associated with words and propositions of mental language. Mental propositions and concepts then play something like the role played by abstract propositions and meanings today. In particular, the truth value of a spoken or written proposition is determined by the truth value of the mental proposition related to it:

> Any sentence signifying by convention is true or false precisely because there corresponds to it a true or false mental [sentence] properly so called. [Peter of Ailly *I* II para 148 (46)]

This means that the analysis of truth above in terms of supposition must apply fundamentally to mental propositions, and indirectly to spoken and written ones. If mental language provides the fundamental ingredients for truth-bearing propositions, it must contain the resources necessary for the job. Ockham suggested that this is *all* that mental language contains. In order to discover the ingredients of mental language, one need only determine what ingredients of natural language can affect the truth of a proposition; mental language contains analogues of these ingredients and nothing more. He concluded, for example, that in addition to terms and a copula, mental propositions must also contain something analogous to case and to number. Case is necessary because changing the case of a word can change truth value: a change of case converts '$\text{Socrates}_{\text{Nominative}}$ sees $\text{Plato}_{\text{Accusative}}$' to '$\text{Socrates}_{\text{Accusative}}$ sees $\text{Plato}_{\text{Nominative}}$' (meaning that Plato sees Socrates). Similarly for number: '*[A] man is [an] animal*' is true, though '*[A] man is [the] animals*' is false. So case and number occur in mental language. But mental language does not contain anything corresponding to the various declensions, because these do not affect truth conditions. Some synonyms differ only in declension, so it appears that the declension is contributing nothing. It appears, in fact, that there are to be no pairs of synonyms of any kind in mental language:

> The multiplicity of synonymous expressions in no way enhances the significative power of language; whatever is signified by an expression is signified equally by its synonym. The point of the multiplicity at work in the case of synonyms terms is the embellishment of speech or something of that nature, so that the relevant multiplicity has no place at the conceptual level. [Ockham *SL* 1.3 (52)]

3.5 Connotative Terms

Ockham divides terms into absolute and connotative. He characterizes connotative terms as follows:

> A connotative name, . . , is one that signifies one thing primarily and another thing secondarily. Connotative names have what is, in the

> strict sense, called a nominal definition. In the nominal definition of a connotative term it is frequently necessary to put one expression in the nominative case, and another one in one of the oblique cases. The term 'white' provides an example. The term has a nominal definition, one expression of which is in the nominative case and another in one of the oblique cases. Thus, if someone should ask for the nominal definition of 'white', the answer would be 'something informed with whiteness' or 'something having whiteness'. [Ockham *SL* 1.10 (70)]

A connotative term has a "nominal definition", which is a phrase which expresses the "what" of a term. [Ibid. Loux translates this as "the meaning of a name".] Examples of connotative terms are: 'just', 'white', 'animate', 'human', and so on. [Ibid] Ockham defines a connotative term as one that signifies something primarily and something secondarily. It appears from his examples that a term signifies a thing secondarily if and only if something in the nominal definition of the term signifies the thing primarily. The adjective '*white*' signifies white things primarily, and it signifies whitenesses secondarily — apparently because the term '*whiteness*' in its nominal definition ("something informed with whiteness") signifies whitenesses primarily.

Examples of absolute terms are: 'man', 'animal', 'goat', 'stone', 'tree', 'fire', 'earth', 'water', 'heaven', 'whiteness', 'blackness', 'heat', 'sweetness', 'smell', 'taste', and the like. [Ockham *SL* 1.10 (69-70)] Absolute terms do not signify one thing primarily and another thing secondarily. Presumably, this is because they do not have nominal definitions. Instead, absolute terms have "quidditative" or "real" definitions. The terminology "real definition" does not mean "genuine definition"; rather, it is a characterization of the thing in reality that the term being defined stands for. For realists, this is typically a form existing outside of the mind. Nominalists don't say this; instead they take it that a real definition characterizes all and only the things that an absolute term supposits for. A real definition is not synonymous with the word being defined, since different non-synonymous real definitions are possible for a term. One can correctly characterize men by saying that a man is any animal that is rational. This, however, does not give the "what" of the linguistic term 'man'.

The fact that all connotative terms have nominal definitions suggests an analysis of connotativeness of a term in terms of complexity of the concept that it is subordinated to. If a connotative spoken or written term directly signifies the same concept as does its nominal definition, then since the latter is a complex concept, connotative terms would all directly signify complex concepts. Buridan [*SD Soph* 1 ninth conclusion (840)] says: "only a spoken term to which there corresponds not a simple but a complex concept is one that has a nominal definition in the strict sense.

Further, if all nominal definitions contained only absolute terms, or contained only absolute terms when all connotative terms were analyzed in terms of their nominal definitions, then all terms would either be absolute or would have complex analyses in terms of absolute terms. Some commentators have speculated that

this is what Ockham intended (or should have intended). It is not clear, however, whether a written or spoken connotative term directly signifies the same (complex) concept as does its nominal definition. Nor is it clear that all connotative terms can be ultimately eliminated from all nominal definitions. Ockham himself suggests otherwise at *SL* III.26.[39]

It is an additional question whether there are simple connotative terms in mental language. If so, they have nominal definitions in mental language which are complex terms. The simple connotative terms must not be synonymous with their nominal definitions, because then distinct terms in mental language would be synonyms, which Ockham rules out. So terms with nominal definitions are not synonymous with their definitions. Commentators disagree about whether this is the right thing to say about Ockham's view.

4 TYPES OF SUPPOSITION

4.1 Univocation as the Source of Types of Supposition

Aristotle uses 'equivocation' for a case in which things are called by the same name but the name does not have the same definition in each case. An example is '*dog*', which can apply to one thing because it is a canine, and to another because it is a certain star. So if you have inferred that something is a dog, and is not a dog, you haven't inferred contradictory propositions[40] unless '*dog*' has the same definition in each proposition.

According to Boethius,[41] there is a similar way in which apparently contradictory propositions are not actually contradictory, even when you use the same word with the *same* definition in each application. His illustration is:

[A] man walks	*homo ambulat*
Man does not walk	*homo non ambulat*

(Recall that there are no articles in Latin, so the displayed Latin propositions are explicitly contradictory in form.) These propositions are not contradictory if '*homo*' in the first proposition stands for an individual man, and in the second proposition refers to the species man. This is not equivocation, because the word '*homo*' does not have different definitions in the two propositions. Boethius calls this phenomenon "univocation".[42] Some medieval treatises were written explicitly about univocation.

[39] See [Panaccio, 2004 4.2 (68-69)] for discussion of this point.

[40] Equivocation is discussed along with other fallacies in Aristotle's *Sophistical Refutations*. A "refutation" is achieved when the respondent admits a proposition that contradicts the proposition being defended. So the relation of contradictoriness is central to that essay.

[41] Boethius's commentary on Aristotle's *On Interpretation*, cited in De Rijk *LM* II.I.XV.1 (492).

[42] In essence the proposal is that univocation occurs when a word is used twice with the same signification, but what it stands for (its "supposition" or its "appellation") changes. This kind of explanation is given e.g. in the *Treatise on Univocation:* "Univocation therefore is when the appellation of a name varies and the signification remains the same." (Second paragraph of

One might naturally dispute the claim that the words have the same definitions in the two cases, but one might also naturally accept this. After all, in order to know which species is being referred to, all you need to understand is the word with its normal definition. Is this a case in which a word is used in two different ways but with the same definition in each case, or is it a case in which two different but related definitions are appealed to? Writers took both sides of this issue.[43]

Consider the realist picture; a meaningful categorematic word has a form that it signifies, and it is commonly used to supposit for the things that fall under the form. There are thus three things prominently associated with a term that it may be used to supposit for: itself, its signified form, and the things falling under the form.

Material supposition is the kind of supposition a word has when it supposits for itself or for a related piece of language. An example is:

Donkey is bisyllabic

Here on the most natural interpretation '*donkey*' is used with material supposition, to supposit for tokens of the word itself.

Simple supposition is the sort appealed to in the proposition:

Donkey is a species

On the most natural interpretation of this proposition, '*donkey*' is used with simple supposition, to supposit for the form that it signifies; in this case for a species of animal.

Personal supposition is the kind of supposition a word has when it is used normally, for the things falling under the signified form. An example is:

Every donkey is running

On the most natural interpretation of this sentence, '*donkey*' is used here with personal supposition — to supposit for individual donkeys.

Writers point out that the terms '*simple*', '*material*' and '*personal*' are technical terms, whose meanings cannot be gleaned from their non-technical uses. For

Elizabeth Karger's unpublished translation of *Tractatus de Univocatione Monacensis*, pp. 333-51 of de Rijk II.2.) In *About Univocation* we read: "Univocation is [*a case of*] *the supposition of a name having varied, the signification having remained the same.* (Translation of a selection from *Fallacie Parvipontane*, pp. 545-609 of De Rijk I. Bottom of second page of translation.) For discussion of the historical origin of *univocation*, see De Rijk *LM* II.I.XV.1 (492).

[43]The author of *About Univocation* gives several arguments in favor of the view that univocation should be grouped with equivocation. The author of the *Treatise on Univocation* explains how univocation differs from equivocation: in equivocation there is a change in the signification of a word, whereas in univocation the signification does not change; only the appellation ($\approx$ the supposition) changes. Buridan *SD* 7.3.2 (516) classifies examples of univocation as cases of equivocation; he defines equivocation in terms of a word in context having different significations, and he holds that in '*homo ambulat*' the word '*homo*' does have diverse significations even though there is no new definition. This is because when '*homo*' is used to stand for the species it is subordinated to a different mental concept than when it is used to stand for persons.

example, personal supposition is not necessarily supposition for persons, as is clear in '*No stone is a crystal*', in which neither '*stone*' nor '*crystal*' supposits for any person.

There are difficulties in *defining* exactly when a certain kind of supposition is present, even when it is clear in practice what the intended answer is supposed to be. For example, you cannot define material supposition as a case in which a word supposits for itself, because of examples like:

Some words are nouns

Here, '*word*' is used personally, not materially, even though it supposits for itself (because it supposits personally for all words, including the word '*word*'). Likewise, you cannot define simple supposition in terms of a word's suppositing for a form, because in

Every species has a genus over it

the word '*species*' is used personally, not simply, even though it supposits (personally) for forms. These were not difficulties in practice, and so I won't dwell on the problem of definability.[44]

4.2 Material Supposition

If you want to say something about a word, you can use the word itself to do so — this is material supposition. An obvious example is "*Homo est nomen*" {"Man is a noun"}. In modern English we usually put a mentioned word in quotation marks, or in italics, or in another font — or we do nothing at all and just rely on context to make it clear that we are mentioning the word. In medieval times there were no italics or quotation marks, and so one needed to rely on context to tell when a word is being taken to stand for itself, or for other words of the same type, or for the spoken version of the word (if it is written), or for words with the same root and a different inflection. Any of these uses are called material supposition.

In contemporary analytic philosophical writing there is a convention that using a word to mention itself is a kind of misspelling — for proper spelling, the word must be lengthened by adding quotation marks to it. This changes it to a different word; for example, these are two completely different words:

[44]See discussions in Buridan *SD* 4.3.2 (257): "Another doubt is: "How does the subject supposit in the proposition: 'A noun is a conventionally significative utterance'?–it appears that it supposits materially, as it supposits for itself and for many other terms." I say that it supposits personally, for it does not supposit for itself because it is such an utterance, but because the term 'noun' along with many other terms are the ultimate significata of the utterance 'noun'. In the same manner also, in the proposition 'Every utterance is a sound' the terms supposit personally, even if they supposit for themselves, for they supposit for themselves only insofar as they are, together with other utterances and sounds, the ultimate significata of the utterances 'utterance' and 'sound'."

Ockham *SL* I.64 Handles cases of the above sort by the addition of the qualification at the end of this account: "a term supposits personally when it supposits for the thing it signifies and does so significatively." He does not define 'significatively'.

donkey
'donkey'

The former refers to donkeys, whereas the latter refers to a word. If there is no spelling error, a word is never used to stand for itself. In the medieval account, however, any word may automatically, as a matter of the ordinary semantics of natural language, be used to stand for itself.[45]

The definition of material supposition has a simple core:

> Supposition . . . is called material when a word itself supposits either for the very utterance itself or for the word itself, composed of the utterance and the signification — as if we were to say 'man is a monosyllable' or 'man is a name'. [Sherwood *IL* V.2 (107)]

Most writers gave definitions similar to this.[46] In order to clarify the number of options for e.g. written words referring to spoken words, some authors reflect this in their account of material supposition. Burley says

> ". . . supposition is material when (a) a spoken utterance supposits for itself spoken, or (b) for itself written, or (c) for another utterance that is not inferior to the former utterance taken in such a way, or also (d) when an utterance taken under one kind of supposition supposits for itself taken under another kind of supposition, or (e) when an utterance taken in one way supposits for itself taken in such a way that it cannot supposit or have supposition at all."[47]

[45]In later writing something akin to quotation marks came into use. Greek grammarians had used a definite article in front of a word to indicate that it is being mentioned; an example would be "*The and is a conjunction*". Sometimes medieval authors used '*this*' for this purpose. But later writers emulated the Greek practice by using the French definite article, spelled 'li' or 'ly'. (Since Latin has no definite article, they had to import a word.) Since 'li' is not Latin, its use in Latin for this purpose is unambiguous. The use of 'ly' goes back at least to Sherwood *IL* V.12 (117).

[46]William Ockham *SL* 1.64 (91): "Material supposition occurs when a term does not supposit significatively, but supposits for a spoken word or a written word. A good example is: man is a name. The word man here supposits for itself but it does not signify itself." (This is Loux's translation with the quotation marks removed.)

John Buridan *SD* 4.3.2 (253): ". . . supposition is said to be material when an utterance supposits for itself or for one similar to itself, or for its immediate significate, which is the concept according to which it was imposed to signify, as the term 'man' in the proposition 'Man is a species'." (Buridan's account intentionally includes what some others would call simple supposition — as he makes clear.)

Albert of Saxony: "Material supposition is the interpretation of a term for itself or for any sign similar to itself . . . and which it was not instituted to signify." *SL* II.3.

Paul of Venice *LP* II.1 (143): "Material supposition is the acceptance of a term in a proposition for its material significate, e.g., "'man' is a noun". It is clear that "man" does not stand for [anything] unless for itself or for its like which are the material significates of this term "man"."

[47]Walter Burley, *Longer Treatise*, 1.1.2 para14 (82). His examples (in sections (15)-(21)) include (a) the use of 'man' in the spoken sentence 'Man is spoken', which is used to describe an utterance of 'man'; then a spoken word supposits materially for itself spoken. (b) In the written sentence 'Man is spoken' used to describe a spoken utterance of 'man', we have a written word

He continues with an intricate discussion of various options and examples; this is where a reader should look for some details.[48] Material supposition is of most importance when an author appeals to it for some ulterior motive — e.g. when an apparent reference to an extra-mental form is taken to be a reference instead to a word. Most of these applications are obvious.

4.2.1 Material Supposition and Quotation

Material supposition differs from the use of quotation marks to mention a word, even though these two uses sometimes satisfy the same purpose. For example, if you wish to say something about a word, you write either:

Donkey has two syllables

with '*donkey*' understood materially, or:

'Donkey' has two syllables

Some words of English seem to force something like material supposition. Consider the following apparently bad argument:

I am called Herman. Herman is my name. So I am called my name.

Suppose that we decide that quotation marks should be used to clarify things. The inference would then have the wording:

I am called 'Herman'. 'Herman' is my name. So I am called my name.

This is a valid inference according to our usual practice of using logic; it is substitutivity of identity. But the conclusion seems to be ill-formed, or it is well-formed and the argument is invalid. Quotation marks are not useful in clarifying this argument. This is because the phenomenon is a case of material supposition. The passive '*be called*' in English forces its object (*without* the addition of quotation marks) to be taken materially. Let me temporarily use underlining to indicate words that are forced to be interpreted as being taken materially. Then the argument is of one of these forms:

I am called Herman. 'Herman' is my name. So I am called my name.
I am called Herman. Herman is my name. So I am called my name.

suppositing materially for itself spoken. (c) In the spoken sentence 'That a man is an animal is a declarative expression' there are two interpretations, both material cases of supposition: one is that the subject of the sentence supposits materially for the sentence 'A man is an animal', in which case the spoken sentence is true; the other is that it supposits for itself, in which case the spoken sentence is false (because no that-clause is a declarative expression). (d) Call the proposition 'Every man runs' A. Then suppose someone says 'Man supposits personally in A'. Here 'man' supposits materially for itself suppositing personally. (e) In the utterance 'White cannot supposit' the word 'white' supposits substantively (even though it is not a substantive) and materially for itself suppositing adjectivally.

[48] In sections (15)-(22), outlined in the last note, and further in sections (23)-(26).

Read in this way, the sentences are all coherent, and it is easy to spot the fallacy: in either second premise the first word refers to a proper name, and the phrase following '*is*' refers to that very same name; thus the identity statement is true. But in the conclusion, '*my name*' does not refer to that proper name; it refers to itself. So there is a kind of equivocation in 'my name', which changes reference (because it changes supposition) from the second premise to the conclusion.

Notice that when an expression is taken in material supposition it only stands for the words themselves; it does not give any additional information, such as how those words are used. So given the truth:

a said b said this doctrine

You may only infer that if *a* is correct then:

b said this doctrine.

You cannot infer from this how *b*'s words were to be taken. In particular, you do not know whether *b*'s words were to be taken in personal supposition, in which case *b* stated a certain doctrine, or whether they were taken in material supposition, in which case *b* said '*this doctrine*'. This openness of interpretation does not occur when quotes are used; the original sentence would have to be one or the other:

a said 'b said this doctrine'
a said 'b said 'this doctrine"

This means that quotation marks are useful when you want your sentence to distinguish these two claims, and material supposition is useful when you don't want to make this distinction (e.g. if you know exactly what words *a* used, but you don't know how they were intended). This is not a defect of either convention, it is just a way in which they differ.

4.3 Simple supposition

When a word stands for its associated form, it has simple supposition. Those who thought that there are no forms were likely to see these cases as ones in which the word stands for the mental concept that the word is a sign of. On either option, this agrees with Frege's view that in certain contexts a word stands not for its ordinary referent, but for something like the meaning that the word has. However the analogy is limited, since simple supposition was not appealed to in order to handle what we nowadays see as paradigm non-extensional contexts. In particular, it was not usually appealed to in analyzing modal contexts, or belief contexts — other analyses were given for such cases.[49]

William Sherwood explains simple supposition as:

> [supposition] is simple when a word supposits what it signifies *for* what it signifies [*IL* V.2 (107)]

[49]For some of these see sections 5.3 and 6.1.

Since Sherwood holds that a word directly signifies a form, this seems to mean that in simple supposition the word supposits for a form.

Lambert gives the condition:

> Simple supposition is the kind according to which a term is interpreted for itself or for its signified thing, without relation to the supposita contained under it [PT3g(ii) (110)]

For Lambert a word's "signified thing" is a form, so in one version of simple supposition a word supposits for the form it is a sign of.

Both writers hold that what a term signifies is a form; so a term in simple supposition stands for a form. Peter of Spain makes this clear, while maintaining his older view that predicates of categorical propositions always have simple supposition:

> Simple suppositions include that of a common subject term, as in 'man is a species' or that of a common term put in an affirmative predicate, as in 'every man is an animal'. The predicate term 'animal' has simple supposition, since it only stands for the nature of a genus. [TVI.6 (70)]

On the nominalist semantics of Ockham and Buridan and their followers, there are no forms. Instead, apparent references to forms are actually references to concepts. A word is used materially to supposit for itself or related expressions, it is used simply to supposit for the concept that it is conventionally imposed on, and it is used personally to supposit for the things that the concept is naturally a concept of. This is how Ockham speaks. Buridan balks at using the term 'simple supposition', which is used by others to classify a relation between words and forms. Instead, since expressions in simple supposition supposit for concepts, and since concepts are words of a mental language, he says that what others call simple supposition is actually reference to words — and thus a kind of material supposition. In '*Donkey is a species*' the word '*donkey*' supposits for the associated mental concept – i.e. for a mental word. So he uses a two-part classification of uses of terms into material and personal.[50]

Even ignoring Peter of Spain's special view that predicates automatically have simple supposition, the topic is full of controversy. Some will be discussed here.

According to William Sherwood, simple supposition comes in three varieties. Our discussion will correspondingly have three parts.

[50] Buridan *SD* 4.3.2. "some people have posited also a third member, which they called 'simple supposition'. For they held that universal natures are distinct from the singulars outside of the soul. And so they said that a term supposits personally, when it supposits for the singulars themselves, that it supposits simply, when it supposits for that universal nature, and materially, when it supposits for itself. But I hold that Aristotle correctly refuted that opinion in the seventh book of the *Metaphysics*, and so this kind of supposition has to be eliminated, at least, according to this interpretation. In another manner, others call supposition 'simple' when an utterance supposits for the concept according to which it is imposed and material when it supposits for itself or another similar to itself. And this can be permitted, but I do not care [about this usage], for I call both 'material supposition'."

Variety 1 of Simple Supposition

The simplest sort of simple supposition is given by the standard example:

> *Man is a species*

As quoted above, Lambert describes this as a case in which a term has no relation at all to "the supposita"; that is, to the things that fall under the form that is signified. William similarly describes this kind of simple supposition as a case in which

> [A] a word is posited "without any connection with things". [*IL* V.7 (111)]

In either case this is pretty straightforward: the utterance is treated just as if the term were a noun standing for the form that it signifies.[51]

Conceptualist alternatives are predictable: in this variety of simple supposition, the word stands for the concept it signifies, either because that concept *is* the form, or because there are no forms, but only concepts.

> Simple supposition occurs when a term supposits for an intention of the soul and is not functioning significatively. For example, in '*Man is a species*' the term 'man' supposits for an intention of the soul, for it is that intention that is a species. [*SL*, 1.64 (190)]

Other varieties of simple supposition are more complex, and require additional discussion. Lambert describes the other cases as ones in which there is a relation to the supposita, "not determinately, but indeterminately"[PT3g(ii)]. William divides these into two types, and I'll follow his classification. The examples that fall under these two additional types were discussed by several authors.

Variety 2 of Simple Supposition

The second variety of simple supposition occurs when

> [a word is posited] for the significatum connected with things insofar as it is actually preserved in every single thing, and is predicable of it [Sherwood *IL* V.7.2 (111)]

The standard example which gives rise to this kind of simple supposition is:

> *Man is the noblest of creatures*

[51] In a very early anonymous writing (*Cum sit nostra*) the author maintains that in a proposition like '*Man is a species*' the word '*man*' is not a common term at all, but rather a discrete name (like a proper name) of the species; this view is attributed to the grammarian Priscian. So it is not a common term used simply. This leaves unaddressed the question of what kind of supposition it has, but it appears that it may have personal supposition here — since the word apparently supposits for what it names, not for what it signifies.

William says that here

> the predicate is not ascribed to the species itself, but to the species insofar as it is in things. [Ibid]

This is because from '*Man is the noblest of creatures*' one can infer:

> *this man, insofar as he is a man, is the noblest of creatures.* [ibid.]

The phrase "*noblest of creatures*" is predicated of individual men, but *reduplicatively*, that is, using the locution '*insofar as*' or just '*as*'. So this is true:

> *Socrates, as [a] man, is the noblest of creatures*

while this is not true:

> *Socrates is the noblest of creatures.*

Unfortunately, none of this was worked out in any detail. So articulating a semantics of such constructions would be an invention that goes beyond currently available theorizing.

Instead of working out details, some other authors rejected the proposal that a new kind of suppositing is needed here. A clear example is Ockham. He disagrees with the appeal to simple supposition to handle these examples. He sets up the issue by reviewing some arguments on the issue:

> First, as follows: 'Man is the worthiest creature among creatures' is true. I ask which kind of supposition 'man' has [there]. Not personal, because each singular [of the proposition] is false. Therefore, it has simple supposition.But if simple supposition were for an intention of the soul, [the proposition] would be false, because an intention of the soul is not the most worthy of creatures. Therefore, simple supposition is not for an intention of the soul. [*SL* 1.66 (193)]

Ockham's own view is:

> Therefore, it has to be said that 'man' [in this proposition] supposits personally, and [that the proposition] is literally false, because each [of its] singulars is false. Nevertheless, it is true according to the meaning of those who maintain [the proposition]. For they do not mean that a man is nobler than any creature in general, but that he is nobler than any creature that is not a man. And this is true among corporeal creatures, although it is not true for intellectual substances. [ibid. (194)]

So the answer is that the sentence means that every man is worthier than any creature other than a man. This is followed by a typical Ockhamian qualification:

> So it is often the case that authoritative magisterial propositions are false literally, and true in the sense in which they were made. That is, [the speakers] meant true propositions by them. That is so in the present case. [ibid. (194)]

This is important to him, since so many of the church authorities were realists, and he wants to agree with them but without a commitment to realism. So he (mis)interprets them creatively.

In the example just discussed the notion of nobility seems to play a special role, since there is indeed some intuitive push to hold that no man is simply noble, but s/he is only noble "as a man". But it would seem odd to say that '*A rose is the reddest of flowers*' means that each rose, insofar as it is a rose, is red. Some authors saw the problem as one that arises with any comparative or superlative form, not just with special cases. But again, the details were not worked out. (If Ockham is right, there are no details to be worked out.)

Variety 3 of Simple Supposition

Sherwood's third case of simple supposition is:

> . . . a word is posited for the significatum connected with things insofar as it is related to anything generally, in an unfixed way, and is not identified with anything in a determinate way [*IL* V.7 (111)]

This case is meant to cover these widely discussed examples:

> *Pepper is sold here and in Rome*
> *This plant grows here and in my garden*
> *Woman who has damned us has saved us.*[52]

The problem is that the particular pepper that is sold here is not sold in Rome; other pepper is sold there, so the word '*pepper*' cannot be interpreted as having personal supposition. And you don't mean that the species pepper is sold here and in Rome, because species cannot be sold — which rules out the first variety of simple supposition. Nor are you saying that any pepper insofar as it is pepper is sold here and in Rome, which rules out the second variety of simple supposition. So something else is meant — a third variety of simple supposition, which William calls "unfixed supposition". The term, insofar as it supposits for a species, does so through undesignated individuals belonging to the species. (This explanation struck some writers as being obscure and uninformative.)

Ockham also disagrees with this analysis; he says about a similar example:

> If it is said that 'This herb grows in my garden' is true, and that nonetheless the subject does not have discrete supposition, the response is that the proposition, taken literally, is false. [*SL* I.70 (200)]

[52]The intent is that the proposition is true because Eve damned us, and Mary, mother of Jesus, saved us.

The term 'this herb' appears to be a paradigm discrete (singular) term. The proposal to assign it simple supposition is a proposal to treat it not as a discrete term (or at least not as a discrete term suppositing for an herb). Ockham's view is that it is indeed a discrete term, and the sentence is literally false. Because the subject is a discrete term, it has discrete supposition, which is a form of personal supposition. So the subject stands for a particular herb present to the speaker. And *that* herb does not grow in my garden. So the sentence is false.

> however, by that proposition one means 'An herb of that sort grows in my garden', where the subject supposits determinately.[53]

Again Ockham thinks that this is a case where what people mean to say differs from what they do say. The speaker means something like "Some such herb grows in my garden".

5 AMPLIATION AND RESTRICTION

5.1 Overview

In a non-modal present tense categorical proposition, common terms typically do not supposit for everything that they indirectly (or secondarily) signify. Instead they supposit only for the presently existing things that they indirectly signify. As a result, in

> *Every donkey is running*

the subject is restricted to suppositing for presently existing donkeys. If all of them are now running, then the proposition is true, even though there were or will be donkeys that are not now running. Similarly, in the proposition:

> *Socrates is a philosopher*

the subject is restricted to presently existing things. Since Socrates does not presently exist, the term '*Socrates*' in this proposition supposits for nothing at all. Since this is an affirmative proposition with an empty subject term, it is false.

This restriction of supposition in a present tense nonmodal categorical proposition to presently existing things is the 'Restriction' in 'Ampliation and Restriction'.

Terms may also supposit for things in addition to their presently existing significates. In

> *Some donkey was running*

[53] Ockham *S.L.*I.70 (200) For Ockham, determinate supposition is a mode of personal supposition; see section 7. Burley 1.1.4 para 81 (102) has a similar reply: "Nevertheless, a true proposition can be understood by means of it, namely *'Such* an herb grows here and in my garden'.

the subject term stands for both present and past donkeys; the proposition is true if one of them was running sometime in the past. This expansion of what the terms may supposit for is the 'Ampliation' in 'Ampliation and Restriction'. It is somewhat arbitrary what is to be called restriction and what is to be called ampliation. So long as terms supposit only for things that they signify, one may call any use of a term a restriction if the term does not supposit for everything that it signifies. And any use of a term to supposit for things outside of some reference class of things, such as the presently existing things, can be called an ampliation. The custom is generally to say that terms are restricted when they supposit exactly for the presently existing things that they presently signify, or for some subset of these, and to call any extension of this range of things ampliation.[54]

Some kinds of restriction and ampliation are:

1. Supposition restricted to presently existing things. (All previous sections have taken this to be the default case.) Examples are:

   Every donkey is grey
   No donkey is a stone

   In these cases the underlined words are "restricted" to the presently existing things that they signify. This restriction is caused by the present tense on the verb.

2. Supposition for past things (for formerly existing things) or for future things. Such supposition is caused by the past and future tenses, as in:

   A donkey was grey
   A donkey will be grey

   In these examples, the underlined words supposit for present or past or future things (details given below). When these tenses are used, subject and predicate terms of the tensed verb are ampliated to include supposition for things outside of the presently existing things. (Ampliation is also caused by past or future participles with present tense copulas.)

3. Supposition for never-existing but possible things, caused by an alethic modal word:

   A mountain can be golden
   Every pink donkey is necessarily an animal.

   Since no past, present, or future mountain is golden, '*mountain*' must be suppositing for possible mountains in addition to actual ones in the first

[54] Words may also sometimes supposit for things that they do not signify at all. Examples are the uses of the word '*donkey*' in the propositions:
Donkey is a species
Donkey is a noun
This type of use, which is not called ampliation, was discussed in section 4.

proposition (for it to be true). Likewise, one must be discussing possible pink donkeys in the second; otherwise the proposition would be false. This ampliation is caused by the modal words '*can*' and '*necessarily*'.

4. Supposition for impossible but conceivable things, caused by words that "pertain to the soul":

   A chimera is believed to be an animal.
   A bishop wants a donkey which is a stone.[55]

Authors disagree about the truth values of these cases, but they seem to agree with the semantic mechanism. Certain verbs such as '*believe*' or '*want*' are naturally used with terms that appear to supposit for things that are not possible. Since it is impossible for a chimera to exist, one must take the first proposition to contain supposition for not just actual or possible chimeras, but for impossible ones (if there are any) too. Some authors reject this ampliation because they hold that there are no impossible things, and so terms cannot supposit for them. For simplicity, I will interpret these latter authors as agreeing with the semantic principle that in such contexts the correct semantics is to let the terms supposit for absolutely all things that they signify,[56] and this will include impossible things if there are any. The sentences will then be taken to have different truth values for those who disagree about whether there are any impossible things. For example, Buridan does not accept impossible beings; so he would consider the first proposition above to be false; since chimeras are impossible, there aren't any chimeras to have beliefs about. The subject term is ampliated to supposit for impossible things, but since there aren't any impossible things, the subject is empty and the sentence is false.

The following subsections will discuss temporal ampliation by tenses, ampliation to the merely possible by (alethic) modal words, and ampliation to all things whatsoever by verbs which indicate an act of the soul. These all have a semantic account that is uniform across tenses and modals. It has two parts.

> One part is how elements of a proposition affect what terms supposit for: A present tense (on an ordinary verb) restricts the terms to supositing for the presently existing things that they presently signify; a past tense on an ordinary verb ampliates the subject term to supposit for both presently and formerly existing things that it signifies or signified, and ampliates the predicate term to supposit for the things that it will signify; and similarly for the future tense, and for modal words, and for verbs pertaining to the soul.

[55] Note that the subject term is not ampliated in this example, since it occurs with a present tense non-modal verb. The verb pertains to an act of the soul, and this affects the supposition of terms that come after it, but the subject of such a verb is not thereby ampliated.

[56] Not just verbs that pertain to the soul do this; semantic words such as '*signify*' and '*supposit*' ampliate terms following them so that they supposit for everything that they signify. See section 5.5.

> The second part of the theory states how supposition bears on truth conditions. As stated in section 3.3, a sentence of the form:
>
> *Every A is/was/will be/can be B*
>
> is uniformly true iff the subject has supposition, and everything for which the subject supposits is (tenselessly) something for which the predicate supposits. Likewise:
>
> *Some A is not/was not/will not be/can not be B*
>
> is true iff either the subject lacks supposition, or it supposits for something that the predicate does not supposit for.

Thus, although the past tense shows up on the copula '*was*' in '*Every donkey was running*', it has no effect on the verb at all; the verb represents tenseless identity in every case. The tense only affects (ampliates or restricts) what the subject and predicate terms supposit for. Likewise (apparently) for modal propositions: '*every donkey possibly runs*' is true iff every possible donkey *is* (tenselessly) a possible runner. And '*every chimera is conceivable*' is true iff every chimera — possible or not — *is* (tenselessly) something able-to-be-conceived. (The term '*conceivable*' does the ampliating here because it pertains to the soul.)

Restriction of a term by other terms

One topic that will not be discussed here is the restriction of a term by another term or by a relative clause modifying it. All writers agree that in this proposition:

> *Every grey donkey is running*

the complex term '*grey donkey*' supposits only for donkeys that are grey, not for donkeys in general, or for grey things in general. Contemporary logicians might suppose that the simple term '*donkey*', which is part of the complex subject, supposits for all presently existing donkeys, and that the supposition of this simple term is unaffected by its being modified. Some medieval writers however hold that the modifier '*grey*' restricts the supposition of the simple term '*donkey*' so that *it* only supposits for donkeys that are grey.[57] The complex term thus supposits for exactly the things that the simple term '*donkey*' supposits for in this context.

If this is so, it is not relevant to any of the truth conditions of sentences containing such constructions. This is because it is the complex term '*grey donkey*' that interacts with the rest of the proposition; '*Some grey donkey is running*' is true if and only if the subject ('*grey donkey*') and predicate have a suppositum in common. The complex term supposits exactly for donkeys that are grey, whether '*donkey*' is restricted by '*grey*' or not. In either case you take the grey things and then intersect these with the donkeys — or with the donkeys that are grey; it

[57] See Peter of Spain *T*XI.4. At XI.5 he also holds e.g. that in '*grey donkey*' the '*donkey*' restricts '*grey*' to suppositing only for donkeys.

makes no difference — and you end up with a complex term suppositing for grey donkeys.[58] Since the issue of restriction by modification has no effect on the truth conditions of sentences containing modified terms, I'll ignore this complexity.

5.2 Ampliation and Restriction by Tenses

5.2.1 A rejected view

Some writers begin their discussion of ampliation caused by tenses by rejecting a view that must have been widely discussed, if not widely held. This is the view that:

> A past tense proposition is true now iff its present tense version was true sometime in the past.

On this view, '*A donkey was grey*' is true now iff '*A donkey is grey*' once was true. This theory is easily shown false by examples like:

> *A white was black*

This is true if something that is now white was black in the past. And this is so even if the present tense proposition:

> *A white is black*

was never true.[59] A similar example for the corresponding modal case is:

> *A white can be black*

which has a true reading, even though

[58] Some authors said that as a consequence of such restriction, a word like '*risible*' "alienates" the supposition of '*donkey*' in:

Every risible donkey is running.

It makes it supposit for nothing. Again, the important point is that the complex subject '*risible donkey*' supposits for nothing, and it doesn't matter whether this is because the intersection of risible things with donkeys is empty, or because the intersection of risible things with no things at all (that is, with risible donkeys) is empty.

[59] In discussing the future tense Buridan says (*SD Sophismata* 4 (881): And from these [considerations] there follows an important point: it is not required for the truth of a past or future tense proposition in which the subject is appellative that the corresponding proposition in the present tense be true sometime in the future. This is so, even assuming that such a proposition will always exist in the future. Here by "corresponding proposition", I mean one that has the same subject and predicate, for although '[A] white [thing] will be black' is true, because it is equivalent to the true [proposition] 'What is or will be white will be black', nevertheless, '[A] white [thing] is black' will never be true, even if it should be uttered. But in reducing a future tense proposition to a present tense proposition, it is necessary to remove the appellation from the subject of the proposition and it is necessary to change the appellative subject into a non-appellative subject that supposits for what the appellative subject supposited for, and in respect of which the proposition was true; as when, pointing to [that thing], we were to change the subject into the pronoun 'this', and say 'This is white', and then it is necessary that, if the future tense proposition was true, then a present tense proposition of that sort will be true sometime.

A white is black

is not possibly true.

5.2.2 A Complexity — ambiguity or disjunction?

A sentence like '*Some bishop was running*' can be true in two different ways. In one way it is true if something that is a bishop now was running in the past, even if it was not a bishop then. The other way is that something which may not be a bishop now (indeed, may not even exist now) was a bishop in the past and ran in the past. All theorists agree that there are at least these two options. They disagree, however, about whether this is because the sentence is ambiguous between these two readings (as Ockham says[60]), or whether the sentence is unambiguous and it has disjunctive truth conditions (as Buridan says[61]), so that it is univocally true in either of these cases. I don't know how to choose between these options. For simplicity, I will discuss the disjunctive truth condition option.

The proposition '*A bishop was running*' must be made true by something that ran in the past, not by something that only runs at present, so this dual interpretation is not available for the predicate. So the subject and predicate of a past tensed sentence are affected differently by the tense. The predicate is ampliated/restricted so as to supposit only for things which it formerly signified. The subject however is ampliated so as to supposit for things which it signifies now or which it formerly signified.

5.2.3 Coordination of Tenses and Complex Terms

How tenses work in natural language is a complex matter, which is made difficult by unclarity in the data — that is, unclarity due to speakers of the language not associating clear truth conditions with tensed sentences. As a result, it is not easy to assess the ultimate success of the medieval theory. It conflicts with some standard paradigms of late 20^{th} century tense logic, but it is often not completely clear which view better matches ordinary language usage. Since the intent is to develop the semantics of a somewhat artificial regimented use of Latin, I will be reluctant to draw many firm conclusions about its adequacy in terms of capturing Latin usage.

Some complexities in how tenses work are discussed here.

5.2.4 Coordination of times between subject and predicate

I begin with what some may consider the clearest flaw in the simplest applications of the account. Consider the sentence:

[60] *SL* II.22 (158): ". . . every past-tense or future tense proposition in which the subject is a common term must be distinguished as equivocal . . . For if the proposition is past-tense, then the subject can supposit for that which is such-and-such or for that which was such-and-such."

[61] *SD* 4.5.2 (293): ". . . a term put before the verb appellates its form in a disjunctive manner, for the present and for the tense of the verb"

A bishop was grey

This is true iff something that is now or was sometime a bishop was sometime grey. One part of these truth conditions is clearly right: if something which is now a bishop was grey previously, whether it was then a bishop or not, the sentence has a true reading. But suppose that something was once a bishop, and was once grey, although that thing was never a bishop while being grey. In this situation the medieval reading comes out true. It is unclear to me whether this accords with how speakers of natural language treat such sentences.[62] In any event, these are the official truth conditions given to the sentence by the theory.

A typical 20^{th} century tense logician might treat the sentence as if it is ambiguous and has the following three readings:[63]

$\exists x(\text{bishop } x \ \& \ \text{Past}\{\text{grey } x\})$
$\exists x(\text{Past}\{\text{bishop } x\} \ \& \ \text{Past}\{\text{grey } x\})$
$\text{Past}\{\exists x(\text{bishop } x \ \& \ \text{grey } x)\}$

The last option is not a reading that the medieval theory attributes to the sentence. The proposition cannot be read so as to entail that there is a time in the past such that the subject and predicate both supposit for the same thing then; this is because the ampliation of the subject and predicate terms are independent of one another. This is a fundamental theoretical limit on the theory as articulated in the texts. To many modern philosophical logicians this will seem wrong. Others will hold that the simultaneity of past times of being a bishop and of being grey is at most a possible implicature of the sentence, and not a constraint on any of its readings.[64]

This lack of coordination of times of subject and predicate was sometimes featured by "truths" like:

A virgin was pregnant
A boy was an old man
An old man will be a boy[65]

[62] I think it is closer to actual current English usage than many present logicians think. Of course, this may be because speakers tend to speak loosely, and perhaps the meanings of their sentences should not reflect that looseness.

[63] These would be read: Some present bishop is such that there is a past time at which it was grey. For some presently existing thing, it was a bishop at some earlier time, and it was grey at some earlier time. At some earlier time it was the case that something was then a bishop and was then grey.

[64] In favor of the latter, one might note that it is quite coherent to say '*A bishop was grey until just before he became bishop*'. However, when ambiguity is at issue, reasoning of this sort is usually inconclusive. This is because of the difficulty in telling whether the additional clause cancels an implicature of a non-ambiguous proposition, or merely eliminates one of the readings of an ambiguous proposition.

[65] These examples are given e.g. by Paul of Venice *LP* II.8 (161), who also gives '*A prostitute will be a virgin*', '*A decapitated person will sing*'. These examples were staples of the theory, and they were used in the early 16^{th} century by Juan Luis Vives [Guerlac, 1979] to ridicule scholastic theorizing.

The first is true because someone who was a virgin sometime in the past was pregnant sometime in the past. Etc.

5.2.5 Tenses with relative clauses

We have not yet said how to handle complex terms with relative clauses occurring in tensed sentences. If we follow our instructions so far, we end up with an ampliated complex term, but with no account of how the complex term is to be ampliated in terms of how its parts work. In the case of modification by a relative clause there is a very simple treatment. We just suppose that ampliating a complex term made with a relative clause ampliates only the simple term that is modified by the relative clause. The relative clause itself is already taken care of, since it has a verb of its own, and the tense on that verb takes care of ampliating the terms in it. This I think is how the medieval theory was intended to work. An example is the following sentence. It has a main verb in the past tense, and its subject term contains a relative clause in the future tense.

A donkey which will be grey was brown

This is true iff some present or past donkey which will be grey in the future was brown at some time in the past. This seems to me to reflect our own usage.

There remains a question about the coordination (or lack thereof) of times within the complex phrase. Consider:

A bishop which was grey was running

This sentence is true according to the present account iff there is some present or past time at which someone was a bishop, and some past time at which that person was grey, and some past time at which that person was running. It is not required that the times of being a bishop and being grey are the same. This may, arguably, be one correct reading of the sentence.

5.2.6 Parasitic Terms — Genitives

In section 2, we were able to include transitive verbs and genitive constructions by introducing parasitic terms — terms that within modern theorizing contain a variable that may be bound by another denoting phrase with the appropriate case. The meaning of a term such as 'donkey-of-x' is something like 'donkey possessed by x'. The time of possession is as relevant to the construction as the time of being a donkey. I am not aware of discussions of this issue.

5.2.7 Coordination of times among parts of the subject or predicate

In parts of the subject or predicate which are disjoint terms Buridan says they must all be treated as simultaneous. [*SD:Soph* 3.1 (887)] An example is *I ate an apple in Paris*, where, he says, there must be some time in the past such that I was in Paris then, and I ate something then, and it was an apple then. This is

a natural intuition, but it is not clear how this idea can be combined with the fundamental theory. The problem is that '*eating-thing*' and '*in Paris*' and '*apple*' are all independent terms. The first stands for all past eating things, the second for all things that were in Paris, and the third for all things that were apples. And they all can be independently quantified: *I ate every apple in Paris*, meaning *I some-past-time-ate every some-past-time-apple that was some-past-time-in Paris.* The issue is how to make this turn into *At some past time I then-ate every then-apple that was then-in Paris.* This would require some development of the semantics; whether it would be a revision or a refinement is unclear.

5.2.8 Subclauses and Sequence of Tense

There is another way in which tenses are coordinated in natural language. This is part of what is informally called "sequence of tense" in grammar. Consider the sentence:

Socrates complained that Plato hit him.

This seems to have a reading in which the past tense in the subordinate clause (the tense of '*hit*') is parasitic on the tense of the main clause (the tense of '*complained*'). On this reading, the sentence alleges that there was a past time at which Socrates made a complaint, and the content of the complaint concerns something that took place prior to the complaining. Not prior to *now*, but prior to another past time.

The medieval semantic account we have of this is that it is true iff Socrates is a present or past thing who is a past complainer of a certain proposition: one that existed in the past and was of the form:

Plato hit Socrates

A proposition of that past tense form evaluated at a certain time would say that Plato hit Socrates at some time before then (before the time at which it was uttered). So the whole sentence is true iff at some past time Socrates complained, and his complaint was the content of a mental proposition existing then saying in the past tense that Plato hit him. Thus the sequence of tense seems to work out exactly right.

The above reading where one takes the past tense in the subordinate clause to be past relative to the past tense in the main clause is sometimes called the "past-under-past" reading. Some sentences with the past tense in both clauses have another reading. For example, this proposition:

Socrates thought that Euthyphro was a fool

has a natural reading according to which at some time in the past Socrates had a thought whose content was that Euthyphro was *then* a fool. (The past-under-past reading would say that the content that Euthyphro was *previously* a fool.) This may be taken care of by stipulating that a past tense in a subordinate clause sometimes behaves semantically as a present tense. That is, the above sentence

is true iff Socrates at some time in the past thought a mental proposition whose content was that of:

Euthyphro is a fool

The interaction of tenses in main and subordinate clauses is not well understood. There is some hope of the medieval account doing as well in capturing the semantics of sentences with embedded subclauses as most modern theories. However, this topic was not discussed in detail by medieval writers.

5.3 Modal Propositions

5.3.1 What are modal propositions?

Buridan [*SD* 1.8.2 (67-70)] discusses three kinds of propositions that other writers call "modal". He considers only one of them to be genuinely modal.

Kind 1: Non-modal Propositions containing Modal Words

Some ordinary propositions contain modal words as parts of their subjects or predicates. Some examples are:

A proposition is possible
No proposition is necessary
Some necessary proposition is false

These are not modal propositions, even though they contain modal words. In each of these cases, the modal word is all or part of the subject or predicate, and thus does not affect the copula. A genuinely modal proposition is one in which the copula changes its mode. If nothing affects the copula, the proposition is by definition not modal. Buridan calls the displayed propositions "assertoric". They require nothing special in terms of their syntax or their semantics. They just happen to quantify over propositions and contain terms that attribute modal properties.

Kind 2: Composite Modal Propositions

These are cases in which a modal word applies to an entire proposition. There are two kinds of such cases. In one kind of case, a modal adverb or prepositional phrase is used with scope over the entire sentence, such as:

Necessarily every donkey is an animal
Possibly, some donkey is grey

In the other kind of case, a modal adjective or related word is applied to a whole embedded clause, such as

That every donkey is an animal is necessary
For some donkey to be grey is possible

In both of these cases the modality is predicated of what Buridan and others called a *dictum* — roughly, of "what is said". In the last two propositions this is clear. These are the sorts of propositions in which a subject or a predicate consists of a sub-clause which apparently supposits for a proposition. These are really assertoric propositions; they predicate a modal predicate of a proposition or propositions, but their own copula is unaffected and so they are not genuinely modal. It is not clear exactly how to formulate a semantics for the first examples with modal adverbs, but they seem to be exact parallels of the second sort. If you change the adverb to the corresponding adjective plus copula, and make the rest of the sentence into a sub-clause, you get propositions of the second sort. The main clause then contains a newly introduced copula which is itself unaffected by the modal words in the proposition.

We already have a syntax and semantics for the next-to-last proposition. For example:

That every donkey is an animal is necessary

is true iff there is a mental proposition of the form

every donkey is an animal

which is necessary. (Whether such a proposition is necessary or not depends in part on one's metaphysical views. Presumably the proposition just given is not necessary because it has existential import, and so it is false if no donkeys exist.)

Kind 3: Modal Propositions

A real modal proposition is one in which a modal word applies to the copula. These come in at least three varieties.

1. Modal adverbs modifying the copula alone

   The least-discussed cases are ones in which a modal adverb or prepositional phrase directly modifies the copula with no other effect on the sentence. Such cases look just like the first class under (2) except they are to be read with the modal expression applying to the copula instead of applying to the whole proposition. Examples are:

   Some animal possibly-is grey
   Every donkey necessarily-is an animal
   Every donkey of-necessity-is an animal
   He possibly-is grey

2. Combinations with the helping verb '*can*'

   The second class uses the verb '*can*' ('*potest*') as a helping verb with the copula. The copula takes its infinitive form; otherwise there is no effect on the rest of the expression. Cases of this sort are:

Some animal can be grey
No donkey can be a stone
He can be grey

3. Combining a modal adjective with the copula

   The third sort is more complex, with a syntax different from that of English. Here a modal adjective together with a new copula combines with the existing copula, which assumes its infinitive form. The subject and predicate terms also take the objective case. Transliterations of some Latin examples are:

   Some animal possible is be grey
   No donkey possible is be a stone
   Him possible is be a stone

   An almost English analogue made by rearranging the words and inserting '*to*' before the infinitive gives

   Some animal is possible to be grey
   No donkey is possible to be a stone
   Him is possible to be a stone

   Klima points out that this is acceptable English if you change '*possible*' to '*able*':

   Some animal is able to be grey
   No donkey is able to be a stone
   Him (he) is able to be a stone

   The syntax is important, and so I will use a transliteration of the Latin forms, preserving the Latin word-order.

My own opinion is that examples of type (3) are really accusative-infinitive constructions with raised subjects, as evidenced by the accusative case of their subjects, and they should be seen as special cases of type (2) above. But this is not the way that Buridan (and probably others) viewed them. So I will treat them as special cases of genuinely modal propositions. These three forms are spelled out in the next section.

5.3.2 Modal Propositions — Semantics

What are the truth conditions for complex modal (alethic) propositions? Discussion in medieval texts leaves the truth conditions for alethic modal propositions somewhat unclear. The unclarity that we face here has to do with the logical forms of such propositions. One option works fairly clearly, and matches most of the discussion in the texts. This is, roughly, to make modal propositions look as much like temporal ones as possible. In particular, in modal propositions we assume that the copula itself is unaffected by the modal sign, and the terms themselves are ampliated to possible things, or to necessary things. In particular:

> The subject is ampliated to possible things (including things that exist at any time).
> The predicate is also ampliated; what it is ampliated to depends on the mode.

There is no other effect.

Examples:

> *Every animal possibly-is a donkey* ⇒ (Every possible animal x)(Some possible donkey y) x is y
> *Every animal necessarily-is a donkey* ⇒ (Every possible animal x)(Some necessary donkey y) x is y

But which things does a term like '*possible donkey*' supposit for? Presumably a possible donkey is an entity that is possibly a donkey. This will include all actual donkeys, past present and future — since modal words are explicitly said to ampliate their supposition to the past, present, and future. It will probably not include any other actual things, since e.g. a rock has an essence of its own which lies outside the category of donkeys. (This depends on what view you have about necessary properties. I am assuming what I think is the most usual view, attributed to Aristotle.) Will it include never-existing things that are possible donkeys? So far as I can see, the semantic theory is neutral on this. If there are such things, then '*donkey*' signifies them, and it supposits for them in modal propositions.

What then about necessary donkeys, which are needed for the second sentence. This will include all actual donkeys if each donkey is necessarily a donkey. (Although there may be none, since something that is necessarily a donkey necessarily exists.) Again, it will probably not include any other actual things. Again, there is a question as to whether there are also non-actual necessary donkeys. Perhaps these coincide with the non-actual possible donkeys (on Aristotle's view — though probably not on contemporary views). Again, that is a metaphysical view that the semantic theory is neutral on.

5.4 Ampliation due to Words which pertain to the soul

This doctrine is simple: the use of words which pertain to the soul ampliate certain terms so that they may supposit for everything that those words signify. For example, in '*The antichrist is opinable*' the adjective permits the subject to supposit for future things, so the subject is not empty. Since one may have opinions about the antichrist, the proposition is true. '*Some person thinks of a gold mountain*' can be true because the verb ampliates the predicate. However, '*Socrates thinks of the antichrist*' is not true, since the verb does not ampliate the subject term, and Socrates does not exist; so the proposition is false.

5.5 Ampliation due to Semantic Words

Semantic terminology ampliates terms, just as do words that pertain to the soul. For example, '*signify*' ampliates terms as widely as they can be ampliated. This is clear from examples like '*In any proposition, 'Dodo' signifies all dodos*', which is intended to apply to all dodos, not just to presently existing ones. This is consistent with no dodo presently existing. Some authors would accept *'chimera' signifies chimeras and no chimera is possible*'. (Those who oppose this view do so because they think that there are no chimeras at all to be signified.) The same goes for 'supposit', as in *'dodo' supposits for dodos in past tense sentences*', etc.

One might want to ask about the ontological commitments of such a theory. The logic and semantics alone don't commit you to much. Saying that '*donkey*' signifies all donkeys is, by itself, a truism. Since it is an affirmative sentence it commits you to there being at least one donkey. But no more. It is possible to state various commitments within the theory. For example, one might say '*Some donkey which will be grey is not [a being]'*. This commits you to future donkeys that do not exist. But only if you endorse it. If you thought that God had just destroyed all donkeys, and would not permit there to be any more, you wouldn't assert that sentence. Likewise '*Some donkey which was not and which is not and which will not be is possible*' commits you to possible donkeys that are not actual at any time; if you don't believe in them, you needn't endorse the proposition.

Some tried to argue like this: Since '*chimera*' signifies all chimeras, every chimera is signified by '*chimera*'. The conclusion is an affirmative sentence, which seems to commit you to chimeras (perhaps impossible ones, since none are possible). Buridan escaped this conclusion by denying that '*chimera*' signifies all chimeras. Since there are no chimeras, the word does not signify anything. This does not make it meaningless, because "chimera" is a complex concept, which is composed of concepts which are themselves non-empty and meaningful. indexampliation and restriction—)

6 NON-EXTENSIONAL CONTEXTS

In contemporary semantics, non-extensional contexts have received a great deal of attention. These are contexts in which interchanging expressions with the same extension can change the truth-value or reference of the whole context. This section covers three types of non-extensional constructions that were widely discussed by medieval logicians.

6.1 Belief Contexts

That-clauses are paradigm non-extensional contexts. Other types of contexts are equally non-extensional. In English, we have "accusative-infinitive" constructions that behave somewhat like that-clauses. An example is the clause in the second of these sentences:

I believe <u>*that he is a cleric*</u>
I believe <u>*him to be a cleric*</u>

An accusative-infinitive construction is so-called because the subject is in the accusative case ('*him*' instead of '*he*') and the verb is in the infinitive ('*to be*' instead of '*is*'). It is apparent that accusative-infinitive constructions behave semantically much like that-clauses; the major difference is that the tense of an embedded accusative-infinitive clause is parasitic on the tense of the main verb, whereas the main verbs of that-clauses have tenses of their own.[66]

In medieval Latin, that-clauses are rare, and accusative-infinitives are the norm. People who translate Latin regularly translate Latin accusative-infinitive constructions into English that-clauses. Scholars who translate semantic essays also generally do this as well. This causes trouble only when the author is explicitly discussing the syntactic form of the accusative-infinitive. I will use the accusative infinitive form.

Accusative-infinitives occur in sentences in positions where regular terms occur with personal supposition. For example, in '*Socrates believes a proposition which Plato utters*' the complex phrase '*proposition which Plato utters*' would normally be taken to supposit personally for propositions that Plato utters. The question naturally arises, then, of whether accusative-infinitives supposit, and what they supposit for. I will discuss three positions on this.

6.1.1 Ockham's view

The most well-known view is that of Ockham. He holds that we tend to interpret accusative-infinitives materially: they supposit not for themselves, but for the corresponding indicative propositions. So '*him to be a cleric*' supposits for a proposition of the form '*he is a cleric*'.[67] The objects of belief, then, are these propositions. If the propositions are the mental ones of this form, then this is not very different from the modern view that propositions are abstract entities that are expressed by written and spoken propositions. (On the medieval view, mental propositions are "signified by" spoken ones, or the spoken propositions are "subordinated to" the mental ones.) If Ockham meant that accusative-infinitive constructions materially supposit for spoken or written propositions, then you get something like the modern view that the objects of belief, etc, are sentences.

[66] Often the "independent" tense of a that-clause is, semantically, interpreted parasitically. For example, in '*She believed that he was mad*' the time of the being mad is usually understood to be the same as the time of the believing, or understood to be previous to the time of the believing. It is not usually just any time prior to the utterance of the sentence. The time of the being mad in accusative-infinitive '*She believed him to be mad*' must be taken to be the time of the believing.

[67] This is part of a general treatment of accusative infinitive clauses: *SL* I.67 (198) "Likewise in 'A man to run is true' the subject 'a man to run' does not supposit for itself but for the proposition 'A man runs', which it does not, nevertheless, signify. [I have changed the translator's that-clauses to accusative infinitives and removed interior quotation marks.]

6.1.2 The non-thing view

In contemporary semantics many people think that a that-clause stands for an abstract proposition, something like a Fregean thought or perhaps a Russellian proposition. Abelard has a view something like this. He holds that an accusative-infinitive construction supposits for "non-things which are not nothing". A "non-thing" is something that is neither a substance, nor a quality, nor any other sort of thing that falls within the Aristotelian categories of what there are. To say that these things are "not nothing" is to clarify that saying that an accusative-infinitive supposits for a non-thing is not a roundabout way of saying that it doesn't supposit for anything. Granted, it does not supposit for any thing, but 'thing' has a restricted use here.

Abelard argues that we should acknowledge such non-things:

> We often call by the name 'cause' what are not any *thing*. For example, when we say "He was flogged *because* he does not want to go to the forum." 'He does not want to go to the forum', which occurs as a "cause" here, is no essence. [Spade 1994, 42]

Although Abelard does not put his view in semantic terms, Adam Wodeham does. He says that an accusative-infinitive can supposit for itself, or for what the corresponding proposition signifies:

> A distinction must be drawn . . . because man-being-an-animal can supposit for and be taken for the dictum of a proposition, and in this sense it is indeed complex or incomplex. Or it can be taken for that which is signified by such a dictum, in which case it is neither complex nor incomplex, but is something signifiable by the complex — for example, by the complex *Man is an animal*.[68]

The expression 'man-being-an-animal' is a translation of a Latin phrase '*hominem esse animal*', which also translates as 'a man to be animal'. This expression can supposit for what is signified by the proposition '*A man is an animal*'. It would also normally be translated by a that-clause, so the view as pertaining to English would be that the that-clause '*that a man is an animal*' supposits for what '*A man is an animal*' signifies. When pressed to say just what [a] man to be [an] animal is, he says (again, the accusative-infinitive is translated as a participial phrase):

> we should say that man-being-an-animal is not a thing (*aliquid*) or a substance, but is instead man-being-something and man-being-a-substance-or-accident. . . . One sign signifies adequately not substance but something-being-a-substance and so forth, whereas another sign signifies something-not-being-a-substance and so forth. . . . You will say: man-being-an-animal is either something or nothing. I say that

[68] In Pasnau [2002, 337]. Wodeham speaks of what is signified by the *dictum* of a proposition, which would be the actual accusative-infinitive clause, but his example makes it clear that he is discussing what is signified by the proposition itself.

> neither should be maintained, and that it is not something but rather man-being-something, as was said. [Ibid]

6.1.3 Accusative-infinitives supposit for whatever their subjects supposit for

Several writers held that the object of knowledge or belief is whatever the subject of the proposition that expresses the belief refers to. This was discussed in non-semantic terms as the question of what is the object of a mental believing or knowing.

Buridan held a version of this view, holding that accusative-infinitives supposit for whatever their subject supposits for, if the accusative-infinitive correctly characterizes the subject; otherwise they supposit for nothing. Buridan illustrates this view as follows (where the accusative-infinitives have been translated as that-clauses):

> To the other [argument] I reply that *that Socrates loves God* is Socrates, if Socrates loves God; but if he does not love God, then *that Socrates loves God* is nothing. In a similar fashion, *that Socrates hates God* is Socrates, if Socrates hates God; but if Socrates does not hate God, then it is nothing; therefore, when you argue thus: '*That Socrates loves God* is Socrates', I concede it, assuming it to be the case that Socrates loves God. And if you say: 'in the same way, *that Socrates hates God* is Socrates', I deny it, for it is nothing, when Socrates loves God. But, perhaps, *that Socrates hates God* was Socrates, if Socrates earlier hated God. Therefore, when it is further said that *that Socrates loves God* is good and commendable, I concede it; for it is a good man who loves God. And when it is said: *that Socrates hates God* is damnable, I deny it, for [*that Socrates hates God*] is nothing. But *that Socrates hates God* was bad, for it was a bad man hating God. And it is not unacceptable that a good man was a bad man; therefore, neither is it impossible that *to love God* was *to hate God*. [*SD*: *Sophismata* 1 (844)]

This seems to be subject to easy refutation. Suppose that God loves all creatures, and that Socrates hates God. Then, since God loves Socrates, God loves Socrates-to-hate-God, which seems wrong. Buridan does not discuss this kind of problem.

6.2 Intensional Transitives

A famous problem occurred in Aristotle [*Soph. Ref.*179a33-b6]. You see someone approaching, too far away to determine who it is. The facts are supposed to be:

> You don't know the one approaching.
> The one approaching is your father.

The apparent conclusion is that you don't know your father. The goal is to say how to avoid this absurd conclusion.

Nowadays, our instinct would be to deny the first premise. We would be inclined to say that we do know the one approaching. Granted, we don't know who the one approaching is, but this is a different claim. Thus the puzzle is easily solved. However, this was not the medieval approach.[69]

One solution offered by Buridan and several of his followers was to say that certain mental verbs relate one thing to another, not absolutely, but in relation to a *ratio*. You can be said to know A if you know A under the ratio A. If B is the same as A, then it does not follow that you know B, for this requires that you know B under the ratio B. And the identity of A with B does not guarantee that the ratio A is the same as the ratio B. This solves the puzzle, since:

> You know your father under the ratio *your father*.
> Your father is the one approaching.

You can legitimately conclude:

> You know the one approaching under the ratio *your father*.

But you cannot conclude:

> You know the one approaching under the ratio *the one approaching*.

There is thus a kind of *de dicto* construal of the locution 'know ___'. There is also a *de re* wording; it is the one in which the term that supposits for the known thing precedes the verb. In this case there is the analysis:

> X you know = you know X under some ratio

So these are both true:

> You know your father
> Your father you know (You know your father under some ratio.)

(The first is taken to be data. The first entails the second, because it entails

> You know your father under the ratio *your father*.)

And if you know the one approaching, this also is true:

> The one approaching you know.

But none of this lets you conclude:

> You know the one approaching.

What, then, is a *ratio*? This is not clear, but it would be natural to take the ratio associated with a term to be the mental concept that the term is imposed on. That would legitimate the above solution.

[69]Nor is it clear whether it avoids the problem. For we now face a related puzzle: "You don't know who the one approaching is. The one approaching is your father. So apparently you don't know who your father is." One can interpret the medieval theorizing as about this latter formulation.

6.3 I promise you a penny

This paradox arises with several wordings, including:

I promise(d) you a penny
I promise(d) you a horse
I owe you a penny/horse
For riding is required a horse

I'll focus on the version "I owe you a horse". The problem is that if I promise to give you a horse, though no particular horse, then it seems as if I owe you a horse, but there is no horse that I owe you.

It seems to be true that

I promised you a horse

doesn't entail

I promised you this horse

for any given horse. If I promise that I'll give you a horse, this particular horse has apparently not been promised, for I can pay off the promise without giving you this horse. Some realists explained this by saying that what is owed is the universal horse.[70] This is indeed given when I pay off the debt; it is given in the only way a universal can be given, by giving one of its instances.

This needs to be supplemented by an explanation of why '*I promised you a horse*' results in your owing the universal horse. Wycliff explains this by holding [Ibid 2.64] that the verb '*promise*' causes the terms following it to occur with simple supposition. So in the first proposition the words occur with simple supposition, and thus supposit for the universal horse. However, singular terms cannot have simple supposition. So the words '*this horse*' in the second proposition occur personally, and supposit for this particular horse. Wycliff goes on to hold that the entailment does hold in the other direction: If I promise you this horse, then I promise you a horse. Given his analysis of the constructions, it is not at all clear why this should be so. He simply asserts that from '*I promise you this [horse]*' it follows: '*I promise you a [horse]*'[Ibid 2.64].

A much-discussed issue is how these two propositions are related:

I promised you a horse
A horse I promised you

Everyone seemed convinced that the latter proposition requires for its truth that there is some particular horse that I promised you. Since the first proposition seems not to require this, one wonders how they are connected. One could just

[70] Wycliff *T* Tract 3, Chapter 3, ed. cit., 2.63, cited in [Read, 1985]. Also Burley *PAL* 1.1.3 para 59 p. 96.

say that neither entails the other, but one wants an explanation of the semantics of the former, if it is not the same as the latter.[71]

Buridan argued that by parity, if I promised you a horse, then either every horse I promised you, or no horse I promised you. He found the latter counter-intuitive, and so settled on the former. This view was not widely accepted.

Ockham proposes what seems to me to be the right thing to say. He says that '*I promise you a horse*' means that this ought to be true: '*I give you a horse*'. He says:

> Thus, 'I promise you a horse', given the force of 'I promise', asserts that 'I give you a horse' (or something similar) will be or ought to be true sometime. The assertion is not that a proposition like 'I give you this horse' (where the reference is to some particular horse) will be or ought to be true. Thus, it does not follow that if I promise you a horse, I promise you this horse or I promise you that horse. [Ockham *SL* 1.72 (207-08)]

Buridan dismisses this, saying

> We should realize that some people, wanting to evade the difficulty straight away, say that this is not a proper locution: 'I owe you a horse' or 'A horse is owed by me to you', but that this locution should be taken in the sense that 'I owe or am bound to bring it about that you have from me a horse'. And so the proposition 'I owe you a horse' or 'A horse is owed by me to you' is neither true nor false, except in this sense.[72]
>
> But this solution seems to me to be superficial, for there are several ways in which [it can be arranged that] this can be brought about: for one way to bring this about would be to give you Blackie, and another way to bring this about would be to give you Tawny. Therefore, the question would recur whether I would be bound to do so in every such way or whether I would be bound to do so in no such way. [Buridan *SD* 9.4 *Fifteenth sophism* (912)]

It is difficult to assess Buridan's criticism, for it seems to miss the mark. His objection is that the proposal does not settle the question of whether to fulfill the obligation I must give you every horse, or no horse. But the proposal seems to be that giving a single horse satisfies the obligation, and any one will do.

Much intricate discussion of this example continued throughout the 15^{th} century and beyond. For more information, see Ashworth [1974; 1976].

[71] Most writers agreed that '*horse*' in the latter proposition has determinate supposition, since it is equivalent to '*This horse I promised you, or that horse I promised you, or . . , and so on for all horses.*" (See chapter 8 for a discussion of determinate supposition.) Many wanted to say that '*horse*' in the former proposition has merely confused supposition; this will be discussed in chapter 8.

[72] For discussion of this "evasive move" by Ockham, as well as in contemporary logic, see [Read, 1985; Biard, 1988; Klima, 1993].

7 MODES OF SUPPOSITION: THE EARLY VIEW

7.1 Modes of Personal Supposition

Personal supposition occurs when a term is used to stand for things that it signifies (according to nominalists) or for things that fall under the form that it signifes. Terms with personal supposition may be divided into those that have discrete supposition and those that have common supposition. Discrete supposition is possessed by singular terms that are used personally. This includes proper names, such as '*Socrates*' or a simple demonstrative such as '*that*' when it is the whole term making up a denoting phrase, or a whole complex demonstrative such as '*that donkey*' when it is the whole term making up a denoting phrase. Common supposition is the kind of supposition that a common term has when it is the whole term making up a denoting phrase (usually) with a quantifier word.

A common term which is used personally has what is called a *mode* of (common) personal supposition.[73] A mode of supposition is something like a kind of quantificational status. It is a status that a term has in a proposition based on where it occurs in the proposition and what quantifier word occurs with it. There are three modes of common personal supposition:

- Determinate Supposition
- (Confused and) Distributive Supposition[74]
- Merely Confused Supposition

Determinate supposition has something to do with a term's being existentially quantified; a paradigm example is '*donkey*' in '*Some donkey is spotted*'. Distributive supposition has something to do with a term's being universally quantified; a paradigm of distributive supposition is '*donkey*' in '*Every donkey is spotted*'. Merely confused supposition is neither of these; it needs to be discussed. (An example of a term with merely confused supposition is '*donkey*' in '*Every animal is a donkey*'.)

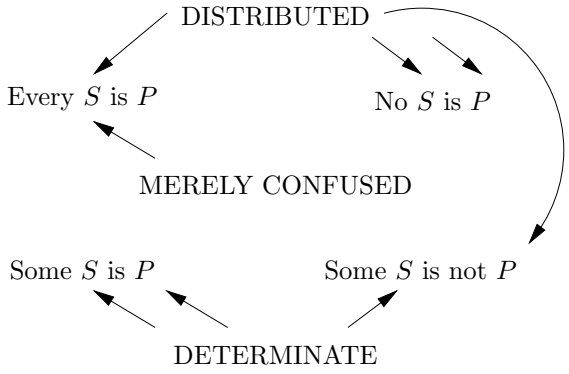
Almost all authors agreed on the classification of terms of the Aristotelian standard forms of categorical propositions:[75]

[73]Some authors (see Marsilius of Inghen 1 (57-59)) believe that terms used *materially* can also be assigned a mode of supposition, since they are common terms. For example, in '*Every donkey in that sentence is bisyllabic*' the word '*donkey*' is a common term that stands for any word spelled d-o-n-k-e-y. On this analysis, '*donkey*' has distributive material supposition in that sentence. Most authors either disagree with this view, or ignore it. According to some, even discrete terms used materially are common terms, and can have modes of supposition, as in '*Every this uttered this morning was the subject of the sentence in which it occurred*'.

[74]Some authors use 'distributed' and some use 'distributive'; I use these interchangeably. (The term 'distributed' may presuppose that a term cannot have that mode of supposition unless it has been distributed by some distributing sign; see section 7.5 below.)

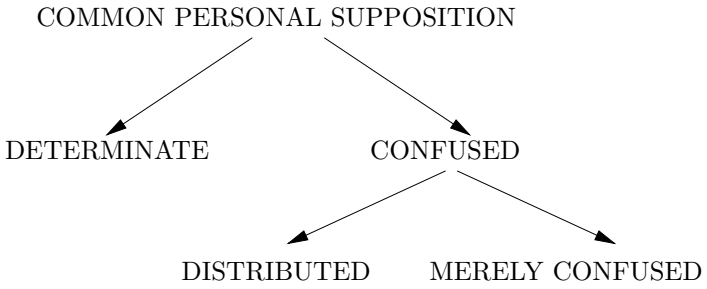
[75]Exception: Peter of Spain *T* VI.6 (70-71) held that all predicates have simple supposition, and do not have any mode of personal supposition. Some authors did not discuss the predicates of particular negatives.

- The subjects of universals and the predicates of negatives have distributive supposition.
- The subjects and predicates of particular affirmatives, and the subjects of particular negatives, have determinate supposition.
- The predicate of a universal affirmative has merely confused supposition.



7.1.1 Deriving the Classification

Usually the three-part classification of terms results from two bifurcations: personal supposition is divided into *determinate* versus *confused*, and *confused* is divided into *distributed-and-confused* versus *merely confused*.[76]



The issue of how to decide whether a term is determinate or confused, and distributive versus merely confused, is a central concern of the theories.

7.1.2 What are the Modes of Supposition for?

What is the point of classifying terms into the above categories? The most popular application of the modes of common personal supposition is to analyze fallacies

[76]So the '*merely*' in '*merely confused*' contrasts with the '*distributed*' in '*distributive and confused*'. The term '*confused*' does not here mean '*bewildered*'; it means something more like '*taken together*'. (The Latin transliterates as '*fused with*'.)

that result due to a term's moving from one mode to another, producing a kind of quantificational ambiguity. An example is the following inference:

Every man sees a donkey
∴ *A donkey every man sees* (= "for some donkey, every man sees it")

In the premise, the term '*donkey*' has merely confused supposition, and in the conclusion it has determinate supposition. The error was identified as the fallacy of moving from a term with merely confused supposition to one with determinate supposition.[77] We would identify the error as involving a change in quantifier scope. Modes of supposition are properties that are affected by the relative scopes of quantifier words in the proposition. The mode of supposition of a term was as important to medieval theorists as quantifier scope is to us today.

The modes of common personal supposition are also used in formulating rules of inference that cover inferences that go beyond those addressed in the theories inherited from Aristotle. (See sections 7.6 and 8.7.)

Once the importance of modes of supposition for inference was recognized, they tended to become a topic of independent study.

7.1.3 Terms functioning as Denoting Phrases

One thing common to all parts of the tradition is that the modes are used to classify *terms*, as opposed to denoting phrases that contain the terms. For example, in '*Every donkey is running*' one classifies '*donkey*' as having distributive supposition, not '*every donkey*'. But the classification of terms is a classification of terms *as affected by quantifier signs*. In the proposition '*Every donkey is running*' the classification of '*donkey*' is sensitive to the '*every*' preceding it. In most cases it turns out that there is no significant difference between classifying denoting phrases such as '*every donkey*' or classifying main terms, such as '*donkey*'. So when we classify '*animal*' in '*Every donkey is some animal*' as merely confused, we can, if we wish, see this as a roundabout way of classifying '*some donkey*' as merely confused in that proposition. (The classification of terms that are not the main term of a denoting phrase needs additional discussion.)[78]

[77] In many cases, confusing one mode of supposition for another was seen as an instance of the Aristotelian fallacy of *form of expression*. Peter of Spain *T*VI.8 (71) says: " In the examples above, ... the term '*man*' stands for every man, running or not, but it makes the utterance true for one man running. That each is determinate is clear, because in *Socrates is an animal, Plato is an animal, Cicero is an animal, etc.; therefore every man is an animal*, there is a fallacy of form of expression, shifting from many determinates to a single one." A change is made in the mode of supposition, but this is concealed because the words used before and after the change seem to have the same forms. (The Latin that is translated here as "form of expression" is '*figura dictiones*'. This is often translated as "figure of speech", but it does not involve any figure of speech as this term is understood today.) The order of the words changes, but there is no grammatical change, such as switching the subject and direct object, as in the fallacy of amphiboly, or of grouping, as in the fallacy of composition or division.

[78] Examples of terms that are *not* the main term of a denoting phrase are '*donkey*' in '*Every non-donkey is a stone*', and in '*Every grey donkey is running*'. These are cases in which the term forms only part of a complex term that combines as a whole with a quantifier sign. Although

7.1.4 Two versions of the theory

Most theorizing about modes of supposition falls under one of two overall versions.

One version, which I will call the **early view**, is espoused by three 13^{th} century figures (among others): William Sherwood, Lambert of Auxerre, and Peter of Spain.[79] On this version, the meanings of the modes are defined quantitatively, in terms of "how many" of its supposita a term is taken for; this is explained in the remainder of this section.

The other version — the **later view** — is developed by some 14^{th} century figures, especially Walter Burley, William Ockham, John Buridan, and Buridan's followers, Albert of Saxony and Marsilius of Inghen. An eclectic version of the theory is given by Paul(s) of Venice, and the theory continues to be studied into the 15^{th} and 16^{th} centuries. For these authors, explanations of the modes are given in terms of certain types of inferences (*ascent* and *descent*) allowed by the contexts of terms. This view is discussed in the next section.

7.2 The Early View: William Sherwood, Lambert of Auxerre, and Peter of Spain

In an early version of the study of modes of personal supposition, William Sherwood has a charming and provocative quantitative account of the modes of supposition:

> Personal supposition is determinate when the locution can be expounded by means of some single thing. . . . [Personal supposition is] confused, on the other hand, when the word supposits for many, and distributive when it supposits for many in such a way as to supposit for any. [Sherwood *IL* V.2 (108)]

The suggestive terms that need elucidating are:

Determinate	. . . expounded by some single thing
Confused	. . . supposits for many
Distributed	. . . supposits for many, so that it supposits for any
Merely Confused	. . . supposits for many, but not for any

Our goal in this section is to make sense of this. We begin with determinate supposition.

7.2.1 Determinate Supposition

William gives the following account of determinate supposition :

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authors often take the official position that their theory does not apply to terms that are parts of other terms, they frequently discuss and classify such terms anyway.

[79]Some earlier anonymous works endorse this version as well: the 12^{th} century *Dialectica Monacensis*, and the 12^{th} century *Properties of Discourse* (with some idiosyncrasies) in [De Rijk, 1967].

> Personal supposition is determinate when the locution can be expounded by means of some single thing, which is the case when the word supposits for some single thing. Therefore in '*a man is running*' it can be true for anyone running. [Ibid]

He soon adds:

> The sentence '*A man is running*' means that the predicate is in some one individual, not in many, even though the predicate *is* in many — for a sentence sometimes *permits* this but it does not *signify* it. [Ibid V.11 (116)][80]

I assume that the basic account is in the first sentence: supposition is determinate when the locution "can be expounded by means of some single thing". This must not be read as saying that in the sentence the word supposits for only one thing, as if '*man*' in '*a man is running*' supposits only for one particular man among men. Apparently such a view was taken seriously, since Peter of Spain takes pains to reject it, but in any event William's view is not of that sort. The added quote makes this clear: The sentence does *not* mean that the predicate is *not* in many individuals. So we should read '*the predicate is in some one individual*' as permitting it to be in more than one. That is, we should read it as '*the predicate is at least in some individual*'.

Generalizing to propositions of various forms, I suggest that we read the condition as saying:

> A term is *determinate* in a proposition $=_{df}$ the proposition means that there is at least one thing x that the term supposits for such that the proposition is true for x.

To say that the proposition is true for x (with respect to a term) is equivalent to saying that another proposition is true, namely, the one you get by replacing the term (and its quantifier sign, if any) by 'x'.

> '$\ldots QT\ldots$' is true for x = '$\ldots x\ \ldots$' is true

where 'Q' is any quantifier sign directly governing 'T'. Example: The term 'donkey' is determinate in 'Some mammal is a donkey' because

> 'Some mammal is a donkey' means that there is at least one donkey x such that 'Some mammal is x' is true.[81]

This seems to me to be a possible reading of what he says. It also makes sense of all of his applications. For example, it is easy to check that this makes both terms

[80] I interpret the first sentence as "The sentence *'A man is running'* means that the predicate is in at least some one individual; it does not mean that the predicate is in many, even though . . .".

[81] Granted, this mixes up use and mention, but I think it is clear enough. We must interpret 'means that' loosely, as something like 'necessarily, is true iff'.

of the particular affirmative, as well as the subject term of the particular negative (ignoring the case where the subject term is empty) determinate, and does not make any other of the terms of standard categorical propositions determinate. I'll assume that this is his view.

Peter of Spain gives a loose characterization,[82] which can be interpreted as equivalent to Sherwood's. The same is true of Lambert.[83]

7.2.2 Distributive Supposition

William defines confused and distributive supposition as follows:

> Personal supposition is confused ... when the word supposits for many, and distributive when it supposits for many in such a way as to supposit for any. [Sherwood *IL* V.2 (108)]

To assess this we need to know first what it is for a term to "supposit for many". I think that this is a very weak claim; it only denies that the term has determinate supposition, i.e. it only denies that it supposits for (at least) *one*. That is, it says that the proposition in question can *not* be expounded in terms of anything involving "(at least) some single thing." It is clear that nothing stronger is intended than this, or, if something stronger is intended, it is soon retracted. For example, William says soon after this definition [*IL* V.12 (116)] that a word has confused

[82]Peter of Spain *T*VI.8 (71): "Determinate supposition labels what a common term has when taken indefinitely or with a particular marker, as in '*man runs*' or '*some man runs*'. Each of these is called determinate, since though in each, the term '*man*' stands for every man running or not, they are true if only one man is running. {changing the translator's 'only if' to 'if only'.} To stand for is one thing, to make a locution true for something, another. In the examples above, ... the term '*man*' stands for every man, running or not, but it makes the utterance true for one man running."

[83]Lambert *PT* 3g(iv) (111): "Determinate supposition is what a common term has when it can be taken equally well for one or for more than one, as when one says '*A man is running*'. In that proposition '*man*' has determinate supposition because it is true if one man is running or if more than one are running. But it is called determinate because for the truth of a proposition in which a common term that has that sort of supposition is used, it is enough that the common term is interpreted necessarily for some suppositum, and it is not required that it be interpreted necessarily for more than one, although in supposition of this sort it can be interpreted for more than one." I take it that the heart of this account is:

A term T has determinate supposition in a proposition when it is sufficient for the truth of that proposition that 'T' be interpreted in it for one of its supposita.

When Lambert talks about *interpreting* a term in a proposition *for one*, it may be that this is the same as when William talks about the proposition *being true for a single thing* (with respect to that term). On this interpretation Lambert's remarks yield a slightly different account than William's, since Lambert's account only talks about a sufficient condition for truth, instead of a necessary and sufficient condition. If we look at the terms of standard form categorical propositions, this yields the standard results except that it makes predicates of universal affirmatives have determinate supposition; it does this since '*Every S is P*' does follow from '*For some x that is P, every S is x*'. This cannot be what Lambert intended, since he says that these predicates have confused supposition. Probably Lambert was taking for granted that in determinate supposition it is also *necessary* for the truth of the proposition that it be true for one of T's supposita. If this is added to the above account, it becomes equivalent to William's.

supposition whenever it supposits *either* for many things or for one thing taken repeatedly. (He is worried about the second '*man*' in '*Every man sees a man*' when everyone sees the same man — so that it is false that many men are seen.) So the '*many*' is merely a way of denying that we have a case of '*one*', that is, of determinate supposition. This reduces the characterization of distributive supposition to:

> A term has distributive supposition when it does not have determinate supposition and it supposits for any.

It is clear in context that '*supposits for any*' means that the term supposits for *any of its supposita*; I will take this limitation for granted. The trick is to see how to get this to mean anything other than a tautology. After all, how could a term not "supposit for any" of its supposita? The answer must be that for a term to supposit *for* something in a proposition requires more than just that the thing in question be among the term's supposita. I suggest that a term "supposits for any of its supposita in a proposition" just in case that proposition's being true entails that it (that very proposition) is *true for* any of the supposita of the term (with respect to the term in question), in the sense of '*true for*' discussed above. Thus, a necessary and sufficient condition for:

> 'T' supposits for any of its supposita in a proposition '$\ldots T \ldots$'

is:

> the proposition '$\ldots QT \ldots$' entails that '$\ldots QT \ldots$' is true for any of T's supposita.

This can then be further spelled out as follows:

> Necessarily, if '$\ldots QT \ldots$', then for any x that is T: '$\ldots x \ldots$' is true

For example, the term '*dog*' has distributive supposition in '*every dog is spotted*' because it does not have determinate supposition there, and:

> Necessarily: if every dog is spotted, then, for any x that is a dog: x is spotted.

(Again, this is an explanation of how to understand William's account, not a proposal for how to replace his account with something clearer. His account was already fully stated above, and I have not suggested revising it.)

One can check that this account also gives the intended results for the subject and predicate terms of universal negatives. It also classifies the predicates of particular negatives as distributed, because

> Necessarily: if some dog is not spotted, then, for any x that is spotted, some dog is not x.

One might wonder why I have formulated the condition for distributive supposition here in terms of a conditional instead of a biconditional. I am guided by the intended applications; in order for the predicate of '*Some S is not P*' to have distributive supposition (which is required by William's rule at V.13.1) one needs a conditional, not a biconditional.

Peter of Spain says:

> Distribution is the multiplication of a common term effected by a universal sign. [*T* XII.1 (185)]

He doesn't say much more. Whatever he has in mind, his account is not the same as William's, for he insists that a universal sign is needed for distribution, and he thinks that negation is not a universal sign. (This will be discussed below.) So the predicate of a particular negative cannot have distributive supposition for Peter; instead, he says that such predicates have simple supposition. This is no surprise since he claims this is true for all predicates. [*T*VI.6 (70-71)]

Ignoring predicates, the best way to read Peter is probably to assume that he means roughly the same as William except that he states the additional condition that distribution occurs only in connection with a distributing sign,[84] coupled with the view that '*every*' and '*no*' are distributing signs but '*not*' is not. He thus agrees with the others for the most part, though he differs regarding some details.

Lambert's account is close to that of Sherwood.[85]

7.2.3 Merely Confused Supposition

The definition of merely confused supposition is the hardest to get clear on. Sherwood's explanation is virtually no explanation at all:

> [Personal supposition] is merely confused [when the word supposits as does] the word 'animal' [in 'every man is an animal']. [*IL* V.2 (108)]

There are a number of additional examples, but little in the way of explanation. However, his initial exposition suggests that merely confused supposition is nothing other than confused supposition that is not distributive. So we can just understand merely confused supposition as any personal supposition that is neither determinate nor distributive, and then use the above explanations for determinacy and distribution. Among standard categorical propositions, this would classify only the predicate term of a universal affirmative as merely confused, which is what most people other than Peter of Spain thought.

Peter of Spain does not define '*merely confused*'.[86] His view seems to be that where others see a need for merely confused supposition (most prominently, in

[84] This appears to be what Peter argues in *T*VI (10-12) and in XII (24).

[85] Lambert *PT* 3g(v) (112): "[Distributive] ... supposition is what a common term has when it is interpreted for all of its supposita necessarily..." Above I suggested that Lambert's '*the term is interpreted for*' is the same as William's '*the proposition is true for*'. If we make this same equation here, Lambert's account is the same as William's.

[86] At VI.10 (72) it appears that he might consider defining merely confused as immobilely confused, but this is in the context of a speculation that he goes on to reject.

the predicates of universal affirmatives) he sees only simple supposition. So Peter parts company with the others on this category. Lambert gives a somewhat obscure explanation that can be interpreted as equivalent to Sherwood's.[87]

7.2.4 Summary of the early theory

Although all of the texts allow of multiple interpretations, in the light of intended applications of the accounts, I suggest the following criteria for the modes of supposition of common terms used personally in simple categorical propositions:

- 'T' has **determinate supposition** in the proposition '$\ldots QT \ldots$' iff Necessarily: '$\ldots QT \ldots$' is true iff for at least one x that is T, '$\ldots x \ldots$' is true
- 'T' has **distributive supposition** in the proposition '$\ldots QT \ldots$' iff Necessarily: if '$\ldots QT \ldots$' is true then for every thing x that is T, '$\ldots x \ldots$' is true
- **merely confused supposition** in the proposition '$\ldots QT \ldots$' iff 'T' has neither determinate nor distributive supposition in '$\ldots QT \ldots$'

More schematically:

- **Determinate:** $\ldots QT \ldots \leftrightarrow$ for at least one x that is T: $\ldots x \ldots$
- **Distributive:** $\ldots QT \ldots \rightarrow$ for every x that is $T : \ldots x \ldots$
- **Merely Confused:** neither Determinate nor Distributive

To have a better feel for what this amounts to, here are how the terms in the following propositions would be classified by these criteria:

[87] The full quote from Lambert *PT* 3g(v) (112) is: "Weak immobile supposition is what a common term has when it is interpreted necessarily for more than one suppositum contained under it but not for all, and a descent cannot be made under it." He uses 'weak' where others use 'merely confused'. He is trying to define both *weak* and *immobile* in the same sentence. (Lambert holds that there is no such thing as mobile weak confusion.)

How this works depends on how it is interpreted. Suppose, for example, that we interpret it as weakly as possible. Then we construe Lambert's '*more than one*' as we did William's '*many*', that is, merely as a way to deny that the term "can be interpreted for (at least) one." This part of the clause then simply denies that the term has determinate supposition. Then we interpret '*but not for all*' as merely denying that the term is "interpreted for all of its supposita," that is, as merely denying that the term is used distributively. The overall result is that a term is defined to have merely confused supposition when it has personal supposition that is neither determinate nor distributive. This is a satisfactory account that coheres with others (except for Peter's). Stronger construals are possible, but I do not see any natural way to produce one that coheres with the application of the theory to the subjects and predicates of simple categorical propositions.

Some A is every B	A is determinate	B is distributive
Every A is every B	A is distributive	B is distributive
No A is every B	A is distributive	B is merely confused
Some A is no B	A is determinate	B is distributive
Some A is not every B	A is determinate	B is determinate
Some A is not no B	A is determinate	B is determinate

7.3 Mobility

Some terms in a proposition are *mobile* and some are *immobile*. A mobile term is one that can be instantiated; that is, the proposition containing it entails the result of replacing the term (along with its quantifier sign, if any)[88] with any discrete term that stands for a suppositum of the original term. For example, '*donkey*' is mobile in the proposition '*Every donkey is running*' because that proposition, together with the information that Brownie is a donkey, entails '*Brownie is running*'. This inference is also called a *descent*, since one moves from a term such as '*donkey*' to a term '*Brownie*' that stands for something that falls under '*donkey*'. Terms that are not mobile are *immobile*. An example is '*donkey*' in '*Every running thing is a donkey*'; from this and that fact that Brownie is a donkey one may not infer (descend to) '*Every running thing is Brownie*'.

Sherwood, and most others see mobility as limited to distributive terms. Sherwood's definition of mobility is:

> [Distributive confused supposition is] mobile when a descent can be made, as in the term '*man*' in ['*every man is an animal*']. It is immobile when a descent cannot be made, as here: '*only every man is running*' (for one cannot infer '*therefore only Socrates is running*') [*IL* V.2 (108-09)]

Lambert's definition is practically the same (he even illustrates it with the same examples). [*PT* 3g(v) (113)][89] The identical examples suggest an identical source for both passages.

Mobility interacts in complicated ways with other notions, as we will see.

[88] If the quantifier sign is negative, such as '*no*', then you need to preserve the negation by inserting '*not*' in its place. For example, from '*No donkey is running*' and '*Brownie is a donkey*' you infer '*Not Brownie is running*'. Since discrete terms commute with '*not*' you can write instead '*Brownie is not running*'.

[89] Lambert "Notice that confused supposition is strong (= distributive) whenever it is mobile, but not conversely — mobile whenever it is strong (= distributive) — for it can indeed be strong (= distributive) and immobile. When one says '*Only every man is running*', '*man*' has strong (= distributive) but immobile confused supposition; for the exclusive word ['*only*'] added to the distributed term impedes it so that a descent cannot be made under it." Lambert uses '*strong*' where others use '*distributed*'.

7.4 Locality

Sherwood and Lambert (and probably Peter of Spain) hold that modes of supposition are determined by how they occur in their local context. That is, a term is classified in terms of what mode it has in a simple proposition, and that classification does not change when the proposition is embedded in more complex propositions. Recall from the previous section that prefixing the word '*only*' to '*every man is running*' impedes mobility without changing the fact that the term to which it is added is distributive. This is implicit in Lambert's wording: the result is a term that has "distributive but immobile" supposition. William comments on this as follows:

> But it seems a descent can be made in every case of distributive supposition, for a word supposits distributively that supposits for many in such a way as to supposit for any; therefore a descent can be made to any. And it must be said that a descent *can* be made in every case of distributive supposition considered in itself *(quantum est de se);* nevertheless, the descent can be impeded by means of an adjunct, as by means of the word '*only*' in the example above. [*IL* V.3 (109)]

Kretzmann explains this (correctly, I think); he says that Sherwood means that the term, which is originally distributed in '*every man is running*', is mobile *there*. But embedding that proposition in a larger one can impede the mobility without affecting the distribution of the term. The test for distributive supposition is apparently applied before the impeding term is added, and adding it does not change this. Additional evidence that Sherwood believes this is that he later pairs the proposition '*Only every man is running*' with '*Not every man is running*', saying that both of them contain immobile distribution. [*IL* V.13.5 (119]

A consequence of this view is that no term that is not distributed in its smallest proposition may become distributive upon embedding. Peter of Spain makes this clear in his discussion of what causes distribution. He notes that from '*Not [a] man is just*' one can infer '*Not Socrates is just*'. That is, '*man*' is mobile in that proposition. However, this is not because '*man*' is distributed there; it is not. So not only is there distribution without mobility, there is mobility without distribution.

It appears as if the mode of supposition of a term is defined in terms of that term's quantificational functioning when that term is in its "minimal position". This is like our defining quantifiers in terms of what they do when they are in canonical position: on the front of a formula with scope over it. When a universally quantified sentence is embedded in something larger, we do not say that the quantifier is no longer universal, even if it becomes immobile (that is, even if universal instantiation is no longer valid for it). In the early theory, modes of supposition work that way. The only difference is that in the medieval theory there are *two* basic minimal positions where a term gets its mode of supposition: subject position and predicate position in a minimal categorical proposition.

The significance of suppositional mode being local will emerge when we discuss rules of inference below, and when in the next section we contrast the early theory with the later, in which modes of supposition are global, not local.

7.5 Causes of Distribution and Confusion

All scholars agree that neither confusion nor distribution can occur unless it is caused to occur. Generally, it was thought that they must be caused to occur by some sign which has scope over the term in question.[90] We look here at some 13^{th} century accounts.

Confusion and distribution need to be caused, but determinacy does not; this is because a common term that is unaffected by any sign is determinate by default. It is also immobile by default. Its status can be changed by the addition of some sign, though not just any old sign; a particular sign such as '*some*' leaves its determinate status unchanged.

> Signs other than particular ones can affect
> Confusion ... causing it or undoing it,
> Distribution ... causing it or undoing it,
> Mobility ... causing it or undoing it.

For example, '*every*' in the universal affirmative form confuses and distributes and makes mobile the subject to which it is adjoined; it confuses the predicate without distributing it and without making it mobile. The '*no*' in the universal negative confuses and distributes both the subject term to which it is adjoined and the predicate term, and makes them both mobile.

Sherwood gives three rules for confusion and distribution [*IL* V.13.1 (117).] (The rules presuppose that a term starts out determinate by default.):

> [A] *every distributive sign confuses the term immediately adjoining it confusedly and distributively.*
> [B] *an affirmative [distributive] sign confuses the remote term merely confusedly.*
> [C] *a negative [distributive] sign confuses the remote term confusedly and distributively.*
> Thus this follows: '*no man is an ass; therefore no man is this ass*'. But this does not follow: '*every man is an animal; therefore every man is this animal*'.

Sherwood's rules say nothing about mobility, which is what is at issue in the examples he gives. (The question is whether one may descend under '*ass*' or '*animal*'.) I think that he is presupposing a rule about mobility, which I will call D:

[90]The notion of scope here is a modern one; I use it here to sum up what seems to me to be the medieval practices. Certain writers thought that confusion or distribution could be caused without a sign, by the will of the speaker (such as giving distributive supposition to the subject of an indefinite proposition such as '*A donkey is an animal*'). Cf. [Barney *et al.*, 1997].

[D] *a distributed term is mobile if nothing impedes its mobility.*

In the first example he gives, the '*no*' distributes the predicate term by rule C, and thus it is mobile by rule D. That is, you can infer an instance under the term '*ass*'. In his second example the '*every*' confuses without distributing the predicate term. This seems to assume an additional principle for mobility, which is something like:

[E] *a term is immobile unless made mobile.*

Since there is nothing in the second example to make the predicate term mobile, the inference fails. (The '*every*' makes the predicate term merely confused; that is, it makes it confused without making it distributive.)

How adequate are Sherwood's rules? They apply fairly well to the terms occurring in standard categoricals.

> In a universal affirmative the subject is (confused and) distributed by rule A, and made mobile by rule D; the predicate is confused without being distributed (it is merely confused) by rule B, and remains immobile by rule E.
> In a universal negative both terms are (confused and) distributed, by rules A and C, and they are both mobile by rule E.
> In a particular affirmative no rules apply, so both terms are determinate by default and immobile by rule E.
> In a particular negative the subject is determinate by default, and immobile by rule E. What about the predicate? The above rules apply to the predicate if '*not*' is a negative distributive sign; otherwise they don't apply. If they do, the '*not*' confuses and makes mobile the predicate by rules C and D.

In all cases but the last, the classification of terms as determinate, distributive, and merely confused agree with the tradition, and the classification with respect to mobility agrees with everyone's intuitions about what instantiations are possible under the terms. The only uncertainty is regarding the predicate of the particular negative predicate. Intuitively, it is mobile because one *can* descend under it, and this would follow if '*not*' distributed it.

William himself seems to be committed to the view that '*not*' is a distributing sign when it occurs directly with the copula. This is because he elsewhere classifies certain terms as distributive which cannot become distributive any other way. For example, he says that the second '*man*' is distributed in '*every man does not see a man*'. The '*every*' cannot distribute the second '*man*' because it is a remote term vis a vis the '*every*'; this leaves only the '*not*' to do the trick. Likewise, he says that '*man*' is distributive in '*Socrates does not see a man*'. [91] So '*not*' is a distributive sign in this position. This would entail that the predicates of particular negatives have distributive supposition — a matter that he does not explicitly discuss. This would also make such predicates mobile (by rule E), which is good, because from

[91] Kretzmann points this out (right after rule C).

'*Some animal is not a donkey*' it does follow that *'Some animal is not Brownie*' given that Brownie is a donkey.

What does '*not*' do when it is not next to the copula? William does not say, but he seems to be committed to the view that it is not a distributing sign in those positions. For example, the following inference is a good one, according to his previously discussed views on equipollent signs (*IL* section 1.7):

> *Some donkey is not running*
> *∴ Not every donkey is running*

If '*not*' were a distributing sign, both of the terms in the conclusion would have distributive supposition by rules A and C, whereas in the premise the subject has determinate supposition. This would then be an explicit counterexample to his rule (discussed in section 7.6) that you cannot infer a distributed term from the same term having determinate supposition.

I suggest that the rules William of Sherwood gives are supposed to apply only within simple categorical propositions, which is where terms get their modes of supposition. When additional signs make propositions more complex, this can affect the mobility of signs, but once a term is given a mode of supposition, that does not change.[92]

Peter of Spain holds a slightly more radical view; he argues that '*not*' is never a distributing sign. This is because '*not*' does not "signify quantity universally". His argument is:

> wherever distribution takes place, a general term is taken universally.
> Therefore, it is fitting that there be a word signifying quantity univer-

[92] William's rules on the modes of supposition and on the causes of distribution were shared by the author of the *Dialectica Monacensis* [De Rijk, pp. 614-15]:

"A universal affirmative sign confuses a common term immediately adjoined to it mobilely and distributively.

"A term is said to be confused distributively and mobilely when any inferior contained under it can be inferred, as is clear in this locution: 'Every human is an animal', where the term 'human' is confused by the sign 'every' mobilely and distributively. Hence it can be inferred: 'Socrates [is an animal], and Plato [is an animal], and so of others'.

A universal affirmative sign meagerly confuses a common term following it mediately.

'A term is called meagerly confused when according to the requirements of the locution it is able indifferently to be taken for one or for several.

"Such a rule is given concerning a universal negative sign:

A universal negative sign confuses a mediately or immediately following common term distributively and mobilely.

"For it has two [effects], namely distribution and negation. For 'no' is the same as 'every not'. By reason of the distribution it confuses the term immediately adjoined to it distributively and mobilely; by reason of the negation it confuses the term mediately adjoined to it distributively and mobilely. For there is a rule concerning negation that a negation confuses any term it confuses distributively and mobilely."

See also *Cum sit nostra* [De Rijk, p. 449].

The writer of *Properties of Discourse* [Barney *et al.*, 1997] writing at about William's time, disagrees with this. He maintains that distribution is always mobile, and he holds that in the example '*Not every man runs*' the negation does not remove *supposition* but it does remove the *distribution*. This seems to look forward to the 14^{th} century view.

> sally. But a universal sign signifies quantity universally, while negation does not. Therefore, negation does not have the ability to distribute.[93]

This leaves Peter to explain the fact that the subject term of '*Not some man is just*' is mobile. His explanation is that this is merely a consequence of the logic of negation. Since '*Socrates is just*' entails '*some man is just*' (on the assumption that Socrates is a man), the contrapositive inference also holds: '*Not some man is just*' entails '*Socrates is not just*', which is what mobility of '*man*' amounts to here. In short, mobility can result from phenomena other than distributing signs.

(Peter can consistently deny that '*not*' distributes, because, as we have seen, he thinks that the predicate term of a particular negative does *not* have distributive supposition — it has simple supposition. Although his view is consistent, he no longer has any account of its mobility that would explain why '*Some animal is not a donkey*' entails '*Some animal is not Brownie*', given that Brownie is a donkey.)

When William's rules for causes of distribution are applied to the examples of almost-standard form propositions listed earlier, we get:

1	Some A is every B	A is determinate	B is distributive
2	Every A is every B	A is distributive	B is ???
3	No A is every B	A is distributive	B is distributive ???
4	Some A is no B	A is determinate	B is distributive
5	Some A is not every B	A is determinate	B is distributive ???
6	Some A is not no B	A is determinate	B is distributive ???

In all of the cases lacking question marks, the entries agree with the results of applying the criteria of section 2 to the whole proposition.

About 2: One question about William's rules regards the interpretation of the rule for remote terms with affirmative distributing signs:

> *an affirmative [distributive] sign confuses the remote term merely confusedly.*

This may be read as saying that an affirmative distributive sign makes the remote term merely confused. On this interpretation the rules conflict, since the cited rule makes the predicate merely confused, though the 'every' preceding the predicate term distributes it. However, the rule may also be interpreted as saying that an affirmative distributive sign merely (= only) confuses the remote term — that is, it confuses it and does nothing else. On this interpretation the '*every*' attached to the subject term confuses the predicate term — which leaves it open as to whether it is distributive or not — and the '*every*' attached to the predicate term distributes the term. If this is so, the result agrees with the criteria from section

[93] Peter of Spain *T*XII.24 (198). Peter also has a less compelling argument; he says "If negation has the ability to distribute, then just as 'Every Socrates' is incongruous, so too is 'Non-Socrates'. Which is false, since although a distributive sign cannot be added to a singular term, nevertheless negation can." This seems to assume that only a quantifier sign can distribute, which is the point at issue.

2 applied to the whole proposition. Probably the latter is intended, though the statement of the rule seems to require the former.

About 3: The rules seem to make the predicate term distributive, since both quantifier signs individually distribute it. This conflicts with the application of the criteria.[94]

About 5 and 6: The same thing can be said for the predicate terms of examples 5 and 6 if '*not*' is taken to be a distributing sign when it occurs within a basic categorical proposition. If on the other hand the '*not*' has no effect at all, the predicate term is clearly distributive because of the '*every*' or '*no*'. In either case the result conflicts with the criteria.

So the parts of the theory do not hang together. If the criteria for modes of supposition are accurately stated above, then the rules for causes of mode of supposition do not work for the applications in question. Perhaps that means that some other criteria are needed, but it is not clear what they might be.

We have been discussing what happens when we look at nonstandard basic categorical propositions. There are other complications as well. If '*not*' is not a distributing sign when it occurs outside of a minimal categorical proposition, then it is clear that modes of supposition in the 13^{th} century theory are not invariant under logical equivalence. In particular, William's rules of equipollence change modes of distribution. Since '*no*' is equipollent to '*not some*', if neither '*not*' (used outside a basic categorical) nor '*some*' can distribute, then '*donkey*' is not distributed in '*Not some donkey is a stone*', though it is distributed in the equipollent proposition '*No donkey is a stone*'. This is not necessarily an objection — indeed, it seems like a fairly natural consequence of the local nature of modes in the 13^{th} century theory. This is like saying that the quantifier in '$\neg\forall xFx$' is universal, even though the quantifier is not universal in the logically equivalent '$\exists x\neg Px$'. So with respect to quantificational phenomena, classifying a term as distributed in the early theory is much like classifying a quantifier as universal. Universal-quantifierhood is not preserved under logical equivalence, nor is distributive-termhood.

Mobility is different, as is the analogue for mobility of quantifiers. The universal quantifier is mobile in '$\forall x\neg Fx$' as is the existential quantifier in '$\neg\exists xFx$'; in both cases the quantifier may be instantiated; that is, you get a logical consequence by erasing the quantifier and replacing the variable it was binding with a name.

(The 14^{th} century theory, unlike the 13^{th} century one, makes the modes of supposition behave logically like mobility.)

[94] It was common to handle such cases by giving a rule that states: *Whatever immobilizes the mobile, mobilizes the immobile, and vice versa.* In that case, one sign mobilizes the word and the other reverses this. This would make the predicate immobile — which is clearly the right result. But the rules we are examining are about modes of supposition, not about mobility, and William has no "reversal" principles for those.

7.6 Rules of Inference and Fallacies

It is clear that the modes of supposition of terms can influence what inferences propositions containing them enter into. Attempts were made to catalogue some principles governing these. A classic sample of these are Sherwood's "rules" in his chapter on kinds and modes of supposition. We have already seen his Rule I, which gave the three principles (A-C) governing the causes of confusion and distribution. His Rules II-V describe some consequences of relations among the modes. I'll first go over them to see how they are supposed to work, and then I'll consider some problems with them.

> Rule II *An argument from merely confused supposition to distributive confused supposition does not follow.*
>
> Thus when every man sees only himself this does not follow: 'every man a man does not see; therefore every man does not see a man'. [*IL* V.13.2 (118)]

The point here seems straightforward: the second '*man*' in the premise is merely confused (by the '*every*', according to rule A) but it is distributed in the conclusion (presumably by rule C, assuming that '*not*' is a distributing sign here). Since it is distributed in the conclusion, it is mobile (by rule D), and so the conclusion entails e.g. 'every man does not see Socrates'. But that clearly does not follow from the premise. So the inference is clearly invalid. Sherwood sees this as an instance of a general pattern involving a term's changing its mode of supposition from merely confused to distributive.

> Rule III *An argument from many cases of determinate supposition to one of determinate supposition does not follow, but [only] to one of confused supposition.*
>
> Thus when every man sees only himself this does not follow: 'a man is seen by Socrates, and [a man is seen] by Plato (and so on with respect to [all] individual [men]); therefore a man is seen by every man'. But this does follow: '. . . therefore by every man a man is seen', for a distribution has force in a succeeding phrase but not in a preceding phrase.[Ibid]

The inference resembles what was elsewhere called "induction": concluding that a universal generalization is true because each of its instances are. For example, if m_1, m_2, . . . are all the men, then from '$m_1$is mortal' and '$m_2$is mortal' and . . . , you can infer 'every man is mortal'. Abstractly put, from '$\phi(m_1)$', '$\phi(m_2)$', etc, infer 'ϕ(every M)'. But this principle holds only when the 'every man' ends up with wide scope, and this is not the case in the bad example that Sherwood discusses. The good example, the one that *does* follow, differs exactly in that 'every man' ends up with wide scope. But this talk of "wide scope" uses modern terminology, and the example was not discussed in these terms. Instead, what is discussed is an effect of scope. For the shift of scope of 'every man' also alters the scope —

and the mode of supposition — of 'a man'; in the good example it has merely confused supposition, and in the bad one it has determinate supposition. So the rule is formulated in terms of whether you move from many cases of determinate supposition to determinate supposition, or to merely confused supposition.

> Rule IV *An argument from determinate supposition to distributive confused supposition does not follow, but only to merely confused supposition.*
>
> Thus this does not follow: 'a man is not seen by Socrates; therefore Socrates does not see a man' — e.g. if Socrates sees one man only. But this follows correctly: 'a man is seen by every man; therefore every man sees a man'. [Ibid][95]

The badness of the first inference and goodness of the second are obvious; the rule identifies the difference in terms of a principle about modes of supposition.

> Rule V *An argument from distributive confused supposition to determinate supposition does follow, but not from merely confused supposition.*
>
> Thus this follows: 'Socrates does not see a man; therefore a man is not seen by Socrates'. But this does not: 'every man sees a man (e.g., every man sees only himself); therefore a man is seen by every man'. [Ibid (119)]

In the first inference '*man*' goes from distributive supposition to determinate supposition, and in the second it goes from merely confused to determinate. The rule blames the goodness/badness of these inferences on the pattern of the modes of supposition.

These rules all seem to have insight behind them, and they look promising as part of an overall theory of inference in terms of modes of supposition. But there are some problems with them.

Problem 1: Embedded propositions

The rules are all illustrated by fairly simple categorical propositions, but they are supposed to hold in general. However, the rules are stated in terms of modes of supposition, which do not change when a proposition is embedded in something more complex — yet the goodness of the inferences can be affected by this. A simple example of an application of Rule IV is this inference:

> *If a man is seen by every man, then a man is seen by every man;*

therefore,

> *If every man sees a man, then a man is seen by every man.*

[95]The translation quoted here has been altered in conformity with discussion in the Preface to [Kretzmann, 1968].

The premise is tautological, but the conclusion would usually be considered to be false. However, the conclusion results from the premise only by changing a determinate '*man*' to a merely confused one. Rule IV says that the inference should be good, but it isn't. The problem arises because putting a term in the antecedent of a conditional can affect its mobility, and thus the inferences that can be drawn using it. If there is no effect on its mode of supposition, then it is not trustworthy to state principles of inference in terms of modes of supposition as in Rule IV.

A parallel example can be constructed for Rule V.

(The 14^{th} century view discussed in the next section holds that the modes of supposition *are* affected by context in the way that mobility is affected in this theory.)

Problem 2: What stays the same?

Here is an easy counterexample to Rule V, which says that an inference from distributive supposition to determinate is valid:

> *Every donkey is an animal.*
> *Some donkey is every animal.*

The reason for the fallacy is blatant: we did not just change the subject from distributive to determinate, we also changed the predicate from merely confused to distributive. Apparently, the rule should specify that nothing else can change in the proposition when the rule is applied. But something else *often* changes; for example, some word switches position with another, and this may affect both of their modes. Perhaps what is meant then is that except for the quantifying sign of the term under discussion, the same words must be used in the premise and conclusion, and their grammatical relations must not change, and the modes of supposition of the other terms must remain the same. William's examples of good inferences obey these constraints. But there are comparable simple cases which are not addressed. Consider the inference:

> *Every girl gives every present to some boy*

so

> *Every girl gives some boy every present*

The constraints are obeyed by that inference (since nothing changes mode of supposition), but it is invalid, indicating that one cannot infer a distributed subject from a distributed subject. In other words, this is an invalid inference in which no term changes mode of supposition.

The following restricted rules seem to be valid:[96]

[96] Karger [1984, pp. 100–102] states some more general circumstances in which versions of rules II, IV and V all hold. She also adds a rule stating circumstances under which two propositions are equivalent when their corresponding terms have the same modes of supposition.

> If a term has distributive supposition in a proposition, then if you change its quantifier sign so as to give it determinate supposition (inserting a negation after the denoting phrase containing the term if it changes from affirmative to negative, or vice versa) then the resulting proposition is entailed by the original.
> If a term has determinate supposition in a proposition, then if you change its quantifier sign so as to give it merely confused supposition (inserting a negation after the denoting phrase containing the term if it changes from affirmative to negative, or vice versa) then the resulting proposition is entailed by the original.[97]

The ideas behind these rules are neat, but a great deal of work remains to be done. When it was done it was within a somewhat different framework — roughly that of the next section.

8 MODES OF SUPPOSITION: THE LATER VIEW

8.1 The Fourteenth Century Definitions of the Modes

In the 14^{th} century, Walter Burley, William Ockham, and John Buridan developed a new approach to the analysis of the modes of common personal supposition. It is not clear whether, or how, they saw their accounts as being different from those given by earlier authors, but the accounts did differ in three important ways. (1) The modes are defined in terms of valid inferences, not in terms of quantities such as 'many' or 'any'. (2) The modes of supposition of terms are defined globally instead of locally, so that embedding a proposition in a larger one can alter not just mobility but also mode of supposition. (3) In earlier accounts some good inferences depend on whether the term in question has or lacks mobility, but rules for these inferences are phrased in terms not of mobility but in terms of modes of supposition. When a term has distributive but immobile supposition, rules based on distributivity tend to fail. Conversely, a term without distributive supposition might have the mobility needed to yield an inference that the rules deny to terms that are not distributive. In the later theory, as a result of (1) and (2) the modes of supposition are brought into conformity with kinds of mobility.

The new accounts define the mode of supposition of a term in a proposition[98] in terms of conditions on ascent and descent under that term in the proposition. (A descent is similar to a quantifier instantiation step in modern logic; an ascent

[97] When the only quantifier signs are '*some*', '*every*' and '*no*', it is not possible to give an example of this second rule. The possibility of giving such an example arises in some post-medieval theories; see Ashworth 1974 IV.II.1 on special invented signs governing supposition.

[98] Some authors held that only main terms have personal supposition in a proposition (where a main term is one that is not part of another term). Burley *DS* para 5-15 argues this. But the definitions to be given of modes of supposition apply meaningfully to any term in a proposition. In practice, authors sometimes applied the definitions to parts of terms, even when denying that this is possible.

is similar to a quantifier generalization step.) Each of the three modes has a distinctive pattern of allowable ascents and descents.

What follows is an account that takes what is most common to the views of Burley, Ockham, and Buridan. The theory consists of three definitions; they give necessary and sufficient conditions for a term's having determinate, distributive, or merely confused supposition. Begin with determinate supposition:

Determinate supposition[99]
A term F has determinate supposition in a proposition P if and only if

> [Descent]: you may descend under F to a disjunction of propositional instances of all the Fs, and
> [Ascent]: from any such instance you may ascend back to the original proposition P

A propositional instance of a proposition with respect to F is the proposition you get by replacing the quantifier word of the denoting phrase containing F by '*this*' or '*that*', and adding a '*not*' if the quantifier word is negative. Descent and ascent are inferences, picturesquely expressed in terms of the directions in which the inferences go. As an example, we validate the claim that '*donkey*' has determinate supposition in '*Some donkey is spotted*' by establishing these two claims:

> DESCENT: You may descend under '*donkey*' in '*Some donkey is spotted*' to a disjunction of instances of all donkeys. That is, from:
>
> *Some donkey is spotted*
>
> you may infer:
>
> *This donkey is spotted or that donkey is spotted or . . . and so on for all the donkeys.*

[99]This account is common to both Ockham and Buridan. Burley omits the ascent clause.

Buridan *SD* 4.3.5 (262-63) says "... there are two conditions for the determinate supposition of some common term. The first is that from any suppositum of that term it is possible to infer the common term, the other parts of the proposition remaining unchanged. For example, since, in 'A man runs', the term 'man' supposits determinately, it follows that 'Socrates runs; therefore, a man runs', 'Plato runs; therefore, a man runs', and so on for any singular contained under the term 'man'. The second condition is that from a common term supposing in this manner all singulars can be inferred disjunctively, by a disjunctive proposition. For example, 'A man runs; therefore, Socrates runs, or Plato runs or John runs ...' and so on for the rest."

Ockham *SL* I.70 (200) says: "... whenever it is possible to descend to the particulars under a general term by way of a disjunctive proposition and whenever it is possible to infer such a proposition from a particular, the term in question has personal determinate supposition."

Burley *PAL* 1.1.3 para.82 (102) says: 'Supposition is determinate when a common term supposits disjunctively for its supposita in such a way that one can descend to all of its supposita under a disjunction, as is plain with 'Some man runs'. For it follows: 'Some man runs; therefore, Socrates runs, or Plato runs, and so on'."

ASCENT: You may ascend back to the original proposition, because from any instance of the form:

This donkey is spotted

you may infer the original proposition:

Some donkey is spotted.

Distributive supposition has a parallel explanation:

Distributive Supposition[100] A term F has distributive supposition in a proposition P if and only if

[Descent]: you may descend under F to a conjunction of propositional instances about all the Fs, and
[Ascent]: from any one instance you may not ascend back to the original proposition P.

So '*donkey*' has distributive supposition in '*Every donkey is spotted*' because

DESCENT: You may descend under '*donkey*' in '*Every donkey is spotted*' to a conjunction of instances about all donkeys. That is, from:

Every donkey is spotted

you may infer:

This donkey is spotted and that donkey is spotted and . . . and so on for all the donkeys.

ASCENT: You may not ascend back to the original proposition. From an instance of the form:

This donkey is spotted

you may not infer the original proposition:

[100]Buridan's account omits the ascent condition. Ockham includes additional provisions for immobile distribution. Burley does not define distributive supposition; instead he defines two kinds of distributive supposition. This will be discussed below.

Buridan *SD* 4.3.6 (264) says: "Distributive supposition is that in accordance with which from a common term any of its supposita can be inferred separately, or even all of them at once conjunctively, in terms of a conjunctive proposition. For example, from 'Every man runs' it follows that therefore 'Socrates runs', [therefore 'Plato runs', or even that] therefore 'Socrates runs and Plato runs ...' and so on for the rest."

Ockham *SL* I.70 (201) says: "Confused and distributive supposition occurs when, assuming that the relevant term has many items contained under it, it is possible in some way to descend by a conjunctive proposition and impossible to infer the original proposition from any of the elements in the conjunction. . . ." (The "in some way" clause is to allow for immobile distributive supposition; it will be discussed in section 8.3.)

Every donkey is spotted.

Finally, merely confused supposition:

Merely Confused Supposition[101]
A term F has merely confused supposition in a proposition P if and only if

> [Descent]: you may not descend under F to either a conjunction or a disjunction of propositional instances about all the Fs, and
> [Ascent]: from any instance you may ascend back to the original proposition P.

The term '*mammal*' has merely confused supposition in '*Every donkey is a mammal*' because:

> DESCENT: You may not descend under '*mammal*' in '*Every donkey is a mammal*' to either:
>
> > *Every donkey is this mammal and every donkey is that mammal and . . . , and so on for all the donkeys*
>
> or to:
>
> > *Every donkey is this mammal or every donkey is that mammal or . . . , and so on for all the donkeys.*
>
> ASCENT: You may ascend back to the original proposition from any instance. From:

[101] The account given is that of Burley. Burley 1.1.4 para 85 (103): "Supposition is merely confused when a common term supposits (a) for several things in such a way that (b) the proposition is inferred from any one of them and (c) one cannot descend to any of them either copulatively or disjunctively. The predicate supposits in this way in 'Every man is an animal', because: (a) the term 'animal' supposits for several things. For if it supposited for some determinate one, the proposition would be false. (b) The proposition is inferred from any of its singulars. For it follows: 'Every man is this animal; therefore, every man is an animal'. And (c) one cannot descend under 'animal' either disjunctively or copulatively. For it does not follow: 'Every man is an animal' therefore, every man is this animal or every man is that animal'. Neither does it follow: 'Every man is an animal; therefore, every man is this animal and every man is that animal', and so on.

Buridan agrees, but omits the ascent condition. Buridan 4.3.6 (264): "But merely confused supposition is that in accordance with which none of the singulars follows separately while retaining the other parts of the proposition, and neither do the singulars follow disjunctively, in terms of a disjunctive proposition, although perhaps they do follow by a proposition with a disjunct term."

Ockham adds a condition on descent which will be discussed in section 8.9.3. Ockham I.70 (201): "Merely confused supposition occurs when a common term supposits personally and it is not possible, without a change in either extreme, to descend to particulars by way of a disjunctive proposition, but it is possible to descend by way of a proposition with a disjunctive predicate and it is possible to infer the original proposition from any particular."

Every donkey is this mammal

you may infer the original proposition:

Every donkey is a mammal.

In illustrating the definitions above we showed that the subjects of affirmative propositions and the predicate of the universal affirmative have the modes that they were assigned throughout the tradition. All of the other terms in the standard categorical propositions are like this. For example, we can show that by these definitions the predicate term of a particular negative proposition has distributive supposition:

The term '*donkey*' has distributive supposition in '*Some stone is not a donkey*' because

DESCENT: You may descend under '*donkey*' to a conjunction of instances of all donkeys. That is, from:

Some stone is not a donkey

you may infer:

Some stone is not this donkey and some stone is not that donkey and ... and so on for all the donkeys.

ASCENT: You may not ascend back to the original proposition; from an instance of the form:

Some stone is not this donkey

you may not infer the original proposition:

Some stone is not a donkey.

The earlier and later theories agree on the terms of the standard Aristotelian categorical propositions, but they disagree in many other cases. Consider the proposition

Not every man is running

The subject of this proposition was taken to have distributive supposition by Sherwood. This is presumably because the proposition is got by putting '*not*' in front of a universal affirmative proposition, in which '*man*' is already distributed. But on the later theory the mode of supposition changes when the '*not*' is put on. It is easy to check that according to the later theory of ascent and descent the term '*man*' in '*not every man is running*' has determinate supposition. This is because from

Not every man is running

one may descend to

> *Not this man is running or not that man is running, etc, for all the men.*

and the original proposition is entailed by any disjunct.

8.2 Clarification of the Definitions

Some clarifications of the definitions are necessary.

8.2.1 The nature of ascent and descent

The first point has to do with the nature of the inferences involved in ascent and descent. Consider the following explanation of why '*donkey*' is distributed in '*Every donkey is an animal*'. From:

> *Every donkey is an animal*

one may infer

> *This donkey is an animal and that donkey is an animal and*... for all the donkeys.

This does *not* mean that if the displayed conjunction contains a term for every donkey then '*Every donkey is an animal*' entails '*This donkey is an animal and that donkey is an animal and*...', for the former sentence does *not* entail the latter. What is meant instead is that from the truth of ' *Every donkey is an animal*', together with the information that '*This donkey is an animal and that donkey is an animal and ...*' contains a term for each donkey, one may infer that the disjunction is true. The inference is from the generalization *plus* the information about the exhaustiveness of the disjunction to the disjunction. This is how the test should be understood.

8.2.2 Occurrences of terms have modes of supposition

Second, the classification into modes of suppositions is a classification of occurrences of terms, not of their types. In particular, in the proposition

> *Every donkey is a donkey*

there are two terms, each of which has its own mode of supposition: the subject term has distributed supposition and the predicate term has merely confused supposition. You can't just ask for the mode of supposition of '*donkey*' in the proposition without specifying which occurrence is meant.

8.2.3 Repeated occurrences must be ignored

Third, the classification is meant to be a classification of an occurrence of a term independent of the particular nature of any other terms in the sentence. In particular, if the same term is repeated, as in the example just discussed, you don't just apply the test for modes of supposition to an occurrence of the term in that very sentence. For if you do, the results may depend on the very same term occurring elsewhere, and that may give unintended results. For example, the following is an incorrect way to try to show that the subject term of '*Every donkey is a donkey*' has determinate supposition:

> The subject term of '*Every donkey is a donkey*' seems to have determinate supposition, because that proposition does indeed entail '*This donkey is a donkey or that donkey is a donkey or*...', and any one of these disjuncts alone entails the original proposition.

But the subject term is supposed to have distributive, not determinate, supposition. To avoid the bad consequence one must ignore the fact that a repeated term is the same term. To test the subject term of

Every donkey is a donkey

you should apply the test to

Every donkey is a blah

And although that proposition does indeed entail

This donkey is a blah or that donkey is a blah or...

the ascent condition fails; the original proposition is not entailed by any of the disjuncts alone. So the subject term does not have determinate supposition. It is straightforward to show that it has distributive supposition.

A similar provision is needed regarding complex terms that contain repeated terms within themselves. Consider what kind of supposition is possessed by the subject term of:

Every donkey which is not a donkey is running

The sentence is logically false, and each instance ('*this donkey which is not a donkey is running*') is also logically false. Logically false propositions entail anything.[102] So if the term is left unchanged, it is possible to descend to a conjunction of instances, and it is also possible to ascend back from any instance. This does not conform to any of the defined modes of supposition, so such a term has no

[102]This was a common view. For example, Buridan *TC* I.8.3 (196): "From any impossible sentence any other follows, ..." Paul of Venice *LP* III.1 (167) "From the impossible follows anything, i.e. from any impossible proposition any other proposition follows."

mode of supposition. On the other hand, the proposition is a universal affirmative proposition, and we said that the subject term of a universal affirmative has distributive supposition. Which is right?

It is not clear from the texts what to say about this. The issue is similar to the modern question of whether one may instantiate '$\exists x(Fx\&\neg Fx)$' to get '$Fa\&\neg Fa$', where a = Socrates. Certainly one may infer the latter from the former, since the first is logically false. But can one infer the latter *by instantiating*? Either answer is possible. My intuition is that we should say that the inference, though valid, is not a case of instantiation. The analogue for the medieval example above is to say that the subject term has distributive supposition, and that the ascent condition is not met because although the original proposition may be inferred from the instance, this is not a proper case of ascent. However, it is not clear how to define ascent so as to rule out this inference. Probably the best approach is to extend the prohibition on repeated terms to include terms that occur within a complex term. For the example above, one would then test:

> *Every donkey which is not a blah is running*

This does indeed entail the conjunction of all instances of the form:

> *This donkey which is not a blah is running and . . . and so on for all donkeys*

and the original proposition may not be inferred from any instance. So the conditions for distributive supposition are met.

8.2.4 Empty terms

There is a problem about how to apply the tests for ascent and descent when the term under which one is ascending or descending is empty. If the tests are taken literally, this seems to require a conjunction or disjunction with no conjuncts or disjuncts. But there is no such thing. For example, one cannot make a conjunction or disjunction of the instances of '*No donkey which is not an animal is living*', because there are no instances.

I am not aware of any discussion of this problem in medieval texts, except possibly for Ockham's wording of his account of distributive supposition:

> Confused and distributive supposition occurs when, *assuming that the relevant term has many items contained under it* [my italics], it is possible in some way to descend by a conjunctive proposition and impossible to infer the original proposition from any of the elements in the conjunction. . . . [*SL* I.70 (201)]

This appears to say that the test is to be applied on the assumption that the term is not empty. Just suppose that there are some donkeys that are not animals, and make a conjunction of instances of these:

> *This donkey which is not an animal is not living and that donkey which is not an animal is not living, and . . . and so on for all the donkeys that are not animals.*

Then if the original were true, the conjunction would be true (and no ascent would be possible). So this device seems to work fine (though it is not completely clear how to spell it out).

8.3 Mobility

In the earlier theory some distribution was mobile, and some immobile. Mobility most often referred to the possibility of inferring from a proposition the result of replacing a common term with any singular term that supposits for one of the things the common term supposits for. On the new theory, this describes the descent condition that is part of the definition of distributive supposition. As a result, mobility collapses into distribution, and there is no room for an immobile distributed term.

This seems to be how Buridan viewed the matter. He does not talk about a distributed term not being mobile. And in at least one place he seems to equate the two notions. For example, in discussion of the effect of infinitizing negation on a term, he says that a term is not distributed *because* it is not mobile:[103]

Marsilius of Ingham also rejects immobile distributive supposition:

> I answer that each term standing confusedly distributively has mobile supposition. And I do not think the term *man* in this proposition *every man runs except Sortes* to have immobile supposition, because, in my opinion, this term stands for all men other than Sortes because of restriction.[104]

However, the notion of immobile distributive supposition was firmly established in the tradition, and others sought to preserve it. Ockham, as quoted two sections

[103]Buridan *SD* 4.3.7.3 (271): " But concerning 'Every non-man runs' we should say similarly that the term 'non-man' is distributed, while 'man' is no longer distributed, for reason of the traditionally posited rule: 'whatever mobilizes the immobilized immobilizes the mobilized, that is, that which has the power to distribute an undistributed term removes the distribution of a distributed one."

The point here is that the prefix '*non-*' by itself would distribute '*man*', but so would the sign '*every*'. The second distribution removes the first, since the first mobilizes the supposition of '*man*' and the second then immobilizes it. (Distribution of terms modified by '*non-*' is discussed later in this section.) Paul of Venice *LP* II.5 (153) makes a similar equation: ". . . according to one rule pertinent to this matter, viz., whatever mobilizes the immobile, immobilizes the mobile. That is: If any sign having the power to distribute some term finds again the same term undistributed, the sign makes the term stand distributively; and if the sign finds again the same term distributed, the sign makes the term stand without distribution, . . . "

[104]Marsilius (63) in [Bos, 1983]. The point seems to be that descent is possible under the term '*man . . . except Sortes*' to every one of the things that it supposits for, namely, for every man other than Sortes. ('Sortes' is a common abbreviation of 'Socrates'.) Also Marsilius *TPT* 1 (63) "each term standing confusedly distributively has mobile supposition."

above, allowed for immobile distributive supposition by including a provision in the descent condition for distributive supposition for altering the wording during descent. If no alteration is necessary, the supposition is mobile; if some is necessary the supposition is immobile. Unfortunately, he did not specify which rewordings preserve distribution, and which do not. For this reason, I have not tried to capture his form of immobile distribution.

Burley also allows for immobile distribution; he says:

> But confused and distributive supposition is immobile when a common term is distributed for its supposita and one cannot descend to those supposita with respect to that with respect to which the distribution is made. For instance, in 'Every man besides Socrates runs', the term 'man' is distributed with respect to an exception, and one cannot descend with respect to the same exception. For it does not follow: 'Every man besides Socrates runs; therefore, Plato besides Socrates runs'. [*PAL* 1.1.4 para 101 (107)]

This seems to define distributive supposition in terms of whether the common term is distributed; the supposition is then mobile if descent is possible. Since Burley offers no account of what makes a term distributed, there is no clear doctrine here. (But the next section may offer an answer in terms of the causes of distribution.)

Paul of Venice perhaps comes closest to capturing the spirit of immobile distribution:

> Immobile distributive personal supposition is the acceptance of a common term standing personally beneath which one does not descend; but if one should descend, he must do so conjunctively. For example from 'necessarily every man is animal', it does not follow with the due mean, 'therefore, necessarily, this man is animal and thus of singulars', because the antecedent is true, and the consequent is false. [*LP* II.5 (152).]

Unfortunately, Paul does not say how to tell how one should descend when "one does not descend".

I will continue to work with Buridan's simpler assumption that no real distribution is immobile; that is, I will assume that all distribution is mobile by definition. Immobile distribution will continue to be a not-very-well-defined special case, which is not actually a subclass of distributive supposition.

8.4 Causes of the Modes

The later theory continues the tradition of the earlier theory that modes of supposition (other than determinate) must be caused. Since the later theory disagrees with the earlier regarding which modes of supposition some terms have, the rules for the causes of modes must be different. And since according to the later theory

embedding a proposition within a larger context can change the modes of supposition possessed by its terms, the rules must allow for a sign to change the mode of supposition that is assigned by another sign. The key to the latter is to formulate rules that can be applied recursively. For example, the '*every*' in '*Every donkey is running*' assigns distributed status to the subject term '*donkey*'; embedding this within a '*not*' allows for the '*not*' to reclassify the mode of supposition of '*donkey*' as determinate in '*Not every donkey is running*'.

Here are some rules that are common to several authors:[105]

> DEFAULT: A main term of a proposition has determinate supposition unless something causes it not to.[106] A particular affirmative sign adjoined to a term makes it have determinate supposition (or, equivalently, has no effect).
>
> UA:[107] A universal affirmative sign distributes the term it is adjoined to and confuses any other main term following it in its scope if that term is not already confused.
>
> UN:[108] A universal negative sign distributes the term it is adjoined to, and also distributes any other main term following it and in its scope:

[105]Paul of Venice *LP* II.5 (152-56) has similar rules plus a host of others. For example, the superlative and comparative constructions distribute their terms; an exceptive expression (as in '*every man except Socrates*') makes its term be merely confused, as does a reduplicative expression (as in '*Every man as a man*'), and terms concerning an act of the mind, such as '*know*', '*believe*'. Paul speaks here of signs "confounding" terms, and thereby affecting their mode of supposition. Marsilius *TPT* 1 (65-71) gives nineteen rules.

[106]Buridan *SD* 4.3.5 (263): "If you ask 'How do I know when the supposition is determinate?. I say . . . you will know if you see that there is no cause for confusion."

Ockham *SL* I.71 (202): "when in a categorical proposition no universal sign distributing the whole extreme is added to a term, either mediately, or immediately, . . , and when no negation or any expression equivalent to a negative or a universal sign is added to a common term, that common term supposits determinately."

[107]Buridan *SD* 4.3.7.1 (265) says: "a universal affirmative sign distributes the term immediately following and construed with it.", and 4.3.8.1 (273): "there are many causes of nondistributive confusion. The first obtains when the universal affirmative sign confuses a common term following upon it, but not immediately, as when in the proposition 'Every man is an animal' the term 'animal' supposits by nondistributive confused supposition."

Ockham *SL* I.74 (213) says: "in every universal affirmative and universal negative proposition that is neither exclusive nor exceptive, the subject has confused and distributive mobile supposition." I.73 (211): "where a common term mediately follows an affirmative sign of universality, it has merely confused supposition."

[108]Buridan *SD* 4.3.7.2 (269-70) says: ". . . a negative universal sign is nothing else but a word that implies in itself a negation with a particular affirmative sign. For 'no-one' is equivalent to 'not one', 'nothing' to 'not something', . . . ". Since Buridan takes the universal negative sign to be equivalent to negations plus the particular affirmative sign, his views must follow in part from his view about these two items. These provisions seem to yield the right answers.

Ockham *SL* 1.74 (213) says: "The first rule is that in every universal affirmative and universal negative proposition that is neither exclusive nor exceptive, the subject has confused and distributive mobile supposition." For the predicate term, Ockham only mentions the first subcase: "The second rule is that in every such universal negative proposition the predicate stands confusedly and distributively." However he probably intends his treatment of negation to apply to the universal negative sign; see below.

> if the term is determinate or merely confused, the negative sign confuses and distributes it.
> if the term is distributed, the negation sign makes it merely confused

> NEG:[109] A negating negation has the following effect on any main term following it and in its scope:

> if the term is determinate or merely confused, the negation confuses and distributes it.
> if the term is distributed, the negation makes it merely confused (according to Buridan)[110] or determinate (according to Ockham)[111]

The standard categorical propositions are correctly classified by these rules:

Every A is B	A is distributed and B is merely confused by UA
No A is B	A and B are both distributed by UN
Some A is B	A and B are both determinate by DEFAULT
Some A is not B	A is determinate by DEFAULT; B is distributed by NEG

Applied to some formulations of nonstandard categorical propositions, almost all classifications agree with the definitions of the modes in terms of ascent and descent:

[109] Buridan 4.3.7.2 (269): "a negating negation distributes every common term following it that without it would not be distributed and does not distribute anything that precedes it." This applies to the first two subcases. For the third case, see the next note.

[110] Buridan 4.3.8.2 (275): "a common term is confused nondistributively by two distributive [parts of speech] preceding it, either of which would distribute it without the other." Examples include the predicate terms of 'No man is no man' and 'No man is every man'. He also (277) cites the predicate of 'No man sees every donkey' as an example, and notes that the predicate term of 'Every man is every donkey' is not an application of the rule, since the first 'every' by itself would not distribute the predicate term. He adds that the rule must have an exception for "two negations taken together, relating in the same way to what follows upon them. For these only cancel each other out, and so the supposition remains the same, as it would be if the negations were removed. Thus, in 'Not: no man runs' or in 'Socrates does not see no man', 'man' supposits determinately".

[111] Ockham *SL* I.74 (214): "if the term were to stand confusedly and distributively when one of these expressions were taken away, then with the addition of such an expression it would stand determinately." His example is 'Socrates is not every man', wherein 'man' stands determinately.

Every A is every B	*A* and *B* are both distributed by UA. (The first '*every*' has no effect on *B* because *B* is already confused by the second '*every*'.)
No A is every B	*A* is distributed by NEG; *B* is distributed by UA but then made merely confused by NEG.
Some A is every B	*A* is determinate by default and *B* is distributed by UA
Some A is not every B	*A* is determinate by DEFAULT; *B* is distributed by UA but this is then made to be merely confused according to Buridan, or determinate according to Ockham.

The question remains how to resolve the difference between Buridan and Ockham concerning the mode of supposition of a term that is apparently distributed by two signs. Buridan says that it ends up merely confused, and Ockham says that it ends up determinate. In the example just given, the right answer in terms of ascent and descent is that it should be determinate. This is also the case for another example that Ockham gives: '*Socrates is not every man*'. Unfortunately the only examples that Buridan gives using '*not*' are examples that he sees as special exceptions of his own rule; the result should be determinate when there are "two negations taken together, relating in the same way to what follows upon them." Examples are

Not: no man runs
Socrates does not see no man

The answer he gives here — that the predicate terms have determinate supposition — agrees with Ockham. So perhaps Ockham is right, and Buridan wrong? No, for there are other examples mentioned by neither author where a negating negation applies to a distributed term and makes it merely confused, just as Buridan says; an example is '*donkey*' in

Not: some farmer sees every donkey

Both the '*every*' and the '*not*' would distribute '*donkey*' on their own; together they make it merely confused. (Descent is not possible, but ascent is possible from '*Not: some farmer sees this donkey*'.)

So the rules are not adequately formulated, and it is not at all obvious how to make them adequate within the theoretical framework given here. I will discuss this further in the next section, where we consider a revision of the theory in which distribution comes in two sorts. (Negating one sort produces determinate supposition, while negating the other sort produces merely confused supposition). But that is a revision that goes beyond medieval theorizing, and so I will not consider the issue of causes of modes of supposition further here, except to note that the rules that were given correctly classify a wide range of cases.

8.5 Modes caused by infinitizing negation

Infinitizing negation was sometimes held to be a cause of distribution.[112] Buridan claims that in '*[A] non-man runs*' the term '*man*' is distributed by the infinitizing negation, but this distribution is cancelled out by the '*every*' in '*Every non-man runs*'. This would fit in with the above rules for the causes of modes of supposition:

> INF-NEG: An infinitizing negation distributes the term to which it is adjoined if that term is not distributed; if it is distributed, it makes the term determinate. It has no effect at all on any other term in the proposition.

This rule predicts the examples given by Buridan. In the case of '*[A] non-man runs*' the term '*man*' is distributed by the '*non*', and no further rules apply to it, so it is distributed. In the case of '*Every non-man runs*' the '*every*' changes the distribution back to determinacy, provided that the rule UA is expanded to say that if it is adjoined to a distributed term it makes it determinate. Since at present rule UA is silent about such a case, this does not alter its other applications. Buridan actually argues for this by appeal to "the traditionally posited rule ..., i.e. that which has the power to distribute an undistributed term removes the distribution of a distributed one." [*SD* 4.3.7.3 (271)]

The rule also insures that iterated infinitizing negations cancel one another, so that the mode of supposition of '*non-non-man*' in any given occurrence would be the same as it would be if the two negations were absent.

There is however something odd about this view. Buridan defends the view by appeal to the definitions of the modes, arguing implicitly that the term '*man*' is distributed in '*[A] non-man runs*' because one may descend to '*Non-Socrates runs*'. The descent is valid, but the application of the definition to this case is quite different from other cases. Typically you produce a descended form by erasing the term in question *along with its quantifier sign* and inserting a singular term. But in the descent above the term is erased and no other changes are made. In particular, the '*non*' is retained, as is any quantifier sign preceding the '*non*'. The context of the term is thus different from that of other terms that we have considered. One might then suppose that the definition of the modes does not apply to a term prefixed by '*non*'. Or one may take the definition to be broader than what was proposed above. The latter view is probably better, not just because some logicians did indeed take this application to be a case of distribution, but also because if we count negated terms as having modes of supposition, then the principles of inference discussed in the section after next appear to apply correctly to these terms. So it is logically useful.

[112] Peter of Spain *SL* XII.24 (198) discusses this as if it were a well-known view, and he rejects it. However, Buridan *SD* 4.3.7.3 (271) writes as if Peter holds the view. This may not be important since Buridan elsewhere says that some of the views he attributes to Peter's work he is actually making up for his own convenience. In any event, Buridan himself holds the view.

8.6 Restricted descent and parasitic terms

Recall that in order to handle transitive verbs and genitive constructions we needed to use parasitic terms; intuitively, terms that contain a free variable to be bound by another term. An example is '*Some farmer's every donkey is running*'.

> *Of-some farmer every donkey is running*

≈

> *(Of-some farmer x)(every donkey-of-x y) y is running*

So far in this section we have not looked at propositions with such terms. When we do, we find they are problematic. For, some of them have distributive supposition according to the rules for causes of the modes of supposition, but they do not satisfy the descent/ascent conditions for distributed supposition. The example given above is of this sort. The term '*donkey*' is distributed by its '*every*' (according to rule UA). However, its mode of supposition is not well defined. The problem is apparent even with the informal statement of the conditions for descent. The test requires that one can descend from the original proposition to:

> *Of some farmer this donkey is running and of some farmer that donkey is running and ..., and so on for all the donkeys*[113].

So stated, the descent fails, for there may be donkeys that are not owned by any farmer, and that are not running, even though all of the donkeys of some specific farmer are running.

In this case it appears as if we should not descend to all of the donkeys that there are, but to only those that are owned by ...? By what? By any farmer? This would make the descent fail, since there may be donkeys that are owned by a farmer and are not running, even though all of *some* farmer's donkeys are running. The term would then not have distributive supposition.

Buridan discusses an example just like this, and he says:

> "But concerning the third proposition, 'A man's any donkey runs', I say that the word 'man's' is not distributed at all, and remains in its determinate acceptation. But the word 'donkey' is distributed, not absolutely, however, but restricted by the word 'man's' taken determinately. And so if we need to make a subsumption under this distribution in a syllogism, then we have to syllogize as follows: 'A man's any donkey runs, Brownie is that man's donkey; therefore Brownie runs'." [*SD* 4.3.7.1 (266)]

There are two problems with his remarks. One problem is that it is not clear how to construe the syllogism as Buridan words it. This question concerns how to

[113]This assumes that when one makes a descent under '*donkey-of-x*' we end up with another '*donkey-of-x*' in which the '*x*' is bound by the original '*of-some farmer*'. I can't think of any other form that would make sense.

understand 'that man's donkey'. The intent seems to be that the second premise must be something like 'Brownie is a donkey of a man whose every donkey runs'. But this is not what is proposed. The example is frustrating because what Buridan actually says ('*Brownie is that man's donkey*') looks pretty straightforward, it just happens to be a form of words for which we have no clear logical form. The second problem is that however plausible his remarks, they do not justify the claim that the descent condition is satisfied in a modified way. For example, the following construal of the conjunction is clearly wrong:

> *Of some man this donkey is running and of some man that donkey is running and . . . , and so on for all the donkeys of that man.*

For here there is clearly no possible antecedent for the anaphoric '*that man*'.

When Buridan says that '*donkey*' is not distributed "absolutely" we must admit that it is not distributed at all, according to the definition of distribution given above. In fact, it has no mode of supposition at all. We can say that it has *restricted* supposition, but this stands for an idea that we have not spelled out, and that perhaps cannot be spelled out.

In general, any parasitic term will have no usual mode of supposition at all. This will not prevent other terms from having modes of supposition; for example, in Buridan's sample sentence '*man*' is indeed determinate, as Buridan says, and '*running*' is merely confused; both of these are predicted by the rules for causes of supposition. But parasitic terms do not have these modes according to their definitions in terms of ascent and descent.

A special case: There is one kind of case in which an indexed term might be said to have a mode of supposition, and this is when its free variable is bound by a singular term. So in the sentence '*Socrates's every donkey is running*' we can say that '*donkey*' is distributed because one can descend to:

> *Of Socrates this donkey is running and of Socrates that donkey is running and . . . , and so on for every donkey of Socrates*

Strictly, this is an extension of our account of supposition, for we had to specify that one is to consider all *donkeys of Socrates*, but it is a natural extension.

As for the rules for causes of modes of supposition, we should confine such rules to non-parasitic terms, or at least to terms that are indexed with a variable that is bound by a DP with a singular main term. Then, as noted at the end of the last section, they are mostly correct.

8.7 Inferences

When the notions of modes of supposition were introduced, they brought along with them new and useful ways to assess inferences. This already occurred with the earlier theory, as discussed above. Many proposals made with the new terminology were vague, and of limited usefulness. But some applications were clear

and compelling. One is the case of inference "from a superior to an inferior" with a distributed term.

> **From a superior to an inferior**: If a (non-parasitic) term T is distributed in a proposition P, then from P together with a proposition indicating that $T*$ is inferior to T, the proposition that results by replacing T by $T*$ in P follows.[114]

A common term $T*$ is inferior to a term T iff '*every* $T*$ *is* T" is true; this is also the condition for T being superior to $T*$. And a singular term t is inferior to a common term T iff 't *is* T" is true; again, in this case T is superior to t.

The simplest illustration of this is Aristotle's syllogism BARBARA:

Every M is P
Every S is M
$\therefore$ Every S is P

The term M is distributed in the first premise. The second premise states that S is inferior to M. The conclusion follows by replacing M with S in the first premise. The same principle also validates CELARENT:

No M is P
Every S is M
No S is P

Again, M is distributed in the first premise, and the second states that S is inferior to M. The conclusion follows by replacing M with S in the first premise.[115]

This principle goes beyond Aristotle's syllogistic. Buridan points out that this inference is a good one [*SD* 4.2.6 (244)]:

Socrates is seeing every horse
Brownie is a horse
$\therefore$ *Socrates is seeing Brownie.*

[114]Buridan *TC* 3.7.8 (283): "For any given proposition with a distributed term, whether nominative or oblique, an acceptable syllogism can be constructed by taking another term under the given term as the minor proposition . . . " Paul of Venice *LP* III.3 (172) "From a higher-level term to its corresponding lower-level term distributed affirmatively the argument is not solid unless with the due mean, because it does not follow: 'every animal runs; therefore, every man runs'. . . . But with the due mean, the argument is solid, . . . " [The "due mean" would be 'Every man is an animal'.]

Parasitic terms are not subject to the rule. Here are two counterexamples (assuming that 'animal' is superior to 'donkey'):

Of some farmer every animal is running
$\therefore$ Of some farmer every donkey is running

Of every farmer every animal is running
$\therefore$ Of every farmer every donkey is running

[115]BARBARA and CELARENT are especially important since Aristotle reduced all of the other moods of syllogisms to them.

The reverse principle also holds for determinate or merely confused supposition:[116]

> **From an inferior to a superior**: If a (non-parasitic) term T has determinate or merely confused supposition in a proposition P, then from P together with a proposition stating that $T*$ is superior to T, the proposition that results by replacing T by $T*$ in P follows.

For example, another of Aristotle's first figure syllogisms follows by this principle:

Darii
Every M is P
Some S is M
$\therefore$ Some S is P

Burley's version of "from a superior to an inferior" is slightly more comprehensive than the version stated above:[117]

> A consequence from a distributed superior to its inferior taken with distribution and without distribution holds good ...

I think that what he means by "and without distribution" is that if the quantifier sign preceding the distributed superior is changed so as to change the term from having distributive supposition to having determinate or merely confused supposition, then the inference is still good. For example, not only is this a good consequence:

	Every A is B	B is superior to A
	Every B is C	B is distributed
$\therefore$	*Every A is C*	A replaces B in the second premise

so is this:

	Every A is B	B is superior to A
	Every B is C	B is distributed
$\therefore$	*Some A is C*	A replaces B in the second premise, with '*every*' changed to '*some*'

Another example would be:

[116] Paul of Venice *LP* III.3 (171): from a lower-level term to its corresponding higher-level term affirmatively and without a sign of distribution and without any confounding signs impeding there is a solid inference. E.g., 'man runs; therefore, animal runs'." [I don't know why Paul does not require a "due mean" here, as he does for the inference from a higher-level term to its corresponding lower-level term.] Ockham *SL* III.3-6 (600) cites this rule: "ab inferiori ad superius sine distributione et affirmative est bona consequentia et simplex." He then gives a number of counterexamples to it, such as examples in which terms do not have personal supposition. He then qualifies the rule (page 601): "ab inferiori ad superius sine distributione et affirmative est bona consequentia si termini supponant personaliter et significative." (cited in [Moody, 1965 p. 288, note 1]). "From a superior to an inferior without distribution and affirmative is a good consequence if the terms supposit personally and significatively."

[117] Burley *Consequences* in [Kretzmann & Stump, 1988 page 300].

	Not some D is E	E is distributed
	Every F is E	E is superior to F
$\therefore$	*Not some D is every F*	F replaces E in the first premise, adding '*every*'

8.8 Exponibles

We saw earlier that it is a general principle that an exponible proposition bears the same logical relations to other propositions as do its exponents. This principle also extends to modes of supposition. Buridan brings up this issue in connection with "words that imply negations in themselves or in their exponents". Examples are "the verbs 'begin' and 'cease', or the words 'without', 'besides', 'only', and several others." He says that such words can bring about distribution. He explains:

> "...if the proposition in which such distributives occur needs to be expounded by other propositions or by another proposition because of its obscurity, a term can be said to have such supposition or suppositions that it is found to have in its exponent or exponents." [*SD* 4.3.7.5 (273)][118]

In some cases Ockham seems to agree with this. For example, in discussing the proposition '*Socrates begins to be literate*', he claims that the subject '*Socrates*' has none of the three modes of common personal supposition; this is because it meets none of the descent/ascent conditions.[119] He goes on to explain:

> The reason why such terms have none of the aforesaid forms of supposition is as follows: the propositions containing such terms are equivalent to conjunctive propositions each composed of two or more propositions. These propositions have the same subject; nevertheless, at least one of them is affirmative and one negative so that one and the same term has different forms of supposition in these propositions. Consequently, these terms do not have any one of the normal forms of supposition in the proposition whose parts are the various exponential propositions. For example, 'Socrates begins to be white' is equivalent to the conjunction 'Socrates was previously not white and now for the first time is white'. In 'Socrates is white' the word 'white' supposits determinately; whereas, in 'Socrates was not white' it has, because of the preceding negation, confused and distributive supposition. [*SL* I.75 (216).]

The idea is a natural one: In the exposition of the original proposition the word in question occurs twice, with different modes of supposition; thus there is no single

[118] As an example of an exponible Buridan (*SD* 4.2.4) gives: " 'Only a man runs' is analyzed as 'A man runs and nothing other than a man runs'".

[119] Actually, his argument at this point rests on his own definition of merely confused supposition, which is slightly different from the one discussed in this chapter. This does not affect the point made here.

mode of supposition that it has in the original proposition. However, Ockham limits this principle. In the paragraph following the cited one he says:

> One might claim that a consequence of this analysis is that the subject in 'Only what is an animal is a man' would not have merely confused supposition. The point would be that since the proposition is equivalent to a conjunction of propositions, one of which is affirmative and the other negative, the subject would have a different form of supposition in each of the elements of the conjunction. The response is that in the exclusive affirmative proposition the subject has merely confused supposition, for although the exponents of that proposition have subjects with different forms of supposition, those subjects are not identical with the subject in the original exclusive proposition; and because the subjects of the affirmative and negative exponents differ from the subject of the exclusive proposition, that subject can have one of the three forms of proposition.

Probably Ockham has in mind expounding 'Only an animal is a man' as 'An animal is a man and every man is an animal'. Here the subjects are clearly not the same, but it is not clear why that should yield any answer at all. I speculate that Ockham's policy is to try to apply the tests for supposition to the original proposition, and if one of them works, then that settles the matter. If none of them work, then the term has no mode of supposition. It is only in the latter case that one looks for an explanation in terms of the term's occurring twice in the exponents of the proposition with different modes of supposition. What is puzzling is why, if this is a good explanation, the same sort of explanation should not be good in other cases.

8.9 Variant Interpretations of the Modes

I have been discussing certain definitions of the modes of personal supposition. No one author used exactly these three definitions. Instead I chose them to represent the "best of Burley-Ockham-Buridan". In this section I discuss variant definitions.

8.9.1 Determinate supposition

We have used this version of determinate supposition:

> A term F has determinate supposition in a proposition P if and only if
>
> > [Descent]: you may descend under F to a disjunction of propositional instances of all the Fs, and
> > [Ascent]: from any such instance you may ascend back to the original proposition P.

Ockham and Buridan agree on this definition.[120] Burley gives the same account but omits the ascent condition.[121] I assume that this is an oversight — if the ascent condition were omitted then the definition would not cohere with many paradigm cases. For example, 'donkey' in 'Every donkey is running' would be determinate because one can descend to a disjunction. The notion of determinate supposition was quite stable from author to author.

8.9.2 Distributive Supposition[122]

Our definition of distributive supposition has been:

> A term F has distributive supposition in a proposition P if and only if
>
> > [Descent]: you may descend under $F$to a conjunction of propositional instances of all the Fs, and
> > [Ascent]: from any one instance you may not ascend back to the original proposition P.

This is essentially Ockham's definition.[123] Buridan and others omit the condition that prohibits ascent from an instance.[124] I am not certain how important this is. That is because the prohibition of ascent may be redundant. For there to be a case in which one can descend to a conjunction, and then ascend from any instance, one would need a proposition and a common term in it such that the proposition is logically equivalent to any of its descended forms. I suspect that it is impossible

[120]Albert of Saxony (*SL* II.4) gives an equivalent account: "Determinate supposition is the use of a general term for each of the things it signifies by its imposition, or which it signifies naturally (if it is a mental term), in such manner that a descent to its singulars can be effected by a disjunctive proposition. In this sentence, 'A man runs', the term 'man' has determinate supposition, because the term 'man' in this sentence stands, disjunctively, for everything which it signifies by its imposition. For it is sufficient, for the truth of the proposition 'A man runs', that this disjunctive proposition be true: 'This man runs, or that man runs', and so on for all singulars." Since any disjunct entails the disjunction, this last sentence gives the ascent condition.

Paul of Venice *LM* 3.8 (89) gives a vaguer account that is the same if one takes for granted that he is discussing disjunctions: "Determinate supposition is the meaning of a term in a proposition beneath which one can infer all of its referents and conversely on the condition of a proper middle."

[121]Paul of Venice *LP* 2.4 (149) gives the same account as Burley.

[122]Buridan's account omits the non-ascent condition. Ockham includes additional provisions for immobile distribution. Burley does not define distributive supposition; instead he defines two kinds of distributive supposition. This is discussed below.

[123]See note 101

[124]See note 101 for Buridan's view. Other authors than Buridan gave equivalent definitions. Albert of Saxony *SL* II.4 states: "Confused and distributive supposition is the interpretation of a spoken or written term, in conjunctive manner, for each thing which it is instituted to signify; or it is the interpretation of a mental term for each thing which it signifies naturally—-such that a descent to the singulars, for which it stands, can be made in conjunctive manner, by reason of that supposition." This account was also given by at least two anonymous early authors (in [De Rijk, 1967]): [UD] "Confused and distributive is when of necessity a descent is made to every inferior." and [CSN] "Confused and distributive supposition is when a common term supposits for its inferiors and it is possible to make a descent under it to any appellated content under the distributive term.".

to formulate a proposition containing a term that works this way, using only the terminology we have at our disposal.[125]

Paul of Venice in *LM* divides distributive supposition into mobile and immobile. His account of mobile supposition is interestingly different from earlier accounts. He says that a term has mobile distributive supposition in a proposition if one may descend under a term to a conjunction of instances of that proposition, and one may also ascend — not from a single instance – but from the whole conjunction.[126] It is easy to check that the subjects of the standard universal propositions have this kind of supposition, as well as the predicate of a universal negative proposition. But the predicate of a particular negative proposition does not have this kind of supposition, since you cannot go from

> *Some A is not this B* and *some A is not that B* and *... and so on for all the Bs.*

to

> *Some A is not a B.*

It is odd for him to call this "immobile" supposition, because one can descend under the predicate term of a particular negative to any instance. Perhaps he sees mobility as requiring a valid inference in both directions. We will return to his particular kind of supposition in a later section. Meanwhile it appears that there was a practical consensus on the account of distributive supposition *simpliciter*.

8.9.3 Merely Confused Supposition

We have been using this definition:

> A term F has merely confused supposition in a proposition P if and only if
>
> > [Descent]: you may not descend under F to either a conjunction or a disjunction of propositional instances of all the Fs, and
> > [Ascent]: from any instance you may ascend back to the original proposition P.

This mode shows the most variation in how it is characterized. I'll look at two options. The first is the most famous one, devised by Ockham, which invokes

[125] Spade [1988] argues this, giving a clear explanation of what is at issue. (Trivial counter-examples to the claim are ruled out by the policy of ignoring the effect of repeated terms in testing for mode of supposition.)

[126] Paul of Venice *LM* 3.11a (89-121): "Distributive general reference is twofold because some is mobile, some immobile. Distributive mobile general reference is the meaning of a common term beneath which one can infer to all of its singulars conjunctively on the condition of a proper middle and, conversely, with the same middle."

a constraint involving descent to a disjunctive term. The other is the option of removing the ascent condition.

Descent to a Disjunctive Term The displayed definition is due to Burley.[127] Ockham added to Burley's conditions a clause which states that although it is not possible to descend to a disjunction of instances of the proposition under the term, one can descend to a disjunctive term.[128]

> A term F has merely confused supposition in a proposition P if and only if
>
> > [Descent]: you may not descend under F to either a conjunction or a disjunction of propositional instances of all the Fs, but you may descend to a proposition with a term in place of F that enumerates all of the Fs disjunctively, and
> > [Ascent]: from any instance you may ascend back to the original proposition P.

For example, in '*Every horse is an animal*' one may not descend either to a disjunction or conjunction of instances of the form '*Every horse is this animal*'. But one may descend to:

> *Every horse is (this animal or that animal or . . . for all the animals).*

Buridan mentions this option but does not endorse it as a requirement.[129]

Ockham's proposal was very popular, and several other writers adopted it.[130] I think that this was not a fruitful development. The reason is that the possibility of descent to a disjunctive term is a grammatical issue, not a logical one. This is because:

> *every P*

can always be paraphrased as

> *this P and that P and . . .*

but not as '*this P or that P or . . .*'.[131]

Further,

---

[127] See note 102.

[128] See note 102.

[129] See note 102.

[130] For example, Albert II.4 says: "Merely confused personal supposition is the interpretation of a term for each thing it signifies by its imposition, or which it signifies naturally (if it is a mental term), in such manner that a descent to its singulars can be made by a proposition of disjunct predicate, but not by a disjunctive or conjunctive proposition. And Paul of Venice *LP* 2.4 (150) gives: "Common mobile personal supposition which is merely confused is the acceptance of a common term standing personally beneath which one descends to all of its referents in disjuncts, as in 'every man is [an] animal and these are all animals; therefore, every man is this animal or that animal and thus of singulars'."

[131] Likewise, '*no P*' can always be paraphrased as
this P and that P and . . . not
but not as 'this P or that P or . . . not'.

some P

can always be paraphrased as

this P or that P or ...

but not as '*this P and that P and ...*'. (Likewise for '*a P*' and so on.)

The paraphraseability of a DP using a disjunctive term is a matter of which quantifier word grammatically accompanies it.[132] But it was generally presumed that the quantifier can be changed (if additional changes are made) so as to preserve mode of supposition. Several of William's equipollences do this. For example, '*donkey*' has determinate supposition in

Some donkey is not spotted

and also in the logically equivalent

Not every donkey is spotted

The former can be paraphrased as

This donkey or that donkey or ... is not spotted

but the latter certainly cannot be paraphrased as

Not (this donkey or that donkey or ...) is spotted.

This might not be important when the term already has determinate or distributive supposition, for paraphrasing a common term into a disjunctive term is not relevant to these classifications. But consider the following example. This inference is a case of a descent under Q to a disjunctive term:

Not some donkey every animal isn't
∴ Not some donkey (this animal or that animal ...) isn't

The inference fails,[133] thus establishing that '*animal*' does not have merely confused supposition in the premise according to Ockham's account. But it should have merely confused supposition. The premise is the result of applying equipollences to a universal affirmative proposition:

[132]This does not contradict [Priest & Read, 1980], who claim that in the notation of the first-order predicate calculus you can always replace a one-place predicate by a new predicate which is defined in terms of a disjunction of its instances. They are right, but this is a completely different idea. A "predicate" in the medieval theory, when it comes to modes of supposition, behaves as a denoting phrase. When you write in Latin (or English) '*this F or that F or ...*' you have produced a full-fledged denoting phrase (one that cannot have a quantifier word such as '*every*' or '*some*' or '*no*' put on front). But a predicate in the predicate calculus does not have the status of a denoting phrase.

[133]In assessing the inference it is essential to keep the scopes straight in the conclusion. The '*not*' in '*isn't*' does not have scope over any DP in the proposition.

every donkey an animal is ⇒
not some donkey not an animal is ⇒
not some donkey every animal isn't

The other modes of supposition are preserved through these equipollences, and merely confused supposition is also preserved on all accounts other than Ockham's. A related example concerns the predicate term in '*No animal is every man*'. Marilyn Adams[134] points out that it does not have either determinate or distributive supposition, and on Ockham's account it does not have merely confused supposition either, because one cannot descend to a proposition with a disjunctive term. From:

No animal is every man

we may not infer

No animal is this man or that man or ...[135]

John Dorp[136] proposes that in order to tell what mode of supposition a term has we should first move the verb to the end, and then move any negation to the right of all of the verbs. (Applications of Sherwood's equipollences given in 1.7 will do this.) This needs to be done before applying Ockham's test. Then, every term in the categorical will end up with '*every*' or '*some*' as its quantifier sign. If it is preceded by '*every* it has distributive supposition. If it is preceded by '*some*', then either it will satisfy the conditions for determinate supposition, or not. If it does not, it may be paraphrased by a disjunctive term. This renders Ockham's test for merely confused supposition accurate. However, it may also render it redundant, for it is not clear that there are any occurrences of terms that are neither determinate nor distributed, and that satisfy the ascent condition for merely confused supposition, but that are prohibited from having merely confused supposition because of failure of the disjunct predicate option.

Some authors[137] proposed a fourth mode, which is possessed by any term which is not determinate or distributive or merely confused. A term has this mode of supposition if one may descend to a conjunctive term. In the example just given, this is possible; from:

No animal is every man

we may infer

No animal is this man and that man and ...

[134]Adams 1987, 366-67.
[135]Buridan *SD* 4.3.8.2 (277) says that '*donkey*' is merely confused in '*No man sees every donkey*'. But '*every donkey*' cannot be paraphrased here as '*this donkey or that donkey or ...*'.
[136]Cited in [Karger, 1993, pp. 418–20].
[137]Cf. [Read, 1991, 77-82].

Still other authors[138] defined merely confused supposition in terms of descent to either a disjunctive or a conjunctive term. Since one or the other of these descents is always possible, this is a redundant provision.

Of course, we are arguing about definitions, and Ockham may have intended to let grammar decide what mode of supposition a term has. But he never says anything like this. I doubt that he noticed this — and likewise for the other authors who followed him in this.

Omitting the Ascent Condition As we have seen, Buridan did not go along with Ockham in requiring descent to a disjunctive term. However, he did not have anything to replace this condition by; in particular, he did not give an ascent condition. This makes his merely confused supposition include *every* term that is not determinate or distributive. This is a coherent way to define the modes, but it seems to lump together quite different phenomena. On this account, all of the underlined terms below have merely confused supposition:

Every donkey is an animal
Maria believes that every animal runs
Every man who sees every animal runs

On the account introduced in section 8.1, the first would be merely confused and the others would lack a mode of supposition.

9 ISSUES ABOUT MODES OF SUPPOSITION

9.1 Refined Modes of Supposition

Various writers, focusing on Ockham's account of merely confused supposition, have noticed that there seems to be a gap in the definitions of the modes. On his account we have:

Descent to a disjunctive proposition
Descent to a conjunctive proposition
Descent to a disjunctive term

But what about

Descent to a conjunctive term?

This possibility has been considered by both medieval[139] and contemporary logicians. In modern times, Paul V. Spade [Spade, 2000] has argued that there aren't any examples of main terms with a mode of supposition outside of the traditionally given ones. Given the modes as described in section 7.1, I agree (assuming that the notation does not include non-extensional contexts).

[138] Maulfelt; see [Read, 1991 77-82].
[139] See discussion in [Read, 1991].

I have argued above that whether one descends to a disjunctive or conjunctive term is a matter of grammar, not of logic. That may be why the addition of a fourth mode in terms of descent to a conjunctive term has not yielded an interesting theory. However, there is a way to add a fourth mode that yields an improved account. This is achieved not by adding a new mode that classifies terms which satisfy none of the conditions for the traditional modes, but rather by subdividing the category of distributive supposition. For reasons to be given later, I will call the one sort of distribution "wide distribution" and the other "narrow distribution". Wide distributive supposition is traditional distributive supposition in cases in which one can ascend back to the original proposition from the *whole conjunction* of propositional instances under the term. (Not from a single instance, but from the whole conjunction.) Narrow distributive supposition is distribution where one cannot make this ascent. The four modes would then be:

Determinate:

> Descent to a disjunction of propositions, and ascent back from the whole disjunction.

Merely confused:

> No descent to a disjunction of propositions but ascent back from the disjunction.

Wide distributive:

> Descent to a conjunction of propositions and ascent back from the conjunction.

Narrow Distributive

> Descent to a conjunction of propositions and no ascent back from the conjunction.

There is precedent for the category of wide distribution; it is what Paul of Venice [*LM:TS* 3.11a (95)][140] proposes for mobile distribution. So far as I know, narrow distribution has not been proposed. (Immobile distribution generally refers to cases in which no descent is possible.)

If we think of denoting phrases as restricted quantifiers, then this is equivalent to saying:[141]

[140] "Distributive general reference is twofold because some is mobile, some immobile. Distributive mobile general reference is the meaning of a common term beneath which one can infer to all of its singulars conjunctively on the condition of a proper middle and, conversely, with the same middle. Thus this follows: 'This animal runs and this animal runs and thus of each individual and these are all animals; therefore, every animal runs'." He correctly includes subjects of universal affirmatives, and both subjects and predicates of universal negatives as having this mode of supposition.

[141] I am speaking loosely here when I refer to quantifier rules such as "Existential Instantiation". Such rules can indeed be invoked, but only if the syntax of the language is enhanced with variables, etc.

Determinate:

> Existential Instantiation holds, and so does Existential Generalization.

Merely confused:

> Existential Instantiation does not hold, but Existential Generalization does.

Wide distributive:

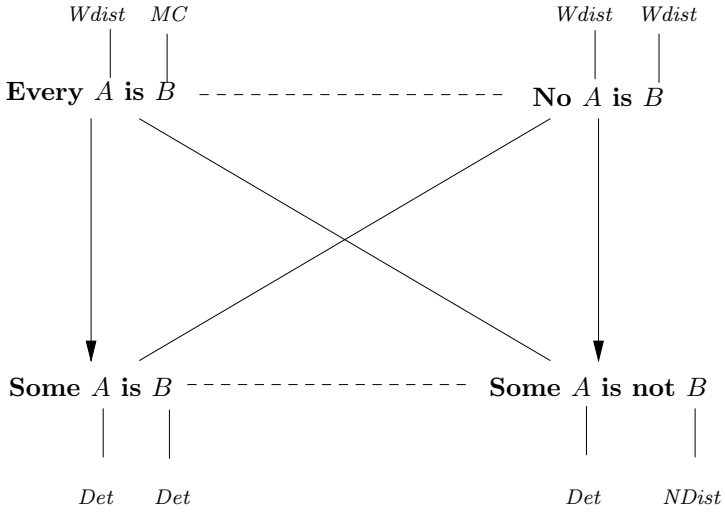
> Universal Instantiation holds, and so does Universal Generalization.

Narrow Distributive

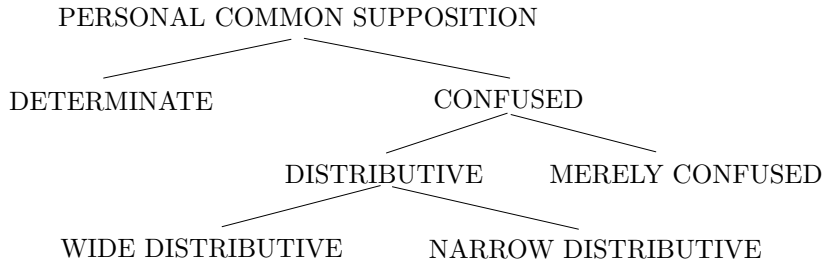
> Universal Instantiation holds, but Universal Generalization does not hold.

9.2 Symmetry

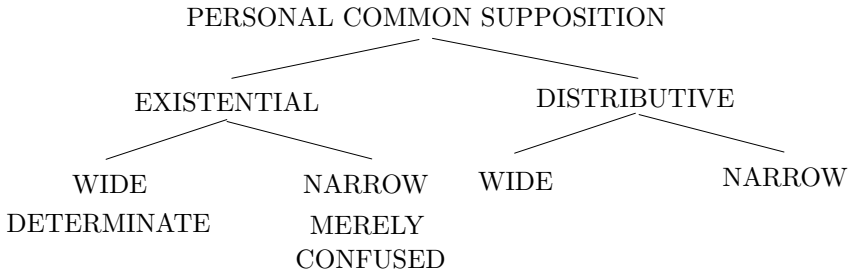
The new classification of terms yields more symmetrical patterns. Let us call Wide Distribution and Determinate supposition *opposites*, and likewise Narrow Distribution and Merely Confused supposition. Then when a proposition is negated, each term has its mode of supposition reversed. Since diagonally opposite propositions in the Square of Opposition are (logically equivalent to) negations of one another, the modes of supposition of the terms in any proposition mirror (in reverse) those in its opposite. The modes of supposition in the traditional square are now:



These definitions call for a new classification of the modes. The standard medieval classification, with our new distinction between kinds of supposition, would look like this:



But a more natural classification would be this:



This omits the general notion of Confused Supposition as the complement to Determinate. Whether this is a good or bad thing is not clear to me. Certainly medieval authors saw something in common to the various sorts of confusion; whether it is merely the absence of determinacy is not clear.

9.3 Causes of the modes

With these new distinctions we can refine the theory of the causes of the modes of personal supposition developed in the last chapter. The principles used there yielded mostly intended results, except for the problem about what mode a term has if it is acted upon by two distributing signs. Ockham and Buridan disagreed about this, and neither of their views are correct. Their theory lacked the resources for getting a correct answer, since it does not subdivide the category of Distributive Supposition.

The revised theory makes a new set of rules possible. The refined rules below are to be applied recursively to the signs of a sentence that have scope over the copula; as before, they apply first to the sign that is rightmost and has scope over the copula. We suppose as usual that verbs have been analyzed in terms of the copula and a participle.

DEFAULT: A main term of a proposition has determinate supposition unless something causes it not to. A particular affirmative sign adjoined to a term gives it determinate supposition (or, equivalently, has no effect). In either case, any terms to the right and within the scope of the denoting phrase containing the term retain the mode of supposition they already have, except that a wide distributed term becomes narrow distributive.
UA: A universal affirmative sign widely distributes the term it is adjoined to and makes any other term to its right merely confused if it is determinate, leaving terms with the other modes unchanged.
UN: A universal negative sign widely distributes the term it is adjoined to; if a term following the universal negative sign has determinate supposition it, becomes wide distributed; if the term has wide distribution it becomes merely confused; if the term has merely confused supposition it becomes narrowly distributed, and if it has narrow distribution it becomes merely confused.
NEG: A negating negation has the following effect on any main term following it and in its scope:

> if the term is determinate it becomes wide distributed, and vice versa;
> if the term is merely confused it becomes narrowly distributed, and vice versa

All provisions except for the first and last are consistent with the earlier rules, though they provide more detailed information. The previous version of the last rule said that if a term has distributed supposition then the negation makes it merely confused (according to Buridan) or determinate (according to Ockham). On the new account if the term has narrow distribution it becomes merely confused, as Buridan said, and if it has wide distribution it becomes determinate, as Ockham said. Two examples we looked at earlier were these:

Not: no man runs
Not: some farmer sees every donkey

The new rules correctly classify '*man*' as having determinate supposition in the first proposition, and they correctly classify '*donkey*' as having merely confused supposition in the second proposition.

9.4 Global Quantificational Import

9.4.1 What are *the modes of common personal supposition?*

This is a long-standing problem in the secondary literature: We have definitions of the modes, but what are we defining?

What we have is a theory of what I call *global quantificational import*. This is the import a quantified DP actually has, described in terms of the import it would have if it had scope over the whole global context (or almost the whole context).

This idea can be made precise within the theory of "prenex forms". In contemporary symbolic logic, if no biconditional sign appears in a formula of quantification theory then you can take any quantifier in that formula and move it in stages toward the front of the formula, each stage being equivalent to the original formula, provided that you switch the quantifier from universal to existential (or vice versa) whenever you move it past a negation sign or out of the antecedent of a conditional, and provided that you do not move it past a quantifier of opposite quantity (i.e. you don't move a universal past an existential, or vice versa). For example, you can take the universal quantifier in:

$$\neg(Gy \rightarrow \underline{\forall x} Px)$$

and move it onto the front of the conditional to get:

$$\neg \underline{\forall x}(Gy \rightarrow Px),$$

and then the resulting universal sign can be moved further front, turning into an existential:

$$\underline{\exists x} \neg(Gy \rightarrow Px).$$

This chain of equivalences can be interpreted as the movement of a quantifier to the front, retaining its identity while sometimes changing its quantity. If you do this systematically to all the quantifiers in a formula, the result is a formula in "prenex normal form," in which the quantifiers are all on the front in a row, each of them having scope over the rest of the formula to its right. In terms of these prenex forms you can define the global quantificational import of any quantifier in a main term in any categorical formula. Let us take this idea and use it to analyze the terminology of supposition theory. The subject matter here is terms, not quantifiers, but each main term comes with its own quantifier, so we can treat the theory as if it is a theory of restricted quantification (with denoting phrases being the restricted quantifiers). We then give this account:

> A prenex string for ϕ is a string of affirmative denoting phrases on the very front of ϕ, with no other signs between them.
> [That is, each is of the form '*Every T*' or '*Some T*' or '*D*', and there are no negations, and each denoting phrase has scope over the rest of ϕ].

An example with the prenex string underlined:

> *Every dog some donkey* not *is*

There is a systematic way to convert any categorical proposition into another one in which all of the main terms in the original one are in prenex position in the new one, and the converted proposition is logically equivalent to the original. Here is the process:

> Change every main denoting phrase of the form '*No T*' into '*every T* not', and every main denoting phrase of the form '*a T*' into '*some T*',.
> This leaves '*every*' and '*some*' as the only quantifier signs on main terms.
> Remove any double *not*'s anywhere in the formula whenever they appear.
> Starting at the left, replace each '*not every T*' by '*some T not*', and each '*not some T*' by '*every T not*', and each '*not D*' by '*D not*'.
> Remove double *not*'s whenever they appear.

Every categorical proposition has a unique prenex-convert produced by these rules. Call the quantifier signs '*every*' and '*some*' opposites. We can then define:

> a main term has (wide)/(narrow) quantificational import in a proposition iff when the proposition is converted into prenex form the term (is not)/(is) preceded by a main term with the opposite quantifier sign
> a main term has (universal)/(existential) global quantificational import in a proposition iff when the proposition is converted into prenex form the term ends up with ('every')/('some') as its quantifier sign

This defines global quantificational import for all main terms in any categorical proposition.

9.4.2 Causes of the Modes

One can now establish the following equivalence between the classifications above in terms of global quantificational import and the refined modes of supposition that are yielded by the rules governing causes of the modes in the last section. If these rules are applied to the forms we have been discussing:

> A term has Determinate supposition according to the rules iff it has wide existential quantificational import
>
> A term has Merely Confused supposition according to the rules iff it has narrow existential quantificational import
>
> A term has Wide Distributive supposition according to the rules iff it has wide universal quantificational import
>
> A term has Narrow Distributive supposition according to the rules iff it has narrow universal quantificational import

Illustration: Let us test '*donkey*' for its mode of supposition in '*Some donkey is a predator*'. '*Donkey*' has determinate supposition here, because it is already in prenex form, existentially quantified:

> *Some donkey is a predator*

It has wide distributive supposition here, for the same reason:

Every donkey is a predator

The term '*predator*' in the sentence just displayed has merely confused supposition because it is existentially quantified with scope inside that of '*every donkey*'.

Now consider:

Some predator is not a donkey

in its equivalent form

Some predator not a donkey is

Here the 'not a donkey' is equivalent to 'every donkey not', yielding:

Some predator every donkey not_ is

The original '*some donkey*' has now become universal, thus classifying it as having distributive supposition. This illustrates the importance of looking at things globally; although '*donkey*' is not preceded by any universal quantifying sign here, it has universal import. You could universally instantiate it. This is why it is classified in this theory as (narrow) distributive.

9.4.3 Parasitic Terms

We have just established that a main term in a categorical proposition has a certain mode of supposition according to the rules for the causes of supposition iff it has a corresponding quantificational import. But, as we have noted above, both of these disagree with the modes of supposition as defined in terms of ascent and descent in the case of parasitic terms. Such terms do not admit of either ascent or descent, and so they have no mode of supposition at all. The situation is summed up by:

> A main term that is not parasitic has the mode of supposition that is attributed to it by the refined rules for the causes of modes of supposition — equivalently, if it has the corresponding global quantificational import.
>
> Parasitic main terms have no modes of supposition, though they are classified as having modes by the rules from causes of modes, and they have global quantificational import.

As noted earlier, the useful rules "From a superior to an inferior" and "From an inferior to a superior" do not apply to parasitic terms.

9.5 Other possible modes of supposition

The fact that all non-parasitic main terms of a categorical proposition have one of the three defined modes of supposition holds in part because of the limited array of quantifier signs that are available: '*every*', '*some*', '*no*' and indefinite. In theory,

other modes can be defined in terms of ascent and descent. Here is a chart laying out the options. The notation in the chart means:

- D&: descent is possible to a conjunction of propositional instances
- D∨: descent is possible to a disjunction of instances, though not to a conjunction of propositional instances;
- D-: neither kind of descent is possible
- A&: ascent is possible from a conjunction of propositional instances, though not from a disjunction of propositional instances
- A∨: ascent is possible from a disjunction of propositional instances
- A-: neither kind of ascent is possible

The definitions of the modes can then be displayed as:

	D&	D∨	D-	A-	A&	A∨
Determinate		X				X
Wide Distribution	X				X	
Merely Confused			X			X
Narrow Distribution	X			X		
[5]	X					X
[6]		X			X	
[7]		X		X		
[8]			X		X	
[9]			X	X		

It is debatable whether any term could have mode [5]. The remaining modes are possessed by terms coupled with the following invented quantifier signs:

[6] *Many-or-all* <u>*donkeys*</u> *are spotted.*
[7] *Some-but-not-all* <u>*donkeys*</u> *are spotted.*
[8] *Every-or-no* <u>*donkey*</u> *is spotted.*
[9] *Maria said that some* <u>*donkey*</u> *is spotted.*

Cases [7] and [8] each have two variants, depending on whether you decide that the term is affirmative, or negative. (If a denoting phrase is negative, then when you erase it for descent you need to insert a negation sign.)

I am not aware of any writer who discussed modes of this sort.

9.6 Modes of supposition for non-main terms

Our discussion so far has been confined to main terms of categorical propositions. There are non-main terms that have modes of common personal supposition according to the definitions in terms of ascent and descent. These are not specified

by the rules from causes, and they do not correspond to any kind of global quantificational import. They are non-parasitic terms that occur inside of complex clauses, or inside of complex terms derived from relative clauses. We focus here on terms that occur within relative clauses.

Can we extend our rules for the causes of modes of supposition? The rules do not presently address non-main terms. They do, however, assign modes of supposition to non-main terms which are main terms of their local clauses, relative to the local clauses, and they classify the complex terms containing them, relative to the main clause. One might hope that these pieces of information would combine to tell us the mode of supposition of the non-main term in the whole proposition. For example, we can indeed predict that if a complex term C which consists of a term modified by a relative clause has determinate supposition in a proposition P, and if a term E has determinate supposition within the clause that forms the relative clause, then E has determinate supposition in P. For example, the complex term '*horse which some donkey sees*' has determinate supposition in:

Some man sees a horse which some donkey sees

And '*donkey*' has determinate supposition in:

some donkey sees ___

Therefore, '*donkey*' has determinate supposition in the whole proposition. (It is easy to check that one may descend to a disjunction, and ascend back from any disjunct.)

This fact has more application than might be thought. For this result also applies to constructions generated from relative clauses, constructions such as adjectives or participles in attributive position. So we can tell instantly e.g. that '*spotted*' and '*running*' have the same modes as that of the complex term containing them in:

Some spotted donkey is running
Some running donkey sees a horse

Anything to which our Fronting rule applies automatically falls into this category, as '*king*' in '*Some king's donkey is running*'.

An additional rule is this: if a complex term C which consists of a term modified by a relative clause has merely confused supposition in a proposition P, and if a term E has determinate supposition within the clause that forms the relative clause, then E has merely confused supposition in P. For example, the complex term '*horse which some donkey sees*' has merely confused supposition in:

Every man sees a horse which some donkey sees

And '*donkey*' has determinate supposition in:

some donkey sees ___

Therefore, '*donkey*' has merely confused supposition in the whole proposition. (It is easy to check that one may not descend to a disjunction, and one may ascend back from any disjunct.)

Are there more rules of this sort? If so, they are few. In a large number of cases the mode of supposition of a complex term in a proposition together with the mode of supposition of a term T in the clause of that complex term do not determine a mode of supposition of T in the whole proposition. In particular, if a term has wide distributive supposition in a clause within a complex term that has narrow distributive supposition in a whole proposition, then the mode of supposition of the simple term is not determined. Consider the following propositions in which '*donkey*' has wide distributive supposition within a clause making up the relative clause portion of a term with narrow distributive supposition. Then:

'*Donkey*' has determinate supposition in:

Some horse isn't an animal that sees every donkey

And it has merely confused supposition in:

Some horse is an animal that sees every donkey

And it has narrow distributive supposition in:

Some horse sees every animal that is every donkey

And it has no mode of supposition at all in:

Some horse sees every animal that sees every donkey

Perhaps there are some subtle rules that will constitute an algorithm for determining the mode of supposition of a non-main term from some information about it that is evident from inspection (which is how the rules for the modes of supposition of main terms work), but I don't know how to construct them.

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THE ASSIMILATION OF ARISTOTELIAN AND ARABIC LOGIC UP TO THE LATER THIRTEENTH CENTURY

Henrik Lagerlund

1 INTRODUCTION

The development of logic from the rediscovery of the complete corpus of Aristotle in the early twelfth century through the thirteenth and fourteenth centuries cannot be characterised as anything else than remarkable. No other time in history, except the late nineteenth and twentieth centuries, has seen such a development of logic.

In this chapter, I will try to give an account of some of the problems and developments from the late twelfth century into the mid-thirteenth century. My focus will primarily be on the commentary tradition of Aristotle's logical works, that is, the *Categories*, *De intrpretatione*, *Prior Analytics*, *Posterior Analytics*, *Topics*, and *Sophistici Elenchi*.[1] Running parallel to the commentary tradition is a less known and much less studied tradition, namely the history of the Arabic logical works available in Latin at this time. Most scholars are of the opinion that Arabic logic had very little if any influence on Western logic, but, although Arabic logic did not revolutionise western logic as was once thought, it certainly is part of the Western logical tradition and as such it had a not insignificant influence; although not in the way previously thought. I will here highlight some of the more interesting things I have noticed about these works.[2]

Alongside the commentaries on Aristotle's works medieval logicians also wrote treatises, summaries, and textbooks of logic. As part of this overview, I will try to say something about this development as well. A large part of this will be a detailed look at what the content of some of the more famous thirteenth century works was. I will for example look closer at Peter of Spain's *Tractatus* and William of Sherwood's *Introduction to Logic*. I will supplement this with notes

[1] I will not say anything about the commentaries on the *Posterior Analytics*. This tradition is hardly studied and its development seems to be a little bit different than the other works. Although it is a work on method it is unclear whether thirteenth century logicians saw it as a work in logic. For an overview of this tradition see [Longeway, 2007, 3–79].

[2] For a more detailed study of this tradition and its influence on Western logic I will have to refer the reader to a forthcoming monograph of mine with the title *Latin Arabic Logic in the Middle Ages: The Doctrines and Influences of Arabic Works on Logic Translated into Latin in the 12th and 13th Centuries*.

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and references to Roger Bacon's *Summulae dialectices* and Lambert of Auxerre's *Logica*. This will give the reader a good sense of what logic was thought to be in the mid-thirteenth century.

At the beginning of the thirteenth century there were two theories that logicians thought about as not deriving from Aristotle. The first was what came to be called the properties of terms and the other was the discussion of syncategorematic terms. Under the heading 'properties of terms' the logicians of the time dealt with signification and supposition of so called categorematic terms. This theory worked its way into the textbooks very early. The treatment of syncategorematic terms developed into a separate literary genre and both Peter of Spain and William of Sherwood wrote treatises on *Syncategoreumata*. I have included a discussion of the content of these works as well.

I have naturally included references to the works I discuss and also to the most important secondary literature, but I have not quoted in original language and I have tried to explain as many technical terms as possible. I have also not used any notation from contemporary symbolic logic, since I do not think it is helpful for understanding medieval logic. In my view it raises more questions than it answers.

The organisation of the sections in this chapter is chronological, which means that I will begin with Arabic logic, since most of it was translated in the mid-twelfth century, and then move on to my treatment of the commentary tradition. I will end with the discussion of the so called textbook authors, which might seem a little backwards since they are usually much less advanced than the commentaries. If someone only wants a quick introduction to thirteenth century logic, he or she can jump directly to that section below, since I am not presupposing the previous discussion.

Even though this chapter is fairly long, I have only been able to scratch the surface of the rich debates about logic and language in the late twelfth and early thirteenth centuries. I hope this can serve as an introduction and appetiser to more independent and deep studies of the primary texts themselves.

2 BACKGROUND

Medieval logicians divided logic into: the old logic (*logica vetus*), the new logic (*logica nova*), ancient logic (*logica antiqua*), and modern logic (*logica moderna*).[3]

(2.1) *Logica vetus*: Before the first decades of the twelfth century medieval thinkers in the Western tradition only had access to the *Categores* and *De interpretatione*. These works together with Porphyry's *Isagoge*, which is an introduction to the *Categories*, and Boethius' commentaries on these three works as well as his logical monographs[4] were called *logica vetus* (or *ars*

[3]For an outline of this see also [de Rijk, 1962, 14–5].

[4]Boethius' logical monographs are: *De syllogismis categoricis*, *De syllogismis hypoteticis*, *De divisione*, and *De differentiis topocis*. See [Jacobi, 1988].

vetus). From 1200 a work attributed to Gilbert de la Porrée (c. 1075-1154) called *De sex principiis* also belonged to this group.

(2.2) *Logica nova*: Around the 1120's the *Prior Analytics*, *Topics*, and *Sophistici Elenchi* in Boethius' translations became readily available. Later in the twelfth century the *Posterior Analytics* was translated and added to the group of works called the *logica nova* (or *ars nova*).

(2.3) *Logica antiqua* (or *antiquorum*): This is the generic name for all the works included in *logica vetus* and *logica nova*.

(2.4) *Logica moderna* (or *modernorum*): This name covers the part of medieval logic that cannot be traced back to any of the writings of the *logica antiquorum*.

As a rough guide one can say that the commentary tradition expands the *logica antiquorum*, while it is in the more independent treatise and summary writings that the *logica modernorum* is developed; of course, the treatises and summaries will also include discussions of the *logica antiquorum*. In the later parts of the thirteenth century the logical theories of the *logica modernorum* had worked its way into the commentaries and was used as tools to expand Aristotle's text, and if one looks into the fourteenth century the commentaries become more and more detached from Aristotle's original text and is more an excuse for writing about a particular topic.

Later medieval logicians considered Peter of Spain (c. 1205-77) to be the founder of the *logica modernorum*. As de Rijk has shown this is quite wrong, but, as we will see below, he gives a very authoritative and extremely influential outline of this part of medieval logic. According to him, *logica modernorum* contains supposition, relatives, ampliation, appellation, restriction, and distribution (see section 5.2 (On Supposition) below for an explanation of these terms). If we add the discussion of syncategorematic (or logical) terms to these, then we get what was, at least at Peter's time, considered to be the parts of the *logica modernorum*. Those pursuing this branch of logic were called the *Moderni* or *Terministae* (hence the popular name terministic logic).[5]

All of the logical works of Aristotle that was included in the *logica antiquorum*, except the *Posterior Analytics*, were translated by Boethius.[6] Although the *Categories* and *De interpretatione* were commented on at least since the ninth century, the others were not studied before 1120, but it was not until the late twelfth century that commentaries on these works started to appear, and in some cases the first commentaries are from the early thirteenth century.[7]

In the early thirteenth century, Aristotle very fast formed the foundation of the teaching in the arts faculty at the newly established university in Paris. Logic was

[5]See [de Rijk, 1962, 15–6].

[6]See *Aristotelis Latinus* I-VI.

[7]See [Ebbesen, 1976; 1979; 1981a], and also [Green-Pedersen, 1984]. The first commentary on the *Topics* listed by Green-Pedersen is from the 1230's.

never very controversial and was largely unaffected by the teaching ban on Aristotle in the 1210-30's. It was the *Physics* and the *Metaphysics* that the authorities of the time found objectionable, but it was only after these controversies were resolved that the commentary tradition really took of, and eventually became the main literary form of writing philosophy in the thirteenth century.

During the time of the controversy most of Averroes' commentaries on Aristotle's works had been translated. They were very helpful for a general understanding of Aristotle's logical works. An important line of interpretation in mid-thirteenth century commentaries is to read in the metaphysics into the logic and this seems to me to be mainly due to influence from Averroes.

With the expansion of commentaries and the growing importance of Aristotle's texts for university teaching the need for new or refreched translations soon became apparent. In the 1260's, William of Moerbeke translates or revises Boethius translations. At the same time he also translates several of the Greek commentators. The most important Latin commentators of the logic works from the mid-thirteenth century that are known by name are Thomas Aquinas (d.1274), Robert Kilwardby (d. 1279), Albert the Great (d. 1280), Boethius of Dacia (d. c. 1280), Simon of Faversham (d. 1306) and Radolphus Brito (d. 1320). The most productive author from this time is, however, the one called Anonymous, since the vast majority of commentaries from this time are anonymous.

It was generally accepted before de Rijk published his groundbreaking study on the *logica modernorum* that this part of medieval logic was primarily an influence from Byzantine and Arabic logic.[8] This was as de Rijk shows completely wrong. He argues convincingly that it is partly due to Aristotle's *Sophistici Elenchi* but foremost it was due to the creative minds of late twelfth century logicians. His judgement of earlier views was so harsh that Arabic logic in the Latin tradition has hardly been studied at all. I will below give a sketch of what was translated and what these works contain.

3 ARABIC LOGIC IN LATIN

The Latin logicians of the thirteenth century had quite a good grasp of Arabic logic. The major source for this knowledge was the *Maqāsid al-falāsifa* ('The Meaning of the Philosophers') by Abū Hāmid Muhammad Al-Ghazālī (1058-1111). It was supposed to be a preparatory work for his later much more well known work *Tahāfut al-falāsifa* ('The Incoherence of the Philosophers'). The whole of the *Maqāsid* contains a presentation of logic, physics, metaphysics and theology, and has been claimed to be an intelligent reworking of foremost Avicenna's *Dānesh-name* ('Book of Science'), which is a compendium of his doctrines written in Persian. Al-Ghazālī's work was translated in full into Latin early in the second half of the twelfth century, which is not more than 50 or 75 years after it was written. In one of the manuscripts the title is *Liber Algazelis de summa theoricae*

[8] See [Prantl, 1867, III, 263–4].

philosophiae, but it was also printed in Venice in 1506 under the title *Logica et philosophia Algazelis arabis*. The twelfth century translation is by Dominicus Gundissalinus and was commissioned by John Archbishop of Toledo (1151-66).[9]

The Latin translation of the *Maqāsid* was very much read and became, at least in the thirteenth century, the basic text from which the Latin authors gained their knowledge of Arabic philosophy. The reason it became so much read was because it mentions some of the 'hot' topics of the period, for example the divisions of the sciences, the distinction between essence and existence, the eternity of the world, the number of souls etc.[10] It was also a very controversial work. In his work the *Tractatus de erroribus philosophorum*, Giles of Rome lists 18 errors of Al-Ghazālī taken from the *Maqāsid*. They later found their way into the *Directorium Inquisitorum* from 1376 by Nicholas Eymerich.[11] The logic became very well known as well. Albert the Great uses it quite extensively and it was used or copied by Ramón Lull.[12]

Althought the *Maqāsid* was a summary of Al-Farabi's and Avicenna's doctrine, the Latin thirteenth century also had some knowledge of Al-Farabi's and Avicenna's logics directly. A twelfth century translation of the beginning, namely the part on Porphyry's *Isagoge*, of Avicenna's encyclopedic worked called in Arabic *Kitab al-Shifa* ('The Book of Healing') was circulating.[13] And also a fragment of the part on the *Posterior Analytics* from the same book existed in Latin.[14] Small fragments of some of Al-Farabi's logic works have also been discovered in Latin.[15]

In the early thirteenth century, some of Averroes' commentaries on the *Organon* were translated in to Latin. William of Luna translated the middle commentary on Porphyry's *Isagoge* and the middle commentaries on Aristotle's *Categories*, *De interpretatione*, *Prior Analytics*, and *Posterior Analytics* between the 1220's and 30's[16], but Latin writers seem to have had knowledge of other logic works by Averroes than these.

Averroes' commentaries were important for a general understanding of Aristotle's very difficult texts. They are generally, as mentioned already, helpful in expounding the text and clearing up mistakes otherwise easily made, but the middle commentaries do not really go beyond Aristotle's own text. If you read them carefully, however, an interpretation is usually indicated, but what influence, if any, they had in the subsequent thirteenth century commentary tradition has not been carefully studied. It seems clear that the strong connection between logic and metaphysics emphasised in these commentaries had some influence, though. In what follows, I will outline the view on the matter and form of a syllogism

[9] See the introduction to Al-Ghazālī, *Tractatus de logica*, 229.
[10] See [Alonso, 1958].
[11] See Giles of Rome, *Errores philosophorum*.
[12] See [Johnston, 1997].
[13] See Avicenna, *Avicenne perypatetici philosophi*.
[14] See D. Gundissalinus, *De divisione philosophiae*, 124-33.
[15] See [Salman, 1948]. In [Salman, 1939] it is argued that there was knowledge of a commentary of the *Posterior Analytics* by Al-Farabi in the thirteenth century.
[16] See [Wolfson, 1963] and [Hissette, 1997]

present in Al-Ghazālī's *Maqāsid*,[17] and some interesting views on *differentia* and modal propositions found in Averroes that seems to have had some influence.[18]

3.1 The Logic of Al-Ghazālī

Al-Ghazālī begins the *Maqāsid* with a distinction between imagination (*imaginatio*) and belief (*credulitas*). An imagination is an apprehension of a thing, he tells us, which is signified by a single mental utterance (*dictio*) in the intellect. Names like 'stone' or 'tree' etc. signify these apprehensions or imaginations. A belief is on the other hand a sentence or a complex utterance, that is, that which says or expresses something, like 'the world begins'. A belief is always a composition of at least two imaginations. The kind of knowledge, which is mediated by imagination, is arrived at through definitions and descriptions, and the kind of knowledge, which is mediated by belief, is arrived at through arguments. It is logic that give the rules by which we are able to give good and true definitions and arguments. Hence logic is the very foundation of knowledge and all science, argues Al-Ghazālī.[19]

The division of logic is done to show the aim or intention of logic.[20] The aim is to define and to prove. He gives the following fourfold division:

(3.1.1) On terms and how they signify understandings (*intellectiones*).
(3.1.2) On concepts (terms) and their divisions.
(3.1.3) On propositions and their composition.
(3.1.4) On proofs, which are subdivided into material and formal.[21]

The major thing to note about this division is the close connection between mind and logic. The view of logic as being about intentions is derived from Avicenna and it was very influential. It gave rise to a major controversy between those that argued that logic was a science of discourse (*scientia sermocinalis*) and those that argued that is was a science of reason (*scientia rationalis*). Albert the Great argues following Avicenna, for example, that logic is a science of mental intentions.

The first part of logic is, according to the division above, about words and concepts. He divides his discussion about these terms into five subparts, namely:

(3:1.5) On terms signification of understandings (concepts).
(3:1.6) On terms division into simple and complex.
(3:1.7) On term division into singular and universal.
(3:1.8) On term division into verbs, names and particles.
(3:1.9) On the intention of terms.

Words signify understandings (or concepts; '*intellectus*' in Latin) in three different ways, argues Al-Ghazālī. First of all, a word can signify a concept directly as for

[17] Parts of this has been published in Lagerlund [forthcoming].
[18] I have written about the modal part previously in Lagerlund [2000].
[19] See Al-Ghazālī, *Tractatus de logica*, 239, 6-13.
[20] Al-Ghazālī, *Tractatus de logica*, 242, 100-102.
[21] Al-Ghazālī, *Tractatus de logica*, 243, 115-120.

example 'house' signifies houses. He calls this kind of signification '*secundum parilitatem*'. Secondly, a word can signify more indirectly as a consequence like for example 'house' also signifies the walls of a house. He calls it '*secundum consequentiam*'. Thirdly, an even more indirect signification as in the case of 'roof' which signify the walls holding it up. This he calls '*secundum comitantiam*'.

Al-Ghazālī mentions that the first two are the most useful and the third much less so. If one were to group these three into distinct kinds of signification I think one would have to say that the first one is primary while the other two are secondary or derivative kinds of signification. The word 'house' is what later Latin logicians (for example William Ockham) would call a connotative term, which then has both a primary signification and a secondary, that is, it signifies both the thing and the persons living in the house, since this is what makes the house into a house.

According to Al-Ghazālī, 'house' has both kinds of signification and probably the word 'house' would not mean what it means for us if it did not have both these significations. He does not use other examples, but it seems natural that he would think that 'human' only has the first kind of signification. Given this reasoning there seems not to be much of a distinction to be made between the second and the third kinds of signification.

In Latin medieval logic, the signification of a word is taken to be that which it makes us think of. Such a characteristic of signification seems particularly appropriate for Al-Ghazālī's use of the term. Many logicians also, as Al-Ghazālī, held that words or terms signify thoughts primarily, but I have not been able to find any evidence that they got this doctrine from Al-Ghazālī. It seems more likely that the source for this doctrine is instead Aristotle. This is clear if one has a look at Lambert of Auxerre's *Logica*, which was written in the 1250's and became an authoritative textbook on logic in the Western tradition.

He begins his very short discussion of signification, which in the thirteenth century textbooks was only a small, but important, step towards explaining supposition theory, with the explanation that "signification of a term is the understanding of a thing".[22] He then continues to say that a sound is imposed by the will on this understanding of the thing. All this is explained by a reference to the famous passage in the beginning of Aristotle's *De interpretatione*. Lambert continues to note that according to this view, which he adheres to, sounds (words) signify the passions of the soul (or concepts) which are in the understanding, and it is then the understanding (concept) that signify the thing. The doctrine is then clear, namely that words primarily signify concepts or something in the intellect. This seems also to be Al-Ghazālī's view of signification.

The difference between simple and complex words or terms is also explained in terms of what the terms signify in the intellect. A term is simple if its parts do not signify anything in the intellect. To explain what he means he uses the term 'homo' ('human being'). If one breaks it apart into 'ho' and 'mo' then those parts do not signify any concept or understanding in the intellect, and hence they are not simple term, but 'homo' of course does and is hence a simple term. A complex

[22]In Latin: "significatio termini est intellectus rei", Lambert of Auxerre, *Logica*, 205.

term is hence a term that signifies two or more concepts or understandings in the intellect. This goes together with his view that signification is additive, that is, the signification of the complex term is a sum of the signification of its simple parts.

The third division of terms (*dictionis*) is into singular and universal terms. A term is singular if its signification prohibits it to stand for more than one particular thing or limits it to only one. His examples are 'this Peter', 'this horse' or 'this tree'. A term is on the other hand universal if its signification does not prohibit it to stand for more than one thing, like 'horse', 'tree' or 'human being'. He seems not to distinguish between proper names and singular terms, and given his example of 'this Peter' he seems not to think that proper names like 'Peter' are what is nowadays often called rigid designators. 'Peter' is just a universal term like 'horse', which can apply to many.

The fourth division (3.1.8) is between verbs, names and particles. He notes first that particles do not appear by themselves in the intellect only in connection with other terms. A verb is different from a name in that it signifies an intention and a time at which it is done. His example is 'He struck', which signifies a striking of something by someone and a past time when this was done. A name does not signify a time. The last division is between different intentions or meanings of terms. He lists five, namely univocal terms, divers terms, that is, many terms for the same thing, multivocal terms, that is, many terms for many things, equivocal terms, and something he calls convenient (*conveniantia*) terms. The last is, he says, something between univocal and equivocal and an example is 'being', which is said of both substances and accidents.

The title of the part of Al-Ghazālī's logic that deals with universal terms is "on the intention of universal terms". In the sentence: 'This human being is white and an animal', the predication of 'white' and 'animal' of 'human being' is quite different. Al-Ghazālī calls the first accidental and the second essential. To explain this difference he first explains what essential and accidental mean in these cases. There are three conditions for essentiality:

> (3.1.10) A is essential to B, if B cannot be understood or thought without A;
> (3.1.11) A is essential to B, if A is necessarily prior to B;
> (3.1.12) A is essential to B, if B cannot be posited to be without A.

It seems that all of these, (3.1.10)-(3.1.12), must be fulfilled for something to be essentially something else. The first condition says simply that if we cannot think about one thing without also necessarily thinking about something else, like in the case of human being and animal. Thinking about a human or a horse brings also the concept of an animal into the mind. This means that these concepts or terms are essentially tied. The second condition is a little bit less straightforward, but he seems to mean, as Aristotle indeed thought, that what is essentially tied to each other is also ordered in a special way, namely in the sense that animal is prior to human being and horse. In the third condition, the idea seems to be that

if something A is essential to something else B, then this means that B cannot be posited to exist without also A being posited to exist. If a human is posited to exist, then an animal is also posited to exist.

Accidents are separated in the standard way into two different kinds, namely separable accidents and inseparable accidents. The examples he gives of separable and inseparable accidents are youth of a human being and blackness of an Ethiopian (a very common example found in Porphyry's *Isagoge*). In one case the accident is separable from the substance, but in the other it is not.

Genus, species and difference (*differentia*) are crucial for definitions, Al-Ghazālī explains, and it is only these three that can be said to fulfill the three criterion above of essential relations or essential predication. A property (*proprium*) is not essential, according to Al-Ghazālī, since 'A human being is able to laugh' is not an essential predication according to (3.1.10)-(3.1.12). It is hence an accidental predication.

The longest part of the *Maqāsid* is on argumentation and it is divided into a discussion of formal and material argumentation (or syllogisms). This distinction is not entirely clear, although directly taken from Avicenna, and I will discuss it in some detail below. I will first shortly deal with formal arguments, which is much more straight-forward.

The part dealing with formal arguments is divided into syllogisms, induction, and arguments by example. The part dealing with the syllogisms is divided into categorical syllogisms and hypothetical syllogisms. Echoing Aristotle in the *Prior Analytics*, he defines a syllogism in the following way:

> A syllogism is a discourse in which statements are posited and granted, and from which a statement other than those previously granted necessarily follows.[23]

This definition applies to both categorical and hypothetical syllogisms, and indeed to all arguments.

The categorical syllogisms are at least initially given a standard presentation. He mentions that it contains two propositions as premises and that the middle term, which connects the two premises, are crucial for the production of a conclusion. The middle term connects the major and minor terms, which are respectively the predicate and subject of the conclusion. It is the order of these terms that give the different syllogistic figures. Al-Ghazālī mentions three figures:

> (Figure 1): The medium term is the predicate of one premise and subject of the other.
> (Figure 2): The medium term is the predicate of both premises.
> (Figure 3): The medium term is the subject of both premises.

[23] Al-Ghazālī, *Tractatus de logica*, 259, 18-9. Compare Aristotle's definition in the *Prior Analytics*: "A deduction (syllogism) is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so." (24b19-20) The translation is by Robin Smith.

He notes further that in none of these figures does a conclusion follow from only negative premises, from only particular premises, and from a combination of the major negative and the minor particular.[24] The first figure is different from the other two in primarily two ways: first of all, the syllogisms in it are not reduced to any other syllogisms, and the conclusions in it are of four different kinds, namely universal affirmative, universal negative, particular affirmative, and particular negative. In the second figure there are no affirmative conclusions and in the third figure there are no universal conclusions.[25]

As Aristotle himself had maintained the first figure is perfect. He, however, says very little about why it is perfect. Al-Ghazālī want to explain this by showing why the first figure syllogisms are "certain", as he says. He says that "when you posit a true affirmative proposition, then whatever is said about all of its predicate the same thing is said about its subject."[26] In a true proposition like 'A human being is an animal' one must think that the subject includes all of the predicate. He thus assumes some kind of inclusive relation between the subject and the predicate and this accounts for the certainty of the first figure syllogisms.[27] This becomes more clear when one has a look at how he presents the four valid syllogisms. He namely presents them with the minor premise first, since he wants to highlight the inclusion relation between the terms. The examples he gives of the four syllogisms are:

> (3.1.13) 'Every body is a composite, but every composite began, therefore every body began.'
> (3.1.14) 'Every body is a composite, but no composite is eternal, therefore no body is eternal.'
> (3.1.15) 'Some thing is a composite, but every composite began, therefore some thing began.'
> (3.1.16) 'Some thing is a composite, but no composite is eternal, therefore some thing is not eternal.'[28]

The first thing to notice is of course that each syllogism is lined up as a conditional with the minor premise first. Putting the premises in this order lines up the terms nicely as minor, middle and major. In the first figure, 'body' includes 'composite' and 'composite' in turn includes 'began', which makes it obvious that 'body' includes 'began' as well. The quantification of the propositions is just expressing how many of the subjects have the predicates. In (3.1.13), all have.

In the second figure, Al-Ghazālī presents the following valid syllogisms:

> (3.1.17) 'Every body is divisible, but no animal is divisible, therefore no body is an animal.'

[24] Al-Ghazālī, *Tractatus de logica*, 260, 61-3.
[25] Al-Ghazālī, *Tractatus de logica*, 260, 64-70.
[26] Al-Ghazālī, *Tractatus de logica*, 260, 76-8.
[27] This was called taking the terms universally in Latin logic. See the discussion about this below.
[28] Al-Ghazālī, *Tractatus de logica*, 261, 88-110.

> (3.1.18) 'No eternal thing is a composite, but every body is a composite, therefore no eternal thing is a body.'
> (3.1.19) 'Something is that which is divisible, but no soul is divisible, therefore something is that which is not a soul.'
> (3.1.20) 'Some thing is not a composite, but every body is a composite, therefore some thing is not a body.'

(3.1.17) is reduced to (3.1.14) by converting the major premise. He proves (3.1.18) by converting the minor premise and then the conclusion of the resulting syllogism to get (3.1.14).[29] In (3.1.19), the major premise is converted to get (3.1.16). The last syllogism in this figure cannot be proved by conversion to a first figure syllogism, and must hence be proved in some other way. Al-Ghazālī gives two different proofs of this syllogism. The second proof he gives is the traditional or Aristotelian one. It is an indirect proof from the contradictory of the conclusion, namely 'every thing is a body', which together with the major premise and (3.1.13) gives 'every thing is a composite'. This, however, contradicts the minor premise, which we have assumed to be true.[30]

The first proof he gives is less straight-forward and not at all traditional. In translation, this is what he says:

> It is posited that: 'Some thing is not a composite'. This 'some thing' is in itself some whole. We therefore posit that this is some whole and give it some name. It is therefore constructed as in the second mood of this figure.[31]

This is rather cryptic, one must admit, but a better explication of what he means can be found if one looks at Avicenna's *Danesh-name*. From what he says one can conclude that the proof goes something like this:

> Take the thing under the subject term in the premise 'Some thing is not a composite' that is not a composite and call it T. Knowing that T is not a composite, then 'No T is a composite' is true. We now also know from the major premise in (3.1.20) that 'Every body is a composite', and from these two it follows by (3.1.18) that 'No T is a body'. Given that T is a thing that is not a composite 'Some thing is T' is obviously true. Now, from 'No T is a body' and 'Some thing is T' it follows by (3.1.16) that 'Some thing is not a body'. Q.E.D.

It is a valid proof, if my reconstruction is correct, but it is not clear from what Al-Ghazālī actually says that this is the way he thinks the proof is supposed to go.

In the third figure, he states the following six valid syllogisms:

[29] Al-Ghazālī, *Tractatus de logica*, 263-4, 165-221.
[30] Al-Ghazālī, *Tractatus de logica*, 264, 213-221.
[31] I have included the Latin so you can read it for yourselves. "Positio est cum dicis: 'Aliqua res est non composita'. Haec autem 'aliqua res' in se quoddam totum est. Ponamus ergo illud esse quoddam totum, et appelletur quomodolibet. Fiet ergo quasi secundus modus huius figurae." (Al-Ghazālī, *Tractatus de logica*, 209-12.)

> (3.1.21) 'Every human being is an animal, and every human being is rational, therefore some animal is rational.'
> (3.1.22) 'Every human being is an animal, and no human being is a horse, therefore some animal is not a horse.'
> (3.1.23) 'Some human being is white, and ever human being is an animal, therefore some white thing is a human being.'
> (3.1.24) 'Every human being is an animal, and some human being is a writer, therefore some animal is a writer.'
> (3.1.25) 'Every human being is rational, and some human being is not a writer, therefore some rational thing is not a writer.'
> (3.1.26) 'Some animal is white, no animal is snow, therefore some white thing is not snow.'

(3.1.21) is proved by conversion of the minor premise to get (3.1.15). The next is proved in the same way by this time to get (3.1.16), and also (3.1.23) to get (3.1.15). (3.1.24) is proved by conversion of the major premise and conversion of the conclusion. He does not mention that he must transpose the premises as well, but perhaps he takes this to be obvious given his conditional reading of the syllogisms. The last syllogism in this figure is proved by converting the minor premise to get (3.1.16).

(3.1.25) is claimed to be proven by supposition again, but no proof is this time given. A proof in the line of the one given above might look like this:

> Take a human being that is not a writer in 'Some human being is not a writer' and call it H. Hence 'No H is a writer' is true. We have posited that 'Every human being is rational' and since H is a human being, it follows that 'Some H is a human being', which together by (3.1.15) becomes 'Some H is rational'. Given this and 'No H is a writer' it follows by (3.1.26) that 'Some rational thing is not a writer'. Q.E.D.

This proof method is very powerful and in fact Al-Ghazālī would need no other to prove the valid syllogisms. It is somewhat reminiscent of Aristotle's proof by echtesis.

Al-Ghazālī ends his discussion of the categorical syllogisms at this point and it is quite strange that he does not mention the other ten valid syllogisms of the Aristotelian system. Avicenna's discussion in the *Danesh-name* is equally incomplete so Al-Ghazālī seems here only to follow his lead.

He then eventually turns the discussion towards the doctrine of the matter of the syllogism. The matter of a syllogism is its propositions or premises. If they are credible and true, then the conclusion will be credible and true, but if the premises are false, then the conclusion will not be credible.[32] He tries to explain what he means by the distinction between the matter and form of a syllogism by an analogy with a coin. The matter of the coin is the gold it is made of and its form is its roundness. If the form is destroyed or falsified in some way, we will

[32] Al-Ghazālī, *Tractatus de logica*, 273, 495-8.

not call it a coin anymore. Sometimes the matter of the coin is also changed, that is, a coin, he notes, might be made from iron or silver. It is then not worth as much, but it is still a coin. In the same way the form and matter of a syllogism can change. If the form is change so that it is not in accordance with any of the figures, then it is no longer a syllogism and presumably not an argument any more, but sometimes the form is the correct one, but the matter is different and the premises are only opinions or even sometimes false.[33]

Al-Ghazālī seems to think that syllogistics provide the form of all argumentations. In a sense one can thus say that the three syllogistic figures give the definition of what an argumentative form is. Given this his analogy with the coin outlined above seems not very well chosen since the roundness of a coin is quite arbitrary, that is, a coin can look in all sorts of ways and still be a coin. The form of a syllogism is, however, necessarily given by the first, second or third figures. The form is, however, even more restricted than this, since all syllogisms in the second and third figure are reducible to the first figure. The form of all arguments is thus given by the first figure and the valid syllogisms in that figure. This is a very restricted view of the form of an argument, that is, to define the form as the first figure and the four moods in that figure. But as we saw above he used the same definition of a syllogism as Aristotle in the *Prior Analytics*. This is on the other hand a very general definition and if it is something like this he has in mind for the form of an argument then he has a rather lose sense of what form is.[34]

He continues his analogy with the gold coin mentioned above to outline five levels of propositions. He says that in the same sense as there are five kinds of purity of gold there are five levels of propositions. These levels are distinguished by the truth or certainty of the kinds of propositions in them. On the first level the propositions are true and believable (assented to) without doubt and deception. These are the propositions used in demonstration. On the second level are the propositions in proximity of truth and of which it can be difficult to judge whether they are false or not, or as he says to spot the fallacy in them, but if scrutinized they might turn out to be false or fallacious. These are the propositions used in dialectics. On the third level we find propositions that can be labeled opinions but which are nevertheless convincing. It is easier to see their possible fallacies though. These are the propositions used in rhetoric and law. On the fourth level the propositions are such that they look like they are true, that is, they imitate truth by being similar to true propositions, but they are really further from the truth than any of the propositions on the levels above, that is, they are neither true, probable or a genuine opinion about something. These are the propositions used in sophistical arguments. At the fifth level we find propositions which we know are false, but which nonetheless move the soul in some way to form opinions. These are propositions used in poetical arguments.[35]

[33] Al-Ghazālī, *Tractatus de logica*, 237, 499-505.

[34] I have argued in Lagerlund [forthcoming] that it must be the stricter he is actually committed to.

[35] Al-Ghazālī, *Tractatus de logica*, 273, 507-29.

These five different levels of propositions are then further subdivided into all in all thirteen different kinds of propositions. The division is the following:

(3.1.27) Demonstrative propositions;
(i) Per se necessary propositions (*primae*);
(ii) Propositions about sense perceptions (*sensibiles*);
(iii) Experimental propositions (*experimentales*);
(iv) Reputable propositions (*famosae*);
(v) Intuitive propositions.

(3.1.28) Topical propositions;
(i) Maximal propositions (*maximae*);
(ii) Granted propositions (*concessae*).

(3.1.29) Rhetorical and legal propositions;
(i) Propositions reporting received opinions (*receptibiles*);
(ii) Apparent maximal propositions (*maximae in apparentia*);
(iii) Believable propositions (*putabiles*).

(3.1.30) Sophistical propositions;
(i) Believable propositions;
(ii) Deceptive propositions (*simulatoriae*).

(3.1.31) Poetical propositions;
(i) Imaginative or transformative propositions (*imaginative sive transformativae*).

The thirteenth kind of proposition which does not fit into the classification above, but which he nonetheless outlines is called estimative propositions. He explains that these are false propositions which somehow have been fixed in our souls and which we cannot doubt. Some of the examples he gives are: 'That which has no parts cannot be anything', 'Nothing is that which is not in the world and not outside it', or 'The totality of the world terminates in a void or in a plenary'.

The first five under (3.1.27) are all true propositions and as such they are all the subject of demonstrative syllogisms or demonstrative science. The first kinds are per se necessary and naturally believed by the intellect. An example he gives is: 'Two are more than one'. These are truths of reason, that is, truths the intellect realizes without any training and education. The second kinds are simple truths based on perception, like 'the sun is shining'. Next are the experimental propositions. While the first was purely based on the intellect and the second only on the senses, these propositions are based on both. An example would be 'Fire ignites'. Reputable (*famosae*) propositions are such propositions which nobody doubt, but where the foundation for their truth is now known. His example is: 'Egypt exists', although no one has actually been there to look. The last kinds of propositions usable in demonstration according to this classification at least

are what I have called intuitive propositions. Al-Ghazālī calls them propositions that include their own proofs. The arguments in them need to be brought out by finding the middle term. He gives the example: 'Two is the middle of four'. The argument for this can be drawn out of the proposition itself to get the following argument:

> 'A middle is a part of a whole equal to the other part, but two is one of two equal parts of four, therefore two is the middle of four'.[36]

The idea seems to be that given propositions like the one discussed here one can just by looking at them see that one can easily formulate an argument for them – one simply has to add a middle term. Interestingly, the argument he himself ends up giving is not a syllogism, but one could easily make a syllogistic argument with the conclusion 'two is the middle of four'.

The doctrine pretty much explicit in the fist book of Aristotle's *Topics* is that the only difference between demonstration and dialectics is the nature of the premises involved. The Arabic or Avicennan doctrine of the matter of the syllogisms seems to take this view to heart. The two kinds of dialectical or topical premises Al-Ghazālī outlines are above called maximal and granted. The maximal propositions are as close to truths or as close to propositions used in demonstrations as one could get. It is quite natural for our minds to accept them, he argues. Among his examples are: 'Lying is shameful', 'Innocent [people] should not be punished', 'Justice is necessary', etc.[37] He also mentions that since these kinds of propositions are not absolute truths their contradictories will not be false, but they will be considered absurd by everybody, he thinks. Granted propositions are not firmly believed or held to be true as maximal propositions. Some accept them and some do not. Otherwise they do not differ much from maximal propositions.[38]

When these two kinds of propositions are used in syllogistic arguments, the argument is called dialectical. He foremost outlines four conceptions of the usefulness of such arguments. First of all dialectics is useful since it will help us to know things we presume to know but does not really know. Secondly, it is useful if we want to learn some true proposition, although we do not want to go into the intricate details of a demonstrative argument and we are not content with a rhetorical argument. The topical argument for the proposition then constitutes a good middle alternative. Thirdly it is useful as an introduction to an art or science, like medicine or geometry. By starting from the topical propositions we can get to know the demonstrative propositions of any given science. Finally, he notes that it is the nature of dialectical arguments that they in the same question can conclude two contradictory extremes. But this is very useful since by making such arguments and then locating the errors made one can often establish what is the truth in the particular question discussed.[39]

[36] Al-Ghazālī, *Tractatus de logica*, 274, 577-81.
[37] Al-Ghazālī, *Tractatus de logica*, 276, 617-21.
[38] Al-Ghazālī, *Tractatus de logica*, 277, 641-55.
[39] Al-Ghazālī, *Tractatus de logica*, 278-9, 688-707.

There are three kinds of propositions that determine whether an argument is rhetorical, namely propositions reporting received opinions, apparent maximal propositions and believable propositions. The first kinds of propositions are propositions held to be true because some authority, written or oral, has uttered it. Many people therefore hold them true. An apparent maximal proposition is a proposition that when we hear it sounds very credible, but which on closer inspection is false. Believable propositions are propositions one can believe, although it is often the opposite that one actually believes, like for example the proposition 'Whoever moves around at night, is up to no good' whose opposite is what is more often believed, namely 'A good person does not move abound at night'.[40] The arguments done in the rhetorical and legal arts are not meant to certify anything only to persuade. Its main usefulness is to turn the soul to pursuing truth and avoiding falsity, but also it is very useful in law.

Sophistical propositions are first of all believable propositions, but most importantly deceptive propositions. The deceptive propositions are propositions that simulate first (or *per se*), experimental, or maximal proposition, and seem to be such propositions but are not. He has a longer discussion about sophistical syllogisms and gives a list of ten things to look out for when compiling a syllogistic argument. This list is taken almost word for word from Avicenna's *Danesh-name*.[41]

The last category of propositions is the imaginative propositions. They are used in poetical argumentations. These propositions are known to be false, or we know that they are false as soon as we hear them. His example is: 'The honey looks like dung'.[42]

It is obvious that the fivefold division of the matter of a syllogism is also a division of five kinds of logical arts. In this respect there is a demonstrative logic, a topical logic, a rhetorical logic, a sophistical logic and a poetical logic. These kinds of divisions into different kinds of logic which are formally identical but distinguished by their material diversity was characteristic of the Arabic logical tradition and particularly of Avicenna. Early in the part of the *Shifā* dealing with logic he writes that: "all of these [logical arts] are like participants, either actually or potentially, in the structure and the form of the syllogism. And the majority of their difference are in their matters."[43] The doctrine of the matter of the syllogisms had some influence on Latin medieval logic, for example, Aquinas takes it over and it can also be seen in the logic textbooks discussed below.[44]

3.2 Averroes' Commentaries on the Organon

The middle commentaries of Averroes were never really written to fully explain or go beyond the thought of Porphyry or Aristotle. Averroes therefore sticks very

[40] Al-Ghazālī, *Tractatus de logica*, 277, 646-67.
[41] Al-Ghazālī, *Tractatus de logica*, 280-1, 743-807. See also Avicenna, *Avicenna's Treatise on Logic: Part one of Danesh-Name Alai*, 46-7.
[42] Al-Ghazālī, *Tractatus de logica*, 278, 670-3.
[43] Quoted from [Black, 1990, 84].
[44] See Lagerlund [forthcoming] for a discussion of its influence on Western logic.

close to the original text and only on occasion deviate in order to explain further or to rule out misunderstandings. He also sometimes says that some argument or view is wrong, but he does not bother to explain why or give the counter argument. As an interpreter one therefore must really drag his view out from the text since the text itself gives us very little help.

Averroes' commentary on the part of *Isagoge* contains an interesting discussion of *differentiae*. It starts by dividing up *differentia* into different senses. He begins by saying that there are three senses of *differentiae*, namely a general, a proper and a most proper. A general *differentia* is when one thing differs from another as Socrates is another substance than Plato and when the same thing differs from itself by a state or a disposition as, for example, the young Socrates differ from the old. A proper *differentia* is when one thing differs from another by an inseparable accident. The most proper differentia is a differentia that produces a difference in species. Averroes calls the latter a species-producing *differentia* and the two former *differentiae* without any qualification. According to standard terminology what Averroes's calls species-producing *differentia* is specific *differentia*.

As he explains, species-producing *differentiae* produce another being while the other only produce diversity and 'it is through those that produce another being that genera are divided into species and that definitions are completed. The other kind of *differentiae* produces diversity only in respect to the states of things, not in respect to their essence.'

There is also another division among *differentiae*, namely they are either separable or inseparable and the inseparable are divided into substantial (*per se*) and accidental (*per accidens*). This division is supposed to map onto the division above so that we get:

> General *differentia* = Separable *differentia*.
> Proper *differentia* = Inseparable accidental *differentia*.
> Most proper *differentia* = Inseparable substantial *differentia*.

This means of course that the inseparable but accidental *differentiae*, like snubness of nose, are not species-producing (specific) *differentiae* and hence not employed in the definition of the essence of a thing. He explains:

> The inseparable, in turn, are of two sorts, substantial (*per se*) and accidental (*per accidens*). An example of the accidental sort is snubness of nose, and examples of the substantial sort are rationality and capacity for science in man. It is the substantial *differentiae* that are employed in the definition of the essence of the thing to which they belong, and, as we have said, it is they that produce another being by the side of the genus to which they are joined. *Differentiae* that are accidental are not employed in the definition of the essence of the thing of which they are predicated, and do not produce another species, but only diversity.[45]

[45] Averroes, *Middle Commentary on Porphyry's Isagoge*, 14.

There is also a further division of substantial *differentiae*, namely into those that divide the genus and those that are constitutive of it. A *differentia* like rational is both divisive and constitutive, since it divides the genus animal, but is constitutive of the species human being. Hence some *differentiae* can be constitutive and not divisive, which means that they are part of the essence but does not divide up the genera. He writes:

> Some substantial *differentiae* constitute intermediate genera, and some divide them. Animal may serve as an example. As an intermediate genus, it has *differentiae* that constitute it, that is, complete its definition and produce it, as, for example, food consumption and sensation; for food consumption and sensation complete the essence of animal, animal being a food-consuming, sensitive body. But animal has other *differentiae* that divide it, such as rational and irrational, since animal is either rational or irrational. The *differentiae* that divide the intermediate genus also constitute the species that are beneath that genus.[46]

All this is, of course, completely standard. Having made these divisions he goes on to discuss some definitions of substantial *differentia* that Porhyry gives. The first definition he examines states that a *differentia* is that whereby species exceeds genus. With this he seems to mean that the *differentia* is that which is present in the species in addition to that which is present in the genus. The *differentia* rational is not actually present in the genus, since it then would also have to contain its opposite irrational, and hence two opposed things would be present in the same thing. Rather the *differentia* is only potentially present in the genus and actually present in the species. In Averroes explanation it goes like this:

> [Substantial] *differentia* has been defined in several ways. According to one definition, *differentia* is that whereby species exceeds genus. For by virtue of its *differentia*, rationality, [the species] man exceeds animal, its genus, since this *differentia* is not actually present in animal. Were it actually present, its opposite would also have to be present, that is, the absence of rationality, which completes the division of animal as when we say that animal is either rational or irrational. Only in the species below animal are these *differentiae* present. This argument contains a fallacy.[47]

Porphyry himself explicitly says that the *differentia* is potentially present in the genus, but Averroes does not because he thinks that the argument contains a fallacy. He does not say which fallacy, but it seems clear that the problem is that *differentiae* cannot even be potentially present in the genus. This is brought out by Gersonides in his commentary on Averroes' commentary on *Isagoge*. Gersonides explains that what is potential is possible and by positing a possibility nothing

[46] Averroes, *Middle Commentary on Porphyry's Isagoge*, 14-5.
[47] Averroes, *Middle Commentary on Porphyry's Isagoge*, 15.

impossible should follow, as Aristotle says in the *Prior Analytics* (32a18-20). If one were to assume that every animal is rational then something impossible would follow, since horse and ass would be rational. A consequence of this is hence that an essential *differentia* is never present in the genus, but it is instead what is added to the genus from outside. How this is supposed to be understood becomes clearer in Averroes' discussion of the next definition proposed by Porphyry. Let me quote this passage in whole before I expand on it. He writes:

> [Substantial] *differentia* has also been defined as that which is predicated of a number of things differing in respect to species, by way of stating which thing it is, not what it is. For when asked what man is, we first reply by giving his genus, saying that he is animal, and then when asked which animal he is, we reply that he is rational. The definition of man is completed only through the whole of this. The reason is that everything is composed either of matter and form, or things analogous to matter and form. Genus is analogous to matter, and differentia is analogous to form, and consequently the unity formed by genus and *differentia* is the substance of the thing insofar as it is in the mind, just as its matter and form are its substance insofar as it exists outside the mind.[48]

Differentiae are predicated of a number of things by way of stating which thing it is and not what it is. So, the *differentia* does not tell us what the thing it is predicated of is; it instead gives us the answer to the question: 'What's it like?', and the answer is: 'It is rational!' It thus gives you the essence of the thing. The question: 'What is it?' refers back to the genus, and the answer to that question is: 'It is an animal!' It is only when we combine the answers to these two question that we get the definition of the thing, that is, rational animal. The reason this is the case, he explains, is because everything is composed of matter and form. In the definition of a species both matter and form are needed. The genus brings the matter and the differentia brings the form and together you get the species. This is the Aristotelian arithmetic of natural things, that is, 'Genus + Differentia = Species' or Matter + Form = Substance'.

As can be noted at the end of the quote above, Averroes implies a distinction between the concepts in the mind and the things in the world they signify. In the mind, we form the definition 'Human Being is a Rational Animal' and in the extra mental world we have particular humans, which are composites of matter and form. The complex term 'rational animal' signifies together the composite, but separately each part of the composite. The *differentia* hence signifies the rational soul, which is the substantial form of the human being. The essence of a human is, of course, its rational soul.

Averroes does not say much about *differentia* in his commentary on Aristotle's *Categories*, which, of course, is because Aristotle himself does not say a whole lot

[48] Averroes, *Middle Commentary on Porphyry's Isagoge*, 15-6.

about *differentia* in the *Categories*. In Part II, Chapter 9, he, however, notes the following:

> We must not let anyone confound us by saying that rationality and *differentiae* in general are present in a subject just as accidents are present in a subject, their subjects being the things whose *differentiae* they are; the analogy would be between the presence of rationality in man and the presence of whiteness in body. For rationality is present in a subject, that is, man, as part of it, while the relation of whiteness to body is different. For this reason our description of accidents as what are predicated in a subject should not be understood as meaning that they are in a subject as a part of it, but rather that the subject can exist without the accident.[49]

Here he specifically takes up problem of whether the *differentia* is a substance or a quality and he answers clearly that it is not a quality. *Differentiae* are not in a subject, that is, the species, since they are part of the subject. A quality like whiteness is in a subject and it is also something that the subject can lose without ceasing to exist, but a *differentia* which is an essential part of it as a substance, it cannot exist without.

The substantial form as the *differentia* is constitutive of the species and the essence of it; something it naturally cannot exist without. One could then say that the substantial form on Averroes view is a complement of the species as an essential part of it. The distinction between divisive and constitutive *differentiae* is going to be very important to understand Averroes' view. The substantial form is the thing in human beings that makes it onto what it is and to what the concept '*differentia*' refers to in the definition of the species human being. In this respect it is in the classification what divides the genus animal and also what constitute the species human being. All other constitutive *differentiae* are going to be derivative upon it as essential characteristics or complements of human beings. The view expressed here can be found in Robert Kilwardby's commentaries on the *Isagoge* and the *Categories* (see below). It can also be read into Aquinas' *De ente et essentia*.

Averroes' treatment of modal syllogistics is very interesting and strengthens his general metaphysical interpretation of Aristotle's logic. It seemed also to have had some influence on the thirteenth century.[50] In his minor and very compact *Quaesita* of the *Prior Analytics*, Averroes focuses his discussion mainly on the modal syllogistic. In its interpretation of the modal syllogistic, this minor work differs from the middle commentary, the *Expositio*. While the *Expositio* gives a straight-forward temporal interpretation of the modalities, the *Quaesita* presents a somewhat different view, one which is more ontological, and based on the nature of the terms involved in the different syllogisms.

[49] Averroes, *Middle commentary on Aristotle's Categories*, II, 9, 41-2.

[50] See Lagerlund [2000, Chapter 2], where the relation between Kilwardby and Averroes is explained.

Chapter 3 of *Quaesitum* IV deals with the mixed necessity and assertoric syllogisms. Averroes first states that the terms of necessity propositions in Aristotle's modal syllogistic are themselves necessary and that such a term is one by virtue of itself (*unus per se*). What does this mean? A necessary term always stands for its subject. It is a name of a thing which is continuously actualised in the world, such as eternal individual beings, or natural species of substances and their essential properties. Averroes also states that there are terms which are accidental. An accidental term does not always stand for the same subject, that is, the term sometimes stands for a certain subject and sometimes not.

Given this, Averroes explains what constitutes a necessity proposition and what in general constitutes a assertoric proposition and a assertoric simpliciter proposition, which he thinks is the right type of assertoric premise in a mixed Aristotelian syllogism with necessity and assertoric premises. It is clear that if both terms in a proposition are necessary and an essential predicate is predicated of a (necessary) subject, then the proposition is necessary. According to Averroes, a proposition can be considered necessary if it consists of a necessary subject and a necessary predicate inhering in it '*connexo ipsi subiecto*'. This probably means that even if the property is not essential, but an inseparable accident (*accidens inseparabile*), such as the whiteness of swans, then the proposition is necessary.

If, on the other hand, both terms are accidental, and refer to properties which are not always actual, then the proposition is an assertoric proposition. Averroes is here interested in a special class of assertoric propositions, namely, in those which are assertoric simpliciter. If they are true, they are true independently of temporal restrictions whenever the subject is actual. His example of such an assertoric proposition is 'Every walking being is moving'. He adds that if the subject is walking it is necessarily moving. One could say that such propositions are conditionally necessary.[51]

The third suggestion discussed by Averroes is the combination of necessary and accidental terms. He first discusses the case in which the subject term is accidental and the predicate term is necessary. His example is 'Every walking being is an animal'. This proposition can be understood in two ways: either the subject term 'walking being' is understood to stand for its supposita as walking, or it is taken to stand for them as those which are walking. In the former case, the proposition is assertoric, and in the latter case, it is necessary. Because the latter reading is indirect, it can be said that the proposition is a assertoric proposition *per se* and a necessity proposition *per accidens*. According to Averroes, in a mixed first figure syllogism with necessity and assertoric premises, the assertoric premise should be assertoric *per se* and necessary *per accidens*. One can ask, of course, why the minor premise is called an assertoric premise and the conclusion necessary. Both are assertoric *per se* and necessary *per accidens*. The second case with necessary and accidental terms is that the subject term is necessary and the predicate accidental.

[51] It was standard practice in Arabic logic after Avicenna to elaborate modal propositions with all kinds of restrictions placed on the terms involved. See [Street, 2004] and [Lagerlund, forthcoming b].

Such propositions are assertoric simpliciter, but never necessary *per accidens*.

According to Averroes, the propositions which are necessary *per se* have necessary terms. There are no problems involved in converting such propositions into necessity propositions. Propositions which are accidentally necessary are not convertible into necessity propositions. Averroes' example of a propositions of this kind is: 'Every writer is a human being'.

Averroes' discussion about syllogistic propositions which are necessary *per se* or necessary *per accidens* is much more detailed than what one finds in for example Robert Kilwardby's short remarks about the conversion of the necessity premises (see below). One striking similarity though is that both claim that the subject term of a necessary proposition *per se* must not stand for its supposita accidentally. A proposition is called necessary *per se* only if both terms are necessary, and it is called *per accidens* only if one term is necessary and the other accidental. There may be necessary connections between the terms of a proposition which do not fulfil these conditions. The proposition is then not necessary. Accidental necessity propositions have an accidental term as a subject term and, therefore, there is no essential connection between the terms as such.

Much more should be said about the logical theories in Arabic logic works accessible to Latin logicians, but this will have to do for now. These doctrines were never as influential as Prantl assumed, but neither did they exert no influence what so ever.

4 THE COMMENTARIES ON ARISTOTLE'S *ORGANON*

There is an unfortunate lack of edited texts from the early commentaries on Aristotle's logic.[52] Albert the Great wrote commentaries on all logic works in the *logica antiquorum* and his are the only ones that have been or are being critically edited. Thomas Aquinas' logic works has also of course been edited. He only wrote commentaries on *De interpretatione* and the *Posterior Analytics*. One of the most influential thinkers from this period was, however, Robert Kilwardby. Between 1235 and 1245, he lectured on most of the *logica antiquotum* and although he was influenced by Averroes he stands out as a very original thinker. Unfortunately, none of his commentaries on Aristotle's logical works have been edited. Albert the

[52]There is no chronological catalogue of all the Latin commentaries on the logical works of Aristotle, but there numerous lists of commentaries available. In [Marenbon, 2000, II, 77–127], all commentaries before 1150 has been listed, and, in [Marenbon, 2000, VIII, 21–49], all commentaries on the *Categories* and *De interpretatione* before Abelard are listed. In Green-Pedersen [1984], all commentaries on the *Topics* are listed. In [de Rijk, 1962] and [Ebbesen, 1981b] many of the early commentaries on *Sophistici Elenchi* are listed. In [Lohr, 1967; 1968; 1970; 1971; 1972; 1973] and [1974] an alphabetical list of all medieval commentaries on Aristotle's logical works can be found. A project in Cambridge run by John Marenbon and Tony Street will substantially enhance our knowledge of the medieval commentary tradition. In the near future M. Cameron and J. Marenbon will publish four edited volumes called *Aristotelian Logic, East and West, 500–1500* and with the respective subtitles, *the Prior Analytics*, *the Peri Hermeneias*, *the Topics* and *Methods and Metodolology*.

Great seems to a large extent to have copied many of Kilwardby's ideas.[53]

4.1 Porphyry's Isagoge *and Aristotle's* Categories

Porphyry's *Isagoge* was written as an introduction to the *Categories* and it was treated as such in the Middle Ages. It was usually the first logic text university students met in the thirteenth century. It was also important because it gave medieval authors an opportunity to comment on universals. This is not because there is an account of universals in *Isagoge*, since there is none, but because the three "hard questions" that Porphyry states in the beginning. He writes:

> For example, I shall beg off saying anything about (a) whether genera and species are real or are situated in bare thoughts alone, (b) whether as real they are bodies or incorporeals, and (c) whether they are separated or in sensibles and have their reality in connection with them. Such business is profound, and requires another, greater investigation. Instead I shall now try to show how the ancients, the Peripatetics among them most of all, interpreted genus and species and the other matters before us in a more logical fashion.[54]

The way one answers these questions will determine what view of universals one ends up with, and just by stating these questions Porphyry had a profound influence on medieval philosophy and logic. It was, however, Boethius that first attempted an answer.[55]

To many scholars disappointment Averroes chose to ignore these questions in his middle commentary on *Isagoge*, and one of the first that commented on the work in the thirteenth century was Robert Kilwardby. Among his teachings is a course on the *logica vetus*, which contains lectures on the *Isagoge*, *Categories* and *De interpretatione*.[56]

Kilwardby addresses Porphyry's questions in the introduction to his commentary. He thinks universals have an existence as forms in singulars, and have a unity based on agreement in essence. He also argues that the ideas in the mind of God are formal causes and exemplars of the universals. In human cognition, the universals (the forms) are abstracted from then individuals they exist in. Universals must be in singulars he argues, since the singulars would otherwise not make any contribution to the signification of a universal.

Following Avicenna, Kilwardby contrasts the metaphysicians approach to universals and the logicians. In metaphysics, universals are studied in abstraction from language, while in logic they are studied as having a nature connected with language. Universals are names that can be predicated for the logician, but their existence is prior to language, and the signification is derived from the real form.

[53] See [Ebbesen, 1981b; Lagerlund, 2000, Chapter 2; Thom, 2007].
[54] Spade [1994, 1].
[55] See [Klima, 2004] for a outline of the history of universals in medieval philosophy.
[56] See [Lewry, 1981] for a discussion of the content of this course.

He says that a universal is a disposition of a reality, and hence it is also a disposition of a name. He expands on this relation by expanding on the relation between thought and reality. Language is a representation of thought, he argues.

Albert the Great defines a universals in his commentary on the *Isagoge* as "that which, although it exists in one, is apt by nature to exist in many." He then also gives the famous division into three kinds of universals, namely those pre-existing the thing (*universale ante rem*), those existing in things (*universale in re*), and those existing in mind (*universale post rem*). He seems to model this on Kilwardby's distinctions and in an Avicennan fashion both of them can be said to unite a Platonic and an Aristotelian account of universals. All these three aspects of universals need to be taken into account when one gives a theory of universals, according to Albert and Kilwardby. In a sense, they seem to be saying that Porphyry's questions cannot be answered with a simple 'Yes' or 'No'. It is much more complicated than he lets on.

The close connection between logic and metaphysics visible in Kilwardby's lectures on Porphyry is further emphasised in his lectures on the *Categories*.[57] He naturally excludes a linguistic interpretations of the categories. The logician is concerned with reality, but not in the same way as the metaphysician. The ten categories or the ten kinds of utterances (*voces*) signify the ten classes of reality, so while the metaphysician studies the ten classes of reality as species of being, the logician studies them as subjects and predicates.

His discussion of equivocation follows along in the same vain. It is not primarily an equivocation of terms but of reality. As part of his treatment of equivocation he outlines a threefold division of signification. First the act and form of signification, secondly, the thing signified, and thirdly, the comparison of the sign with what is signified. The first is the perfection of the word and every word has only one perfection or form. In the second and third sense of signification an utterance might have several significations. As a meaningful utterance, a name is one in the first sense with one perfection, but it may have several meanings in the sense of what is signified and in the relation of sign to signified.

Kilwardby's whole treatment of signification and definition in terms of perfections or forms and matter is strikingly metaphysical. It reminds very much of the discussion above about Averroes' view of definition (see below for further discussions about this). Both are part of the thirteenth century trend to enforce the strict connections between logic and metaphysics and natural philosophy more or less explicit in Aristotle's own works. This strong relationship was the main target of criticism for later preferably fourteenth century logicians.[58]

[57] This connection is expressed in his commentary on the *Prior Analytics* as well. See section 4.3 below and [Lagerlund, 2000, Chapter 2].

[58] For a more careful outline of the tradition of the *Categories* in the Middle Ages see the articles in [Biard and Rosier-Catach, 2003].

4.2 The De interpretatione

The *De interpretatione* commentary of Kilwardby in the same series of lectures as mentioned above is also very interesting. Aristotle begins his work by saying that "spoken words are signs of mental affections and written words are signs of spoken words." These lines were very influential and Kilwardby starts his commentary by expanding on them. Thought are likenesses of reality, he claims, and he identifies the meaning (signification) of an utterance with the intelligible species (form) in the intellective soul. It is hence the form in singulars or the essence in reality that correspond to the meaning of words.

In his comments on predication in the *De interpretatione* commentary, he explicitly compares predication, that is, the relation between the subject and the predicate, with the relation between form and matter. He uses the terminology '*forma predicati*' or 'the form of the predicate' to explicate how the predicate inheres in the subject. The proposition 'Socrates in white' is true because whiteness inheres in Socrates, and in the linguistic expression the ontological relation is mirrored. As the form of white informs a substance or matter the predicate informs the subject. The whiteness in Socrates is an accidental form, since he in only contingently white, so the predication expressing this truth is only an accidental predication, but supposedly in regards to essential or substantial forms the corresponding predication is essential or substantial, as in 'A human being is rational', since 'rational' is in this case a reference to the human soul. Kilwardby brings these thoughts into his analysis of Aristotle's modal syllogistics as well (see the discussion below).

One of the most interesting and also one of the most discussed parts of Aristotle's *De interpretatione* is, of course, Chapter 9. Albert the Great's and Thomas Aquinas' treatments of this chapter in their respective commentaries are very similar and obviously influenced by Averroes discussion in his middle commentary on *De interpretatione*.[59] I will here present some of the most interesting parts of Aquinas' discussion.

In *Lectio* 13, Aquinas begins by dividing propositions (*enuntiationes*). The first division is according to unity, that is, into simple and conjunctive propositions; the second division is according to quantity, that is, into affirmative and negative propositions; the third division is into quality, that is, into universal, particular, indefinite and singular propositions. The forth division is according to time and then propositions divide into propositions about the past, present and the future. The fifth and final division is in regards to matter. The matter of a proposition is taken from the relationship of the predicate to the subject. If the predicate is *per se* in the subject, then the proposition is said to be in necessary or natural matter. The examples he gives are 'A human being is an animal' and 'A human being is risible'. The the first case the relation is a genus to a species and in the second it is a property (*proprium*) to a species. If the predicate is repugnant of the subject as in 'A human being is an ass', then the proposition is in impossible or remote

[59] See [Knuuttila, 2006].

matter. If the relation is neither of these, then the proposition is in possible or contingent matter. The way Aquinas outlines the modal propositions is very close to the way Robert Kilwardby did it (see the discussion in the next section).[60]

In necessary matter, universal propositions are determinately true, claims Aquinas, and their negations are determinately false, since they are in impossible matter. This holds for past, present as well as future tensed propositions. He then turns to contingent matter and notes that all universal propositions are false in contingent matter while particular and indefinite propositions are true for past, present and future tensed propositions. The reason for this is reviled by the square of opposition, since in a contingent proposition both the affirmation and negation are true and that is impossible for universal propositions.

The problem for Aquinas is where to locate singular propositions in this division. He writes:

> In singular future propositions, however, there is a difference. In past and present singular propositions, one of the opposites must be determinately true and the other false in any matter whatsoever, but in singulars that are about the future, it is not necessary that one be determinately true and the other false. This holds with respect to contingent matter; with respect to necessary and impossible matter the rule is the same as in propositions about the present and the past. Aristotle has not mentioned contingent matter until now because those things that take place contingently pertain exclusively to singulars, whereas those that per se belong or are repugnant are attributed so singulars according to the notions of their universals. Aristotle is therefore wholly concerned here with this question: whether in singular propositions about the future in contingent matter it is necessary that one of the opposites be determinately true and the other determinately false.[61]

For Aquinas this is the problem Aristotle addresses in Chapter 9. The rest of Aquinas' discussion is about whether singular contingent propositions about the future are determinately true or false, or not. He explains the problem like this:

> If it is necessary that every affirmation or negation about future singulars is true or false, it is necessary that everyone who affirms or denies, determinately says what is true and false. From this it follows that it is necessary that everything be or not be. Therefore, if every affirmation or negation is determinately true, it is necessary that everything determinately be or not be. From this he concludes further that all things are of necessity. That would exclude the three kinds of

[60] See also [Knuuttila, 1981, 208-17; 1993, 129-32].

[61] See Thomas Aquinas, *Expositio peryermeneias*, I.13, n. 6, for the Latin, and see [Oesterle, 1962, 103-4], for the translation. I have slightly modified it, though.

contingent things.[62]

The three kinds of contingent things mentioned in the last sentence is a reference to a doctrine about contingency that became standard in the thirteenth century. According to this doctrine, there are three ways something can be contingent, namely that which happens infrequently (*ut in paucioribus*), that which is indeterminate (*ad utrumlibet*), that is, that which is not inclined to one part more than the other, and that which happens for the most part (*ut in pluribus*).[63] The two extremes drops out of the discussion and the problem of future contingents becomes a problem foremost about the middle, indeterminate, propositions.

It is obvious that future singular contingent propositions are not determinately true or false, since this implies that whatever they are about will determinately be or not be. Such consequences are absurd, Aquinas thinks, and hence these propositions are dissimilar to all other propositions, by not being determinately true or false.

It is now determinately true that 'This cup is white', and if truth and falsity are related in like manner in present and future propositions, then 'This cup will be black' is also determinately true or false. When basis for the truth or falsity of this future proposition is in the causal relations among things in the world. The something is in the present, it exists in a determinate way and can hence be said to be truly this or that. If it is in the future, it does not yet exist in a certain way, since it only exists in its cause. It may exist in its cause in three ways, Aquinas thinks. It might exist in its cause by necessity, and as such it has being determinately in its cause, but in another way it might exist in its cause with an inclination to be caused yet it might still be impeded. It is determined by its cause, but not with complete certainty. In a third way something may exist in its cause only as a potency. In this case, the cause is not yet determined more to one way than to another, and consequently it cannot be said of these that it is going to be, but only that it is or is not going to be. Proper singular future, indeterminate, contingent propositions are hence indeterminately true or false, according to Aquinas.[64]

4.3 The Prior Analytics

The first known commentary on the *Prior Analytics* in the Latin West can be found in an anonymous work, which has not yet been edited and thus not properly studied. The author has been called Anonymous Aurelianensis III by Sten Ebbesen, who has studied parts of the work.[65] He dates it to c. 1160-80. The theory of assertoric syllogism was repeated and summarized in almost all logic works

[62]See Thomas Aquinas, *Expositio peryermeneias*, I.13, n. 8, for the Latin, and see [Oesterle, 1962, 105], for the translation.

[63]See [Lagerlund, 2000, 44-6], for a discussion of this division in Kilwardby.

[64]See also [Normore, 1982, 366-7], for an outline of Aquinas view about God's knowledge of future contingents.

[65]See [Ebbesen, 1981a].

from the early thirteenth century, but there are no major commentaries until the 1240's when Kilwardby writes a commentary called, in Latin, *In libros Priorum Analyticorum expositio*.

Kilwardby has nothing of substance to add to the theory of assertoric syllogism[66] (see my outline below of the textbook presentations of logic for a description of the theory of the assertoric syllogisms), but his interpretation of the modal syllogistic is quite remarkable and highly interesting. It was also very influential in the thirteenth century, for example, Albert the Great, Simon of Faversham and Radulphus Brito all follow him in their interpretations, which means that all the major commentators of the *Prior Analytics* in the thirteenth century followed Kilwardby.

He assumes from the beginning that Aristotle's theory is correct and makes it his project to find the interpretation that shows this. He begins by considering a counter-example to the accidental conversion of necessity propositions, namely:

(4.3.1) 'Every literate being is necessarily a human being'.

According to the conversion rules accepted by Kilwardby and Aristotle (1) should convert to:

(4.3.2) 'Some human being is necessarily literate',

(4.3.1) is obviously true while (4.3.2) is false.

Kilwardby seems to assumes that Aristotle's modal syllogistics is a logic for *de re* propositions.[67] He proceeds, however, and gives two separate solutions to this puzzle. The first is based on a distinction between different readings of (4.3.1). Kilwardby explains, that the subject term of a sentence can stand for the subject of the inherence (the *suppositum*), or for the qualification through which the subject is specified (*qualitas/forma*). If the term 'white' stands for *suppositum*, it refers to a thing that is white, or to 'that which is white', but if it stands for the quality (or form), it refers to the whiteness that inheres in that which is white and not to the thing in which it inheres. Kilwardby says that in (4.3.1), 'literate being' stands

[66] See [Thom, 2007] for a different view.

[67] Kilwardby here falls back on a distinction introduced by Peter Abelard (1079-1142). According to Abelard, modal terms are, properly speaking, adverbs, expressing the way in which the decisive thing said of the subject is actual, for example, 'well' or 'quickly' or 'necessarily'. Adverbs which do not modify an actual inherence, such as 'possibly', are secondary modal terms, due to their position in a proposition. Abelard also noticed that in *De interpretatione* 12-13, Aristotle does not operate with adverbial modes, but with nominal modes, such as 'it is necessary that' or 'it is possible that'. He seems to have assumed that Aristotle did this because the nominal modes involve many more problems than simple adverbial modes. According to Abelard, this is more clearly seen from the fact that propositions including nominal modes, such as 'Necesse est Socratem currere', can be understood either adverbially, 'Socrates runs necessarily', or, as suggested by the grammar, 'That Socrates runs is necessary'. He calls these two alternatives *de re* necessity propositions and *de sensu* (or *de dicto*) necessity propositions, respectively. Abelard seems to be the first to employ this terminology. A *de re* modal proposition expresses the mode in which a predicate belongs to a subject. The mode is, therefore, associated with a thing, while the mode in the *de dicto* or *de sensu* case, as he also calls it, is said of that which is expressed by a non-modal proposition. See further [Knuuttila, 1993, 82-96], and [Lagerlund, 2000, 35-39].

for *suppositum*, and this is why (4.3.1) is true, while in (4.3.2) the term is taken differently and now it stands for the quality or form. According to Kilwardby, the meaning of the original subject term is changed when it no longer stands for the *suppositum* (the literate being), but for some abstract quality of being literate, and it is this change that prevents the conversion. If (4.3.2) is read in the following way then the converted proposition is true:

> (4.3.3) 'Something which is a human being is necessarily that which is literate'.

Kilwardby, however, preferred another solution to the alleged counter-examples to the conversion rules of necessity propositions. The second solution is based on a distinction between propositions that are necessary *per se* and those that are necessary *per accidens*. He writes (I, fol. 7rb):

> When it is said: 'Every literate being is necessarily a human being', this subject is not something which can be said per se of this predicate, but since 'literate being' is not separated from that which belongs to a human being itself, the proposition is conceded to be necessary, but when a proposition is necessary in this way it is necessary *per accidens*. Therefore, when Aristotle says that necessity propositions are convertible, he means that only the propositions that are necessary *per se* are convertible.

The idea is here that since 'human being' is not predicated *per se* of its subject 'literate being', the proposition (4.3.1) is not a *per se* necessity proposition and, therefore, not convertible. (4.3.1) is a necessity proposition, not of the *per se* type, only of the *per accidens* type, since it is necessarily true only in the sense that being a human being and being literate are not separable. Kilwardby implies that the relation between subject and predicate term must be of a special kind if a proposition is to be called necessary *per se*. In (4.3.1), 'literate being' and 'human being' do not have the close *per se* relation Kilwardby demands of a convertible proposition.

Kilwardby thinks that propositions *per se* should be understood in the way Aristotle explains them in *Posterior Analytics* I, 4-6. Aristotle discusses four different notions of *per se* (*kath' hauto*) predication, but Kilwardby seems only to have the two first in mind when referring to *per se*. Aristotle says that the first type of *per se* predication (*per se primo modo*) occurs when the definition of the subject includes the predicate. The second type of *per se* predication (*per se secundo modo*) occurs when the definition of the predicate includes the subject. The best characterization of the first relation is the genus-species relation, that is, that the definition of a species includes its genus. The second relation is often characterized by a *proprium* (property), since a *proprium* is included in the definition of a subject; for example, in 'a human being is able to laugh'. The term 'human being' is included in the definition of the predicate 'being able to laugh'. A proposition is *per se* necessary if it includes any of these two predications, according to

Kilwardby. Necessity *per accidens* belongs to all other necessity proposition that do not have the intrinsic relation between subject and predicate.

He also stresses that in a *per se* necessity proposition, the subject must be '*per se aliquod ipsius predicati*'. He seems to mean that in such a proposition, the subject has the predicate as an essential property; in other words, the subject has the predicate as a necessary property through itself and not through something else. A syllogistic necessity proposition is then understood as a proposition expressing essential properties of things located in a genus-species structure. He seems to assume that in a necessity proposition *per se*, the subject term is not an accidental term, but an essential or a necessary term, and that the subject is essentially (*per se*), and not only through inseparability, linked to the predicate. Consequently, if the subject term is necessary and the link is necessary, it follows that the predicate term cannot be a contingent (accidental) term. It must be necessary as well. The Aristotelian theory of necessity syllogistic is thus limited to a special class of terms, which all stand for substances. The same terminology is also used to explain the syllogistic for contingency propositions, which means that Kilwardby is trying to develop a uniform and highly original interpretation. There are several contemporary scholars that have developed a similar interpretation of Aristotle.[68]

In interpreting Aristotle's modal syllogistics, most medievals saw the need to introduce a distinction between different kinds of assertoric proposition. In the mixed syllogism *L–L*, the assertoric minor premise cannot be any kind of assertoric sentence, since the terms could just be accidentally connected. Kilwardby therefore introduces a distinction between simply (*simpliciter*) and as of now (*ut nunc*) assertoric propositions. The seed of this distinction can already be found in Aristotle (*Prior Analytics* I, 15, 34b7-18), but Kilwardby of course uses his *per se/per accidens* distinction to spell out the difference. A simply assertoric proposition involves a *per se* predication while an as of now assertoric proposition involves a *per accidens* predication. In such a way he can ensure that an essential connection between the terms in the valid *L–L* syllogisms are preserved to the conclusion. This is not unproblematic,[69] but the distinction between different assertoric propositions needed somehow to be made and it remained a problem all the way through the Middle Ages.

In the end, however, Kilwardby does not quite succeed in getting just the moods accepted by Aristotle. He for example accepts *–LL* for the first figure, which is not accepted by Aristotle, and does not manage to get *–CC* and *LCC* for Disamis in the third figure.[70] There are also some other moods that he does not succeed in validating and some others which he get but which are not accepted by Aristotle, but perhaps he gets as close as one possible can get to making Aristotle's system consistent.[71]

[68] [van Rijen, 1989; Patterson, 1995; Thom, 1996; Nortmann, 1996].
[69] See [Lagerlund, 2000, 39-42].
[70] For the explanation of these names see below.
[71] See [Knuuttila, 1996; Lagerlund, 2000; Thom, 2004].

4.4 The Topics

Before the thirteenth century the discussion of Aristotle's *Topics* was completely dominated by Boethius' *De differentiis topicis*, which is not really a commentary of Aristotle's work, but a mix of Aristotle and Cicero. The first extant Latin commentary on Aristotle's *Topics* is from c. 1235. Naturally, Boethius remained influential well into the thirteenth century; although they gradually stopped commenting on his work.

In the first chapter of the first book of the *Topics*, Aristotle writes (100a10-20):

> The goal of this study is to find a method with which we shall be able to construct deductions (syllogisms) from acceptable premises concerning any problem that is proposed and – when submitted to arguments ourselves – will not say anything inconsistent.

This is the method of dialectics. It is an argumentative method in dialogue form between a questioner and an answerer. These two first agree upon a problem (a question), which usually is a proposition expressing a well accepted opinion. Next the questioner puts forward questions to the answerer, which he/she answers with 'Yes' or 'No', that is, he/she accepts the propositions proposed or denies them. The accepted question or proposition then becomes premises in the argument. The purpose of this exercise is to examine the agreed upon problem and show that it leads to inconsistencies or at least to opinions not acceptable to the answerer.

Aristotle never tells the reader what a topic (*topos*) is, but he gives examples in Books II-VII of the *Topics*, but as many commentators have observed it is not always helpful to have lots of examples. The problem has been to understand what they are examples of. In the *Rhetorics*, Aristotle gives us some help. There he notes that the *topoi* are "that under which many arguments fall" (1403a17-8). This suggests that the *topoi* are classifications of arguments, that is, they are argumentative forms.

The Latin name for a *topic* is a *locus*. Boethius defines a *locus* as: "that by which the argument is settled, or that by which from a proposed question the argument is drawn out."[72] According to Boethius, it is hence about finding arguments from particular questions (or premises) to conclusions, which means that it is by finding the *locus* or the middle term that we can find the argument that gets us from question to conclusion.

Strictly speaking a question is of the form 'Either A is B or A is not-B', and the problem is which disjunct should be accepted? The question is answered by an argument, and Boethius understands it as being syllogistically structured, but what he presents are not valid syllogisms. In syllogistics, the problem is, once one knows the system, finding the middle term, and the theory of the topics systematises the process of finding the middle term. There is no standard way of doing this though, since they will always depend on the particular argument.

[72] Boethius, *De differentiis topicis* in [Stump, 1978, 30]. The translation is here slightly modified.

Instead Boethius provides us with a systematic way of searching for the middle term.

A syllogism contains three terms called a major extreme, a middle and a minor extreme, and for the syllogism to be valid the two extremes need to bare the right relation to the middle term. The topical *differentiae* provide a classification of these relations. The *differentiae* are the distinguishing features of these relations. For example, species of such relations are 'is the definition of' and 'is the meaning of', and their *differnetiae* are 'from a whole' and 'from a sign'.

An example that Boethius has is the following: I want to answer the question: Are trees animals? I run through the different sorts of relations in the list of topics or loci for one suitable for relating the extremes 'tree' and 'animal'. Once I come to the relation of definition I find that the definition of animal is 'living substance with sense perception' and I immediately see that I can deny this of trees, which means that I have shown that trees are not animals.

The practical question for a theorist of the topics is of course how to organise the list of *differentiae* and there are various lists in the commentary tradition. Boethius' list is based on Cicero's and the tradition after Boethius follow him closely. The character of the discussion did not chance when one started to comment on Aristotle's original book in the thirteenth century instead of commenting on his.

The status of the *loci*, however, gradually became the overarching problem, and it is also in that discussion that some new problems and developments can be seen in the thirteenth century.[73] Most authors from this period described the *locus* as a relation, but what is it a relation between? It is not a relation between terms taken as term, but between the things that the terms stand for. This, of course, touches upon a much larger question, namely what is logic about? The discussion we find in thirteenth century commentaries on the *Topics* is highly interesting.

In 1240 to 1250, in Paris a consensus about the status on the *loci* start to emerge. These authors, one of which is Robert Kilwardby, define a *locus* as a relation of one thing to another (*habitudo rei ad rem*). The logician here meets the metaphysician, which so often is the case in thirteenth century logic as we have seen, that is, logic is about things and their relations.

The *loci* are not just any relation between things, but they are of a general character and could hence equally well apply to various things. This means that they are external and accidental in comparison to any particular thing to which they apply. They do not belong essentially to any kind of species of a thing. This explains the difference between dialectics and demonstrative science. Demonstrative science regard the thing which is the cause or is defined as the thing which it actually is. Dialectics only applies to the general concept without specifying which cause, but concerns causes in general.

[73] The discussion in the rest of this section is foremost based on [Green-Pedersen, 1984, 223-40]. See also [Stump, 1982] for a careful overview of the tradition of the *Topics*.

4.5 The Sophistici Elenchi

The Latin version of *Sophistici elenchi* by Boethius began to circulate in the 1120s.[74] It was retranslated from the Greek a little later by James of Venice and later in the thirteenth century a third Latin translation appeared in 1269 by William of Moerbeke. It relatively quickly became a central part of the new logic developing in the late twelfth century. As with the other logic works of Aristotle the earliest commentaries were literal commentaries, that is, they proceed by first quoting the text from Aristotle and then expanding on it before turning to the next bit of text. Towards the end of the thirteenth century so-called question commentaries started to appear. These were more problametising by asking specific problematic questions based on Aristotle's text.

Although the *Sophistici Elenchi* belongs to the *logica nova* parts of it was well known before the 1120's. The reason for this is a line in Chapter 6 of *De interpretatione*. In his discussion of contradictions, Aristotle notes that the subject and the predicate of the contradictory propositions must be identical, and then he adds that "the identity of subject and predicate of the contradictory propositions must not be 'equivocal'. Indeed there are definite qualifiers besides this, which we make to meet the cauistries of the sophists" (17a35-7). This note makes Boethius outline parts of Aristotle's doctrine in the *Sophistici Elenchi* in his second, more detailed, commentary on the *De interpretatione*. Boethius mentions six cases in which the contradictions of propositions are frustrated by fallacies, namely in the fallacies of (i) equivocation, (ii) univocation, (iii) diverse parts, (iv) relatives, (v) diverse times, and (vi) diverse modes.

The fact that this discussion is included in a commentary on *De interpretatione* is historically very important. *De interpretatione* is about terms in propositions and the discussion of fallacies forces twelfth century logicians to think about ways in which terms can be taken in a proposition. Very early in the twelfth century one can therefore find formulations like the following in commentaries on *De interpretatione*, namely that "the interpretation of a term is determine by its use in a proposition."[75] This is the fundamental idea behind supposition theory and the *logica modernorum*.

The discussion of the fallacies did not single handily generate the doctrine of supposition, since, as de Rijk shows[76], the development of grammar in the twelfth century was also essential to this development. It is clear though that early commentaries on the *Sophistici Elenchi* in the later twelfth century played a very important part in formulating supposition theory. I will not go in to this further since this falls outside of the scope of the present chapter.

It is towards the latter half of the twelfth century that one can find the first Latin commentaries on the *Sophistici Elenchi* that are based on Aristotle's original text. It is also then that the discussion of fallacies are formed and divided in the

[74]See Aristotle, *De sophisticis elenchis*, xi-xv.
[75]See [de Rijk, 1962, 48].
[76]See [de Rijk, 1967, I, 95–125].

way that one later finds in the thirteenth century textbooks on logic. The division between the linguistic and non-linquistic fallacies is expressed and also the number of fallacies are settled, that is, six linguistic and seven non-linguistic (see below for a discussion about some of these).

The *Sophistici Elenchi* has been the inspiration of much original work in logic. For example, a section of Aristotle's work was throughout the Middle Ages very important for discussing so called insolubles or paradoxes. One of the most famous insoluble is the Liar-paradox: 'This proposition is false'. In *Sophistici Elenchi* 25 (180a27-b7), Aristotle discusses the fallacy of confusing things said "in a certain respect" (*secundum quid*) and things said 'simply' or 'absolutely' (*simpliciter*). Aristotle discusses an example of a man that takes an oath to become an oath-breaker and then breaks his oath. Such a man is on the whole or simply an oath breaker, but in the particular case he is in a certain respect an oath-keeper. Having said this Aristotle adds that "the argument is similar too concerning the same man's lying and speaking the truth at the same time". This was taken as a reference to the Liar-paradox and hence it was seen at least in the beginning of the thirteenth century as an instance of the fallacy of *secundum quid et simpliciter*. All early attempts to solve the Liar-paradox in this way were completely mistaken and it was soon realised that Aristotle's discussion was really about something else. Perhaps this is the reason the textbook authors dealt with below did not at all treat insolubles.[77]

Disputing so-called *de sophismatibus* or sophisms was a standard part of mid-thirteenth century logic teaching. An example of a sophistical proposition that was disputed is 'Every human being in necessarily an animal'. This is an example of an ambiguous proposition, which is true or false depending on the interpretation given to it. It is for example false when no humans exist, but if there are humans it is true. Some dealt with such propositions by simply outlining under which conditions they are true, while others felt that a proposition like this one must always be true. Kilwardby, for example, argued that one can truly say that 'Every human being is necessarily an animal' even when there are no humas, because the species or intelligible form of human still remains and it is that which the utterance 'human being' signifies.[78]

5 TREATISES ON LOGIC

5.1 The Tractates and Summaries

Special textbooks on logic began to be widely used already in the early twelfth century.[79] Garlandus Compotista's (d. early twelfth century) *Dialectica* and Abelard's *Dialectica* are two of the more famous ones. Towards the end of the

[77] See [Spade, 1973] for a history of the *Insolubilia*-literature, and see [Spade, 2005] for a clear exposition of the medieval discussion in general.

[78] See [Lewry, 1981, 382].

[79] See [de Rijk, 1967, I, Chapter 3].

twelfth century and the early thirteenth century some substantial and very important anonymous treatises on logic appeared. One of these is the so called *Ars Meliduna*, which contain very detailed discussions of terms and propositions.[80] Another interesting book is the *Dialectica Monacensis*, which is interesting since the author obviously had access to all of Aristotle's writings on logic. de Rijk dates this work to the early years of the thirteenth century. The layout of *Ars Meliduna* is very different from for example Peter of Spain's famous *Tractatus*, which later were called *Summule Logicales*, but around the turn of the century logical treatises with the same structure and content as Peter's work started to appear. In the anonymous *Summa Metenses* all the parts of logic discussed by Peter is present.

Gradually the *logica nova* worked its way into the the treatises and summaries of logic in the first half of the thirteenth century. Before 1200 the textbooks foremost included discussions of the *logica vetus* and some discussion of fallacies apart from the new developments in the *logica modernorum*, but in the early decades of the thirteenth century a standard format and some canonical topics had been settled on, which included both the *logica antiquorum* and the *logica modernorum*. It is these themes one can find in the famous mid-thirteenth century textbooks.

The most famous logic treatises and summaries from the thirteenth century was written by William of Sherwood (d. 1272), Peter of Spain (d. 1277), Roger Bacon (d. 1294) and Lambert of Auxerre (fl. 1250). Peter's *Tractatus* is by far the most famous and influential; fourteenth century as well as later logicians wrote commentaries on it. William's work, which is called *Introduction to Logic*, was also very much read. Roger Bacon's *Summulae Dialectices* and Lambert of Auxerre's *Logica* were not as influential, but they were also wildly read. All of them contain the same material, but they differ slightly in how they treat this material. Their terminology also differs. See the subsequent discussion below for more details.

On Propositions

The *Tractatus* is divided into twelve treatises.[81] The first is an introduction dealing with the subject matter of dialectics (or logic) and the different kinds of propositions. He defines dialectics in the following way:

> Dialectics is the art that holds the road to the principles of all methods, and therefore dialectics should be first in the acquisition of all the sciences.[82]

[80] See [de Rijk, 1967, I, Chapters 7–10].

[81] These twelve are: (I) On propositions (or statements); (II) On predicables; (III) On categories; (IV) On syllogisms; (V) On topics or *loci*; (VI) On supposition; (VII) On fallacies; (VIII) On relatives; (IX) On ampliation; (X) On appelation; (XI) On restriction; (XII) On distribution. My treatment below will also be divided into these paragraphs; except towards the end where I will treat (IX), (X), (XI) and (XII) under the heading of 'supposition'. William of Sherwood's *Introduction of Logic* is divided into six chapters, namely (1) On statements; (2) On predicables; (3) On syllogisms; (4) On topics; (5) On properties of terms; (6) On sophistical reasoning. Most of the same material is, however, dealt with in these two books.

[82] Peter of Spain, *Tractatus*, I.1, 4-6.

This definition reflects the absolute fundamental aspect of logic for the scholastics. Logic is a tool or an art for them presupposed in all science. All study must start with logic.[83]

Dialectics or argumentation presupposes discourse (*sermone*), and discourse presupposes utterance (*vox*), which in turn is sound (*sono*) of a special kind. A sound is either an utterance or not. If it is not an utterance, then it is not of interest to logic. An utterance is either significative or not. An utterance which is significative is a sound which represents something, like for example 'human being'. A non-significative utterance does not represent anything, like 'buba'. A significative utterance is either a name (*nomine*) or a verb (*verbo*). It is a name if it signifies conventionally something without a time, while a verb signifies conventionally (*ad placitum*) something with a time. The difference is simply that since a verb signifies a doing of something, it requires time in which the doing takes place, but a name is atemporal; it does not require time for naming something.

An expression (*oratio*) is a complex significative utterance, that is, it is a sentence like 'The human being is white'. There are also two kinds of expressions, namely perfect or proper expressions and imperfect or improper expressions. An improper expression is, for example, 'white human being'. It is improper since it does not contain both a name and a verb. There are, of course, different kinds of expressions according to the different grammatical modes, like indicative, imperative, imperative, perfect etc., but only indicative expressions are called proposition and hence only they are important for logic.

A proposition is hence a perfect or proper expression in the indicative mode, which is either true or false. There are furthermore two kinds of propositions, namely categorical and hypothetical. The two kinds of propositions also divide into two kinds of logics; one for categorical propositions and the other for hypothetical. Let us first deal with the categorical propositions, since they are the most important ones.

A categorical proposition is a proposition that has a subject and a predicate as its principle parts. In the proposition 'A human being runs' ('*homo currit*'), 'human being' and 'runs' are respectively the subject and the predicate. The logical form of this proposition is not 'A human being runs', however, since there is a copula or something binding the subject and predicate (or verb) assumed in between them. The logical form is therefore 'A human being is running'.

Categorical propositions are divided into universal, particular, indefinite, and singular. The most familiar ones are the following:

Universal: 'Every human being is running'
Particular: 'Some human being is running'
Indefinite: 'A human being is running'
Singular: 'Socrates is running' or 'That human is running'

[83]Logic is part of the science of discourse (*scientia sermocinalis*), which has three parts, namely grammar, rhetoric and logic. This view was defended by both Peter and William.

Categorical propositions are furthermore either affirmative or negative.

Given this we are now in a position to outline the basic relationship between these propositions. This is done by stating the traditional square of opposition, which include the four basic relations; contrary, subcontrary, contradiction and subalternation.

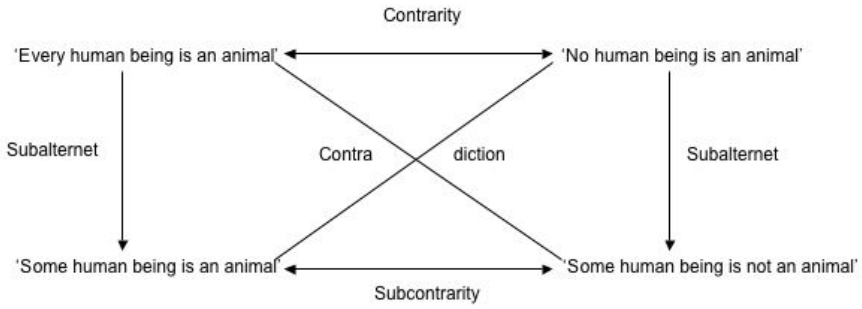

Figure 1. The Square of Opposition

William also presents a hexagon of opposition.

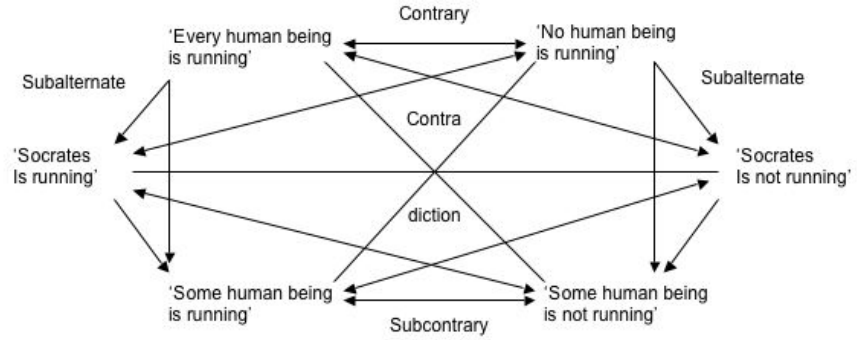

Figure 2. The Hexagon of Opposition

Having presented the fundamentals all textbooks introduce the doctrine of the matter of propositions. This is a theory that has Ancient sources, but was very typical for thirteenth century logic, as noted several times already, and assumes a close relation between logic and metaphysics. The three kinds of matter are

natural (or necessary), contingent, and remote (or impossible). A proposition is in natural matter when the predicate is in the subject or it is a proprium of the subject. The examples Peter gives are propositions like 'A human being is an animal' or 'A human being is rational'. In both these cases there is a strong essential connection between the subject and the predicate. A proposition is in contingent matter when the predicate can both be in and not in the subject like in 'A human being is white' or 'A human being is not white'. Obviously, if a proposition is in remote matter then the predicate cannot be said truthfully of the subject like in 'A human being is a donkey'.

As we have seen earlier in this overview the doctrine of the matter of the proposition was connected with modal propositions and modal logic, but this is not explicitly done by any of the textbook authors. They treat modal propositions quite independently. This particular doctrine seems more connected with truth conditions for modal propositions than with modal propositions in themselves, which might reflect an attempt to loosen the connection between logic and metaphysics. They immediately go onto present equipollence rules and later embark on a discussion of modal propositions.

Hypothetical propositions need to be dealt with before the basic rules of equipollence can be stated, however. There are three kinds of hypothetical propositions, namely conditional, copulative and disjunctive. A conditional proposition is made up of categorical propositions conjoined with the words 'If...then'. A conjunctive proposition conjoins two categorical propositions with 'and', and a disjunctive proposition does the same thing with 'or'. It is the also common to state the conditions under which these hypothetical propositions are true. A conditional proposition is true, if, given the truth of the antecedent, the consequent is also true. A copulative proposition is true if both conjuncts are true, and a disjunctive proposition is true, if at least on of the disjuncts are true.

The part on equipollence usually begins by stating the conversion of categorical propositions. There are three kinds of conversion, namely simple, accidental and contra position. A simple conversion occurs when the subject and the predicate changes place, but the quantity and the quality of the proposition are the same. For example, the proposition 'No human being is a stone' converts truthfully to 'No stone is a human being'. An accidental conversion is then when the subject and predicate changes place, but the quantity and not the quality is changed. As in the conversion of 'Every human being is an animal' to 'Some animal is a human being'. A contra position is when the subject and predicate change place and the quantity and quality are the same, but the terms are changed from finite to infinite ones. An example would be the conversion of 'Every human being is an animal' into 'Every non-animal is a non-human being'.

Before turning to modal proposition some further rules of equipollence are presented. Peter presents these:

> E-rule 1: If a negation precedes a proposition (P), then it is equivalent with P's contradiction.
> Example: 'Not Every human being is running' is equivalent to 'Some

human being is not running'.

E-rule 2: If 'Every' or 'Some' are preceded and followed by a negation in some proposition, then it is equivalent with its subalternate proposition.
Example: 'Not Every human being is not running' is equivalent to 'Some human being is running'.

E-rule 3: If two universal negative signs are posited in the same proposition, and one is in the subject's position and the other in the predicate's position, then, firstly, the proposition is equivalent to its contrary, and, secondly, to its contradictory.
Example: First of all, 'Nothing is nothing' is equivalent to 'Something is something', and, secondly, 'Something not' is equivalent to 'Nothing', and 'Every not' to 'No'.

The last rule looks a little bit strange, but it is only meant to express for example how the quantity of the proposition is changed by adding a negation in front of it or after it. As is clear all tree of these rules are about the way negation effect propositions.

After these simple equipollence rules the first treatise ends with a discussion of modal propositions. The mode is treated by these authors as adjectives and as such they affect the way or the manner in which the proposition are taken. Adjectives might be taken nominally or adverbially and hence modes are explained in that way as well. They explain this in some detail, but first they note that the four modes are necessity, possibility, contingency and impossibility. They usually add also truth and falsity, but note that they do not change the truth value of a proposition as the others do.[84]

The mode of a proposition is either taken nominally as in 'It is necessary that Socrates is an animal', or it is taken adverbially as in 'Socrates runs contingently'. This is the same division as Abelard had made a century earlier.[85] Peter stresses that only the adverbial reading makes the proposition into a proper modal proposition, since only on such a reading is the composition of the subject term and the predicate term modified. The proper way to write a modal proposition is thus: 'Socrates is contingently running', and hence it is the copula that is modified in such propositions, which means that it is really the relation between the term that is qualified. Somewhat confusingly he insists on writing the modal propositions like this: 'Socrates runs is possible'. On such a reading, it looks as if the mode 'possible' is taken nominally and hence function as an ordinary predicate, which

[84]William notes that 'necessarily' and 'impossibly' are used in two ways. Somethings are 'necessary *per se*', which cannot be false now or in the future or past, like 'God is', and other propositions are 'necessary *per accidens*', which cannot be false now or in the future, but might have been in the past, like 'I have walked'. This distinction seems inspired by Averroes mentioned above in section 3.2, but less ontological.

[85]See above and also [Lagerlund, 2000, 35-9].

would make the proposition non-modal. He has, however, made it clear that it is the Abelardian reading he adheres to (without mentioning Abelard, of course).

Having presented the form of a modal proposition, Peter states some initial rules of equipollence.

> ME-Rule 1: 'If an affirmative proposition is possible or contingent and not impossible, then its contradictory is not necessary'.
> Example: 'A is possible B' contradicts 'A is necessarily not B'.
>
> ME-Rule 2: 'If a negative proposition is possible and contingent and not impossible, then its contradictory is not necessary'.
> Example: 'A is not possibly B' contradicts
>
> ME-Rule 3: 'If an affirmative proposition is not possible, not contingent, and impossible, then its contradictory is necessary'.
> Example:
>
> ME-Rule 4: 'If a negative proposition is not possible, not contingent, and impossible, then its contradictory is necessary'.
> Example:

Given the modalities and negation it is possible to state some equivalences between the modalities. It was common to present four groups and say that within each group the modalities are all equivalent.

> I.
> Possibly
> Contingently
> Not-impossibly
> Not-necessary not
>
> II.
> Possibly not
> Contingently not
> Not-impossibly not
> Not-necessary
>
> III.
> Not-possibly
> Not-contingently
> Impossibly
> Necessarily not
>
> IV.
> Not-possibly not
> Not-contingently not

Impossibly not
Necessary

Given these it is not difficult to present the square of opposition for singular modal propositions. It is the following:

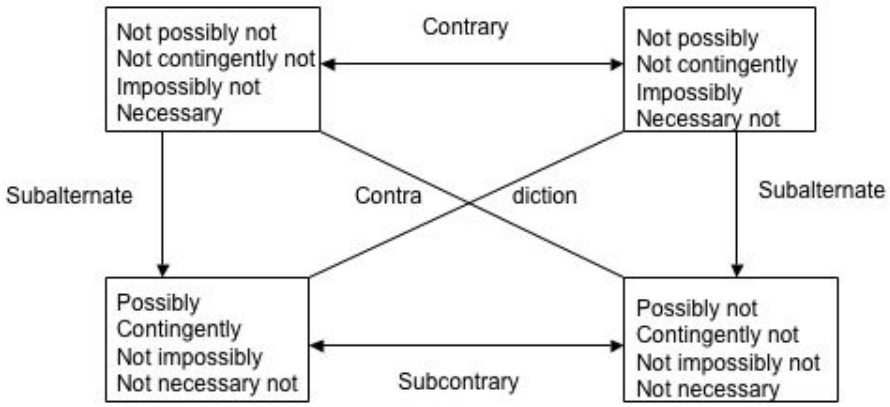

Figure 3. The Modal Square

Notice that the modal square is only for singular propositions. No one at this time presented a square of opposition for quantified modal propositions, which, of course, made is completely useless for modal syllogistics.[86] A theory for quantified modal propositions was not developed until the mid-fourteenth century.[87]

[86]None of the textbooks contain any discussion of modal syllgistics; except a discussion of the modal conversion rules in Roger Bacon and Lambert of Auxerre's books. They, however, closely follow Kilwardby's treatment discussed above (section 4.3). Roger Bacon follows the first of Kilwardby's solutions to the conversion of necessity propositions, while Lambeert of Auxerre presents both solutions. Bacon's continued discussion of contingency propositions is rather confusing, since he uses contingency for what is usually calls possibility. He therefore divides contingency into that which is necessary, possible and non-necessary. It is contingency in the last sense that is usually calls contingency. He then proceeds to make a two fold division of contingeency in this sense into contingency *ad utrumlibet* and contingency *natum* (compare Aquinas threefold distinction above). Propositions taken in none of these senses of contingency convert as necessity or possibility propositions. See Roger Bacon, *Summulae Dialectices*, III, 199-200.

[87]See [Lagerlund, 2000, Chapter 5 and Appendix IX.3].

On Predicables

After having outlined the different distinctions between various propositions, the textbooks return to the traditional order, which means that the next part of these books deal with the predicables, that is, the topic of Porphyry's *Isagoge*. The predicable are universals, which means that they are what can be predicated in the most general sense. The division is the traditional one, namely between genus, *differentia*, species, property (*proprium*) and accident.

A genus, in the relevant sense here discussed, is what is predicated of several different species, such as animal is predicated of horses, humans and lions as different species. There are three ways things can be similar and hence also three ways that they can be different. Things can be the same in genus, species and number. For two things to be the same in genus means that they are truly predicated of the same genus, like 'human' and 'donkey'. Hence two things are the same in species if they are truly predicated of the same species, like 'Socrates' and 'Plato'. Things are the same in number in four ways, namely in name, definition, property, or accident.

All of these are strightforward. Two persons can have the same name and one person can have several names. Some things are the same by definition, like 'mortal rational animal' and 'human being'. 'Human being' and 'able to laugh' are the same in property, since only humans are able to laugh, according to Aristotle. 'Socrates' and 'white' are the same in accident, since white inheres in Socrates. Building on this it is clear how things may differ; for example, things are different in genus if they fall under different genera, like humans and trees, and consequently things are different in species if they fall under different species, like Socrates and Brunellus the donkey. Lastly things are different in number if they are in fact distinct, like Socrates and Plato.

A genus can be characterised in two ways. First of all, a genus is an answer to a 'What is' question; for example, 'What is a human being?' The answer is the genus ' animal'. Secondly, a genus can also be specified as that of which a species supposits. Genus is also divided into the most general genus (*genus generalissimum*) and subalternate genera. The most general genus is the genera above all other genera, that is, substance, while a subalternate genus is a genus below the most general, and they can also be species. These are ten in number.

A species can also be predicated of several things which are below it. 'Human being' is predicated of Socrates and Plato. A species is also an answer to a 'What is it' question, namely for example the question: 'What is Socrates?' The answer is the species a 'human being'. Species are also divided into the most special species and subalternate species. A most special species is a species that cannot also be a genus, like 'human being' and 'horse' Consequently, a subalternate species is a species that can be a genus, like 'body' or 'animal'. This is explained further in the so-called Porphyrian tree.

There are three kinds of *differentiae* outlined in these textbooks, namely common, proper and more proper. A common difference is when something differs

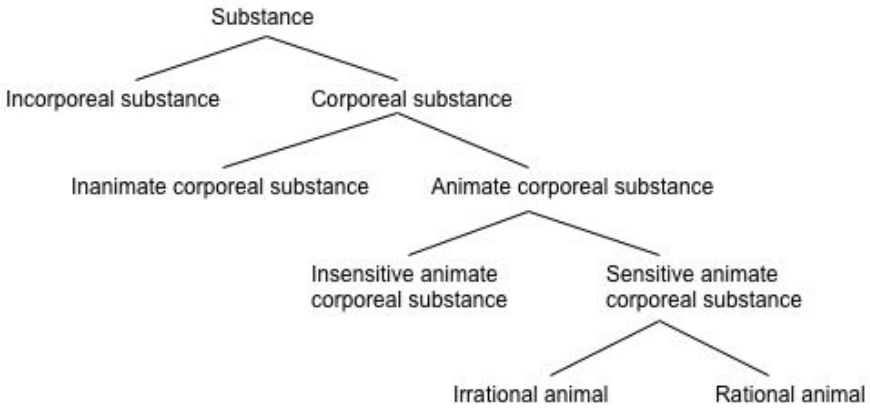

Figure 4. The Porphyrian Tree

from something else by an separable accident, like Socrates sitting differs from Socrates standing. A proper difference is the difference between things through an inseparable accident, like 'snub nosed'. Finally, a most proper difference is when something differs from something else by a specific difference, like humans differ from horses by rationality. This is of course the most important notion if difference. A difference of this kind is, furthermore, both dividing and constitutive, since it divides the genus and is the constitutive part of the species. The genus 'animal' is divided by 'rational' and 'rational' is what constitutes the species 'human being'. This is an identical treatment to what we saw above in Averroes' commentaries (section 3.2).

A property (*proprium*) is always associated with every individual belonging to a given species, as for example 'being able to laugh' is always associated with every human being. Although a human being does not always laugh, it will always be able to. If there are humans then there will also be the property, 'able to laugh'.

An accident is something which can be in a subject and missing from a subject without its corruption, like white, black, and sitting. Accidents can, however, be separable and inseparable. An inseparable accident is an accident which as a matter of fact is never separated from its subject, like the blackness of an Etiopian (the example stems from Porphyry himself).

Based on these some similarities and differences between the predicables are usually observed. One major difference between the first four and accidents is that genus, species, *differentia* and property all are either in the thing they are predicated of or not, that is, they either participate fully or not at all in the subject

of their predication, while this is not true of accidents, since they can begin or stop being part of a subject. There is hence a question, which caused lots of debate in later medieval philosophy, of the intension and remission of accidents, which does not occur for the other four. They are predicated univocally, while accidents are predicated denominatively.

On Categories

According to the textbooks[88] the *Categories* is about that which can be predicated and that which something can be predicated of. They begin, however, by outlining ways of predication. Peter of Spain explains that categorical propositions express something of something else, and there are several ways that something can be in something else as a predicate is in a subject, that it, there are several modes of being in something. He distinguishes eight such modes. Something is in something else:

1. As a part is in a whole.
Example: The fingers of the hand.
2. As a whole is integrated with its parts.
Example: The house with its walls, roof and foundation.
3. As a species is in its genus.
Example: Human being in animal.
4. As a genus is in its species.
Example: Animal in human.
5. As form in matter.
Subdivided into:
5'. As a substantial form in matter.
Example: The human soul in its body.
5". As an accidental form in matter.
Example: White in human beings.
6. As its first efficient cause.
Example: Reign in ruling.
7. As its final cause.
Example: Virtue in beatitude.
8. As in a vessel.
Example: A thing located in place.

These are taken from Aristotle's *Physics* IV.3 and they are also repeated and elaborated in Boethius' commentary on the *Categories*. These divisions are used to explain in what way something is 'said of a subject' and 'in a subject'. According to Peter, on must understand 'said of a subject' as something said of an inferior, that is, it is used as in 3 above, and being 'in a subject' is taken as in 5", as a accident is in a subject.

[88] William does not deal with the categories in his book, but both Roger Bacon and Lambert of Auxerre do.

Things said of a subject, but which are not in a subject are thing like substantial genera and species, and substantial *differentiae*, that is, human being, animal and rational. Things like substantial individuals, like Socrates, are neither said of a subject nor are they in a subject. Things said of a subject that are also in a subject are the genera and species of the other nine categories. They are said of inferiors and they are in a subject as accidents are in a subject, for example, as colour is said of whiteness as of an inferior and it is also in a body as in a subject. Finally there are also things which are in a subject, but not said of any subject, like a particular piece of knowledge is in the soul, or this individual colour of the wall, for example, is in a subject.

After this Peter turns to a discussion of the individual categories and first is, of course, substance. Traditionally, it is divided into primary and secondary substance. Primary substance is what is most properly called substance, and as mentioned above it is neither said of a subject not in a subject, like individual humans (Socrates, Plato etc.) and horses. Secondary substances are species and genera, since they have primary substances under them. A secondary substance can be used as a predicate, but a primary cannot, like 'Socrates is a human being', but one cannot say 'A human being is Socrates'. A primary substance is also a *hoc aliquid*, that is a thing or an individual of which there is only one.

A quantity, which is the second category, is either discrete or continued. Number or discourse are discrete quantities. There is no common terms to which either numbers or discourse can be linked. They are separate and a multitude brought together into a unit. Lines, body, time and place are continuous quantities. A line is joined to a common term, namely a point, and hence continuous.

Relation is the next category treated. Contrariety is a relation, and more or less as well as equal and unequal. Every relation is convertible like father and son are convertible, that is, if A is the father of B, then B is the son of A. This means that in a relation like father to son, which is asymmetrical, then one can always find another asymmetrical relation that together makes up a symmetrical relation.

The fourth category discussed by Peter is quality. There are four subdivisions of quality, namely, (i) habits and dispositions, (ii) capacity, (iii) passion, and (iv) form. A habit differs from a disposition by being more permanent. Knowledge is a habit while virtue is a disposition. Some have a natural capacity for health while others have a natural capacity for sickness; some can play the piano while others cannot. (iii) is a passivity or a quality of being affected, like sensations. The fourth kind of quality is the form or shape of something stable, like the triangularity of a body. The other categories are only mentioned by Peter.

On Syllogisms

After their treatment of the categories comes a discussion of syllogisms (William jumps from the discussion of predicables to the syllogisms). The theory of the syllogisms is the main part of logic for a thirteenth century logician and it is what comes closest to what we now call logic. The other parts of medieval logic would

nowadays sort under semantics, linguistics, or philosophy of language.

In Peter's case the discussion starts with an outline of two rules of class inclusion called '*dictum de omni*' and '*dictum de nullo*'.

> *Dictum de omni*: Nothing is subsumed under the subject term which is not also said of the predicate term.
> (In contemporary logical terms: $A \supseteq B$)
> *Dictum de nullo*: Nothing is subsumed under the subject term which is not also removed from the predicate term.
> (In contemporary logical terms: $A \not\subseteq B$)

These rules are taken to be absolutely fundamental in determining the validity of the first figure moods and they are said to explain the perfection of the first figure moods. It is only after having clarified them that they go on to define syllogisms. The two mentioned definitions are:

> A part of discourse in which it is the case that from somethings posited something else necessarily follows,

and:

> From two posited premises another necessarily follows, which is the conclusion.

The two premises, that is, the major and the minor premise, talked about in the second definition include three terms, which are the logical subjects and predicates of the two premises. The terms are called middle, major extreme and minor extreme. The major premise include the middle and the major extreme terms, while the minor premise include the middle and the minor extreme. The conclusion contains the two extreme terms.

The syllogisms are divided into three figures based on the organisation of the terms involved in the premises.

> First figure: Middle and major extreme
> Minor extreme and Middle
> Example: Every animal is a substance
> Every human being is an animal
>
> Second figure: Major extreme and middle
> Minor extreme and middle
> Example: Every human being is an animal
> No Stone is an animal
>
> Third figure: Middle and major extreme
> Middle and minor extreme
> Example: Every human being is an animal
> Every human being is rational

Given these combinations they present some general rules for the theory. First of all there are five rules:

1. There are no syllogisms from only particular, indefinite or singular premises.

2. There are no syllogisms from only negative premises.

3. If one of the premises is particular then a particular conclusion follows, but not its converse.

4. If one of the premises is negative then a negative conclusion follows, and also its converse follows.

5. The middle term must not be part of the conclusion.

The first figure has nine valid moods or forms of syllogisms. Four of these have direct conclusions and five are indirect. In a direct conclusion the major extreme is predicated of the minor extreme and in an indirect conclusion the converse holds. It became standard in the thirteenth century to treat the fourth figure introduced by Galen as an indirect form of the first. The reason for this was to give a more faithful interpretation of Aristotle who does not mention the fourth figure.

There are two rules that govern the first figure:

6. If the minor premise is negative, then no conclusion follows.

7. If the major premise is particular, then no conclusion follows.

The four first syllogisms with direct conclusions are not given proofs, since they are perfect. Their validity is shown by reference to the *dici de omni et nullo* rules mentioned above. *Dictum de omni* governs the first mood (Barbara):[89]

I.
Every B is A
Every C is B
Every C is A

Dictum de nullo governs the second mood (Celarent):

II.
No B is A
Every C is B
No C is A

The two other moods (Darii and Ferio) are reduced to these two by the square of opposition:

[89]For an explantion of the names of the valid syllogisms see below in this section.

III.
Every B is A
Some C is B
Some C is A

IV.
No B is A
Some C is B
Some C is not A

The five syllogisms with indirect conclusions are in various ways reduced to the first four by using the conversion rules stated above. The five syllogism are:

V.
Every B is A
Every C is B
Some A is C

VI.
No B is A
Every C is B
No A is C

VII.
Every B is A
Some C is B
Some A is C

VIII.
Every B is A
No C is B
Some A is not C

IX.
Some B is A
No C is B
Some A is not C

(V) is reduced to (I) by simple conversion of the conclusion, since by the square of opposition the universal conclusion in (I) implies a particular. (VI) is reduced to (II) by simple conversion of the conclusion, and (VII) is reduced to (III) is the same way. (VIII) is reduced to (IV) by accidental conversion of the major premise, simple conversion of the minor premise, transposing the premises and renaming the terms. (IX) is reduced to (IV) in the same way.

The second figure is governed by three rules added to the ones above, namely:

8. If the major premise in a second figure mood is particular, then no conclusion follows.

9. If the premises are both affirmative in a second figure mood then no conclusion follows.

10. In the second figure only negative conclusions follow.

There are only four valid moods in this figure.

X.
No A is B
Every C is B
No C is A

XI.
Every A is B
No C is B
No C is A

XII.
No A is B
Some C is B
Some C is not A

XIII.
Every A is B
Some C is not B
Some C is not A

(X) is reduced to (II) by simple conversion of the major premise. (XI) is also reduced to (II) but this time by simple conversion of the minor premise and the conclusion, by transposing the premises and renaming the terms. (XII) is reduced to (IV) by simple conversion of the major premise. (XIII) cannot b reduced and is hence proved through impossibility.

The proof through impossibility is very simple and proceeds from positing the negation of the conclusion and deriving a contradiction. The negation of the conclusion is: 'Every C is A', and together with the major premise 'Every C is B' follows, which is the negation of the minor premise.

The third figure is also governed by two rules, namely:

11. If the major premise is negative in a third figure syllogism, then no conclusion follows.

12. Only particular conclusions follow in the third figure.

There are six valid moods in this figure.

XIV.
Every B is A
Every B is C
Some C is A

XV.
No B is A
Every B is C
Some C is not A

XVI.
Some B is A
Every B is C
Some C is A

XVII.
Every B is A
Some B is C
Some C is A

XVIII.
Some B is not A
Every B is C
Some C is not A

XIX.
No B is A
Some B is C
Some C is not A

(XIV) is reduced to (III) by accidental conversion of the minor premise, and (XV) is reduced to (IV) in the same way. (XVI) is reduced to (III) by simple conversion of the major premise and transposing the premises. (XVII) is reduced to (III) by simple conversion of the minor premise, and (XIX) is reduced to (IV) in the same way. (XVIII) is proved by impossibility again. The negation of the conclusion is: 'Every C is A', and together with the minor premise through (I), 'Every B is A' is derived, which contradicts the major premise.

There are two more rules that govern the theory of the syllogisms. They are:

13. In no syllogism with a particular negative indirect conclusion does also a direct conclusion follow, and vice versa.

14. From a universal conclusion there also follows a particular.

From the second rule five additional moods can be inferred bringing the total number of valid syllogisms up to 24.

A well known part of the theory of the syllogism in the Middle Ages was the mnemonic names of the valid moods. These were created so that it would be easier for students in the early universities to learn the valid moods.[90] The 19 moods presented above was summarised in the following way:

> BARBARA, CELARENT, DARII, FERIO, BARALIPTON,
> CELANTES, DABITIS, FAPESMO, FRISESOMORUM,
> CESARE, CAMBESTRES, FESTINO, BAROCHO, DARAPTI,
> FELAPTO, DISAMIS, DATISI, BOCARDO, FERISON.

The first one needs to know to decipher them is that A stands for a universal affirmative proposition, E for a universal negative proposition, I for a particular affirmative proposition, and O for a particular negative proposition. In Celarent, the three letters e, a, and e indicate the the two premises and the conclusion. They also tell us that the major premise is a universal negative proposition, that the minor premise is a universal affirmative proposition and that the conclusion of this syllogism is a universal negative proposition. The identification of the propositions involved in each valid syllogism is, however, not the only use of these names. For example, the first figure direct syllogisms begin with the letters B, C, D, and F, and by looking at the beginning letters of the names of the other syllogisms one can know which of the first figure syllogisms the others are reduced to. Datisi in the third figure is reduced to Darii and Cesare in the second figure is reduced to Celarent etc.

These names also include information about how the syllogisms are reduced. If an S follows one of the letters indicating a premise, then that premise converts simply as part of the reduction. In Cesare in the second figure, the S before the E indicates that the major premise is simply converted to get Celarent. If the letter following is a P, however, then the premise converts accidentally, like is Felapton where the minor premise converts accidentally to reduce the syllogism to Ferio. An M in the name indicated that the premises should be transposed, like in Cambestres, which reduces to Celarent by a simple conversion of the minor premise and a transposition of the premises. If there is a C in the name, like in Bocardo, then that syllogism is proved through impossibility.

On the Topics or On Loci

The text book authors do not deal with demonstrative arguments at all. In fact they skip the whole of the *Posterior Analytics*. I do not know the reason for this. They seem not to think it belongs to logic. William mentions that syllogisms must be divided into demonstrative, dialectical and sophistical. He writes:

> A demonstrative syllogism is one that produces scientific knowledge on the basis of necessary [premises] and the most certain reasons for

[90] These names were first introduced in logic textbooks in the first half of the thirteenth century. See de Rijk [1967, I, 401].

> the conclusion. A dialectical syllogism, however, is one that produces opinion on the basis of probable [premises]. Finally, a sophistical syllogism is one that either syllogizes on the basis of seemingly probable [premises] or seemingly syllogizes on the basis of probable [premises]; in either case it is strictly aimed at glory and victory.[91]

This division is very similar to Al-Ghazālī's discussed above. William never deals with demonstrative syllogisms, however, but, as Peter, turns to the dialectical ones after having outlined the theory of the syllogisms.

The treatment of the *loci* (or *topoi*) in thirteenth century textbooks deals with the same issues as Aristotle dealt with in the *Topics*, but the medieval discussions of the *Topics* had changed quite a lot, however, and it was Boethius that set the subject matter of this part of logic for the Middle Ages. The textbooks author's discussion of *loci* is hence based on Boethius' discussion in *De differentiis topicis*.

Peter of Spain defines an argument in the following way:

> An argument is the *ratio* that makes a doubtable thing credible.

The *ratio*, or the reason', is the medium for deriving or reaching the conclusion of the argument. The notion of a medium or a middle is brought out most explicitly in the theory of the syllogisms where the middle term is the link between the extreme terms. The conclusion of an argument is an approved proposition and before it was proved it was doubtable. This way of describing arguments from premise to conclusion is explicit in Aristotle's *Topics* where the starting point is a question that is tested by the argument.

An argumentation is an argument made explicit by a discourse, that is, the discourse that explicates the argument. An argument differs from both a medium and an argumentation, Peter explains. A medium is called a medium because it has two extremes, while an argument adds to the medium the power of proving the conclusion. In order for there to be an argument there must be a medium able to prove the conclusion. The argumentation, however, is the whole discourse composed of premises and conclusion, and in an argumentation the power of an argument is manifested. The argumentation is thus the whole uttered discourse of premises and conclusion. It is an argument, which is proved by a medium from the two extremes in the premises.

There are four kinds of arguments, according to Peter, namely syllogisms, induction, enthymems, and examples. He does not discuss syllogisms, but gives short examples of the other three. An induction is an argument from a particular proposition to a universal. An enthymem is an imperfect syllogism, like:

[91] William of Sherwood, *Introduction to Logic*, 69. This division is also explicit in Roger Bacon's textbook. He also has a long discussion of demonstrative syllogisms, which is very interesting and seems influenced by Al-Ghazālī. He also discusses something he calls *syllogismus falsigraphicus* and also dialectical and sophistical syllogisms. All these are, according to him, divisions according to the matter of the syllogism. See Roger Bacon, *Summulae Dialectices*, III, 207-20.

> Socrates is a human being
> Therefore: Socrates is an animal

It can be made into a syllogism by adding the major premise: "Every human being is an animal". An enthymem can always be reduced to a syllogism by adding or making explicit the missing premise. An argument by example is based on the similarity between things expressed by the terms. The argument:

> It is not good to drink too much wine
> Therefore: It is not good to drink too much whiskey

This is a valid argument because wine and whiskey are both alcoholic beverages.

An argument is confirmed by a *locus*, which is defined in the following way:

> A *locus* settles an argument or it draws out the argument from the proposed question.

A *locus* is hence what makes something into an argument.

The *loci* are divided into maximal *loci* and maximal different *loci*. The first is identical to the maximal propositions. They are the most prior propositions and the best known, since given the terms included in them they are necessarily true. Peter gives the following examples: "Every whole is greater than its parts", "Whatever the definition is predicated of, the defined thing is as well", and "Whatever a species is predicated of, its genus is as well".

The maximal different *loci* are that by which one maximal proposition differ from another. In the examples: 'whatever a definition is predicated of the thing defined is as well' and 'whatever a species is predicated of, the genus is as well', the components that differ in these are 'definition', 'defined', 'species', and 'genus'. These terms are the maximal different *loci*. These are further divided into intrinsic, extrinsic, and middle.

The intrinsic *loci* are the ones involving arguments that are substantive of a thing, like a definition. The extrinsic *loci* are the ones involving arguments completely separated from a thing's essence, like arguments from opposites. The example Peter gives is if we question whether Socrates is white and conclude that 'Socrates is black, hence he is not white'. The maximal proposition in this case is: 'If one contrary is posited, then the other is excluded'. A middle *locus* is taken from things partly agreeing with the terms posited and partly differing. An example would be querying whether justice is good and concluding that 'a just thing is a good thing, hence justice is good'. The maximal proposition is 'what inheres is one conjugate is in the other thing as well'. The discussion of the division of *loci* is ended by a list of *loci* and maximal propositions, which does not differ from the list Boethius gives in *De differentiis topicis*.

This is Peter's list of different *loci*:

> I. Intrinsic *loci* from the substance of a thing:

(1) *Differentia*: From the definition.
Maximal proposition: About whether the definition is denied of the thing.
(2) *Differnetia*: From the description.
Maximal proposition: About whether the description is denied of the thing.
(3) *Differentia*: From the explanation of the name.
Maximal proposition: From whatever the explanation is removed, also the thing explained is removed.

II. Intrinsic *loci* from things which accompany the substance of the thing.

(4) *Differentia*: From the universal whole (from the genus).
Maximal proposition: If the genus or universal whole is denied of something, the species or the subjective part is denied.
(5) *Differentia*: From integral whole.
Maximal proposition: If the integral whole is posited, each of its parts is posited.
(6) *Differentia*: From a species (or a subjective part).
Maximal proposition: Whatever is predicated of a species, is predicated of a genus.
(7) *Differentia*: From an integral part.
Maximal proposition: If an integral part is destroyed, its whole is destroyed.
(8) *Differentia*: From an efficient cause.
Maximal proposition: If an efficient cause is posited, its effect is immediately posited.
(9) *Differentia*: From a material cause.
Maximal proposition: If the material and permanent cause is removed, its effect is removed.
(10) *Differentia*: From a final cause.
Maximal proposition: That whose end is good, is itself good.
(11) *Differentia*: From a formal cause.
Maximal proposition: If the formal cause is posited, its effect is posited.
(12) *Differentia*: From generation.
Maximal proposition: That whose generation is good, is itself also good.
(13) *Differentia*: From corruption.
Maximal proposition: That whose corruption is bad, is itself good.
(8-13 also have counterparts.)
(14) *Differentia*: From uses.

Maximal proposition: That whose use is good, is itself also good.
(15) *Differentia*: From associated accidents.
Maximal proposition: If the secondary of associated accidents inheres, so does the primary.

III. Extrinsic *loci*.

(16) *Differentia*: From an authoritative statement.
Maximal proposition: Any expert ought to be believed within his science.
(17) *Differentia*: From a similar thing.
Maximal proposition: The same estimation obtains about similar things.
(18) *Differentia*: From the more.
Maximal proposition: If what seems the more to inhere does not inhere, neither will that which seems the less.
(19) *Differentia*: From the less.
Maximal proposition: If what seems the less to inhere inheres, then will also that which seems the more.
(20) *Differentia*: From proportion.
Maximal proposition: The same estimation obtains about proportional things.
(21) *Differentia*: From contraries.
Maximal proposition: If one of the contraries is ascribed to something, the other is denied of the same thing.
(22) *Differentia*: From privative opposites.
Maximal proposition: If one of the privative opposites is ascribed to something, the other is denied of the same thing.
(23) *Differentia*: From relative opposites.
Maximal proposition: If one of correlated things is posited, the other is posited.
(24) *Differentia*: From contradictory opposites.
Maximal proposition: If one of the contradictory opposites is true, the other is false, and vice versa.
(25) *Differentia*: From transumption.
Maximal proposition: What belongs to something under a more well-known name, also belongs to it under a less well-known name.

IV. Intermediate *loci*.

(26) *Differentia*: From inflection.
Maximal proposition: What belongs to one inflected form, also belongs to the other.
(27) *Differentia*: From coordinates.

Maximal proposition: What belongs to one of the coordinates, also belongs to the other.
(28) *Differentia*: From division.
Maximal proposition: If two things divide something between them, then if one is posited, the other is denied.

On Supposition

The properties of terms, as this part of logic was called, contains discussion about signification, supposition, copulation, appellation, ampliation, restriction and distribution. To draw the distinction between signification and supposition, Peter notes that some things are said with construction, like 'A human being runs' or 'a white human being', and things said without construction, like 'animal'. Unconstructed terms have signification and they signify something in one of the ten categories.

A terms signification is, in Peter's words, the conventional representation of some thing by a vocal expression. Sherwood writes that signification is a presentation of the form of something to the intellect. Signification is hence when we take something, typically a word (a sound or an inscription), to stand in for something else. That word will then conventionally represent the thing or it will function as a presentation of that thing to a mind. It will convey meaning by being taken as a sign for something else.[92] Things in the world are either singular or universal, according to Peter, and hence a term that does not signify a particular or universal thing does not signify at all. These are terms like 'every', 'not' etc.

There are two sorts of signification either of a substantive thing or by an adjective or verb. Terms like 'human being' that signify a substantive thing have supposition while terms like 'white' or 'run', that is, adjectives or verbs, are said to couple. Copulation plays no role at all in their discussions.

Supposition is defined by Peter as the acceptance of a substantive term for something. Signification is prior to supposition in the sense that signification is what a term has all by itself, while supposition is what a term has in a composition. The division of supposition differs a little bit between the textbook authors, but it is foremost only a difference in terminology. I will follow Peter of Spain's terminology.

Supposition is either common or discrete, according to Peter. Common supposition is what common terms like 'human being' have, while discrete supposition is what discrete terms like 'Socrates' and 'this human being' have. Common supposition is divided into natural and accidental supposition. The natural supposition of 'human being' includes all it is naturally apt to stand for, that is, all humans that has been, are and will be. Accidental supposition is the supposition a term like 'human being' has in conjunction with something else like 'a human being

[92] This is the origin of the contemporary notion of a mental representation, which can be traced from the twelfth century logical tradition, into early modern philosophy. See [Lagerlund, 2007].

exists'. In that case 'human being' stands for presently existing humans.[93]

Accidental supposition is divided into simple and personal supposition. A common term has simple supposition when it stands for a universal, like in 'human being is a species'. Personal supposition is the acceptance of a common term for its inferiors. It is divided into determinate and confused supposition. A common term has determinate supposition if it is taken indefinitely or with a particular sign; as for example in 'a human being runs' or' Some human being is running'. Confused supposition is the acceptance of a common term for several things by means of a universal sign. In 'Every human being is running', 'human being' has confused supposition.

Peter, and William in more familiar terms, makes a further division of confused supposition. There is confused supposition by the necessity of the mode or sign ("*necessitate signi vel modi*") and confused supposition by necessity of the thing ("*necessitate rei*"). In 'Every human being is an animal', 'human being' is by the necessity of the mode taken confusedly and distributively for all its supposita. Given that all humans have their own essence, the copula 'is' is taken by the necessity of the thing for all essences that 'human being' supposits for, and therefore 'animal' is taken by the necessity of the thing for all animality in each human being. By this reasoning 'human being' is said to supposit confusedly. mobilely, and distributively. It supposits confusedly and distributively, because it holds for all humans, and it supposits mobilely because descent can be made from it to any of its supposita. 'Animal' is said to supposit confusedly and immobilely, since descent is not allowed.

In relation to supposition Peter also discussed notions like ampliation, appellation, restriction and distribution. Ampliation and restriction is a division of personal supposition. Restriction is the narrowing of a common term from a larger to a smaller supposition. In 'A white human being is running', 'white' has a narrowing effect on the supposition of 'human being'. Ampliation is the opposite, that is, the broadening of the supposition of a common term. In the proposition 'A human being can be the Antichrist', the term 'can' extends the supposition of 'human being' to future things. Tensed or modal proposition tend to have ampliated supposition.

Appellation is related to these two notions. Appellation is the acceptance of a term for an existing thing. It is distinct from signification and supposition, since it only concerns existing things while signification and supposition is wider than that and includes also non-existence. The appellata of a term are the existing things it stands for. Distribution, which has already been mentioned, is the multiplication of a common term by a universal sign. In 'Every human being', 'human being' is distributed for all humans.

[93]William of Sherwood starts his division by a distinction between formal and material supposition. Material supposition is "when a word itself supposits either for the very utterance itself or for the word itself, composed of the utterance and the signification — as if we were to say 'man is a monosyllable' or 'man is a name'. It is formal when a word supposits what it signifies." (William of Sherwood, *Introduction to Logic*, 107.) William's division is the one that is more known, but he mentions Peter's division as an alternative.

On Fallacies

The discussion of fallacies is by far the largest part of both Peter's and William's logic books. Their discussion begin by a treatment of disputations. A disputation is an activity of one person "syllogising" with another to reach a conclusion. Peter explains, that five things are needed for a disputation, the originator or the opponent, a respondent, a disputed proposition, the act of disputing and the instruments of the disputation.

There are four kinds of disputations, namely didactic (doctrinal), dialectical, probative (temptative) and sophistical. The didactic disputation comes to its conclusions from premises peculiar to each discipline and not from what seems true to the respondent. The instruments of such a disputation is demonstrative syllogisms. A dialectical disputation draws contradictions from probable premises. Its instruments are the dialectical syllogisms. Probative disputations argue from what seems true to the respondent. Sophistical disputations argue from what seems probable but is not. The difference between the last two is that the first disputation from what seems true or probable to what is true, but the latter from what seems true or probable but is not.

There are five kinds of sophistical disputations, which derive from the five goals of such disputations, namely refutation (*redargutio*), falsity (*falsum*), paradox (*inopinabile*), babbling (*nugatio*), and solecism (*soloecismus*). Refutation is the denying of what was previously granted or granting what was previously denied. Falsity occurs in the case when a proposition does not conform to reality. A paradoxical disputation is when the conclusion is contrary to the opinion of the many or the wise. Babbeling is simply the repetition of the same thing, and finally solecism is a disputation developing a discourse contrary to grammar.

Peter divides the different fallacies into two major groups and calls the first group linguistic fallacies and the second group non-linguistic fallacies. The subdivision is the following:

Linguistic fallacies of:

Equivocation,
Amphibology,
Composition,
Division,
Accent,
Figura dictionis (Figure of speech).

Non-linguistic fallacies of:

Accident,
Secundum quid et simpliciter,
Ignorantia elenchi,
Begging the question,

Consequence,
Non causa ut causa,
Many qustions as one.

Many of these are further subdivided into different cases. Most of the fallacies in this list are well known and I will not go through them all here. Peter's and William's discussions are very detailed. The fallacy of equivocation occurs when a word (name) has diverse signification and this is not realised, as for example in: 'Every dog runs, one of the heavenly constellations is a dog; therefore one of the heavenly constellations runs'. Amphibology is the diversity of judgement based on the unity of expression, as when one argues that: 'Whoever ploughs the seashore moves sand, this man is ploughing the seashore (that is, indicating that he is working in vain); therefore this man moves sand'.

The non-linguistic fallacy of accident is said to occur when something is attributed to a thing as a subject and subsequently also attributed to an accident. An example would be when it is argued that: 'Socrates is esteemed by you, but Socrates is your father's murderer; therefore your father's murderer is esteemed by you'. The fallacy of begging the question ("*petitio principii*") occurs for example when the thing to be proved is assumed as a premise. The fallacy of treating what is not a cause as a cause occurs when in some cases when a syllogism leads to an impossibility, because something impossible follows one of the premises is rejected on the grounds that it was the cause of the impossibility (although it was not). Much more could be said about these fallacies, but I will leave it at this.[94]

5.2 The Syncategoremata

Among the textbook authors there seem to be a general agreement that there are two branches of the *logica moderna*, namely the properties of terms ('*propreitates terminorum*') and an account of the function of syncategorematic terms ('*syncategoremata*'). These two branches are supposed to be theoretically exhaustive of the kinds of terms occurring in various roles in propositions. The first part was dealt with in the textbooks of logic we have seen above, but the second part was usually given a separate treatment in books devoted to the syncategorematic terms. Both Peter of Spain and William of Sherwood wrote such treatises.[95] I will here give an outline of the content of these works.

The division and content of the chapters of William's work is the following:

(1) 'Every' or 'All' (*Omnis*)
(2) 'Whole' (*totum*)
(3) Number words (*Dictiones numerales*)
(4) 'Infinitely many' (*Infinita in plurali*)

[94] See [Ebbesen, 1981b] for the history of the discussions of fallacies. See also [Grabman, 1940] and [Pironet, 2005].

[95] The discussion of syncategorematic terms was absorbed into the *Sophismata* treatises in later medieval logic. See [Pironet, 2005].

(5) 'Both' (*Uterque*)
(6) 'Of every sort' (*Qualelibet*)
(7) 'No' (*Nullus*)
(8) 'Nothing' (*Nihil*)
(9) 'Neither' (*Neutrum*)
(10) 'But' (*Praeter*)
(11) 'Alone' (*Solus*)
(12) 'Only' (*Tantum*)
(13) 'Is' (*est*)
(14) 'Not' (*Non*)
(15) 'Necessary' (*Necessario*) and 'Contingently' (*Contingenter*)
(16) 'Begins' (*Incipit*) and 'Ceases' (*Desinit*)
(17) 'If' (*Si*)
(18) 'Unless' (*Nisi*)
(19) 'But that' (*Quin*)
(20) 'And' (*Et*)
(21) 'Or' (*Vel*)
(22) 'Whether' or 'Or' (*An*)
(23) The particle '*Ne*'
(24) 'Whether ... or ...' (*Sive*)

William begins his discussion of the syncategorematic terms by stating that if one is to understand anything one must understand its parts and this goes for statements (*enuntiationes*) as well. The approach to language is thus intrinsically compositional. He continues by pointing out that there are two parts, namely principal and secondary. The principal parts are the substantive terms and verbs, that is, the part that has supposition. The other part includes adjectival names, adverbs, conjugations and prepositions. Some secondary parts of language is determinations of principal parts in respect of the things belonging to them, like 'white' in 'a white man', and such parts are not syncategorematic terms. Other secondary parts are determinations of principal parts insofar as they are subjects or predicates, like 'every' in 'every human being is running'. 'Every' does not signify that something belonging to 'human being' is universal, but rather that 'human being' is a universal subject. The secondary parts of this kind are syncategormatic terms. I will present some of William's discussion of these terms below.

The first syncategorematic term treated by William is 'Every' or 'All'. He divides his discussion of this term respectively with regard to its signification and to its function. 'Every' or 'All' signifies universally either as a disposition of a thing, as in 'the world is all', in which case it is not a syncategorematic term, or as a disposition of a subject in which case it is a syncategorematic term. In 'Every human being is running', 'every' signifies that the term 'human being' is universal with respect of serving as a subject and standing in a relation to a predicate.

The function of the term 'Every' or 'All' is to divide the subject in respect of the predicate. This means that the term gives the condition for attaching the

predicate to either (i) at least one thing of each kind under the subject, or (ii) to each individual thing belonging to the subject. William is not explicit as to which of (i) and (ii) he prefers. The example 'Every dog is an animal' is analyzed in (i) into a conjunction of 'A collie is an animal', and 'A puddle is an animal', etc. In (ii), it is analyzed into 'Fido is an animal', and 'Spot is an animal', etc. William's discussion of 'Every' or 'All' is very detailed and goes through all cases and examples in which this term functions as a syncategorematic term.

William seems to be one of the first logicians to treat 'is' as a syncategorematic term. The first question he addresses is whether 'is' pertains to the composition of the subject and the predicate or if it pertains to the predicate. To explain this he recalls the distinction between 'signification' and 'consignification', and explains that 'is' is not really a third part of the proposition, that is, a composition, since it signifies that which is said of something other than itself and this is the predicate, but it does consignify the composition, which is the function of the copula.

'Is' is also equivocal for sometimes it indicates actual being (*esse actuale*) and at other times conditional being (*esse habituale*), which is something with some nature, but which does not actually exist although it could. If in 'Every human being is an animal' 'is' is taken in the first way, then it is false when no humans exist. If it is taken in the second way, then it is true no matter whether there are any human or not. In this way, William commits himself to possibilia or to things that could exist but does not. He notes himself that this amounts to saying that things that do not exist but could have a diminished kind of being (*esse diminutum*).

I will end by noting some interesting things said by William about modal terms like 'necessarily' and 'contingently'. These can be taken both as categorematic and syncategorematic terms. If they are taken in the first sense, then they determine the predicate and if they are taken in the second sense then they determine the composition of the subject and the predicate, but as we just saw, if something determines the composition then it also determines the predicate. He writes further that:

> If necessarily' is an adverb, then it determines something that is an act and is in the manner of an act. But the composition belonging to the verb in a sentence is not consignified in the manner of an act; therefore an adverb does not determine it. The assumption is clear, for the thing belonging to the verb to which the composition belongs *per se* is signified not in the manner of a substance but in the manner of an act proceeding from the subject, while that to which the act belongs is always signified in the manner of a substance. And thus it seems that 'necessarily' determines the predicate in the second case just as in the first.[96]

The argument is really interesting. In the proposition 'Socrates runs', 'runs' signifies the act of running, but it consignifies the composition of the act with the

[96]See William of Sherwood, *Introduction to Logic*, 100-1.

substance 'Socrates'. If 'necessarily' is to function like other adverbs, it will not determine the composition, but only the act. The adverbial analysis of modal terms was standard, as noted above, and argues here that such an analysis actually implies that the modality determines the predicate and not the composition. The reason this is important is that it seems to imply that modal propositions are reducible to non-modal propositions with funny predicates. If the modality is not a syncategorematic part of a proposition, then it must be a categorematic part and as a consequence we do not really have any modal propositions.

William will next try to preserve the distinction by saying that a modal term taken syncategorematically determines the predicate, but in a different way than the same term taken categorematically. He writes that "in the first case [taken categorematically] it determines the verb in respect of the thing belonging to it, in the second case [taken syncategorematically] in respect of the composition belonging to it, or insofar as it is a predicate."[97] He then proceeds to indicate what this means by solving the sophisma that 'The soul of the Antichrist will be necessarily'. The soul of the Antichrist will have necessary being because when it exists it will have unceasing and incorruptible being, but the soul of the Antichrist is contingent, since it is possible that it will not exist. If the term 'necessarily' is taken categorematically, the sentence is true, but if it is taken syncategorematically, then it is false. If the term determines the predicate in respect of the composition, then 'The soul of Antichrist contingently will be' is true.

6 CONCLUSIONS

In this chapter, I have tried to give an overview of the development of and doctrines present in Western logic from 1150 to 1250. I have focused on how Arabic and Aristotelian logic was incorporated into an existing tradition, how this changed the tradition and led to new developments in the *logica modernorum*. The chapter ends with an outline of the logic of the thirteenth century as present in some influential textbooks. I am here not going to summarise what has been said above. Instead I want to highlight a few common themes and important developments.

The first and most important thing to note is the development of the *logica modernorum* from the intensive discussions of fallacies and grammar in the late twelfth century. This new branch of logic feeds into the thirteenth century and forms a central part of the most important textbooks of the time. It involves two parts, namely the properties of terms, which is foremost constituted by supposition theory, and the theory of syncategorematic terms. I also noted that this part of logic played an insignificant role in the commentary tradition in the thirteenth century.

The second thing to highlight is the growing commentary tradition on the *logica antiquorum*. It is in the early thirteenth century that this tradition really gets going as part of the teaching practice in the newly established universities. I

[97] See William of Sherwood, *Introduction to Logic*, 101.

also emphasised that there is a tendency in the early commentaries to interpret Aristotle's logic in a strongly metaphysical way. This made modal logic very important and fused logic and science/metaphysics. I also suggest that this way of understanding Aristotle was at least partly influenced by Arabic thinkers.

The third main aspect of this overview has been an outline of some of the logical theories present in Latin translations of Arabic logical texts. I spent some time outlining the logic of Al-Ghazālī and the Aristotelian commentaries of Averroes. I noted that both these seem to have been influential, but also said that it is difficult to judge exactly to what extent. Al-Ghazālī's division of the matter of the syllogism can be found in several logic texts from the thirteenth century. His and Avicenna's view that logic is about intentions in the intellectual soul and hence a science of reason was also influential. A third aspect is the heavy metaphysical understanding of Aristotle already mentioned.

Overviews like the present one are difficult to write and there are many things I have missed and also consciously skipped. I have said nothing about the works on *Insolubilia* and *Obligationes* that began to appear at this time. I have also skipped the commentary tradition on Aristotle's *Posterior Analytics*. Although scholars of medieval logic have come a long way towards understanding the development of logic in the thirteenth century there are still lots of things left to do. None of the hugely influential commentaries on Aristotle by Robert Kilwardby have been edited, and most of the early commentaries on the *logica nova* have not been studied. I hope this overview will work as an inspiration to know more about thirteenth century logic. It holds lots of riches for anyone interested in Aristotelian logic and also anyone interested in the philosophy of language and logic in general.

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LOGIC AND THEORIES OF MEANING IN THE LATE 13$^{\text{th}}$ AND EARLY 14$^{\text{th}}$ CENTURY INCLUDING THE MODISTAE

Ria van der Lecq

INTRODUCTION

The scope of this chapter is the period between 1270 and 1320, of which Gordon Leff made the following observation:

> "No phase of Scholasticism, certainly in terms of our present knowledge, remains more confused than the events in the decades immediately each side of the year 1300." [Leff, 1976, p. 32]

Obviously, a considerable amount of research has been done since Leff wrote these lines, but it is still not so easy to draw a clear picture of this period. At first sight, the image of what happened just before and after the year 1300 looks more like a battlefield of competing metaphysical theories than like an orderly development of the ideas of earlier logicians. The contextual approach of logic seems to be abandonned and the discussions are dominated by epistemological and ontological questions. Nevertheless, as I hope to make clear, some of the 14$^{\text{th}}$ century innovations cannot be appreciated without the knowledge of the work of the late 13$^{\text{th}}$ and early 14$^{\text{th}}$ century philosophers.

Late 13th and early 14$^{\text{th}}$ century logic may be characterized by its interest for grammatical and metalogical problems. Grammar and logic were two of the traditional seven liberal *arts*, but the works of the logicians and grammarians in this period reflect an interest in grammar and logic as sciences, rather than as arts. There was a common opinion that every science has to have an immutable object, but the texts show a considerable disagreement about the origin, status and ontological foundation of this object in the science of logic.

In the debates about the meaning of words we see that referential problems were solved in terms of signification rather than supposition. Beside 'signification' the key notion was 'imposition'. The basic function of language is to signify things, and imposition is the way words acquire their meanings. This is supposed to work as follows: a first *impositor* investigates things and their properties and then decides which sound (*vox*) should be used to signify that object. When this sound has been imponed to signify some thing, it becomes a sign and it has acquired

signification. In principle a word cannot loose its signification. However, by the same imposition it has acquired certain properties (modes of signifying), by which it can fulfill different functions in different contexts.[1]

The approaches of logic and semantics in this period are further characterized by a close connection with epistemology and ontology, to such an extent that some scholars speak of an 'epistemological turn' around 1270.[2] The developments were determined by the growing influence of the 'new' Aristotelian works: *Posterior analytics, Metaphysics* and *De anima,* and also the Arabian philosophers had their influence on the philosophical discussions in the 13$^{\text{th}}$ century.

In Paris the grammarians of the modistic school (*modistae*) dominated the discussions, but many of them also wrote works on logic. Logicians as well as grammarians were interested in language and meaning, but from a different point of view, as shall be explained below. Their common object was the so-called second intention. For that reason, the authors that will pass by in this chapter are called *intentionalists* as opposed to the terminists of the previous and following periods. The intentionalists include well-known philosophers as Thomas Aquinas (d. 1274) and Duns Scotus (d. 1308), but also less known authors as Radulphus Brito (d. 1300), Simon of Faversham (d. 1307), Hervaeus Natalis (d. 1320) and Peter Aureol (d. 1322). Some of them wrote logical treatises, such as commentaries on the *Organon*, logical questions and sophisms, but more often we have to extract their opinions from theological and epistemological works.

The most striking characteristic of this period is perhaps the fact that the language of supposition is virtually absent.[3] The focus was on signification and modes of signifying rather than on supposition. When terms of terminist logic appear, they appear in the framework of the modes of signifying, modes of understanding and modes of being. There was a terminist tradition at Paris from 1250 up to 1270-5[4] and a continuous tradition of terminist logic in Oxford,[5] but as far as I can see there were no new developments until the beginning of the 14$^{\text{th}}$ century when Walter Burley wrote his *De suppositione*. His work will be discussed in one of the next chapters.

Therefore, I shall leave terminism aside. What I would like to show, however, is how the discussions about the foundations of logic and semantics paved the way for the important innovations of the 14$^{\text{th}}$ century. In this context, I shall also pay attention to Roger Bacon's theory of signs.

I have tried to capture the most important developments by dividing this chapter in two main parts: (1) theories of meaning and (2) the logic of intentions. The first part concentrates on theories concerning the relations between language, thought and reality; in the second part I shall focus on discussions about the proper subject of logic.

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[1] Ebbesen [1979, p. 46].

[2] E.g. De Rijk [2005, 79 ff].

[3] Pinborg [1979, p. 21].

[4] De Libera [1968]. Braakhuis [1997, p. 138] argues that the handbooks of the *Logica moderna* were being used in preparatory teaching in schools other than universities.

[5] See Pinborg [1979].

PART I: THEORIES OF MEANING

Names, concepts and things: the semantic triangle

The question 'what does a word signify?' has puzzled philosophers throughout the ages. It had been a subject of discussion in the Latin west ever since Boethius gave his interpretation of the famous passage of *Peri hermeneias* chapter 1, where Aristotle gives his view on the relation between language, thought and reality, or so it seems:

> Now spoken sounds are symbols of affections in the soul, and written marks symbols of spoken sounds. And just as written marks are not the same for all men, neither are spoken sounds. But what these are in the first place signs of – affections of the soul – are the same for all; and what these affections are likenesses of – actual things – are also the same.[6]

Thus, Aristotle sees written words as signs of spoken words and spoken words as signs of 'affections of the soul'. These affections or passions of the soul are the same for every human being, because they are likenesses of extra-mental things, which are also the same for everybody. This passage of Aristotle's has been the subject of numerous commentaries, ancient as well as modern, and may well be considered as one of the basic texts in the history of semantics.

Boethius' first important remark concerns the interpretation of the term 'affections of the soul'.[7] Boethius makes it very clear that what Aristotle meant by an 'affection of the soul' is a concept or understanding (*intellectus*)[8]. The relations between word (*vox*), thing (*res*) and concept (*intellectus*) are formulated as follows: the thing is *conceived by* the *intellectus*, the spoken word *signifies* the conceptions of the soul.[9] Thus the relation between concept and thing is a relation of conceiving, whereas the relation between spoken word and concept is characterized with the term 'signify' (*significare* or elsewhere: *designare*). The concept seems to be the *only* thing that is signified,[10] although this exclusive position of the concept is moderated by the statement that the *intellectus* is the primary significate of the noun, the thing (*res*) being the secondary significate. The relationships between word, thought and object, according to Boethius, can be visualized in a triangle, the so-called semantic triangle:

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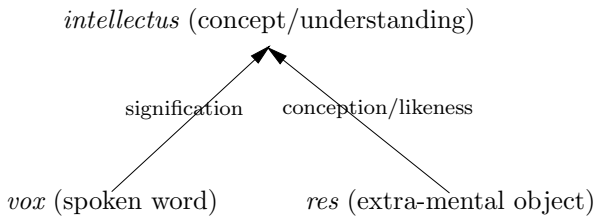
[6] Aristotle, *Peri hermeneias*, 16a3-8. Transl. Ackrill, in [Ackrill, 1963, p. 43].

[7] Anicii Manlii Severini Boetii *Commentarii in librum Aristotelis Peri Hermeneias*, rec. C. Meiser, pars posterior, secunda editio [Leipzig, 1880].

[8] Boethius, *op.cit.,* p. 12, line 28. *Intellectus* is an ambiguous term, like the German term 'Begriff' or the Dutch 'begrip'. It signifies the act of conceiving as well as the product of that act.

[9] Boethius, *op.cit.,* p. 20, lines 17-20.

[10] Boethius, *op.cit.* p. 21, lines 4-5.



Now, why is it that Aristotle speaks about the affections of the soul as likenesses of things? Boethius' answer is that the mind, when it understands, grasps not the object itself, but an image, i.e. a likeness of the object.[11] The conception is, as it were, affected by its object, the image of the thing, whereby it becomes itself a likeness of the thing that is represented by the image.[12] In other words, the *intellectus* is called an affection of the soul because it is affected, or rather impressed,[13] by the image, so that eventually it bears a resemblance with the thing in reality.

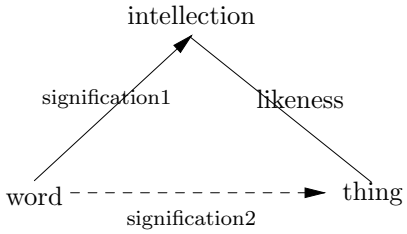
So far the most important points from Boethius' commentary. His influence is clear in Peter Abelard's earlier works, but, more than Boethius, Abelard makes his readers aware of the fact that, beside the concept, there is another candidate for the role of primary significate of a spoken word: it is the extra-mental object itself. In his view words signify things (*res*) as well as acts of the intellect (*intellectus*). The *intellectus* is the primary significate of the spoken word, whereas the thing in reality is the secondary significate. In reality, however, since something has to exist before it can be known, the thing is prior to the concept, so that naturally speaking the signification of the thing comes first.[14] The signification of things is the subject of the *Categoriae*, whereas the signification of concepts is discussed in the *Peri hermeneias.*

Abelard gives three reasons why the *Peri hermeneias* is about the relationship between words and concepts rather than about the relationship between words and things. First, in the *Peri hermeneias* Aristotle makes a distinction between the parts of a proposition: the noun and the verb. Well, the difference between a noun and a verb does not correspond with a distinction between different things in reality. For example, the words 'driver' and 'driving' signify the same object under different modes of conceiving (*modi concipiendi*). The second reason why the *Peri hermeneias* is about the signification of concepts rather than about the signification of things is that there are no things signified by a proposition. A proposition consists of a noun and a verb, and in the understanding of this proposition the understandings of the parts are united. Finally, things are transitory, whereas concepts are permanent, in the sense that the constitution of a concept does not depend upon the actual existence of the object. Thus, in forming a proposition

[11] E.g. Boethius, *op.cit.,* p. 35, lines 3-6.
[12] Boethius, *op.cit.,* p. 35, 6-8.
[13] Boethius, *op.cit.,* p. 34, 13-4.
[14] Peter Abelard, *Logica ingredientibus*, 112,31 - 113,3.

the mental act is important, not the naming of things.[15]

Thus, for Abelard the semantic triangle could be drawn as follows:



Now, Boethius had said that an understanding is empty if there is no thing in reality of which it is a likeness. In other words, if we are thinking, there must be something to think about. Our naming and thinking have to have a foundation in reality. This is no problem with individual names, but the question is: what is the extra-mental 'thing' when a word is a common name, since there are no common 'things' in reality. This is the well-known problem of universals, for which Abelard had an original solution.[16]

Until the end of the 13th century signification is considered to be primarily a relation between word and concept/species or between spoken word and extra-mental thing. The common opinion was that words primarily signify concepts and only secondarily, through the conception of the mind, extra-mental objects, but the question 'what does a term signify?' was always about the *spoken* word. In the 14th century, however, the situation seems to have changed. For example, in Ockham's opinion the concept (mental term) itself is considered a sign, the extra-mental thing being its significate. The relation between word and thing is dependent upon the relation between concept and thing: whenever the signification of the mental term changes, the signification of the vocal name changes as well.[17] This important change (from vocal to mental term as point of departure) has been prepared by the 13th century Oxford philosophers and grammarians, who were the first to develop an original theory of signs.[18] Among these authors the most important one is Roger Bacon (d. ±1294), an Englishman, but working for an important part of his life in Paris as an arts teacher. We shall pay ample attention to his theory of signs, but first we shall have to look at other important developments in this period.

The younger Thomas Aquinas: Primary signification of the species

In his younger years Thomas Aquinas was an advocate of the common opinion, i.e. words primarily signify concepts and only secondarily, through the conception of

[15] *op.cit.*, 308-9.
[16] See chapter 3, pp. 83–155.
[17] Ockham, *Summa logicae*, *Opera Philosophica I*, p. 8, line 30-35.
[18] Joel Biard [1989].

the mind, extra-mental objects. In fact, he held this position all his life. But before the 1260s he seems to identify concept and intelligible species (mental image). In his commentary on the *Sentences* Aquinas distinguishes between three kinds of names. First, there are names for extra-mental things. When the intellect conceives a likeness of an extra-mental thing, for example a stone, the word 'stone' signifies that mental image, which is elsewhere called 'intelligible species'. This image is not simply a picture of the extra-mental object, but an abstraction: it does not contain individual features. Second, there are names that signify intentions like 'genus' or 'species', with only a remote foundation in reality. Finally, there are names that signify concepts without foundation in reality, like the concept of 'chimera'. In the first and the third case names signify concepts in their function of images of existing or non-existing things; in the second case they signify concepts of concepts.[19]

The older Thomas Aquinas: Primary signification of the concept

In his more mature writings[20] Aquinas does not identify intelligible species and concept any more.[21] The concept — or 'the thing conceived', also called *intentio intellecta*[22] — is the essence of the extra-mental object as it is abstracted by our intellect from the object's being. For example, the concept 'man' is the result of an abstraction of the essence of a man, leaving his particular existence out of consideration.[23] This concept is signified by the spoken word 'man'. It is also that which is understood. Individuals cannot be primarily signified, because they cannot be primarily understood. Unlike the sensitive faculty of the soul, the intellective faculty does not have a direct cognition of extra-mental objects: it understands by virtue of an intelligible species and its object is the concept or mental word (*verbum mentis*).

In the scholarly literature there is a discussion about the interpretation of Aquinas's mature theory. Some authors state that the mental word (also called 'intention') should be identified with the extra-mental thing insofar as it is understood; others say that it is a product of the intellect completely distinct from the extra-mental thing.[24] As we shall see below, Duns Scotus seems to refer to the first interpretation: words signify things insofar as they are understood, but both interpretations have their advocates.[25] It should be noted, however, that in medieval philosophical writings the terms 'concept' and 'intention' are semantically ambivalent in the sense that a concept always includes what it intends to signify.[26] Concepts always have a foundation in reality, directly or indirectly. The

[19]See the paragraph on second intentions in this chapter.
[20]Especially his commentary on *Peri hermeneias*, the *Summa theologiae* and *De potentia*
[21]E.g. Thomas Aquinas, *Summa theologiae* I.82.2.
[22]See below, p. 372.
[23]This is the so-called 'non-precisive abstraction'. Precisive abstraction is the abstraction of the form; the result would be the concept 'humanity'.
[24]Cf. Pini [1999, p. 49], who speaks of a 'realistic' vs. a 'mentalistic' interpretation.
[25]See Pini [1999, pp.49–50] for examples.
[26]De Rijk [2005, pp. 24-5] and *passim*.

intellect grasps extra-mental things through concepts that designate these things, including their being taken qua conceived of. The thing understood is the *thing* understood as well as the thing *understood.* Thus, concepts have a mental side and a 'thing' side, so that it is not meaningful to speak of a 'realistic' vs. a 'mentalistic' interpretation in this context.

Grammatica speculativa: the Modists on modes of signification

Medieval speculative grammar has its origin in ancient Latin grammar as it has been transmitted in the works of Donatus (4th century) and Priscian (6th century). In the early Middle Ages grammar was one of the seven liberal arts, in which the arts that were organised in the so-called *trivium* (grammar, logic, rhetoric) took an important place. Grammatical discussions were influenced by the logicians, and in their turn the grammarians played an important role in the emergence of terminist logic, e.g. in the development of the theory of supposition.

The grammarian of the 12th century was an artist and grammar was an art, but the situation changed under the influence of the recovery of the Aristotelian corpus, especially the *Posterior Analytics,* with its emphasis on universal and immutable entities as objects of science. For the grammarians, who wanted to give grammar the status of a science, it meant that vocal expressions, which differ from one language to another, could no longer constitute the true objects of grammar. Thus, grammarians became philosophers speculating about meaning, not the meaning of individual words, but meaning in general, the semantic component of language, which was considered a universal feature of language. The scientific approach to grammar was accordingly called 'speculative grammar'.

Around 1270 a new theoretical framework was established, based on the concept of meaning (*significatio*), or rather the concept of 'mode of signifying' (*modus significandi)*; the grammarians who wrote within this theory were accordingly called 'Modists' (*modistae*). The Modists were mostly Parisian masters, who wrote not only on grammar, but also on logic and metaphysics. The first representatives are Martin and Boethius of Dacia and their Danish compatriots John and Simon (all between 1255 and 1270).[27] The most important author is perhaps Radulphus Brito (± 1290), who provided the most comprehensive discussion of speculative grammar after Boethius of Dacia.[28] The last significant member of the group was Thomas of Erfurt (shortly after 1300), whose work is considered the most complete modistic treatment of grammatical theory available.[29]

Representing the "second generation of speculative grammar"[30], the Modists codified and refined the pioneer work of scholars such as Peter Helias, Robert Kilwardy and the early Roger Bacon. As a result of their philosophical background, they believed that there is a structural parallel (although not a one-to-one correspondence) between language, thought and reality. Since reality is the same for

[27]For more information about these modists and their works, see Rosier, [1983, pp. 18–21].
[28]Pinborg [1982, p. 256].
[29]Thomas of Erfurt, *Grammatica speculativa*, in: Bursill-Hall [1971].
[30]Bursill-Hall [1972, p. 21].

everyone, they claimed that there has to be one universal grammar with rules that are independent of any language in which they might be expressed. Not surprisingly, their work has often been compared with Chomsky's generative grammar.[31]

Thus, at the end of the 13th century the goal of grammar was no longer to describe how to write and speak well, but to explain the conditions for correct discourse in an idealised, perfect language. The Modists stressed the universal deep structure as opposed to the surface differences between languages. In their view grammarians study linguistic expressions, but not in the same way as logicians. Logicians study propositions as the bearers of truth and falsity. They are interested in the logical form of sentences and argumentations. Medieval grammarians are not interested in truth conditions, and not at all in argumentation, but in the linguistic form (*congruitas*) of sentences. Grammarians analyse the structure of language, not its content, although the relation between language, thought and reality has to be taken into account. The proper object of modistic grammar is the construction (*constructio*) of linguistic expressions.[32]

Modus significandi, modus intelligendi, modus essendi

A construction in grammar is a union of two parts of speech[33] (e.g. a noun and a verb, or a noun and an adjective), each of which has its own mode of signifying (*modus significandi*). Modes of signifying (*modi significandi*) are the principles of grammar. According to modistic analysis a word consists of a phonological component (*vox,* an articulated sound) and two levels of semantic components. On the first level a *vox* has a semantic meaning (*ratio significandi*) by which it is associated with an object or content, and on the second level it has a grammatical meaning on which the syntactical component depends. This grammatical meaning is called *ratio consignificandi* or *modus significandi.* The semantic meaning turns a *vox* into a *dictio* and is responsible for the relation between a *vox* and something outside the mind (*res*). This 'object' outside the mind is not a particular thing, but a so-called *essentia*,[34] a content not yet determined in a category. It could be a substance, but also an accident. So, a *dictio* is not a term in the logical sense, but something more abstract. Let us take as an example the English word 'drive'. In modistic terms this would be a manifestation of the *dictio* driv*, which includes all occurrences of the words 'drive', 'driver', 'driving', 'driven', and their derivations. And since the Modists use Latin as their model language, it includes also flexional forms.The *dictio* is the highest common factor of these words and word forms.[35] For a *dictio* to become a linguistic expression and a particular

[31] See e.g. Enders & Pinborg, Introduction to Radulphus Brito, *Quaestiones super Priscianum minorem*, [1980, p. 49].

[32] Rosier [1984, p. 30].

[33] Radulphus Brito, *Quaestiones super Priscianum minorem*, qu. 8, 1980.

[34] Pinborg [1972, p. 113].

[35] Pinborg [1972, p. 115] proposed to use the term 'lexeme' for *dictio,* in the sense given to the term by John Lyons. For a discussion on the applicability of this term, see: Marmo [1994, p. 127].

part of speech, one or more *modi significandi* are necessary, e.g. the modes of noun, nominative case, singular. The *dictio* itself does not appear in linguistic expressions, only its realisation as a particular part of speech. Thus, when the *dictio* driv* possesses the *modi significandi* of noun, nominative case and singular, the word 'driver' appears as a part of speech. The process can be stated in the following manner:

sound (*vox*) + semantic meaning (*ratio significandi*) > dictio
dictio + grammatical meaning (*ratio consignificandi, modus significandi*) > part of speech
part of speech + other part of speech > construction

For a construction to be well formed (congruous) it is necessary that the mode of signifying of one part of speech is compatible with the mode of signifying of the other part.[36] The following example from the English language may help to make this clearer. For the expression 'playing children' to be well-formed, it is necessary that 'playing' possesses the mode of signifying of a participle and 'children' the mode of signifying of a noun (substantive), of number (plural) and of case (nominative). In the expression 'playing games', on the other hand, the mode of signifying of an accusative case is required. However, for the expression 'logic matters' to be well-formed, there are two possibilities: either 'logic' possesses the mode of signifying of a noun and 'matters' the mode of signifying of a verb, or 'logic' possesses the mode of signifying of an adjective and 'matters' that of a noun. The difference between the two interpretations can be visualized as follows: 'logic <- matters' and 'logic -> matters' respectively, the arrows denoting the so-called dependency. A grammatical construction will only be perfect, if its parts match not only syntactically, but also semantically. The expression 'crying toothbrushes', for example, would be syntactically well-formed (congruous) according to the grammar check of my computer, but in most cases we would agree that it is not well-formed semantically,[37] because the expression doesn't make sense, except perhaps metaphorically or in sentences like 'I was dreaming of crying toothbrushes'. Nevertheless, the grammarians were primarily concerned with congruity of speech, not with truth. What mattered was the way reality was described, not reality itself,[38] but they needed to take the structure of reality into account, if only as a warrant for the scientific and universal status of their doctrine. Therefore, the *modes of signifying* (*modi significandi*) had to have an ontological counterpart, which the Modists called: the *modes of being* (*modi essendi*). Furthermore, since the modes of being cannot be signified without being understood, the Modists also needed a mental counterpart: the *modes of understanding* or conceiving (*modi intelligendi*).

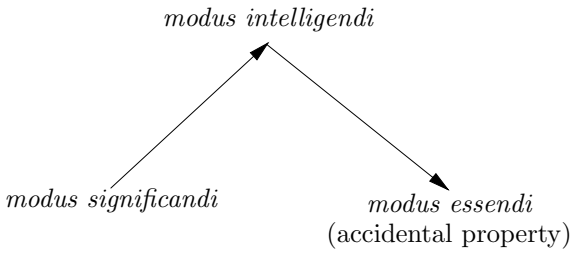
The modes of being were considered accidental properties of the objects outside the mind. The intellect knows the objects through these accidental properties. In

[36] Radulphus Brito [*op.cit.*, qu. 12] speaks of "proportion and similitude" being required for a construction.

[37] According to Thomas of Erfurt [c. 53, Bursill-Hall, 1972, p. 309], the expression *cappa nigra* (white hood) is congruous and proper, and a construction such as *cappa categorica* (categorical hood) is improper but congruous.

[38] Pinborg [1982, p. 261].

other words, the accidental properties give rise to our concepts: the *modi intelligendi* (modes of understanding). Thus, the *modi significandi* correspond with the *modi intelligendi* and through these they find their foundation in the reality of the *modi essendi*, according to the following triangle,[39] which, of course, mirrors the semantic triangle on p. 351 above:



The triangle reflects a parallel between semantics and epistemology: the accidental properties (as distinct from the substantial form) of an object are the basis of our knowledge of the essence of that object, and words (co)signify objects through concepts.

The mode of being (*modus essendi*) is the property of the thing considered absolutely, i.e. without any consideration of the way it is signified or understood. However, because objects can be understood and signified, the mode of being is materially the same as the passive mode of understanding (*modus intelligendi passivus)* and the passive mode of signifying (*modus significandi passivus).*[40] There is, of course, a formal difference: the passive mode of understanding (the mode of being understood) is that same accidental property as understood by the mind and the passive mode of signifying (the mode of being signified) is that property as signified by the vox.[41] It is the same accidental property, in three different forms. On the active level, the modes differ also materially.[42] The active mode of understanding (*modus intelligendi activus*) is a property of the intellect, the ability to understand; the active mode of signifying (*modus significandi activus*) is a property of the *vox*, the ability to signify.

The active mode of signifying is the fundamental element in grammar.[43] Every part of speech has its own modes of signifying, which are derived eventually from the modes of being of the object. The noun 'man', for example, has a mode of signifying corresponding to the mode of being human, as well as modes of signifying derived from the mode of being an autonomous entity and the mode of being predicable of different individuals.[44] The modes of being (the accidental properties of the object) give rise to concepts such as 'man', 'substance' and 'species'. So,

[39]Marmo [1994, p. 144].
[40]Radulphus Brito, [*op.cit.,* qu. 18].
[41]Bursill-Hall [1971, p. 91].
[42]Radulphus Brito, [*op.cit.*, qu. 19].
[43]Thomas of Erfurt, *Grammatica speculativa*, c. vii [Bursill-Hall, 1972, p. 148].
[44]Pinborg [1982, p. 263].

the modes of signifying are derived from the modes of being in their function of passive modes of understanding, i.e. as they have been understood by the mind.[45] As soon as a concept (i.e. an active mode of understanding) has been formed, it can be expressed by a articulated sound (*vox*) in the form of an active mode of signifying. As a result, the grammatical signification takes place and the part of speech is produced.[46]

Notwithstanding their close relationship to the objects in reality, the modes of signifying only reflect our way of imagining such objects, so that they may denote non-existent objects, like blindness or darkness,[47] as well. A mode of signifying need not have an ontological counterpart, as long as the object is not inconceivable.[48]

Now we need to explain how the Modists take account of semantic signification. A word like 'woman', for example, not only has various modes of signifying, like the mode of having a stable disposition (*modus habitus*, which makes it a noun), the mode of independence (which makes it a substantive noun) and the mode of singularity (which makes it a singular substantive noun), it also has a semantic content. Modes of signifying are derived from the accidental properties of the object, but the semantic content of the word is derived from the object itself. Thus, we have two parallel operations: one starting from an accidental property of an object and the sound (*vox*), resulting in a part of speech with an active mode of signifying, and another starting from the object itself and the sound, resulting in a vocal sign or *dictio* with signification in the classical, Aristotelian sense. Grammatical and semantic signification is united in a linguistic unity, e.g. 'woman'. Co-signifying the properties of rest, independence and singularity, 'woman' signifies an extra-mental woman, or in other words, 'woman' signifies its extra-mental object under the mode of rest, independence and singularity.[49]

The question remains how words acquire the meaning they have. According to the Parisian masters of the end of the 13th century the meaning of a word depends on the imposition. When a word has been imposed to signify something – some masters think that this has been done by a person: the impositor – it cannot stop to signify that particular object, since the imposition concerns the common nature of the object, which was thought to be eternal. This becomes a problem in expressions like 'dead man', since 'man' signifies a living being. Consequently, the expression would contain a contradiction. In order to avoid this consequence, a distinction between primary and secondary signification was introduced. The distinction concerns so called 'analogical' terms, one of which is 'man'. The primary significate of this term is man; the secondary significate is corpse. When it stands in isolation, 'man' signifies just one significate, but when an adjunct is added whose sense is incompatible with the primary significate, the

[45] Bursill-Hall [1971, p. 92[.
[46] Bursill-Hall [1971, p. 103].
[47] These terms denote a so-called privation. See: Radulphus Brito, *op.cit.*, qu. 21, 165.
[48] Pinborg [1982, p. 263].
[49] Rosier [1983, pp. 53–5].

term represents the secondary significate only. In this way the theory of imposition was saved.[50]

It should be noted that earlier and later philosophers found the solution of problems like this in the theory of supposition. The Modists, however, neglected the terminist approach, probably because their main interest was in statements about eternal entities and their essential properties, whereas supposition theory was useful and interesting for the analysis of propositions about contingent objects and states of affairs.[51]

Modistic grammatical theory results from a complete interdependence between the structure of reality and the operations of the mind. The Modists, as moderate realists, assumed that reality had a definite structure that was mirrored in cognition and in language. Nominalists like Ockham and Buridan rejected the framework of the theory because it confuses linguistic distinctions with real ones.[52] Nevertheless, the importance of the modistic approach has often been recognised,[53] if only because of the fact that modistic grammar was the first serious attempt to treat linguistics as a science. The foundation of grammar in reality was one of the consequences of their Aristotelian concept of science. As such it is "an expression of the intellectual climate which produced it."[54]

Primary signification of the thing: Roger Bacon on signs

The life of Roger Bacon (ca. 1214–1292) covered a large part of the 13th century. Born in England and having studied in Oxford, he began his career as an arts teacher at the University of Paris in the early 1240s. In this period he wrote works on metaphysics, natural philosophy and grammar. In one of these works, the *Sumule dialectices* (ca. 1250), Bacon proposes that as such names are names only of presently existing entities, a claim which is not considered a common opinion.[55] Although Roger Bacon shows some familiarity with modistic grammatical theories, he does not apply the concept of *modus significandi* in the same way, not even in his grammatical work. He also seems to be acquainted with terminist logic, but the terminist notions do not have a central place in his work.[56]

In 1262 he wrote a remarkable piece of work: *De multiplicatione specierum,* and in the years 1267-1270 he developed a theoretical framework for language studies, a theory of signs, partly adapted from Augustine's doctrine in *De doctrina christiana*, but nevertheless original in its presentation. This theory appears for the first time in the treatise *De signis*[57] (ca. 1267) and is best known from his

[50]For a full account of this problem, see Ebbesen (1979), 43–70.
[51]Ebbesen [1983, pp. 73–4].
[52]Pinborg [1982, p. 257].
[53]See e.g.: [Bursill-Hall, 1972, p. 26; Rosier, 1983, pp. 202–3; Covington, 1984, pp. 126–32].
[54]Bursill-Hall [1971, p. 36].
[55]Maloney [1988, p. 3].
[56]Biard [1989, pp. 29–30].
[57]Edited by Fredborg *et al.*, [1978].

Compendium studii theologiae (1292),[58] his last, uncompleted, work. Especially the last two treatises justify a treatment of Roger Bacon's semantics in a chapter on logic and theories of meaning in the late 13th century.

Signification is usually treated as a property of words or concepts. Bacon, however, places the signification of words in a broader context of a theory of signs. In *De signis* as well as in the *Compendium*[59] Bacon gives a classification of signs. In the *Compendium* he claims that it is original, although he admits that Augustine has a similar classification in *De doctrina christiana*.[60] A sign, he says in *De signis*, is "that which once presented to a sense faculty or intellect designates something to that intellect."[61] This definition is similar to Augustine's except for the fact that Bacon's definition allows for the case where something imperceptible by a sensory faculty is a sign, whereas Augustine's definition refers to sensible signs only: "For a sign is a thing which of itself makes something other come into thought beside the impression it makes upon the senses." Bacon defends his choice for a broader notion by referring to Aristotle who says that passions of the soul (mental presentations) are signs of the things outside the mind.[62]

According to Bacon, a sign is either (1) natural or (2) given by a living being (*data ab anima*). (1) Natural signs occur in various modes, depending on the kind of relationship between sign and significate. For example, in the (1.1) *inference* 'she has milk; therefore she has given birth', there is a (1.1.1) *necessary* relationship between sign (having milk) and significate (having given birth). Likewise, dawn is a natural sign of imminent sunrise through necessary inference. In other cases, for example in 'the ground is wet; therefore it has rained' there is a (1.1.2) *probable* relationship between sign and significate. In yet another sense, a footprint in the snow is also a natural sign: it signifies the person whose it is.[63] This kind of sign is (1.2) an expression of *likeness*. In this mode all artefacts are signs of art in the mind of the artist.

(2) Signs that are given by living creatures (*data ab anima*) are divided into two sub-modes: they can signify (2.1) *naturally*, e.g. in the sense that crying is a sign of grief, but they can also signify (2.2) conventionally (*ad placitum,* litt.: at pleasure), as is the case with spoken and written words, gestures of the deaf and signs of monks. Other examples we could think of are traffic signs and the signs of the conductor of an orchestra. This completes Bacon's division of signs in the first chapter of the *Compendium*.

Bacon is aware of the equivocity of the term 'natural' in his classification. The first sense is related to 'nature' as opposed to 'intellect'. Bacon refers to Aristotle's *Physics* for this distinction.[64] In the second sense, 'natural' means 'arising from the

[58] Edited and translated by Thomas Maloney [1988]. See for more information about Roger Bacon's life and works Maloney's introduction to this work.
[59] *Compendium* [25].
[60] Rosier [1997, p. 94] argues that this originality claim is improbable.
[61] *De signis* (2), ed. Fredborg *et al.*, p. 82
[62] *Peri hermeneias*, 16a3-8.
[63] See for the sources of these examples: Marmo [1997, p. 145 ff].
[64] *De signis* (12).

nature of a species'. Natural signs in this sense come from living creatures (men and animals) but "without purposeful deliberation."[65] No inference is required to see the relation between sign and significate. Bacon argues that non-human animals may well act by intent in making certain vocal sounds that are signs, but they do this from a kind of natural instinct. Only the human intellect can employ signs intentionally with deliberation. The rational soul itself establishes a relation between a sign and what is signified. In this way, language becomes a set of arbitrary symbols.[66]

But now the question arises: what exactly does a word signify? According to Boethius and other commentators on Aristotle's *Peri hermeneias*, a spoken word signifies primarily a concept in the mind and secondarily the corresponding object in reality.[67] The signification of words is conventional (*ad placitum*) and established by imposition, i.e. an act by which a name is applied to an object for the first time and by intent. Concepts, according to this interpretation of Aristotle, are natural signs or likenesses of things in reality. Names signify things only by way of the concepts of the things. The process of naming parallels the process of intellection. This was the dominant view on the relation between language, thought and reality in Bacon's time. Bacon, however, rejects this idea without hesitation. According to him "... a vocal sound imposed to signify a thing outside a mind signifies only that thing by reason of the imposition."[68] In other words, names are invented to signify extra-mental things and for that reason they primarily signify those things. They do also signify species and cognitive habits, but only as a natural consequence of their being imposed to signify the extra-mental object, inasmuch as a thing cannot be understood without them.[69] When we use a word to (conventionally) signify a mental species (concept), there must be a second imposition, this time on the species, and the new word will signify only equivocally.[70] Therefore, words are *natural* signs of species according to the first mode of natural signs mentioned above, because there is a necessary relationship between sign (word) and significate (species): whenever there is a word, there is also a species. Words are *conventional* signs of extra-mental things and occasionally and equivocally of mental species, if the names are chosen by intent to signify those objects.

In the foregoing account of spoken words as natural signs, the word 'species' occurred several times, but with different meanings. In Bacon's view a spoken word necessarily generates a kind of image (*species vocis*) in the mind of the speaker.[71] But at the same time the extra-mental object to which the word refers generates a species of itself (*species rei*) in the mind of the speaker. Both species are natural significates of the same spoken word, but as a sound it naturally signifies the

[65] *Compendium* [35].
[66] Maloney [1983, p. 190].
[67] See above, p. 349.
[68] *Compendium* [59], transl. Maloney.
[69] *Compendium* [60].
[70] *Compendium* [62].
[71] *De signis* II.1 (18).

species vocis and as a name it naturally signifies the *species rei*. It conventionally signifies the extra-mental thing.

Thus, in Bacon's view, the relationship between a name and what it names exists as a result of the deliberating free choice of the name giver. When we give the name 'man' to an existing man, this name can be univocally used for all existing men, but not for the concept or species 'man' in our mind. The main reason Bacon gives for this opinion is that a word's signification of the concept is not a question of free choice. The relation between name and concept is a necessary one, since we cannot name an object before we know it.[72] However, there may be another reason why Bacon so clearly deviates from the common opinion, although he never expressly makes the point in either the *Compendium* or *De signis*. He could have drawn an argument from his Neo-Platonist theory of the multiplication of species. According to that theory all objects produce species (images, likenesses) of themselves in the air. When such a species reaches the eye it has been multiplied several times and with every multiplication it has lost some of its intensity, as is the case with the light that comes from the sun. As a result, the species in our intellect is an incomplete image of the object of which it is a likeness. Consequently, it has incomplete being, according to Bacon. Since names are imposed upon objects in complete being (*in esse completo*), the species of man cannot be named 'man' in the same sense as the object in complete being is a man, and the same account holds for an embryo in the mother's womb. In fact, Bacon argues, the nobler the things are, the more incomplete are their species.[73] The reason that, nonetheless, we have a rather accurate knowledge of things is that our knowledge is mainly knowledge of universals. Now, in Bacon's realistic ontology universals are real parts of the nature of individuals. Thus, when we see an individual, "the universal species is repeated in the medium and sense and the intellect when it comes with the species of each singular; and thus it is fixed in the soul more powerfully than is the species of any singular."[74] Therefore, the universal species, which is the concept, of man is a reliable medium of knowledge, but not the right candidate for the name 'man' that has been imposed on the individual man in complete being.

When a name has been imposed upon a being, it signifies that being by convention, but it also signifies things that 'follow' from it, but this is an implied or secondary meaning that naturally and necessarily accompanies the conventional meaning.[75] In the *Compendium* (chapter III) Bacon speaks about things co-understood or connoted in this context. The number of things connoted is infinite and in *De signis* he shows his enthusiasm about this phenomenon by saying: "And to consider this is of wondrous usefulness and beauty."[76]

Thus, according to Bacon, the name 'man' signifies existing men only, but what

[72] Cf. Maloney [1983, p. 210].
[73] *De multiplicatione specierum*, Pars I, cap. 1, ed. Lindberg, p. 10.
[74] *De multiplicatione specierum*, Pars I, cap. 2, transl. Lindberg, p. 43.
[75] *De signis* [102–104].
[76] *De signis* [103].

about men in the past and future? Terminist logicians introduced the concept of ampliation and restriction (of supposition) to account for this phenomenon, but Bacon rejects this approach.[77] Ampliation and restriction are caused by the human intellect and, therefore, the result is equivocation. He argues that a name cannot signify anything univocally common to an entity and a nonentity, whether present, past, or future.[78] Likewise, the name 'man' does not signify univocally a living man and a dead man. When we talk about a dead man, we are in fact renewing the imposition of the name 'man' during use, perhaps even without noticing it. If the object signified perishes, the word looses its original meaning and a new imposition is required. By this secondary imposition the word acquires a new meaning.[79]

The works of Roger Bacon contain a remarkable theory of signs and an original view on the relation between language, thought and reality. Central in his theory is the role of the speaker, who freely has words at his disposal and is constantly renewing the signification of the words he uses. He does not use the methods and concepts of the terminist logicians of the earlier part of the thirteenth century, who take signification for granted. On the contrary, Roger Bacon takes signification of words as an object of reflection, embedded in a larger theory of signs. His theory of signification shows that signification and supposition cannot "walk hand in hand".[80]

Although Roger Bacon was quite unique in defending existing objects as the *only* significates of spoken words, his reflection on the role of words as signs fits very well in a tendency towards analysing the concept of signification. This tendency is dominant in Paris, but can also be seen in the work of two of Bacon's English contemporaries: Ps.-Kilwardby and John Duns Scotus. Both authors did not confine themselves to explaining the relations between language, thought and reality; they also reflected on the various ways in which words signify.

In a work entitled "The Commentary on Priscianus Maior", that was traditionally but unjustly ascribed to Robert Kilwardby, the author gives a general definition of a sign, inspired by Augustin, underlining its sensible aspect, but at the same time he adheres to Aristotle's definition of words in *Peri hermeneias*: words are conventional signs of passions in the soul, representing those passions. According to the author, words are objects of a science insofar as they imply an intellection. The intellection is the form of the signification, the word is its matter.[81] This approach makes it possible to give the study of signs a scientific character, since now signification is studied as a property of signs in general, not of a particular sign.

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[77] *De signis* [89–99].
[78] *Compendium* [88].
[79] For the Bacon's primary interest in imposition rather than in supposition, see: Fredborg [1981, pp. 172–3]. Cf. also Biard [1989, p. 41], and Ebbesen [1979, p. 53].
[80] Ebbesen [1979, p. 47].
[81] Biard [1979, pp. 44–5].

Duns Scotus on signification and understanding

John Duns Scotus (1266–1308) never wrote a handbook on logic or grammar. Nevertheless, he played an important role in the debates about the semantic issues of his time.[82] The modes of signifying play an important role in his semantics, but he cannot be considered a Modist himself, because he does not defend the modistic thesis that there is a strict correspondence between the modes of signifying, modes of being and modes of understanding.[83]

In his *Peri hermeneias* commentary, probably written in Paris in the 1290s, Scotus gives an overview of the competing opinions regarding the relationships between the three angular points of the semantic triangle. He distinguishes two opinions:[84] one opinion holds that a (spoken) name signifies an intelligible species primarily and an extra-mental thing secondarily; the other opinion holds that a name primarily signifies a thing as understood.[85] Both opinions hold that we need an intelligible species in understanding extra-mental things, but only in the first opinion the species also plays a role in the signification of names. But how should this role be described in the first opinion? Scotus explains that the intelligible species is not in itself, as mental entity, the object of signification. It is in its role of *representation* or mental sign of the extra-mental object (*signum rei*) that the species is primarily signified, in the same way as a photograph of my grandmother represents my grandmother and is signified by the name 'grandmother' when I show someone this picture and say: "this is my grandmother". The primary significate would be my grandmother as she appears on the photograph; the secondary significate would my grandmother herself, but since she died long ago, there is no secondary significate in this case. This example shows that in this opinion the signification of names like 'chimera' (the so-called empty names) is no problem. The name simply signifies the content of the intelligible species. It is not necessary for a thing to exist in order to be signified. This opinion can be regarded as the dominant position until the 1260's and can be found in the work of Albert the Great and the younger Thomas Aquinas, be it in a less elaborated form.[86] Roger Bacon opposes this view in his works on signs.

Scotus mentions an interesting objection the supporters of the theory of the primary signification of the species have to face. If names primarily signify species, a proposition like 'man is an animal' is always false, for the intelligible species of man is not the same as the intelligible species of animal. Now, Scotus makes clear that the notions 'truth' and 'falsity' imply a reference to the extra-mental world. The subject and predicate term of a proposition do not signify the corresponding intelligible species in itself as signs, but that of which the species are signs.[87]

[82] The question of his originality has been the subject of debate among scholars. See esp. Pini [1999] and [2001] and Perler [1993] and [2003].
[83] Perler [2003, p. 173].
[84] For a detailed account of the two positions, see: Pini [1999].
[85] See also Vos [2006, pp. 159–160].
[86] See: Pini [1999, pp. 35–9].
[87] *Super Peri herm.* 1.2, n. 2-6.

The second opinion mentioned by Scotus holds that a name signifies the thing insofar as it is understood, i.e. the thing itself insofar as it is present to our intellect, as a concept produced by our intellect.[88] In this view the intelligible species is not that which is understood, but that in virtue of which the extra-mental thing is known to the intellect. The concept signified is not the intelligible species, but the essence of the extra-mental thing. This is the object as stripped from its individual features. The thing as individual cannot be primarily signified, because it cannot be primarily understood by the intellect.[89] As we have seen, the later Aquinas defended this position. It has no problems with non-existing objects either. When a thing ceases to exist, there is still its essence: the thing as far as it is understood. The signification of a word remains the same, no matter what happens to the extra-mental object.[90]

At first sight the difference between the two opinions is not very big. In the first opinion names signify species in their representational function and the second opinion holds that names signify concepts produced by the intellect. Neither of the two implies that names signify species as mental entities or things as they are something in itself. There is, however, an important ontological difference between the two opinions. An intelligible species, whether it is considered as a mental entity or as a representation of an extra-mental object, is a real quality of the soul caused by the object, whereas the thing insofar as it is understood, at least in the opinion of Scotus and the later Aquinas, is an intention, which has, in Scotus' view, diminished (i.e. not real) and objective being.[91] Thus, in the second opinion understanding as well as signifying is an intentional act of the intellect. This opinion would correspond very well with the epistemology of Duns Scotus.

Nevertheless, Scotus does not commit himself to the theory of the primary signification of things understood.[92] In his commentary on the *Sentences* he even objects to the identification of concepts with things as understood, on which that theory is based. In this work, which is of a later date than the *Peri hermenieas* commentary, he proposes a new version of the theory, according to which concepts are acts of the intellect and not to be identified with things signified.[93] Spoken words signify extra-mental things immediately, as do written signs and concepts. So, spoken words, written words and concepts are all directly related to signified reality. This is a rather remarkable opinion, because it implies that the parallelism between signifying and understanding, an essential feature of the theory of the primary signification of the thing as understood, does not hold. This is indeed what Scotus concludes:

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[88] Elsewhere Scotus calls this concept a first intention (see below, p. 381).

[89] In an act of intuition the intellect can only know the existence of an individual, not its essence.

[90] All the relevant texts can be found in Scotus' commentary on the *Peri hermeneias* and quoted in Pini [1999] and [2001].

[91] See below, p. 380.

[92] This paragraph is based on Pini's interpretation in Pini [2001]. Cf. also Perler [2003, pp. 178–182]. For other interpretations, see Vos [2006, pp. 157–168].

[93] *Ordinatio* I 27, 83. See Vos [2006, p. 164].

> It can be briefly said to the question that the contention common to many opinions, i.e. that 'as something is understood, so it is also named', is false if this is understood precisely, because something can be signified more distinctively than it is understood.[94]

From this passage it is clear that, according to Scotus, naming and understanding do not always correspond to each other. We can name an object without having a precise understanding of it. What a name signifies is to a considerable extent dependent upon the intention of the speaker. It does not mean that speaking occurs without understanding, but a speaker may successfully refer to an object, even if his understanding of this object is incorrect or vague.

With his theory of immediate signification of the thing Scotus was the first to reject the parallelism between understanding and signifying, which makes the conclusion justified that he made an original contribution in the history of semantics.[95] In fact, he can be seen to have found a way to account for the distinction between meaning and reference, in the sense that meaning has to do with understanding and reference with naming. Shortly after he died, philosophers like Ockham and Buridan showed that supposition taken as a property of mental terms is a much better tool to solve referential problems.[96]

Concrete and abstract terms

In the previous paragraphs I have examined the various answers to the question 'what does a name signify'?, where the names considered were concrete general terms, like 'man' and 'animal'. But in the late 13$^{\text{th}}$ century many authors had a particular interest in the semantic function of other terms, such as abstract essential terms (for example 'humanity'), and abstract accidental terms (for example 'whiteness'). I shall discuss the headlines of the modistic approach to this subject in the second part of this chapter when we speak about terms of second intention, but here I shall pay attention to Scotus' view.[97] Scotus' rejection of the parallel between signifying and understanding and the acceptance of signification as a direct relation between word and thing could easily lead to a theory of strict correspondence between language and reality, so that for example the concrete term 'white' and the abstract term 'whiteness' signify numerically different things. Scotus' account of concrete and abstract terms shows that this can be avoided. In short, his opinion is that the concrete term 'white' and the abstract term 'whiteness' signify the same white thing, but in different ways (modes of signifying).[98]

[94] Duns Scotus, *Ordinatio* I, d. 22, q. un., n. 4. Cited from Pini [2001, p. 39] in his translation.

[95] Pini [2001, p. 51].

[96] Marmo [1983, pp. 170–4] shows that there are some affinities between Scotus' terminology and that of the Oxford tradition of the theory of supposition.

[97] For a thorough discussion of the modistic approach, see: Ebbesen [1988].

[98] *Rep. Par.* IV, d. 12, q.1, n. 10, *In Praed.*, q. 8. Marmo [1983, n. 47], notices that Scotus' definition of *modi significandi* is different from the Modists's. For Scotus their only function is the regulation of the well-formation (*congruitas*) of propositions. I can add that this use of *modi significandi* is common even in the fourteenth century, e.g. in the work of John Buridan.

Both refer to the same form, nature or essence of the colour white under different aspects. The abstract term 'whiteness' signifies the nature on its own (*sub propria ratione* i.e. qua colour) and the concrete term 'white' signifies the nature insofar as it 'informs' the subject, but at the same time it *connotes* (or co-signifies) the object in which whiteness inheres. In the same way, 'humanity' signifies human nature *qua* human nature, and 'man' signifies human nature in its contraction to an individual and at the same time it connotes that individual.[99]

Because of the different ways of signifying of concrete and abstract terms one cannot simply substitute one term for the other in a proposition. The proposition 'Socrates is white' is well formed and possibly true, but the proposition 'Socrates is whiteness' is not. 'Whiteness is a colour', on the other hand, would be perfectly well formed and true, even if no white object would exist.

Since 'white' in 'Socrates is white' signifies the nature whiteness as it inheres in the subject, Scotus has to defend an inherence theory of predication, which means that a proposition is true if and only if the accident (such as whiteness) signified by the predicate term (such as 'white') *inheres in* the object referred to by the subject term (substratum).[100]

Scotus' semantic distinction between a concrete term and its abstract counterpart reflects an ontological distinction between a formality in its absoluteness and its accidental or essential inherence. This theory of concrete and abstract names shows that Scotus did not reject the parallel between language and reality.

Equivocation and analogy

A special problem in medieval semantics was the signification of equivocal terms,[101] but in order to make this clear we have to start at the beginning. Medieval discussions on analogy and equivocation go back to Boethius' translation and interpretation of the opening words of Aristotle's *Categories*, where Aristotle had made a distinction between equivocal terms and univocal terms. An equivocal term is a name like 'man' referring to both a man and a picture of that man; an example of a univocal term is 'animal' as a name for both a man and an ox.[102] An equivocal term signifies two concepts, whereas a univocal term is associated with one concept or nature. Boethius subdivided equivocals into two main groups: chance equivocals (*aequivoca a casu*) and deliberate equivocals (*aequivoca a consilio*). Chance equivocals are not interesting or problematic from a semantic point of view: the concepts signified are totally unrelated. The debates concern the deliberate equivocals. Boethius divides these in four groups, according to the cause of

[99] *In Praed.* q. 8. See also: Marmo [1983, pp. 165–8].

[100] Perler [2003, p. 173].

[101] There is an enormous amount of literature on this topic. For this paragraph I rely mainly upon Ashworth [1991; 1992; 1996]. Other important publications on the theme can be found in her footnotes.

[102] Aristotle, *Categories*, 1a1-8: "When things have a name in common but are different beings (*ousiai*), they are called equivocals. Thus, for example, both a man and a picture are animals [...] When things have a name in common and their being is the same, they are called univocals. Thus, for example, both a man and an ox are animals."

the equivocation: (1) resemblance (*similitudo*), (2) proportion (*proportio*, Greek: *analogia*), (3) "of one origin" (*ab uno*) and (4) "in relation to one <term>" (*ad unum*). Boethius' example of a term that falls within the second group is 'principle' (*principium*), which designates unity in relation to number or point in relation to line.[103] This example shows that *analogia* in the Aristotelian sense refers to a similarity of two proportions involving at least four items. In the thirteenth century, however, what came to be called *analogia* (Latin: *proportio*) covered what Aristotle called *pros hen* equivocation, which corresponds to the last two items in Boethius' division, with the result that the term '*analogia*' became ambiguous.[104] In the first sense the analogy consists in a comparison of two proportions (as in: A is to B as C is to D). Late 13$^{\text{th}}$-century logicians do not seem to be interested in this kind of analogy, also called 'analogy of proportionality'.[105] The focus is solely on the second kind of analogy, or 'analogy of proportion', that occurs when one term signifies two or more objects in a prior and a posterior way (*secundum prius et posterius*) and according to attribution (*secundum attributionem*), because of a relationship between the primary and secondary significate.[106]

Another framework for discussion was Aristotle's *De sophisticis elenchis*, especially the passage where he distinguishes between three modes of equivocation and amphiboly: "(1) When either the expression or the name has strictly more than one meaning (...); (2) when by custom we use them so; (3) when words that have a simple sense taken alone have more than one meaning in combination."[107] In one of the logical handbooks that were used in the thirteenth century, Peter of Spain's *Tractatus*,[108] Aristotle's division is interpreted as follows:

1. the first mode of equivocation occurs when one word signifies several things, for example 'dog' signifying a barking animal, a marine animal and a constellation of stars,

2. the second mode occurs when one word signifies one thing primarily and another secondarily (*secundum prius et posterius*), for example 'healthy' can be said of a person and in a derivative way of urine,

3. the third mode occurs when a word has several consignifications. In this case, the noun or expression signifies several things in a context, isolated however, only one. The standard example of the third mode of equivocation is that of *laborans*, as it appears in the syllogism: "Whoever was being cured is healthy, the sufferer (*laborans*) was being cured; therefore the sufferer is healthy." A standard example of the third mode of amphiboly is: "The bishops (*episcopi*) are priests, these asses are the bishop's (*episcopi*); therefore these asses are priests." In Peter's view the definition of equivocation applies more in the

[103] For references, see Ashworth [1996, pp. 234–6].
[104] Ashworth [1991, p. 41] and [1996, p. 236].
[105] Thomas Aquinas, *De veritate*, qu. 2, a. 11.
[106] Ashworth [1996, p. 237].
[107] 166a15-20.
[108] Peter of Spain, *Tractatus* VII, De fallaciis, [ed. De Rijk, pp. 99–104].

> first mode than in the second, and more in the first two modes than in the third, because in the third mode the diversity concerns the consignifications or modes of signifying (*modi significandi*) of one significate, whereas in the first two modes there are several significates involved.[109]

Now, it should be noted that the notion of consignification (*consignificatio*) was used in at least three contexts.[110] First, it was said that syncategorematic terms, like 'every', 'all', 'both', 'only' and many others,[111] don't have signification, but only consignification. Second, verbs were said to consignify time. Third, in grammatical discussions (see above, p. 355) 'consignification' was another term for mode of signifying. Essential modes of signifying included noun, verb or adjective; accidental modes were such features as case, number, and gender. The *laborans* syllogism is an example of equivocation caused by consignification of time, and in the *episcopi* syllogism the cause of equivocation is consignification of case. An important distinction between the two is that, according to many authors, consignification of time could not be influenced by context, whereas consignification of case could. Many 13$^{\text{th}}$-century logicians believed that the signification of words and many of its modes of signifying were fixed by imposition, which means that they cannot be altered by context or speaker intention.[112]

From the discussion in a treatise called *Summe metenses*, ascribed to Nicholas of Paris and dated around 1250, it becomes clear that analogical terms are said to be equivocal in the second sense of equivocation mentioned by Peter of Spain. For example, being (*ens*) is first (*per prius*) said of substance, and afterwards (*per posterius)* of accidents, and 'healthy' is first said of an animal, in which health inheres, and afterwards of urine, which is a sign of health.[113] In his work on fallacies Thomas Aquinas also reduces analogical terms to the second mode of equivocation.[114] Late 13$^{\text{th}}$-century authors usually adopt the standard division of equivocation, as given by Peter of Spain. The first mode involves a plurality of significates that are equally represented. In the second mode these significates are unequally represented; the secondary signification takes place through attribution or transference (*transumptio*). The third mode involves a plurality of modes of signifying.[115] Some others give a somewhat different account.[116]

The discussions on analogy were influenced by the recovery and translation of Aristotle's *Metaphysics* and by Arabic sources. In these works analogy was related to the question how being (*ens*) could be predicated of both substance and accidents. The answer was that 'being' was neither a univocal nor an equivocal term,

[109] *ibid.*, p. 105.
[110] For details see: Ashworth [1991, pp. 53–61].
[111] The standard textbook for this topic was William of Sherwood's *Treatise on Syncategorematic Words*.
[112] Ashworth [1992, pp. 106–7].
[113] For references, see Ashworth [1992, p. 112].
[114] *De fallaciis*, p. 406.
[115] E.g. the commentaries in *Incerti auctores*, ed. Ebbesen.
[116] See for instance the *Elenchi* commentaries of Simon of Faversham and Duns Scotus, referred to in Ashworth [1992, p. 118].

but something in between univocal and equivocal: being was said to belong to accidents *secundum prius et posterius*.[117] The discussion was further complicated by a passage in Aristotle's Physics where he says that "equivocations are hidden in genera".[118]

The result is that we find a threefold division of analogy in the late 13$^{\text{th}}$ century *Elenchi* commentaries.[119] In all three types speaking according to priority and posteriority is involved:

1. The first type of analogy occurs in the relation of a genus to its species. For example, 'animal' is an analogical term that can be predicated of rational and non-rational animals alike. The two species participate equally in the *ratio* of genus (i.e. 'animal' means the same in both cases), but since one species is more noble than the other (rationality is a habit and non-rationality the absence of a habit), one species is said to fall under the genus primarily (*per prius*) and the other secondarily (*per posterius*). This type of analogy is called univocation, because both analogates participate equally in the same characteristic.

2. The second type of analogy occurs when the analogates participate in the same characteristic (*ratio*), but not equally. This is the case with being (*ens*) in relation to substance and accident. Substances participate directly and primarily (*per prius*) in being, whereas accidents do so only immediately and secondarily (*per posterius*), i.e. by means of substances, since accidents do not exist separately from the substance in which they inhere. This type of analogy is neither strictly univocal, nor strictly equivocal, but something in between, because they share more than just a name.

3. The third type of analogy is identical to deliberate equivocation. It occurs when the analogates share only a name, as is the case with 'healthy' said of an animal, urine, a diet and a medicine. 'Healthy' signifies primarily (*primo et simpliciter*) a healthy animal and it signifies the other things secondarily through attribution (*per attributionem*) to the first thing.

This threefold division of analogy and the language in which it is put suggests that the authors were interested in an ontologically based division. In their view, the hierarchy of analogical terms is related to types of participation. Also Aquinas held that analogical predication depends on natures being shared in prior and posterior ways.[120] Duns Scotus criticizes this view in his commentaries on the *Categories* and the *Sophistici Elenchi*. Scotus argued that analogy is possible in the order of things, but that words signify according to their imposition. The process of imposition is voluntary and does not allow for an ordering among significations

[117] See e.g. Algazel's logic, as cited in Ashworth [1992, p. 108].
[118] Aristotle, *Physics* 7 (249a22-5).
[119] The division presented here comes from *Incerti auctores*, qu. 823, [ed. Ebbesen, pp. 314–317]. The examples are mine. See also: Ashworth [1992, pp. 119–120].
[120] Ashworth [1992, p. 127].

that is not the result of this imposition. Furthermore, he rejected the notion of a common *ratio*, with the result that there is, in his view, no room for analogical predication of the first and second type. The first type of analogy –Scotus uses 'cause, 'end' and 'principle' as examples – really involves univocal predication. Terms like 'being', as applied to substance and accident, are really equivocal. Terms belonging to the third type are really univocal. For example, 'healthy' is a univocal term, because it only signifies health: in the animal as in a subject, in urine as in a sign and in diet as in a conserving agent.[121] Thus, from a logical point of view there is no need for something in between univocal and equivocal predication. In one of the previous paragraphs we saw that Scotus rejected the parallel between understanding and signifying. Now it is clear that he also rejected the strict correspondence between language and reality.

PART II: THE LOGIC OF INTENTIONS

Second intentions: the primary subject of logic

The second half of the 13$^{\text{th}}$ century is characterized by the rise of the theory of second intentions as the primary subject of logic. Second intentions are concepts of a certain kind, which give logic its status as a science, because they are supposed to be universal and objective, as will be explained below. All the authors writing in this period regard second intentions as mental entities, which somehow have a foundation in reality (*fundamentum in re*), but the way they see this foundation differs. They agree, however, that one should distinguish between the mental existence of a concept, which is studied by psychology, and its content, which is the proper subject of logic.[122]

Examples of terms signifying second intentions are 'universal', 'genus', 'species', 'proposition', and 'syllogism'. Some authors, Thomas Aquinas for example, consider second intentions as second order concepts, representing not things but other concepts, but other authors, e.g. Simon of Faversham and Radulphus Brito, assume that second intentions are representations of things as related to other things and that they are founded upon special properties of extra-mental objects. Finally, Duns Scotus, Hervaeus Natalis and Peter Aureol ascribe to second intentions a special kind of being: objective being, to be distinguished from material being on the one hand and purely mental being on the other. As we shall see below, the answer to the question: what do second intentions represent, is related to epistemological and ontological issues. But first we need to understand what an intention is.

When I think, I think about something: a book, a child, my holidays last summer, my future plans etc. Sometimes it is clear what the thing is I am thinking about, sometimes it is not, but in any case my intellectual act is *intentional,* i.e. it is directed to something. One might even say that there is a relationship between

[121] For references and more details especially concerning Aquinas, see: Ashworth [1992, pp. 121–131].
[122] Pini [2002, p. 13].

my act and the object intended. But what and where *is* this object exactly and how is the cognitive relationship established between this object and me? These are the questions that arise in discussions about intentionality. The intentionality debate is mainly an ontological and epistemological debate about the relation between our thoughts and the things we think of.

Discussions on the nature of intentions arise around the middle of the 13$^{\text{th}}$-century in connection with the influence of Avicenna.[123] The Latin term 'intentio' has two Arabic counterparts: *ma'qul* and *ma'na*, which makes it semantically ambivalent. *Ma'qul* means concept or thought. Thoughts are related to extra-mental things as well as to words. Not surprisingly, the term *ma'qul* occurs in Alfarabi's commentary on Aristotle's *De interpretatione*, where the relation between thought, language and reality is discussed, as a translation of the Greek *noêma. Ma'na*, at the other hand, seems to be an epistemological term; it refers to the intelligible species that is present in the mind when an extra-mental thing is known. According to De Rijk the semantic ambivalence of the term 'intentio' is at the basis of the distinction between first and second intention: the notion of first intention has to do with the *thing* apprehended, whereas second intentions arise from the object's being *apprehended.*[124] Generally speaking, when the object of an intention is an extra-mental thing, the intention is a first intention and when the object is itself an intention, the intention is a second intention. Second intentions are the subject of logic, according to Avicenna. We call this kind of logic 'intentionalistic', as opposed to terminist logic, which was primarily concerned with language, not with concepts. For intentionalists logic is a *scientia rationalis*, whereas for the terminist logicians it is an *ars* or *scientia sermocinalis.*

Thus, an intention is a concept as well as the foundation of its content.[125] This ambivalence reflects the medieval conviction that it is possible for the human intellect to discover the truth about the extra-mental reality, because of the fundamental parallelism existing between the structure of the outside world on the one hand and the ways in which we understand things on the other hand. For the Medieval mind, however, there is no 'on the one hand' and 'on the other hand': the human intellect grasps the extra-mental things through concepts that designate these things *including* their being conceived of.[126] This is the reason why in intentionalist logic questions about logic and meaning are frequently interwoven with epistemological questions. A good example of this interconnectedness can be found in the logic and epistemology of Thomas Aquinas (d. 1274). His ideas about the representativeness and reliability of our thinking about reality can be seen as the foundation on which later thinkers build their theories.[127] Although his direct influence is difficult to assess and although there is no specific work on intentions from his hand, he seems to have set the philosophical agenda for the authors of

[123]For the Arabic sources of the concept of intention, see Knudsen [1982, pp. 479–80]. For the Aristotelian and Augustinian roots, see De Rijk [2005, p. 30 ff].
[124]De Rijk [2005, p. 23].
[125]See also: Knudsen [1982, p. 480].
[126]De Rijk [2005, p. 24].
[127]De Rijk [2005, p. 348].

the second half of the 13th-century in Paris. Therefore, his ideas deserve ample attention.

Thomas Aquinas on the subject of logic

Logic, according to Aquinas, can be considered in two ways: as an art and as a science.[128] An art, in the Aristotelian sense, is an act of reason, which produces something new, something that didn't exist before; it is a creative act. But in order to proceed methodically and without error, reason needs guidance. This is the business of logic, which is a kind of super-art (*ars artium*). Arts usually produce something: an instrument or a work of art (in the modern sense). Logic, however, is distinguished from other arts insofar as its production (definitions, propositions, syllogism) is immaterial. It is, therefore, not a mechanical art, but a liberal art. Liberal arts are ordained to the intellectual act of the (free) soul, whereas mechanical arts belong to man as a result of bodily activity. Since the body is subordinated to the soul, mechanical arts are called *servilis,* i.e. not free.[129]

Logic is also a science. But whereas other sciences are directed to the knowledge of certain things, logic is concerned with knowledge itself. As a science it must have some definite subject,[130] it must proceed by demonstration from first principles and it must yield certain knowledge of necessary conclusions. In relation to the other sciences logic is instrumental, because it provides the instruments (syllogisms, definitions etc.) with which they work, and it is methodological, since it teaches the methods of other sciences.[131] All the other sciences are about things, whereas logic is about science itself, i.e. about intellectual intentions. Logic is a methodological, rational science.

The guidance of logic is needed for three different acts of the intellect: simple apprehension, judgment and reasoning. For each of them there is a distinct part of logic. The logic of reasoning is divided as follows: demonstrative, analytical or judicative logic leads to certitude; dialectical or inventive logic leads to probability, and sophistical logic leads to the mere appearance of truth.[132] Dialectics and demonstrative logic are said to be distinct (rational) sciences. Demonstrative logic is the logic of presentation: it establishes a truth already discovered, by reducing it to its principles. Dialectics is the logic of discovery: it searches for a particular truth, which it does not yet possess; it argues from general or common principles.

Aquinas makes a distinction between pure logic *(logica docens)* and applied logic *(logica utens)*. Demonstrative logic, being pure logic, studies the instruments and procedures of thought and the norms for their application theoretically, but

[128] The following remarks concerning Aquinas derive mainly from Schmidt [1966].
[129] Aquinas, *Summa theologiae*, Ia-IIae, 57, 3 ad 3; Schmidt [1966, p. 8].
[130] Aquinas calls it a subject, because it is subjected to investigation. See: Schmidt [1966, p. 11].
[131] Schmidt [1966, p. 26].
[132] *In I Post. Anal.*, 1, n.6, as in: Schmidt [1966, p. 32].

the application of demonstrative logic belongs to the particular science, not to logic itself. With dialectics things are different: dialectics does participate in the investigations in the other sciences.[133] Therefore, dialectics is theoretical as well as applied. Theoretical dialectics is a science, but applied dialectics is rather an art: the art of reasoning or what we would nowadays call 'critical thinking'.

In determining what the domain of logic is, we have to distinguish between its *matter* and its subject. Logic is concerned with the operations of reason: simple apprehension, judgment and reasoning. These three operations of reason constitute the *proper matter* of logic. In the first operation the intellect grasps simple things such as a man or a stone; in the second operation the objects thus grasped are combined or divided, so that a proposition can be formed, in this case: 'a man is not a stone'. The third operation is the process from the known to the unknown. The result or product of the operations of reason is *intentions*. Intentions are the subject of logic and should not be identified with the acts of reason, which produce them. Logic is *concerned with* acts of reason, but has intentions as its subject.[134]

Now we have to return to the question what an intention is. The word 'intention' basically means a tendency to something else. If we leave the intentions of the will out of consideration, Aquinas uses the word 'intention' sometimes to designate the intelligible species by which the intellect is informed in the act of cognition. Thus informed, the knower expresses an internal word, the so called 'intellected intention (*intentio intellecta*).[135] This intention expresses the essence of the thing known, which then exists not only in reality in the object, but also in the intellect of the knower. It has a relation of likeness to the thing known. Intentions, according to Aquinas, exist only in the intellect - Aquinas calls them "beings of reason" (*entia rationis*)[136] – but it is important to see that the *intentio intellecta* has a foundation in reality (*fundamentum in re*) and that it has a semantic as well as an epistemological function.[137]

An intention is first and foremost a likeness of an external object by which that object is known. The intellect needs the external object to initiate its act of knowing. Thus, this first intention has an immediate foundation in extra-mental reality. In a second moment the intellect can also reflect upon its operation and the media involved. As a result of the first act of knowing the nature of an extra-mental object exists in the intellect. When the intellect is aware of how that nature exists in the intellect, a logical intention is formed. For example, when the intellect knows that a man and a horse are animals, it knows the nature of 'animal' as a genus. This is the way in which the intention of genus is formed. Because this logical intention can only be formed after the thing itself is known, later authors call it a second intention. Aquinas himself does not use the terms 'first' and

[133]Schmidt [1966, p. 41].
[134]Schmidt [1966, p. 40 ff].
[135]For the relation between intelligible species and mental word, see Pasnau [1997, p. 256 ff].
[136]Translated by Schmidt as 'rationate beings', see Schmidt [1966, p. 52, n. 15].
[137]De Rijk [2005, p. 120 ff].

'second intention' but speaks of immediately and mediately founded concepts,[138] or of 'first things understood' and 'second things understood', which "are called intentions consequent to the mode of understanding."[139] Second intentions are not directly founded on reality, but on the concepts of the things as they are in the mind, the things as understood. Therefore, the foundation of second intentions is called 'remote'.[140] Only second intentions are the subject of logic.

The intellect attributes second intentions to the nature of an object, as it exists in the soul. For example, the intellect attributes the property of being a genus or species to the thing understood. Thus, second intentions are accidents (or properties) of beings or natures as they are known. In the extra-mental world nothing corresponds to these properties, "for in the things that are outside the mind there is nothing that is a similitude of the notion of genus or species."[141] Therefore, their only mode of being is 'rationate being'. Nevertheless, they can only be considered with reference to their real subject.[142]

The foregoing remarks about the object of logic according to Aquinas were meant to show how logic and epistemology are interconnected in the 13th century. Aquinas was neither a nominalist, nor an extreme realist. He shared the common view that there is an extra-mental reality that can be known in a natural way. In his view, intellectual knowledge is directed to the natures of those extra-mental objects. The forms of the things known, the so-called first intentions, have rationate being. Logic too, is intentional. Its objects are the relations between first intentions in thought. Those relations are called second intentions. Through the first intentions second intentions are directed to the extra-mental reality, upon which they remotely depend. Ultimately, all intentions are ways of knowing real things.

In Aquinas' account of intentions, (second) intentions are produced by the reflection of the intellect on its operation, for example when it compares the concepts *man* and *animal*. By comparing those concepts, the intellect forms the intention of a species correlated to a genus. The relation between those concepts is a rational relation, i.e. a relation caused by reason.[143] Other authors maintain that the relation holding between a species and a genus is founded on the nature of the things themselves and that the intellect is not the cause of intentions, properly speaking.[144] As examples of this more realistic approach I shall take into account the positions of Robert Kilwardby, Simon of Faversham, the Modists and Radulphus Brito.

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[138]Thomas Aquinas, *De potentia*, q. 1, a. 1 ad 10, as in: Schmidt [1966, pp. 118–9].
[139]Thomas Aquinas, *De potentia*, q. 7, a. 9.
[140]Schmidt [1966, p. 118].
[141]Thomas Aquinas, *De potentia*, q. 7, a. 6, as in Pini [2002, p. 56].
[142]Schmidt [1966, p. 129].
[143]Pini [2002, p. 60].
[144]See Pini [2002, pp. 64–7] for Peter of Auvergne's criticism on Aquinas.

Intentions as (founded upon) real properties of things.

In *De ortu scientiarum* (ca. 1250) Robert Kilwardby explicitly distinguishes first and second intentions. Things by themselves are first intentions, and the nouns signifying those things, e.g. 'substance' or 'quantity', are nouns of first intention.[145] Now, the focus of logic, being a *scientia rationalis*, is not on things by themselves, but on what Kilwardby calls *rationes rerum*, i.e. aspects of things that make us think about them in a certain way, and on the nouns referring to those aspects, e.g. 'universal', 'particular', 'antecedent', 'consequent' (the last two, when the 'thing' considered is a composition). Those aspects (or properties) are second intentions and the nouns referring to them are nouns of second intention. Things by themselves are called 'first intentions', because they are primarily comprehended, while second intentions come to the fore when things are not regarded by themselves, but according to their mutual relationships.[146]

With Kilwardby second intentions, even abstract second intentions like 'universality', 'particularity' and so on, are regarded real properties of things, which they possess as soon as they are considered by our intellect according to certain logical relationships.[147] Logicians study those properties because they play a role in the process of reasoning, which is the proper subject of logic, according to Kilwardby.

Simon of Faversham also discussed second intentions and their foundation in reality. At the end of his life (1306) Simon was chancellor of the University of Oxford, but it is considered most likely that his logical writings derive from his arts teaching at Paris around 1280.[148] He wrote various logical commentaries, but not on the logic of terms. He also can be considered an intentionalist. He held the common opinion of his time that second intentions are the proper subject of logic. First and second intentions are different ways to understand extra-mental things. When an object is conceived in its own right, leaving aside its individuating conditions, a first intention is produced. When the object is considered according to the logical relationships it has to other things of the same kind, a second intention is produced.[149] In causing second intentions the intellect is moved by the so-called 'appearances' of the extra-mental objects. These appearances represent the object's real properties.[150] The question is whether Simon of Faversham's view is in line with the common, i.e. Aquinas', doctrine of the foundation of second intentions. To my mind, he deviates from this doctrine in a more realist direction.[151] According to Aquinas second intentions are *remotely*, i.e. not directly, depending upon real objects; they are founded on the concepts of things in the mind (first intentions). In Simon's view logical intentions seem to be attributed by the intellect to things, not to concepts, for he maintains that the relation established by

[145] Knudsen [1982, p. 484].
[146] *De ortu scientiarum*, cap. XLVIII, [ed. Judy, p. 459].
[147] De Rijk [2005, p. 20].
[148] For more information about Simon of Faversham, see: Ebbesen *et al.* [1984, *Introd.* pp. 3–6].
[149] De Rijk [2005, p. 189].
[150] Simon of Faversham, *Quaestions on Porphyry*, q. 2, cited by De Rijk [2005, p. 178].
[151] See, however, De Rijk [2005, p. 180] for another evaluation.

the intellect is founded on real properties of things, that move the intellect to form intentions. Thus, when the intellect judges that a man is an animal, it does so on the basis of a relationship between a real essence and the things sharing in it. The predication itself is an act of the intellect, but a man is also an animal when the intellect is not in act. Therefore, the rational relationship between *man* and *animal* is founded upon real properties of extra-mental essences.[152]

The role of intentions in speculative grammar

The special ontological interpretation of the *modi significandi* also determined the modistic interpretation of logic. Unlike the terminist logicians, the Modists were mainly interested in metalogical questions about the status of logical laws and concepts. This approach of logic is so different from the terminist one, that one could easily speak of an interruption in the history of logic.[153] As has been said above, the Modists seldom applied the key concept of terminist logic, supposition. It may be that terminist logic, with its contextual approach and its role in argumentation, was too much an art instead of a science. In the Modists's view, what holds for grammar also holds for logic: if it has to be a science, it has to fulfil the Aristotelian requirement that its formal object is eternal. This eternal object appeared to be the second intentions.

Partly in connection with grammatical analysis the Modists developed their theory of intentions. Every object in its being known can be a first intention. The proper objects of logic, however, are not the first but the second intentions (*intentiones secundae*), i.e. special logical concepts resulting from the three operations of the intellect: apprehension (*apprehensio*), judgement (*compositio*) and reasoning (*ratiocinatio*). To the act of apprehension belong concepts (intentions) like *genus, species, predicate* and *subject*; to the act of judgement belong concepts like *per se* and *per accidens* (kinds of predication), and to the act of reasoning belong intentions referring to a set of propositions, for example *syllogism*.[154]

According to the Modists, first as well as second intentions are caused by the active intellect together with the properties (modes of being) of the object known and signified. First intentions are drawn from a proper mode of being, second intentions from a common mode of being, or rather from the object under a common mode of being.[155] Second intentions are secondary in so far as they presuppose first intentions, not in the sense that they are concepts of concepts. They are concepts of extra-mental objects, conceived and signified under a common mode of being.

Grammarians as well as logicians have a professional interest in the common properties (*modi essendi communes*: common modes of being), from which second intentions and modes of signifying are drawn. Thus, not man or animal as such is object of study, but the logical and grammatical relations that are connected with

[152]Pini [2002, pp. 77–]9.
[153]Pinborg [1975a, p. 40].
[154]Pinborg [1975a, p. 45].
[155]Radulphus Brito, soph. 'Aliquis homo est species', 52, ed. Pinborg [1975b, p. 144].

these objects, i.e. second intentions for the logician and modes of signifying for the grammarian.[156] There is, however, not a complete analogy between logic and grammar. For the grammarian, the congruity of a linguistic construction depends upon the modes of signifying of the elements of the construction (see above, p. 356). If the modes of signifying are combined in the right way, the construction is congruous, although it may not be perfect. For the logician, however, second intentions being combined in the right way do not make a proposition true. For a proposition to be true there has to be a correspondence with reality.[157] Therefore, for further analysis of a proposition, the logician has to go back to the primary meaning of the words, i.e. the concepts and their reference.

It may be clear now that in the Modists's view every intention draws on some property of an object. At the same time it signifies that object. Every first intention signifies and corresponds with a proper mode of being of an object (*proprius modus essendi rei*); every second intention and mode of signifying corresponds with a common mode of being of an object (*modus essendi communis rei*).[158] First as well as second intentions have their direct foundation in reality.

Radulphus Brito on second intentions

An application of modistic analyses can be found in the work of Radulphus Brito, who taught at the Sorbonne during the years 1295-1305.[159] His works contain the first extended and systematic treatment of intentions. His views on intentionality had a great influence upon the intentionality debate in the early fourteenth century. According to Radulphus and the common opinion, an intention is that by which the intellect directs itself to an object.[160] Unlike Thomas Aquinas, Radulphus does not distinguish between intention, act of the intellect, intelligible species and cognition; in his view they are identical. Intentions are the result of abstraction. So far his system seems rather straightforward, at least from an ontological point of view. This is, however, only one side of the story, because objects can be known in two ways and therefore there is also a twofold abstraction at the level of apprehension.

1. In the first place objects are known (and signified) according to a *proper* mode of being. In this way we conceive, on the level of apprehension, of man, horse etc. The act of the intellect (the cognition) itself is called 'first abstract intention' (*prima intentio in abstracto*), whereas the object that is known is called 'first concrete intention' (*prima intentio in concreto*). The name corresponding with the first concrete intention is 'man' or 'horse', and the name corresponding with the first abstract intention is 'humanity'

[156]Pinborg [1975a, p. 47].
[157]Radulphus Brito, cited by Pinborg, [1975a, p. 49]. For an extensive account of Radulphus Brito on intentionality, see De Rijk [2005, chapter VI].
[158]Pinborg [1975a, p. 49].
[159]Pinborg [1975b, pp. 119–120].
[160]Radulphus Brito, soph. 'Aliquis homo est species', paragraph 49 [Pinborg, 1975b; 1974, p. 51].

or 'horseness'. Concrete and abstract intentions are formally identical, but they differ materially.[161]

2. Secondly, whenever an object is known under the aspect of a common mode of being, i.e. something which the thing has in common with other things, this cognition is called a 'second abstract intention' (*secunda intentio in abstracto*). The object itself is called 'second concrete intention' (*secunda intentio in concreto*). Examples of second concrete intentions are genus, species, difference, universal; examples of second abstract intentions are universality, speciality etc., i.e. terms that indicate what it is that makes a universal a universal, or a species a species etc.

The same account holds for the other acts of the intellect: composition and reasoning. Some examples: a first concrete intention on the level of composition is 'man is an animal' or any other composition. The composition is the object of knowledge on this level. The reflection on that particular composition is the first abstract intention on the level of composition. Names of second concrete intentions are 'conclusion', 'proposition', 'question'; an example of a second abstract intention would be the knowledge of a composition as far as it is 'doubted' (in case of a question), or 'affirmed' (in case of a proposition) or 'concluded' (in case of a conclusion of an inference).[162] A first concrete intention on the level of reasoning would be 'every man runs, Socrates is a man, therefore Socrates runs'; a corresponding first abstract intention would be my cognition of the difference between this syllogism and the syllogism 'every donkey runs, Brunellus is a donkey, therefore Brunellus runs.' A name of a second concrete intention would be 'syllogism' or 'deduction'; but when I consider e.g. the difference between inductive and deductive reasoning, this act of my intellect is a second abstract intention. The diagram below may help to make this complicated theory more clear.

Thus, every object can be known in two ways: either according to its proper mode of being or under its common mode of being. In the first case we have an absolute or primary cognition, in the second case a relative or secondary cognition. The second intention is called 'relative' because there is always a relation with other objects, and it is called 'secondary' because an absolute cognition is presupposed.[163]

First as well as second intentions are caused by a mode of being of the object in co-operation with the intellect. On the level of apprehension the object is an extra-mental thing, for example a person named John. John is known in a first cognition as a man. The concept and first concrete intention *man* is not generated by the intellect, but abstracted from the extra-mental object. This kind of abstraction could be compared with what Thomas Aquinas calls total or non-precisive abstraction,[164] allowing one to say 'John is a man'. A second kind

[161] The distinction between concrete and abstract intentions had been introduced by Henry of Ghent (d. 1293). See for an account of his theory: Pini [2002, pp. 68–72].
[162] Radulphus Brito, *op.cit.*, paragraph 50.
[163] Pinborg [1974, p. 53].
[164] Thomas Aquinas, *De ente et essentia*, 3. 68–70.

level of cognition	**intention**	**concrete**	**abstract**
apprehension	first	man (according to his proper mode of being)	humanity
	second	universal, genus, species, differentia	universality, generality etc.
composition	first	man is an animal	reflection on this composition
	second	conclusion, proposition, question	this composition as far as it is concluded, affirmed/denied or doubted
reasoning	first	every man runs, Socrates is a man, therefore Socrates runs	reflection on this reasoning, e.g. that it is deductive reasoning
	second	syllogism, enthymeme, inference, deduction	

of abstraction results in the concept and first abstract intention *humanity*. In Aquinas' terms this would be called formal or precisive abstraction. It is obvious that 'humanity' cannot be predicated of John. Only concrete intentions can be predicated, but it is not the intention *man* that is predicated, because John is not a concept, but the real man under the aspect of humanity (*sub ratione humanitatis*). The predication implies a description that corresponds with the mode of being of the object.[165]

Radulphus pays ample attention to the question of the origin and foundation of second intentions.[166] On the level of apprehension the foundation of a cognition of second intention (e.g. when species is predicated of man) is the same as that of a cognition of first intention (e.g. when animal is predicated of man), namely a real man; only the mode of being under which the object is known is different.[167] Thus, second intentions on the level of apprehension are caused by a real object under a common mode of being (*sub modo essendi communi*). Radulphus concludes that these second intentions are real (but of course not extra-mental) habits of the intellect.[168] Logic considers things according to their (common) modes of being

[165]Pinborg [1974, p. 53].
[166]Radulphus Brito, soph. 'Aliquis homo est species', paragraph 52–59. See also Pinborg [1974, pp. 53–4].
[167]Pinborg [1974, p. 53].
[168]paragraph 56, [ed. Pinborg, p. 146].

that are real and not caused by the intellect. Predicates of second intentions (genus, species, universal) are the object of study of logicians, not, however, in their role of real habits, but in their function of concept and likeness.

On the level of the second and third mental operations (composition and reasoning) it is less obvious how the connection between thought and reality can be preserved, since syllogisms and propositions etc. seem to be mental constructions. Nevertheless, Radulphus argues that all second intentions have a relation to the real world, because ultimately "all logical operations have to be interpreted and verified of the things."[169] In another version of his theory, however, Radulphus admits that second intentions of the second and third kind are entirely caused by the intellect. Thus, these second intentions seem to be less real than the second intentions belonging to the level of apprehension. Since logic is mainly about this kind of intentions, it is called a rational science rather than a real science.[170]

Radulphus' opinions were criticized and modified by Hervaeus Natalis and Peter Aureol, but first we will turn our attention to his contemporary Duns Scotus.

Duns Scotus on intentional being

Duns Scotus (1266–1308) is the first to extend the realm of being to intentional being (*esse intentionale, esse diminutum*). Because even something that does not exist can be the object of our cognition, he concludes that that object cannot be a real extra-mental entity. Therefore, it is a concept or intention, not to be confused with the object of intention in reality (*ratio formalis in re*), upon which it is founded when the object exists.

In the cognitive process Duns Scotus distinguishes between two acts: the sensitive act of perceiving the extra-mental object, and the intellective act of representing it. This intellective act produces an intelligible species. By producing an intelligible species one is able to abstract from the particular circumstances in which the perception occurs. The species, however, is not the object of the intellect. It is merely an instrument, a device we need in the cognitive process. It cannot be the object of knowledge, because it is private to one's individual intellect, whereas an object of knowledge is common to several people.[171] The primary object of knowledge is the intention, or in Scotus' words: intelligible being (*esse intelligibele*). The species has real being (existence) as a quality in the intellect, but the intelligible being has only diminished or *objective* being, i.e. as an object of knowledge. It is not only the object of a human intellect, but also of the divine intellect.[172] This guarantees the reliability of our intellectual knowledge of the world. In producing the intelligible species, the human intellect has access to the realm of intelligible being. In the species the intellect sees the object with intelligi-

[169] Knudsen [1982, p. 489].
[170] paragraph 58-59, [ed. Pinborg, p. 147].
[171] Perler [2001, p. 208].
[172] *Opus Oxoniense*, I dist. III, q. iv. For the complex problem of the ontological status of the intelligible being, see Perler [2002, pp. 217–230].

ble being: the (first) intention, which is its primary object.[173] Thus, the intellect does not produce the intention; it finds it by way of an intelligible species.

Although the intelligible species is not the primary object of knowledge, it can become the secondary object when someone reflects upon the way he cognizes a thing. The result is a second intention. According to Scotus, second intentions are notions caused by the intellect, when it considers properties pertaining to things insofar as they are understood, the so-called intentional properties, for example 'being predicable of many'. Like Aquinas and unlike Radulphus Brito, Scotus maintains that second intentions are created by the intellect as tools for grasping first intentional objects. The intellect is moved to cause a second intention when it considers an essence as a universal. An essence is in itself neither individual nor universal. It is the intellect that gives universality to that essence. Universals (first intentions) are produced by the intellect as rationate relations, but they have their genetic occasion in the essence of the object of which they are predicated. Thus, intellect and object work together in producing first intentions: the intellect as the primary cause and the extra-mental object as the occasion. Although there has to be an extra-mental object for the intellect to cause a second intention, this object plays no role in producing the second intention; the intention itself is founded on a property of an object as understood by the intellect, i.e. a mental entity. For example, the intellect considers an essence *man* as universal and is moved by its property of being predicable of many, so that the second intention *species* is formed. The result is not that the extra-mental object itself becomes universal but that the intellect attributes universality to the way it understands the extra-mental object. First intentions represent extra-mental objects, whereas second intentions represent first intentions.[174]

Like Radulphus Brito, Scotus distinguishes between concrete and abstract intentions, but he gives a different interpretation to the terms. For Radulphus a concrete intention is caused by an object under its proper mode of being and an abstract intention is caused by that same object under its common mode of being. In other words: an abstract intention is a concept, whereas a concrete intention is an aggregate of thing and concept. For Scotus all intentions are concepts. Concrete and abstract intentions are just different modes of signifying the same thing, in the same way as 'white' and 'whiteness' both signify the form whiteness: 'white' signifies the form as attributed to a white thing and 'whiteness' signifies the form itself.[175]

Unlike his intentionalistic predecessors, Scotus does not hold that second intentions are the subject of logic. Logic *deals* with second intentions, but its subject is syllogism. Only syllogism satisfies all three conditions that should be met for something to be the subject of a science: (1) its nature and existence must be known, (2) its properties (e.g. validity) are demonstrated in the science of which it is the object, and (3) everything considered in that science can be reduced to

[173]This is Scotus' implementation of the Augustinian concept of illumination.
[174]Pini [2002, pp. 107–112]. See also Pini [2002] for the relevant texts.
[175]Pini [2002, p. 128]. For Duns Scotus on concrete and abstract names, see above p. 365.

it.[176] And what about the other possible subjects of logic: second intentions, rationate being and the acts of the intellect? The nature and existence of second intentions are not known in logic, so second intentions cannot be its subject. The same account holds for rationate being and the acts of the intellect. Scotus also rejects the old opinion according to which logic is a *scientia sermocinalis*, which means that it is concerned with speech. Demonstrations are not concerned with speech, according to Scotus, and what is studied in logic cannot be reduced to speech. Finally, Scotus denies that reasoning is the subject of logic, since it does not satisfy any of the three conditions.[177] Second intentions do satisfy the third condition: everything considered in logic can be reduced to it, and therefore logic can be said to deal with second intentions.

Reactions to Radulphus Brito: Hervaeus Natalis and Peter Aureol

In the writings of Hervaeus Natalis and Peter Aureol the theory of Radulphus Brito was criticized and modified. The distinction between proper and common modes of being of an object seems to be abandoned. Instead, they introduce a new mode of being of the object, the *esse obiective*.

Hervaeus' views can be found in his treatise *De secundis intentionibus*, dated ca. 1313,[178] which was in fact the first 'stand alone' treatise about this subject. Like many others before him Hervaeus defines an intention as a tendency to something.[179] The word 'intention' has two senses, he says. Taken from the viewpoint of the intellect it means: that which, by way of representation, leads the intellect to the cognition of something, whether it is an act of the intellect, a species or a concept. In this sense an intention is always a real entity, because species or acts of the intellect are real (be it not extra-mental) entities.[180] Taken, on the other hand, from the viewpoint of the object known, 'intention' means: that to which the tendency is directed, the intellective content.[181] It is in this last sense that Hervaeus speaks about intentions. Like Radulphus, he distinguishes between 'concrete' and 'abstract' intentions, but in his application of this distinction he deviates from Radulphus. According to Hervaeus, *concrete first intentions* are real (extra-mental) entities *taken as formal objects of the intellect*, whereas for Radulphus concrete first intentions were extra-mental objects taken absolutely, i.e. not related to an act of the intellect. For Hervaeus there is no intention without an act of knowing, because 'intention' is not part of the definition of extra-mental objects like 'man' or 'stone' (57). *Abstract first intentions* are merely rationate entities (*entia rationis*), since an abstract first intention is a relation between the

[176] Duns Scotus *Super Porph.*, 3, n. 20 (*Oph*, I, 17), as in: Pini [2002, p. 35].
[177] Pini [2002, p. 35].
[178] De Rijk [2005, p. 252].
[179] Hervaeus Natalis, *De secundis intentionibus*, qu. 1, 14. (prel. ed. Judith Dijs. The numbers refer to the paragraph numbers in this edition.) See for an extensive treatment of this treatise: De Rijk [2005, chapter VII].
[180] Qualities and relations were considered real entities by many medieval thinkers.
[181] De Rijk [2005, p. 295].

object known and the act of knowing (*habitudo rei intellectae ad actum intelligendi*). Second intentions are always rationate entities, because *concrete second intentions* are things that are secondarily known, and *abstract second intentions* are relations between the objects known and the act of knowing (18).

According to Hervaeus, first as well as second intentions, whether concrete or abstract, exist *obiective* (i.e. as objective contents) in the intellect (17), since everything that is known exists as an object in the intellect. Thus, when the intellect knows a man or a horse, man or horse is said to exist objectively in the intellect as first concrete intentions. Second intentions are founded upon first intentions and therefore, they exist objectively in the intellect as well (23).

Intentionality exists on two levels, according to Hervaeus.[182] First concrete intentions are founded upon the objects intellected that possess real being outside the mind. The intentionality itself, i.e. the relationship between object intellected and the intellect, is a first abstract intention or *primary intentionality*. If this primary intentionality is itself the object intellected, it is the foundation of another relationship to the intellect: a *secondary intentionality*. In this case the first abstract intention is at the same time a second concrete intention.[183] So, either mediately or immediately, all second intentions are founded upon a first one, which in its turn is founded upon extra mental reality.

Hervaeus deviates from Radulphus on several points. I mention the most important ones:

1. A first intention is not identical with the act of intellection. This is only the case from the viewpoint of the intellect, which is not of interest for the logician.

2. A concrete first intention is not an extra mental object taken absolutely, but rather that object as known by the intellect.

3. First and second intentions are not founded upon different modes of being (properties) of the object known. First intentions have their foundation in reality, and second intentions are founded upon first intentions.

4. All intentions exist objectively in the intellect.

Thus, like Radulphus, Hervaeus turns out to be aware that our intellect makes distinctions that are ultimately founded upon reality, but, unlike Radulphus, he does not think that every abstract intention corresponds with a (common) mode of being in reality.

Although the French Franciscan Peter Aureol (working in Paris ca. 1320) was a theologian and not a logician, he nevertheless deserves a place in these paragraphs on the intentionality debate, because he opposed the views of Radulphus Brito and Hervaeus Natalis and is primarily known as one of Ockham's forerunners as a conceptualist, although recent studies have shown his originality and historical

[182] *De secundis intentionibus*, dist. III, q. 3c, quoted by De Rijk [2005, p. 282].
[183] De Rijk [2005, p. 280].

significance.[184] In Peter Aureol's theory the distinction between concrete and abstract intentions has been abandoned. In Peter's view, intentions are rationate entities (*entia rationis*), to be distinguished from the act of the intellect and existing in the intellect as objective contents of thought (*obiective*). A first intention arises when the object of the intellect is a real being; when the object is itself a rationate being, a second intention is formed. First intentions are products of the intellect (concepts) in which a real being has obtained a new mode of being, i.e. intentional (objective) being. Second intentions are formed by reflection on first intentions. Peter Aureol himself gives the following account of his theory of intentions:

> An intention is the objective concept formed by the intellect containing in itself indistinguishably the passive conception and the thing conceived. 'Intention' is the same as 'concept', and 'first intention' the same as 'concept of the first order', which the intellect forms of the things without reflecting on its own concepts. Second intentions are concepts of the second order, which the intellect constructs by reflection on the first concepts: *universality, predicability*, and the like concerning the simple mental operation of apprehension; *affirmation* or *negation* concerning the second mental operation of judgement; and *the connection of the extremes of a syllogism* concerning the third mental operation, the discursive and ratiocinative act.[185]

Peter disagrees with Radulphus who identified intentions with acts of the intellect, and he rejects Hervaeus' distinction between the object intended (concrete intention) and the relation (*habitudo*) between the intended object and the intellect (abstract intentionality). In Peter's view object and relation are the same. An intention is just an obiective concept, an intentional being, formed by the intellect and to be distinguished from the act itself. It is the mode of being of the object as it appears in the intellect. Object and relation are comprehended in the concept. The intellect can conceive existing and non-existing objects alike.

Logic is concerned with second intentions, but, unlike his intentionalist predecessors, Aureol does not think that second intentions are its proper subject. The logician studies second intentions not as rationate entities – that aspect belongs to the domain of metaphysics – but as far as they are reduced to terms, propositions and syllogisms. Thus, what Aquinas had called the proper matter of logic is for Aureol the proper subject of logic.

The new ontological status of concepts gave rise to severe criticism by Walter Chatton, a contemporary of Aureol and Ockham. Although Ockham defended the objective status of concepts himself in his so-called *fictum*-theory, he abandoned this theory on grounds of parsimony. Like Chatton he saw the *esse obiectivum* as

[184] Russell L. Friedman & Lauge O. Nielsen [2000], Peter Auriol: Introduction, in: *Vivarium* XXXVIII, 1, 1.

[185] Translation Knudson [*op.cit.*, p. 491]. The corresponding Latin text can be found in De Rijk [2005, pp. 738–9].

an unnecessary mediator between object and cognition. As a terminist logician he was more interested in the question if intentions are to be identified with mental terms, a question which will be discussed in one of the next chapters.

Conclusion

The foregoing pages were devoted to a period of only fifty years, which in the course of the history of logic could be considered as an intermezzo. The developments in terminist logic seem to be interrupted by a period of metalogical concerns, as if the authors needed a moment of reflection, concentrating for a while on the scientific foundations of the art of logic. In my view, the most important results of these reflections were Roger Bacon's formation of a theory of signs, Duns Scotus' rejection of the parallelism between signifying and understanding and Peter Aureol's clean way of applying the parallelism between thought and reality. These philosophers paved the way for the innovations of 14th century logic.

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THE NOMINALIST SEMANTICS OF OCKHAM AND BURIDAN: A "RATIONAL RECONSTRUCTION"

Gyula Klima

SOME HISTORICAL-PHILOSOPHICAL PRELIMINARIES

This paper is going to outline the innovative semantic theories of the two great 14^{th}-century nominalist thinkers whose work eventually gave rise to the quasi-institutional separation of the nominalist *via moderna*, the "modern way", from the realist *via antiqua*, "the old way" of doing logic, science, philosophy, and theology in the late Middle Ages.[1] The person who initiated these changes was the English Franciscan theologian, William Ockham. However, the person who was primarily responsible for establishing Ockham's nominalism as a genuinely viable theoretical alternative was the French secular Master of Arts, John Buridan.

The historical significance of Ockham's innovations is that they were the first to introduce a radically new type of theoretical disagreement into scholastic discourse, a type of disagreement with which we, as heirs to these historical developments, are all too familiar, namely, the conflict between proponents of paradigmatically different conceptual schemes. In the case of conflicts of this type, the disagreement is not merely over different possible answers to the same questions answerable within the same conceptual framework. For in a conflict of this type there is no uniformly shared conceptual framework that would fix the commonly recognized rules of the "language games" to be played by the disputants in discussing their disagreements. Therefore, what becomes at stake in these conflicts is rather the very legitimacy of some of the questions and rules of the opposing camp. For example, after Ockham, the old *metaphysical* question "What are the common natures signified by our common terms and how are they related to the singulars?" has to yield to the new *semantic* question "Do our common terms signify some common natures in the first place?" Ockham's and Buridan's resounding 'no' to this new question and their relentless pursuit of the implications of this answer radically transformed late-medieval theoretical discourse.

The importance of Ockham's and especially Buridan's work consists in their presenting a consistent, alternative way of construing the fundamental semantic

[1] For a detailed historical discussion of the late-medieval separation of the *via antiqua* and the *via moderna* see: Moore, W. L. 1989 "Via Moderna", in: J. R. Strayer: *Dictionary of Middle Ages*, New York: Scribner, 1989, vol.12. pp. 406-409.

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relations between language, thought and reality. Indeed, the real strength of their nominalism is not so much their criticism of the older, realist way of construing these relationships as their detailed, systematic account of how a philosophy of language based on a strictly nominalist ontology (denying any form of extramental universals and keeping the number of distinct ontological categories at a "bare minimum" of two or three)[2] is possible.

SOME METHODOLOGICAL PROVISOS

Since this paper is going to focus exclusively on Ockham's and Buridan's *semantic* ideas, it will *not* provide a comprehensive discussion of their logic. Logic in the Middle Ages was a much more comprehensive subject than we conceive of it nowadays, for both theoretical and historical reasons. The primary *theoretical* reason why medieval logic comprised subjects that we would recognize as falling under such varied subjects as metaphysics, cognitive psychology, linguistics, the philosophy of science, and epistemology is the medieval conception of logic as a universal theoretical tool (*organon*) of reason in its pursuit of truth and avoidance of error. The main *historical* reason is the development of medieval logic as a largely conservative enhancement and systematization of Aristotelian logic, in combination with available elements of Stoic logic.

Accordingly, even if the medieval logical output is recognizably about *logic* even to the modern reader, it would appear to be mingled with considerations pertaining to various, sometimes from our perspective somewhat loosely related subjects. But even *within* what we would recognize as strictly pertaining to logical theory itself, we would find an interesting mixture of what we would regard as syntactic and semantic considerations, forming only partial theories of various types of natural language reasonings. The theory of syllogistic, for instance, is a syntactical "validity-checker" for a certain limited type of two-premise inferences,

[2]The denial of the existence of real universals is one of the fundamental theses of nominalism. However, we should keep in mind that medieval *moderate realists*, which means practically *everybody* after Abelard or maybe even after Boethius, also denied separate Platonic Forms, just as they would deny most modern analytic metaphysicians' abstract properties. Medieval nominalists after Ockham would therefore distinguish themselves from moderate realists by denying the existence of inherent common natures distinct from their particulars posited by moderate realists (such as Aquinas or especially Scotus), as well as by reducing the number of distinct ontological categories to two, namely, substance and quality (Ockham and his followers) or three, namely, substance, quantity and quality (Buridan and his followers). However, quite interestingly, the same sort of ontology was also accessible in the older framework, and was in fact proposed by later moderate realists, such as the 15^{th}-century Dominican, Domingo Soto. So the fundamental difference between medieval nominalists and moderate realists lies not so much in their respective ontologies, but rather in their different semantics, in the different ways in which they explain the relationships between language, thought, and reality. For further details of this sort of comparison, see Klima, G. (1999) "Ockham's Semantics and Ontology of the Categories", Spade, P. V. (ed.), *The Cambridge Companion to Ockham*, Cambridge: Cambridge University Press, pp. 118-142; Klima, G. (1999) "Buridan's Logic and the Ontology of Modes", in: Ebbesen, S. – Friedman, R. L. (eds.), *Medieval Analyses in Language and Cognition*, Copenhagen: The Royal Danish Academy of Sciences and Letters, 1999, pp. 473-495.

whereas the theory of supposition (together with the theories of *ampliation* and *appellation* and other *properties of terms*)[3] is a philosophical-semantic theory of reference, occasionally used to justify certain rules of inference and falsify some apparent, fallacious rules of inference, as part of the theory of fallacies.

Therefore, when we are discussing characteristic semantic ideas of our medieval colleagues as part of their logical theory, we should be constantly aware of the rather different theoretical context in which these semantic ideas functioned. Thus, for example, even if the idea of semantic compositionality was definitely present in medieval authors in some form,[4] we should not expect them to provide recursive definitions allowing the effective computation of semantic values of complex expressions as functions of the semantic values of their components in the way *we* would construct a semantic theory. Nevertheless, this fact does not exclude the possibility of a *rational reconstruction* of their ideas in the sense that following *their* intuitive clues, *we* may still provide such definitions that could constitute what *we* would recognize as a full-fledged semantic theory for a certain class of expressions, culminating in a semantic definition of logical validity. At any rate, the subsequent discussion will provide the outlines of a rational reconstruction of this sort, in the hopes that this approach will not only shed some light on certain intriguing features of medieval semantics, but that it will also facilitate comparisons between medieval and modern ideas, pointing to such features of the medieval ideas that we also can (and *should*) take seriously in our own thinking about the semantics of natural languages.

But apart from the potential fruitfulness of this approach from a contemporary perspective, there is another consideration that necessitates it in this discussion, namely, the immense variety of the relevant semantic ideas in the medieval output. In order to understand the importance and character of Ockham's semantic innovations and their further development by Buridan, we have to contrast their ideas with "the former paradigm". But in order to do so, we have to *reconstruct* that "paradigm" as such, i.e., we have to provide a certain schematic summation of those common features of the semantic ideas of earlier authors that Ockham and Buridan abandoned in their paradigmatically different semantic construction.

The rational reconstructions sketched in this paper, therefore, ought not to be regarded as attempted answers to the *factual*, *historical* question: what was the logical semantic theory of this or that medieval author like? Rather, they should be

[3]Cf. Read, Stephen, "Medieval Theories: Properties of Terms", *The Stanford Encyclopedia of Philosophy (Spring 2006 Edition)*, Edward N. Zalta (ed.), `http://plato.stanford.edu/archives/spr2006/entries/medieval-terms/`

[4]See the following interesting remark by G. Nuchelmans: "... the signification of the whole complex was commonly held to be of a compositional nature and to be determined by the signification of its parts. As Pardo put it, only incomplex expressions have been given conventional meanings in a primary and immediate way; a propositional complex, such as *Homo est animal*, on the other hand, has been destined to signify its meaning only in a mediate, consequential and secondary manner, since its signification can be derived from the significations of the incomplex parts." *Late-Scholastic and Humanist Theories of the Proposition*, North Holland Publishing Company, Amsterdam-Oxford-New York, 1980. p.45. Cf. Hieronymus Pardus: *Medulla Dialectices*, Parisiis 1500 (1505), fol.1.V.

regarded as attempted answers to the *counter-factual*, *theoretical* question: what would a semantic theory be like *if* we constructed it on the basis of the semantic intuitions of such and such medieval authors (rather than on the basis of our own post-Fregean/Tarskian intuitions)?

Thus, the discussion will begin with a *rational reconstruction* of what, for want of a better phrase, I will somewhat anachronistically call *via antiqua semantics*, to provide the theoretical contrast for the reconstruction of Ockham's and Buridan's semantic ideas, as they were responsible for the emergence of what might be termed *via moderna* semantics. Next, I will proceed to a brief discussion of what appear to be the main motives and reasons for Ockham's abandonment of the *via antiqua* semantics, and an outline of his alternative semantics, devised to achieve his program of "ontological reduction", to remedy what he perceived as the unjustifiable ontological "excesses" of the older theory. The discussion will then move on to a reconstruction of Buridan's ideas, consciously framed by Buridan in a token-based, nominalist semantics, under the conditions of semantic closure characteristic of natural languages.[5] The concluding section will consider in some detail what sort of modifications the standard semantic construction of modern quantification theory would have to undergo in order to faithfully reflect the two competing medieval conceptions.

VIA ANTIQUA SEMANTICS

Signification

The common starting point for all medieval semantics is the Aristotelian "semantic triangle", the idea based on Aristotle's remarks in his *On Interpretation* to the effect that words signify things not immediately, but with the mediation of the concepts of the mind. There was some disagreement among authors as to whether words signified *primarily* concepts and only *secondarily* the things conceived by means of these concepts or *vice versa*,[6] but since words in themselves were regarded as mere articulate utterances or strings of letters corresponding to such utterances, it was generally agreed that these utterances and strings are significative only in virtue of being subordinated to some acts of understanding. Clearly, if I utter the

[5] A token-, as opposed to type-, based semantics is a semantic system in which each symbol is regarded as a single, individual occurrence to be evaluated as such, as opposed to standing in for any similar token of the same type. To a consistent nominalist, only a token-based semantics is ontologically acceptable. Semantic closure according to Tarski is the condition of a language that contains its own semantic predicates and means of referring to its own items. For further discussion, see Klima, G. (2004) "Consequences of a Closed, Token-Based Semantics: The Case of John Buridan", *History and Philosophy of Logic*, 25(2004), pp. 95-110; Priest, G. "Semantic Closure", *Studia Logica* 43(1984), pp. 117-129.

[6] On what Scotus calls a "magna altercatio" over this issue, see Pini, G. "Signification of Names in Duns Scotus and Some of His Contemporaries." *Vivarium* 39 (2001), pp. 20-51, Pini, G. "Species, Concept, and Thing: Theories of Signification in the Second Half of the Thirteenth Century", *Medieval Philosophy and Theology* 8, 2 (1999), pp. 21-52, and Perler, D. "Duns Scotus on Signification", *Medieval Philosophy and Theology*, 3 (1993), 97-120.

articulate sound 'biltrix' or write down the corresponding string of letters as I just did, anyone who hears or reads it literally has *no idea* what it means. By contrast, if I utter the sound 'man' in English or the corresponding term 'homo' in Latin, anybody who understands these languages will have an understanding of human beings in general, i.e., the word will activate a concept in his or her mind whereby they conceive of human beings in a universal fashion. At any rate, after Boethius, this is the common medieval idea behind any explanation of why Aristotle said that words signify things with the mediation of concepts.

But among "*via antiqua* authors" there was also a further important consideration to motivate the same point concerning common categorematic terms in particular. Commenting on Aristotle's relevant remarks, Thomas Aquinas wrote the following:

> ... names, verbs and speech signify [...] conceptions of the intellect immediately according to the teaching of Aristotle. They cannot immediately signify things, as is clear from their mode of signifying, for the name 'man' signifies human nature in abstraction from singulars; hence it is impossible that it immediately signify a singular man. The Platonists for this reason held that it signified the separated idea of man. But because in Aristotle's teaching man in the abstract does not really subsist, but is only in the mind, it was necessary for Aristotle to say that 'vocal sounds signify the conceptions of the intellect immediately and things by means of them.'[7]

According to this explanation, one can find a proof for Aristotle's claim in the mode of signification of common terms. These terms cannot function in the same way as singular terms naming a certain thing, for there is no such a thing that they could plausibly be taken to name. In accordance with the abstractionist cognitive psychology of medieval Aristotelians, a common concept is formed by abstracting the natures of individual things from their individualizing conditions, by thinking of the natures informing these individuals *not insofar as* they inform *these* individuals.[8] Thus, what a common concept obtained by abstraction directly represents is the nature of *any* thing informed by this nature, whereas the things themselves, the bearers of this nature, each having an individualized instance of it, are represented only indirectly. Therefore, what the corresponding term directly signifies is also this nature itself, whereas the thing bearing this nature is signified only indirectly. But the nature represented by the concept is signified by the term only through the mediation of the concept, *immediately* signified by the term. This semantic idea is spelled out even more explicitly in a 13^{th}-century logic text, the so-called *Summa Lamberti*, in the following fashion:

[7] Aristotle: *On Interpretation: Commentary by St. Thomas and Cajetan*, trans. J. T. Oesterle (Milwaukee: Marquette University Press, 1962), p. 25.

[8] For a more detailed discussion of the medieval Aristotelian conception of abstraction in this connection, see Klima, G., "The Medieval Problem of Universals", *The Stanford Encyclopedia of Philosophy (Winter 2004 Edition)*, Edward N. Zalta (ed.), `http://plato.stanford.edu/archives/win2004/entries/universals-medieval/`.

> . . . it is essential to know that four things are required for an utterance to be significant: a thing, a concept [or some understanding, *intellectus*] of the thing, an utterance, and the union of the utterance with the concept of the thing. What we are calling the thing is something existing outside the soul, which is apprehended by the soul by means of an idea of it — e.g., a man, or a stone. What we call the concept of the thing is the idea [*species*] or likeness of the thing, which exists in the soul; for according to Aristotle in the third book of *De anima* (III, 8, 431b30-432a1), not the stone but rather an appearance [*species*] of the stone is in the soul; and it is by means of the appearance that the soul grasps the thing. The utterance is that which is put forward along with the concept [or understanding] of the thing; in that case [i.e., when the utterance is made with some understanding of a thing] a signification is united to the utterance and the utterance is made significant. And although both the concept of the thing and the utterance are natural in the same way (since they are formed by natural sources), the utterance is nevertheless said to signify by the will of the person instituting it, because the union of the concept of the thing with the utterance is effected by the will, and it is in that [action] that the imposition of the utterance consists. In this way, therefore, an utterance is primarily — in itself — and directly the sign of a concept of the thing; but in addition it is indirectly the sign of the thing. For just as we say that whatever is a cause of the cause is a cause of the thing caused, so we can say that in its own way whatever is a sign of the sign is a sign of the thing signified. Thus, since an utterance is a sign of a concept, and a concept is a sign of a thing, in this way [the utterance] is a sign of the thing as well. An utterance that is a sign of a sign —- of the concept —- will be a sign of the signified — i.e., of the thing; it is, however, a sign of the concept directly but a sign of the thing indirectly.[9]

It is important to note here, as this is going to be one of the sticking points for Ockham, that the term signifies the thing it signifies only by virtue of signifying the concept; still this does not mean that the word is imposed to signify the concept only, because the concept naturally signifies the thing conceived by it. The other important point, again, sternly opposed by Ockham, is that the "thing" signified by a common term is not any of the singular things one can ultimately conceive of by means of the corresponding concept. For the concept directly represents in an abstract universal fashion the nature existing individualized in its particulars. The particulars themselves will therefore not be strictly speaking the *significata* of the

[9] "The *Summa Lamberti* on the properties of terms", *The Cambridge Translations of Medieval Philosophical Texts*, ed. N. Kretzmann-E. Stump, tr. N. Kretzmann, Cambridge University Press: Cambridge, 1988, pp. 104-110. For the same type of reasoning concerning the signification of common terms used by Burley against Ockham, see his *On the Purity of the Art of Logic*, tr. P. V. Spade, Yale University Press: New Haven, 2000, pp. 87-88 (33). For further references to other *via antiqua* authors, see Spade's note 32 on p. 87.

term, but rather its *supposita*, i.e., the things the term can be used to stand for in a proposition. Lambert makes this clear in the immediately following paragraph:

> Now signification differs from supposition in that signification is prior to supposition. For the signification is the concept of the thing represented by means of the utterance, and before the union of it with the utterance there is no term; rather, a term is constituted in the union of that concept of a thing with an utterance. Supposition, on the other hand, is a certain property of a term that has been constituted in that way. There is another difference, because signification extends only to the thing the term is imposed to signify; supposition, however, extends not only to the thing signified by means of the term but can extend to supposita contained under that thing. For example, the signification of 'man' extends only to *man,* not to the things contained under *man;* for 'man' signifies *man,* not Socrates and not Plato. 'Man' can, nevertheless, supposit for Socrates, and for Plato, and for *man.*[10]

The "thing", therefore, that a term on this conception signifies is not any of the ordinary things we would normally use the term to stand for in the context of a proposition. For the thing in question is what the concept directly represents, namely, the nature of the individuals abstracted from its individuating conditions in the formation of the concept. Thus, on this conception, what the term *ultimately* signifies is determined by the *representational content* of the concept *immediately* signified by the term in the mind. That representational content, in turn, is determined by the process of concept-formation, namely, abstraction.

This description of what determines the ultimate signification of a term, however, renders the issue somewhat murky, insofar as it is not exactly clear what we are talking about when were are referring to what a term ultimately signifies. To be sure, it is clear enough what it is *not* supposed to be. It is not supposed to be either the concept that the term immediately signifies or any of the individual things whose nature the concept represents, which are the *supposita* of the term according to Lambert's description. What the term ultimately signifies therefore is either some intermediary representation "between" the concept and the individuals universally represented by the concept or it is the nature of the individuals *as represented* by such an intermediary representation. But even this description of the situation needs some further sorting out. Because we also have to clarify what we are supposed to be referring to when we are talking about "the concept" or understanding that the term immediately signifies "in the mind", and what we

[10]Ibid. Clearly, *man* here is the universal nature signified by the term 'man' and represented by the concept that this term directly signifies, as opposed to the individual humans, such as Socrates or Plato. So, the signification of this term extends only to this universal nature, the direct object of the concept of humans, although, on account of this signification, the term can be used in a sentence to stand for the individual humans who have this nature. The function of the term of standing for these individuals in a sentence is its property that is called 'supposition' (which is why this property is often compared to the modern notion of reference, as it is contrasted with meaning).

are supposed to be referring to when we are talking about "the nature" of the individuals represented by the concept.

To cut a long story short, by the 13^{th} century there was general agreement that there are *no universal things* in reality in the sense of some numerically one entity common to many numerically distinct particular things constituting the substance of each and all at the same time, for the assumption of the existence of such a thing would lead to numerous inconsistencies.[11] To be sure, this did not prevent various authors from talking about common natures or even universal things, but with the understanding that the "things" talked about in this way are not to be understood to be things of nature existing in their universality apart from any consideration of the intellect, rather, they should be regarded as objects of our understanding owing their universality to the abstractive activity of our minds, and having as their foundation in reality their individualized instances constituting the individuals in their specific and generic kinds.

Thus, we should say that what common categorematic terms ultimately signify are these ultimate objects of the intellect, namely, the individualized natures of individual things, the actual presence of which verifies these terms of these individuals. Such an individualized nature, however, is conceived by an act of our understanding in abstraction from its individuating conditions: not *qua* the nature of *this* or *that*, but *qua* that nature, regardless of whether it is of this or of that individual; although everyone agreed that it *could not exist* without being of *some* individual or another. The act of understanding whereby this ultimate object is conceived is an individual concept of *this* individual mind, and the ultimate object is an individualized nature of *that* individual thing. However, since the act of the mind represents this individualized nature in a universal manner, on account of which it can equally represent any other individualized instance of the same nature, the representative content of this act was characterized as the *direct* and *immediate* object of the act, a universal object of the mind, *by means of which* the mind conceives not only of individuals from which it obtained its concept, but also of ones it has never encountered. In late-scholastic terminology, therefore, authors distinguished the *formal* or *subjective concepts*, the individual mental acts of individual minds, from their *objective concepts*, the universal, immediate objects of the formal concepts, whereby the mind conceives of the individualized natures of individuals, its ultimate objects. Some authors even distinguished the objective concept from the common nature itself, i.e., the nature considered in abstraction not only from its existence in individuals, but also from its existence as the object of any intellect.[12]

But we need not go into further details of this rather complicated and difficult psychological as well as ontological doctrine in order to understand its *semantic* import. The upshot of all these considerations, despite variations in finer details in various authors, is that our common categorematic terms ultimately signify

[11] For a discussion of some of these inconsistencies, see again Klima, G., "The Medieval Problem of Universals".

[12] See Klima, G., "The Medieval Problem of Universals", sect. 7.

the ultimate, direct objects of our individual (formal) concepts, namely, the individualized natures of individual things, whereas they immediately signify the immediate objects of these formal concepts (through signifying these formal concepts as well),[13] namely, the objective concepts, and through these, the common natures themselves.

What renders the doctrine difficult to swallow is not only the dubious ontological status of these quasi-entities, these objects-of-the-mind (which are not to be regarded as full-fledged objects in their own right),[14] but also the relative obscurity of their conditions of distinction and identity. In fact, as we shall see, this was precisely one of the main motivations for Ockham to break with this doctrine.

But before dealing with its genuine or perceived problems, we should round out the doctrine insofar as it can serve as the starting point for a full-fledged semantic construction. (Not surprisingly, though, in a proper set theoretical reconstruction, in which these obscure entities are represented by well-behaved set theoretical objects, many of the perceived obscurities of the doctrine almost "miraculously" disappear.) Taking our cue from the foregoing discussion as well as from Peter Geach's seminal paper "Form and Existence",[15] in a formal semantic construction we may represent the signification of a common categorematic term by means of a semantic function that assigns individualized natures, forms or "property-instances" (the terminology is of no importance) to individuals at different times.

Accordingly, if F is a common term, u is an individual element of the universe of discourse U, and t is a time-point or interval (we do not have to determine that in advance), then let $SGT(F)(u)(t)$ be an element of U, representing the ultimate significate of F in u at t, the individualized F-ness of u at time t. The signification of F itself, $SGT(F)$, on the other hand, is the function itself assigning these ultimate significata to F in respect of u and t.[16] The function itself, therefore, can also be regarded as a representation of the objective concept, or the common nature itself, what is immediately signified by F in abstraction from the individualizing conditions of the nature ultimately signified by F. In fact, if we distinguish a special subclass of individuals in U, namely, individual minds designated by m, then we can take $SGT(F)(m)(t)$ to represent what F signifies immediately in mind

[13]See Cajetan: *Commentary on Being and Essence*, tr. by F. C. Wade, SJ, L. H. Kendzierski, Marquette University Press: Milwaukee, 1965, pp. 67-68.

[14]For a more detailed discussion of the ontological issues involved, see Klima, G. (1993) "The Changing Role of *Entia Rationis* in Medieval Philosophy: A Comparative Study with a Reconstruction", *Synthese* 96(1993), pp. 25-59.

[15]P. T. Geach, "Form and Existence," *Proceedings of the Aristotelian Society*, 55(1955), pp. 251-272, reprinted in his *God and the Soul* (London: Routledge & Kegan Paul, 1969), pp. 42−−64.

[16]To be sure, technically speaking, $SGT(F)$ is a function from individuals to functions from times to individualized forms. There are certain technical advantages to working with compounded monadic functions instead of polyadic functions in the semantic construction, but we need not get into those here. These advantages should be obvious in the semantics presented in Klima, G. "Aquinas' Theory of the Copula and the Analogy of Being", *Logical Analysis and History of Philosophy*, 5(2002), pp. 159-176.

m at t, namely, the formal or subjective concept m has of the nature objectively signified by F in abstraction from any mind or any thing that is or can be F.[17]

Furthermore, distinguishing subjective concepts as the immediate significata of F in individual minds may give us the opportunity to make better sense of the distinction between objective concept and common nature made by later Thomists such as Cajetan, taking his cue from Aquinas' remarks in his *De Ente et Essentia*.[18] For if the common nature is the object of the abstractive mind considered in abstraction from both its individuating conditions in extramental things and from its being the object of any mind, whereas the objective concept is the same nature considered insofar as it is the object of some mind or another, then we may say that the common nature is best represented by the signification function, abstracting both from external individuals and minds, whereas the objective concept is best represented by the same function restricted to extramental objects, i.e., the ultimate objects of the formal concepts of individual minds.

But regardless of these somewhat obscure details, the emerging semantic conception is clear enough, and relatively easy to approach from a modern, post-Fregean angle. The ultimate significata of common terms are "trope-like" forms or properties individualized by the subject they inform and by time. What verifies the term of an individual at a given time in a simple predication is, therefore, the actuality, i.e., actual existence of this *significatum*. Thus, for example, 'Socrates is wise' is true because of the actual existence of Socrates' wisdom, namely, Socrates' actual, individualized quality, signified by the predicate 'wise'. Correspondingly, 'Meletus is wise' is false, because of the non-actuality of Meletus' wisdom, provided these sentences are uttered, say, during Socrates's trial. To be sure, if Meletus still *can* be wise at that time, the term 'wise' can be taken to signify his potential wisdom. But in regard of a thing that simply cannot have wisdom, such as a rock or a color, the term 'wise' just signifies nothing. So, the signification of a concrete common term is best represented by a semantic function that takes individuals and times as its arguments, and yields actual or potential individualized properties for these arguments or nothing, in case it is undefined for those arguments.[19]

A simple predication, therefore, in general, yields the combination of the signification of the predicate with its appropriate arguments, provided by those semantic values of the other syntactic components of the predication that determine the individualized significata of the predicate. These "individualizing factors" are the individual thing provided by the *suppositum* (i.e., referent) of the subject, and

[17]This suggestion is worked out in some detail in a formal semantics presented in Essay V of Klima, G. *Ars Artium: Essays in Philosophical Semantics, Medieval and Modern*, Budapest: Institute of Philosophy of the Hungarian Academy of Sciences, 1988.

[18]See n. 13.

[19]These remarks also indicate how we should treat the semantics of the corresponding abstract terms. They signify the same as their concrete counterparts; however, because of their different mode of signification, when they are made the subject of a proposition, they stand for their significata, and not for the things bearing their significata in the way the concrete terms do. Again, the formal semantics for these is worked out in my *Ars Artium*. See also Klima, G. "The Semantic Principles Underlying Saint Thomas Aquinas's Metaphysics of Being", *Medieval Philosophy and Theology*, 5(1996), pp. 87-141.

the time provided by the tense of the verb and the context (of the utterance or, for example, the actual interpretation of a written predication). This last remark, however, introduces two other essential elements into the picture, namely, the verb needed for the predication, i.e., the *copula*, along with the time it co-signifies, and the *supposition* (reference) of the subject.

Supposition

As we have seen, the *via antiqua* analysis of signification provides a number of different semantic values for any common term: their immediate significata (whether those should be identified as the formal concept, objective concept, or the common nature represented by these concepts, or all these in a sequence), their ultimate significata (the individualized natures, forms or "property instances" of individuals), and the merely indirectly signified individuals themselves. Ordinarily, we use concrete common terms to talk about these last-mentioned items. When we in fact use a term in a proposition to talk about these, namely, about the individuals in which the ultimate significata of the term are actual, then the term is said to be in personal supposition, standing (or to use the common neologism, *suppositing*) for its *personal supposita*. When it stands for its (various) immediate significata (or even perhaps its ultimate significata or its significata in relation to its supposita, etc.),[20] then it stands in *simple supposition*. And when it is used to stand for itself (or similar tokens of the same type), then it is said to be in *material supposition*. For example, 'man' in 'Man is a species' stands for the objective concept of humans, i.e., it has simple supposition, but in 'A man is a rational animal' it has personal supposition, and in 'Man is a noun in English', it has material supposition.

Attributing supposition, i.e., a context-dependent referring function to common terms stands in stark contrast with the Fregean conception, on the basis of which Peter Geach has repeatedly criticized the medieval idea of common personal supposition.[21] However, one can clearly reconstruct this semantic function of common terms by using *restricted variables*, i.e., variables that are exactly like the variables of standard quantification theory, except they range not over the entire universe of discourse but only over the extension of their matrix. For instance, the sentence 'A man is a rational animal' in this reconstruction would not have to be formalized by using an unrestricted variable bound by an existential quantifier, forcing us to introduce a conjunction to provide the correct truth conditions of the original. In-

[20] Walter Burley, for example, presents an elaborate system of distinctions for various sorts of simple supposition in his *Tractatus de suppositionibus*, in S.F. Brown, "Walter Burleigh's Treatise *De suppositionibus* and its Influence on William of Ockham", *Franciscan Studies*, 32 (1972), pp. 15-64. he also changed his mind about parts of his doctrine in his *On the Purity of the Art of Logic*. For details, see Spade, P.V., "Walter Burley on the Kinds of Simple Supposition", *Vivarium*, 37(1999), pp. 41-59. But we find similar distinctions in other "older" authors as well.

[21] See Geach, P. T., *Reference and Generality*, Cornell University Press: Ithaca, NY, 1962, c. 2, sect. 28, pp, 34-36; Peter Geach, "Nominalism," *Sophia* 3(1964), pp. 3-14, reprinted in his *Logic Matters*, Berkeley and Los Angeles: University of California Press, 1980, pp. 298-301.

stead, representing the subject term by the quantifiable restricted variable '$x.Mx$', we get a formula that need not contain a conjunction: '$(\exists x.Mx)(Ax.)$'.[22]

In fact, since the syntax of this formula is much closer to the original in this regard than formulae of standard quantification theory, it should be clear that just as changing the determiner in the English sentence (using 'every', 'the', 'this', 'no', or, switching to the plural, 'two', 'five', 'most', 'twenty percent of', in the place of the indefinite article, etc.) does not introduce new conjunctives into the sentence, so the addition of the corresponding (non-standard, numerical, and even "pleonotetic" quantifiers) to this sort of formula need not introduce different logical connectives into the syntax of this formula, as it does in the case of standard quantificational formulae (as when we have to switch from conjunction to implication when replacing the existential quantifier with the universal quantifier — but we cannot do that with the pleonotetic quantifiers).[23]

Thus, using restricted variables to represent common terms in personal supposition clearly has the advantage of providing a better "match" with the syntax of natural languages than the formulae of standard quantification theory can provide. However, with the appropriate *semantic* interpretation restricted variables can do even more.

If restricted variables are used to represent common terms in their referring function, then the *supposition* of these terms can best be interpreted as the value-assignment of such variables. For example, in accordance with the doctrine of personal supposition, 'Every man is an animal' was analyzed by the medievals in terms of the conjunction 'This man is an animal and that man is an animal ...', where the demonstrative pronouns pick out all individuals falling under 'man'. But if we represent this sentence as '$(\forall x.Mx)(Ax.)$', then the restricted variable in this formula, '$x.Mx$', does exactly the same thing, namely, it takes its values from the extension of its matrix. Thus, we can justifiably define a *supposition function* for this variable analogously to the *value assignment function* of ordinary variables of standard quantification theory, with the only difference that whereas ordinary variables range over the entire universe of discourse, restricted variables range only over the extension of their matrix.

However, there can obviously be cases when the extension of the matrix of a restricted variable is empty, namely, when the common term represented by the variable is true of nothing. In such a case we may assign the variable some artificial value, whatever that may be, of which no simple predication is true.[24] This move at

[22]This is the approach to the reconstruction of certain features of medieval supposition theory I first presented in my *Ars Artium*. But there are a number of other, basically equivalent approaches in the literature e.g. by G. Englebretsen, D. P. Henry, A. Orenstein and T. Parsons.

[23]For a discussion of this observation see essay III of my *Ars Artium*. The impossibility of representing pleonotetic quantifiers in standard quantification theory was first proven (for 'most' interpreted as 'more than half the') in J. Barwise and R. Cooper, "Generalized Quantifiers and Natural Language", *Linguistics and Philosophy*, 4(1981), pp.159-219, pp.214-215. (C13), setting off a whole cottage industry of generalized quantification theory in the eighties.

[24]Again, I took this approach in Essay II of my *Ars Artium* as well as in "Existence and Reference in Medieval Logic", in: A. Hieke — E. Morscher (eds.): *New Essays in Free Logic*, Kluwer Academic Publishers, 2001, pp. 197-226.

once yields the result that universal affirmative propositions will have existential import, which in turn restores both the traditional Square of Opposition and traditional, Aristotelian syllogistic invalidated by the conditional interpretation of universal affirmatives necessitated by standard quantification theory.[25]

Such results are interesting and may be regarded as forming a certain conceptual bridge between modern and medieval theories. But we must not forget that the medievals themselves did not analyze sentences in terms of a quantifier-analysis, not even in terms of restricted variables. Rather, they made distinctions *equivalent to* distinctions *we* would draw in terms of quantifier scopes, using their own theoretical devices to distinguish different *modes of personal supposition*, namely the so-called *suppositional descents*. Since these are discussed elsewhere in this volume, and they are not directly relevant to the contrast between the two *viae* I am dealing with here, I will not go into the details of this theory. All that is relevant from our present point of view is that according to the *via antiqua* conception, the two terms of a categorical proposition have radically different semantic functions. The subject term supplies its supposita to fill in the argument places of the abstract signification-function of the predicate, thereby determining *which* ultimate significata of the predicate need to be actual to render the proposition true. The *signum quantitatis* (the "quantifier word") of the subject term will determine *how many* of these significata will have to be actual, whereas the tense of the copula will determine *when* these significata will have to be actual. But the copula on this conception will do actually much more, namely, it will signify the actuality of these ultimate significata, effect the combination of subject and predicate, and signify the existence of the resulting propositional complex, while co-signifying the time when this complex needs to be actual for the truth of the proposition.

The copula and the significata of propositions

If we look at the foregoing suggestions for reconstructing the semantic functions of subject and predicate, it will be obvious that the function of the copula is not represented in these suggestions at all. However, if we want to take the reflections of medieval authors on the issue seriously, we have to acknowledge that the copula in their analysis is not just a mere syntactical marker of the application of predicate to subject (to distinguish a predication from a mere list), but it actually has the genuine semantic function of predicating existence. As Aquinas explains:

> The reason why [Aristotle] says that the verb 'is' co-signifies composition is that it does not principally signify composition, but secondarily; for it primarily signifies what occurs to the mind in the way of actuality absolutely: for 'is', uttered absolutely, signifies being in act, and hence it signifies as a verb. But since actuality, which the verb 'is' principally signifies, is in general the actuality of every form, whether

[25] Cf. Parsons, Terence, "The Traditional Square of Opposition", *The Stanford Encyclopedia of Philosophy (Winter 2006 Edition)*, Edward N. Zalta (ed.), `http://plato.stanford.edu/archives/win2006/entries/square/`.

> it is a substantial or an accidental actuality, this is why when we want to signify any form or act to actually inhere [*inesse*] in a subject, we signify this by means of the verb 'is', either absolutely [*simpliciter*] or with some qualification [*secundum quid*] ...[26]

That is to say, according to Aquinas, the reason why we use the verb signifying existence to indicate the application of the predicate to the subject is precisely because in any act of predication we actually predicate existence: either the existence of the thing supposited for by the subject absolutely, or the existence of the form signified by the predicate in the subject.

Thus, the copula with respect to a suppositum of the subject and with respect to the ultimate significatum of the predicate in that suppositum signifies the existence of this ultimate significatum, which can be compositionally determined as the value of the signification of the verb 'is' and its equivalents. However, depending on the nature of the ultimate significata of the predicate, the existence of these ultimate significata may be radically different. This is the clearest in the case of the ultimate significata of a privative predicate, such as 'blind', and the corresponding positive predicate, such as 'sighted'. Clearly, for the ultimate significata of 'blind' to exist is for the ultimate significata of 'sighted' not to exist. Therefore, since nothing can be existence and non-existence in the same sense, we cannot say that in 'Homer is blind' and 'Socrates is sighted' the copula would signify existence in the same sense. So, the significata of the copula in respect of the ultimate significata of the predicates of these sentences in the supposita of the subjects cannot be said to be acts of existence in the same sense.

However, at the same time, with regard to the immediate significatum of the predicate (the common nature signified by the predicate) the copula also signifies the existence of some other type of entity uniformly, in the same sense, according to Aquinas, namely, of the entity signified by the proposition as a whole, the so-called *enuntiabile*.[27]

The conception of propositional signification involving such entities crops up quite early in the history of medieval logic, and recurs in different guises time and again.[28] It is present in Abelard's theory of *dicta*, and it is worked out in greater detail by the anonymous author of the 12^{th}-century tract *Ars Burana* as follows:

> Note that whether we speak about the *dictum* of a proposition or of the *significate* of a proposition or of an *enuntiabile* it is the same. For an enuntiabile is what is signified by a proposition. For example: 'A man is an animal', this proposition is true, because what it signifies is true;

[26] *In Perihermeneias* lb. 1, lc. 5, n.22

[27] For a detailed reconstruction of Aquinas' ideas on the signification of the copula along these lines, see Klima, G. "Aquinas' Theory of the Copula and the Analogy of Being", *Logical Analysis and History of Philosophy*, 5(2002), pp. 159-176.

[28] Cf. Nuchelmans, G. *Late Scholastic and Humanist Theories of the Proposition*, Amsterdam: North Holland Publishing Co., 1980, and *Theories of the Proposition: Ancient and Medieval Conceptions of the Bearers of Truth and Falsity*. Amsterdam: North Holland Publishing Co., 1973. Perler, D. "Late Medieval Ontologies of Facts", *The Monist* 77(1994), pp. 149-169.

> and that true thing that you in this way understand is the enuntiabile, whatever it is. Similarly, when I say: 'Socrates is an ass', this proposition is false, because what it signifies is false, and the false thing that you conceive in this way is the enuntiabile. And this cannot be seen, nor heard or sensed, but it is only perceivable by the intellect. If you ask in which category of things it belongs, whether it is a substance or an accident, of the enuntiabile we have to say that it is neither a substance nor an accident nor does it belong to any of the categories. For it has its own peculiar type of existence. And it is said to be extrapredicamental, not because it does not belong to any category, but because it does not belong to any of the categories distinguished by Aristotle. Therefore it belongs to some category that can be called the category of *enuntiabilia*. And in this category the most general item will be that consignified by the term 'enuntiabile'. And this can be divided further as follows. Some enuntiabilia are of the present, some are of the past and some are of the future. Furthermore, some enuntiabilia are true and some are false. And further: of the true ones some are necessary and some are not necessary, and of the false ones some are possible and some are impossible. So it is to be understood what an enuntiabile is.[29]

"At the other end of the story", we have, for example, Aquinas' famous 16^{th}-century commentator, Thomas de Vio Cajetan declaring the following:

> And note that Aristotle's maxim posited here: 'a sentence is true according as the thing is or is not' is to be understood not of the thing which is the subject or the predicate of this sentence, but of the thing which is signified by the whole sentence, e.g., when it is said 'a man is white', this is not true because a man or a white thing is, but because a man's being white is, for this is what is signified by this sentence.[30]

And of course we must not forget various "major players" in the meantime, such as Walter Burley, proposing *real propositions* corresponding to ordinary written or spoken propositions, or Adam Wodeham and Gregory of Rimini positing *complexe significabilia* in the same role, evoking the relentless criticism of John Buridan.[31]

What all these in their details rather disparate views seem to boil down to is that these authors, having already been committed to a wealth of semantic values

[29] De Rijk, L. M. (ed.), *Logica Modernorum*, Vol. II, Part II, Van Gorcum: Assen, 1967, pp. 357-359.

[30] Cajetan, Thomas de Vio, *Scripta Philosophica: Commentaria in Praedicamenta Aristotelis*, ed. M. H. Laurent, Angelicum, Romae, 1939, p. 87.

[31] Cf. Zupko, J. "How it Played in the *rue de Fouarre*: Reception of Adam Wodeham's Theory of the Complexe Significabile in the Arts Faculty at Paris in the Mid-fourteenth Century", *Franciscan Studies* 54(1994), pp. 211-25. A good collection of relevant 14^{th}-century texts can be found in Perler, D. *Satztheorien. Texte zur Sprachphilosophie und Wissenschaftstheorie im 14. Jahrhundert*, (Texte zur Forschung Bd. 57), Darmstadt: Wissenschaftliche Buchgesellschaft 1990.

of various ontological statuses on the level of the semantics of terms, apparently had no trouble with adding "another layer" of semantic values on the level of the signification of propositions. Thus, on top of the supposita as well as the immediate and ultimate significata of categorematic terms, they would have the significata of the copula, signifying the existence of the ultimate significata of the predicate in the supposita of the subject, as well as the existence of the significatum of the proposition as a whole. In a compositional semantics reconstructing these ideas, therefore, the copula would have to be assigned a semantic function taking the various semantic values of the categorematic terms as its arguments, and yielding the existence of the semantic values resulting from the combination of the values of these terms.[32]

A survey of "via antiqua semantics"

The resulting semantic theory is complicated and unwieldy, but one that is not necessarily inconsistent and has a number of advantages in logic itself as well as in metaphysics. In the first place, it is clear that the apparently boundless proliferation of various semantic values assigned to both categorematic and syncategorematic terms[33] yields a very fine-grained semantics, capable of making distinctions that more coarse-grained theories cannot make. For example, it is well-known that modern intensional logics, defining intensions in terms of functions from possible worlds to extensions or truth-values, cannot distinguish necessarily co-extensional predicates or propositions, such as the predicates 'trilateral' and 'triangular', or two tautologies. But on the *via antiqua* conception, as sketched here, it is easy to see that the two predicates can have distinct significations, yielding distinct ultimate significata for the same individuals, even if those significata are necessarily co-actual in any possible situation. In the same way, the significata of two non-synonymous tautologies can be two distinct, yet always necessarily co-actual *enuntiabilia* or *complexe significabilia* determined compositionally in terms of the distinct semantic values of the components of these distinct propositions.

Another logical advantage of this sort of construction is a simple, uniform theory of truth, consonant with Aristotle's simple definition cited by Cajetan: a proposition is true just in case its significatum exists, which in turn is conditioned on both the way things are and on the way the compositional structure of the proposition determines the existence-conditions for the propositional significatum. For example, the proposition 'Homer is blind' on this conception is true, provided the *enuntiabile* or *complexe significabile* it signifies, namely, *that Homer is blind*, exists. But this entity, which is the value of the application of the semantic function signified by the copula to its arguments, namely, the suppositum of the subject

[32] For technical details, see Klima, G. "Aquinas' Theory of the Copula and the Analogy of Being", *Logical Analysis and History of Philosophy*, 5(2002), pp. 159-176.

[33] For a discussion of this medieval distinction and its relation to the modern distinction between logical connectives and non-logical terms, see Klima, G. "Syncategoremata", in: *Elsevier's Encyclopedia of Language and Linguistics*, 2nd Ed. Edited by Keith Brown, Elsevier: Oxford, 2006, vol. 12, pp. 353-356.

and the signification of the predicate, exists just in case the ultimate significate of the predicate in the suppositum of the subject, i.e., Homer's blindness exists. The existence of this, however, is conditioned on the non-actuality of the ultimate significatum of the corresponding positive predicate, namely, the non-existence of Homer's sight. Thus, the simple fact of the truth of the proposition 'Homer is blind' will consist in the simple fact of the existence of the significatum of this proposition. However, the existence of the significatum of this proposition is conditioned on the existence or non-existence of several "layers" of other items, assigned as the various semantic values of the syntactic components of the proposition.

And herein reside both the strength and the difficulty of this semantic construction. The primary strength of this construction consists in the expressive power of the system, which renders it capable of making the most refined distinctions, and thus allowing the formulation of the most abstruse metaphysical questions concerning the various semantic values of any phrases in any syntactical category. On the other hand, this is also the difficulty with this construction. It not only allows the formulation of abstruse metaphysical questions: it makes them *inevitable*, especially concerning the identity and distinction of these semantic values and the determination of their nature, their precise ontological status.

OCKHAM'S COMPLAINTS

In fact, the apparently inevitable obscurity of the issues concerning the multitude and distinctions of these semantic values was the best motivation for Ockham to get rid of them in one fell swoop, by discarding the semantics that engendered them in the first place. Ockham's main complaints against the semantics of "the moderns" as he rather tendentiously refers to the representatives of the older theory[34] fall precisely into these two categories: those concerning the sheer multitude of entities (or quasi-entities) implied by this construction, clearly offending a nominalist's "taste for desert landscapes" (as Quine famously put it), and those concerning the obscurity of the nature and distinctions of these entities. The arguments supporting these complaints are also of two main sorts: there are those that argue for the absurdity of the ontological commitments of his opponents, and those that argue for the eliminability of those commitments.

The *first strategy* points to a number of logical and physical absurdities that seem to follow if we allow "multiplying entities with the multiplication of terms", as Ockham claims the "moderns" do.[35] To be sure, as we have seen, Ockham's characterization of his opponents' position is not entirely fair, for they are *not*

[34]In the medieval intellectual context, to be "modern" is to carry lesser weight than the established older authorities. Thus, by referring to his opponents as "moderns" Ockham was trying to "sell" his innovations as being just an attempt to restore a simpler, original Aristotelianism distorted by the unnecessary additions of the "moderns".

[35]Ockham, W. *Summa logicae* (SL), Philotheus Boehner, Gedeon Gál and Stephen F. Brown, ed., ("Opera philosophica," vol. 1), St. Bonaventure, NY: The Franciscan Institute, 1974, p. 171.

committed to positing distinct entities in distinct logical categories. However, it is true that since their semantics left the question of the distinctness or identity of these items open, the semantic framework inevitably leads to a number of apparently intractable metaphysical problems. For instance, if Socrates is similar to Plato in wisdom, and he is similar to Plato in wisdom if and only if his similarity to Plato in wisdom exists, then it seems that he may be similar to Plato in wisdom even if one (or each) of them is not wise, provided this similarity is distinct from Plato's and Socrates's wisdom. For if they are distinct, then God can create any one or two of these things without the others. So, Plato and Socrates can be similar in wisdom without being wise, if God sustains their similarity in wisdom while annihilates their wisdom; and they can both be wise without being similar, if God sustains their wisdom, but annihilates their similarity, which is certainly absurd.

Ockham actually uses somewhat more complicated arguments against distinct "relation-things",[36] but the point is the same in all these, namely, that these distinct relation-things can exist logically independently from their foundations, and so their presence or absence may verify or falsify relational predications logically independently from the existence of those foundations, which is patently absurd. To be sure, "*via antiqua* authors" were quite aware of these difficulties and they had their own *metaphysical* solutions to them.[37] However, Ockham's *logical* "solution" will simply eliminate these "apparent" metaphysical problems.

But besides logical absurdities of this sort, Ockham also considers certain physical absurdities derivable from positing such distinct entities, such as the generation of an infinity of distance-things in the fixed stars by a fly, as its distance from them changes by its flight here on earth.

Finally, Ockham also points to the absurdities deriving from attempts to overcome such and similar difficulties by tweaking the notion of the very distinction of such items, such as Scotus' *formal distinction*. For instance, in attacking Scotus' claim that the common nature of any individual is *merely formally* (and not really) *distinct* from the individual difference that individualizes it, he argues that if there is any distinction between two things, then it must be a real distinction, whence Scotus's notion of a mere formal distinction is vacuous. For, he argues, *this* individual difference is not formally distinct from *this* (very same) individual difference (for nothing is formally distinct from itself), and this individual difference is formally distinct from this common nature; therefore this common nature is not this individual difference, that is to say, they are *really* and not *merely formally* distinct, despite Scotus' claim.[38]

To be sure, as I have indicated, Ockham's arguments are nothing his opponents

[36]Cf. Klima, G. "Ockham's Semantics and Ontology of the Categories", Spade, P. V. (ed.), *The Cambridge Companion to Ockham*, Cambridge: Cambridge University Press, pp. 118-142.

[37]For excellent recent discussions in English, see Henninger, M., SJ. *Relations: Medieval Theories 1250-1325*, Clarendon Press: Oxford, 1989; Brower, J. 'Medieval Theories of Relations', in *The Stanford Encyclopedia of Philosophy* (Fall 2005 Edition), ed. E. N. Zalta, `http://plato.stanford.edu/archives/fall2005/entries/relations-medieval/`.

[38]SL, I, 16, p. 54.

could not handle; in fact most of these were raised and solved by themselves, although at the expense of introducing further refinements and complications into their own framework.

Ockham's *second strategy*, however, undermines precisely this tactic of "adding epicycles", by showing and alternative way of constructing semantic theory in which such complications need not emerge at all.

OCKHAM'S ALTERNATIVE

Ockham was thinking of semantic relations in terms of the Aristotelian semantic triangle, just as much as his predecessors were. However, he radically reinterpreted the idea of what concepts are and how they represent their objects.[39] Consequently, since in his view the semantic properties of written a spoken terms are inherited from the concepts to which they are subordinated, no wonder Ockham's semantics of terms will be radically different as well.

But, apparently, even apart from the differences in his conception of concepts, he has a more directly *semantic* bone to pick with his predecessors. For in an apparent stark contrast to what we have seen in Lambert and Aquinas, Ockham insists that what is primarily signified by a term is not a concept, but the thing conceived by the concept. He argues for this claim as follows:

> ... an utterance primarily signifies that for which on account of its institution it primarily supposits. However, names of first intention supposit for things, and names of second intention for concepts, and names of second imposition for nouns or [other] utterances, as is clear by induction. The major premise is obvious, because that is primarily signified by the utterance, for which the person imposing the name uses it. But it is clear that this is what the utterance primarily supposits for. Thus, we use the term 'man' principally for men.[40]

To be sure, Ockham's "*via antiqua* opponents" would not deny that an utterance (ultimately) signifies not the concept, but the object of the concept which it immediately signifies, i.e., to which it is subordinated. But they would not agree with Ockham what that object is and they would not identify that object in the way Ockham does here, with reference to (personal) supposition and *use*. Walter Burley explicitly attacked Ockham's position on the apparently purely semantic issue, trying to separate it from the connected ontological and cognitive psychological issues. To see the real contrast between their views, it is worth quoting Burley's argumentation at some length:

> Again, the name 'man' signifies something first. And it does not first signify Socrates or Plato, because in that case someone hearing the ut-

[39]For an excellent, detailed discussion of Ockham's doctrine, see Panaccio, C. *Ockham on Concepts*, Aldershot: Ashgate, 2004.

[40]Ockham, *Ordinatio,* Opera Theologica, vol. IV, lb. 1, q. 22, pp. 48.18 - 49.4.

> terance and knowing what was signified by the utterance would determinately and distinctly understand Socrates, which is false. Therefore, the name 'man' does not first signify something singular. Therefore, it first signifies a common entity. And that common entity is a species. Therefore, what is first signified by the name 'man' is a species. (34) I do not care at present whether that common entity is a thing outside the soul or a concept in the soul. Rather it suffices merely that what the name first signifies is a species. Thus 'man is a species' will be true insofar as 'man' is taken for its significate. This is confirmed, because a name is not imposed except on the known, according to the Commentator, Metaphysics VII, 36 and also according to Boethius, who says, 'One imposes names on the things one sees'. But he who imposed the name 'man' to signify did not know me or John who is now present. Therefore, the name 'man' does not signify me or John who is now present. Consequently, the name 'man' does not signify me or John, etc., and yet supposits for me and for John when it supposits personally. Therefore, it is not true that a term supposits for its significate or significates whenever it supposits personally.[41]

As can be seen, the main difference from Ockham's position here is *not* that for Burley a term would signify a concept *and not* the thing conceived by means of the concept. When he says, "I do not care at present whether that common entity is a thing outside the soul or a concept in the soul", he is obviously trying to separate the semantic issue from the related ontological issue of what sort of entity it is that the term primarily signifies. The point for him is that it cannot be any individual, but has to be something common, whether in reality or merely "in the mind" (as its object), to those individuals which the term, on account of this signification, can be used to stand for. But that "common entity" (whether it is some common thing existing in reality regardless of any mental activity, or an "objective concept", i.e., the direct object of an abstractive mental act, representing something individual in abstraction from its individuating conditions) is obviously the direct, immediate object of a mental act. So, for Burley, what the term first, primarily signifies is also the object of some concept, but that object is not any individual thing as such, but rather something that any individual falling under the concept has to "match" by being informed precisely by the form that is abstracted in the concept.

For Ockham, on the other hand, especially after abandoning his earlier *fictum*-theory, and adopting his mature, mental-act theory of concepts, what is conceived by a concept is just any individual thing *indifferently* represented by the concept. Therefore, if a term is supposed to signify what is conceived by the corresponding concept, then it has to signify the individuals themselves, which of course are the personal supposita of the term in a propositional context. So it seems that even what appears to be a purely *semantic* difference between Ockham and Burley (and "the *via antiqua* view" in general) ultimately boils down to a fundamental

[41] Burley, *Purity*, pp. 87-88. (33)-(34)

difference in their cognitive psychology, a difference in their theory of concept-formation. Thus, anyone trying to evaluate the position of each author would have to deal with the details of their cognitive psychology. But regardless of those foundational issues in their respective cognitive theories, it is easy to see the resulting differences in their semantic theory.

For Ockham, the significata of a common categorematic term are the individuals represented by the corresponding concept indifferently, regardless of their existence and non-existence. Thus, a term such as 'man' will signify all men, not only those who exist, but those as well who existed, will exist, or can (or could or would) exist. Thus, in a formal reconstruction of Ockham's semantics the signification of the term 'man' would not be a function assigning individualized humanities to individuals (as it would be in the *via antiqua*), but rather a subset of the domain of discourse, comprising both actual and non-actual elements. Correspondingly, a significate of this term would be just an element of this signification, and thus a personal suppositum of the same term would be any such significatum, provided it is actual relative to the time connoted by the copula of the proposition in which the term is suppositing.

By this move, Ockham could at once get rid of a number of unwieldy problems of the *via antiqua* semantic construction. In the first place, the entire "shady business" of the ontological status of a "common nature" is gone, along with the problems of its distinction from its individuating conditions as it exists in the singulars. Indeed, the entire multitude of the instances of all these "common natures" and the problems of the identity and distinctness of these instances are gone as well. Furthermore, the multitude of these instances in several distinct *ontological categories* may be eliminated, provided Ockham can plausibly apply his semantic model in all the Aristotelian *logical categories* he is still committed to. Showing he can do so is what we usually refer to as Ockham's program of "ontological reduction", namely, the project of showing how all the distinct logical categories distinguished by Aristotle can be mapped onto a parsimonious nominalist ontology, having only two distinct ontological categories, namely, those of substance and quantity.

Ockham's program has been much discussed in the literature, so here I only want to focus on what is semantically relevant from it. To see a typical case of Ockham's "ontological reduction", let us consider the issue of distinct qualities in the species of shape, which Ockham would want to eliminate. Take for instance, a straight piece of wire. On the *via antiqua* conception, this wire is straight on account of being informed by an inherent shape-thing, named "straightness". When the wire is bent, this straightness perishes and gives way to another shape-thing, which may be called "curvedness". Apparently, the only way to account for this change is precisely by means of assuming the existence of distinct shape-things, which may come and go, while their subject, the wire, remains in existence. Ockham, however *can* provide us with a way of accounting for this change without assuming the existence of such distinct shape-things in terms of his theory of connotation. For according to Ockham, when the piece of wire is straight, this

does not have to mean that the wire is informed by a "straightness-thing". All this means is that the ends of the wire are maximally distant, that is to say, the term "straight" does not signify a straightness distinct from the straight thing; rather, it *signifies* the straight thing itself, while *connotes* the maximal distance between its ends. The term is true of this wire, just in case its ends are maximally distant, and becomes false of it, when they are not. Thus, on account of the meaning of the term "straight", which can properly be expressed by means of a nominal definition, Ockham can eliminate the apparent need for a "shape-thing", which was demanded only by what he takes to be a mistaken semantic conception. For, as the nominal definition makes explicit, the term "straight" simply signifies elongated substances (i.e., ones whose length is considerably greater than their width and depth) connoting the maximal distance of their extremes along their length. But the things signified and connoted in this way are only substances and quantities, (which Ockham identifies with substances or qualities as measurable in certain ways).

However, regardless of the details of the ontological project it intends to realize, Ockham's semantic idea is clear. In order to have a nominalist semantics that admits in its domain of interpretation only entities of a certain limited number of categories, one needs to do the following:

1. Take simple absolute terms in the "permitted" categories to be "primitive", in the sense that they are semantically simple, undefined, and they define others.

2. Take other syntactically simple terms that appear to involve commitment to entities in other than the permitted ontological categories to be compositionally dependent for their semantic evaluation on their nominal definitions, which in the last analysis will only contain "primitive" (undefined) terms and syncategoremata (i.e., terms that signify only syncategorematic concepts, which in turn are entities in the "permitted" ontological category of quality).

Thus, Ockham's semantic idea in connection with his program of ontological reduction boils down to the strategy of using "eliminative definitions" to show that ontological commitment to entities in the "non-permitted" categories is merely apparent, and can be avoided in terms of providing the *right* sort of semantic analysis.

There are two logical problems with this idea. The first might be termed "the problem of the primitive vocabulary" of Ockham's semantics, and the second "the problem of a merely programmatic semantics"

The first problem generated much controversy in the recent literature after the appearance of Claude Panaccio's important study challenging "the received view"

on Ockham's ontological program.[42] According to "the received view",[43] Ockham's primitive vocabulary would contain only syncategorematic terms (roughly, logical connectives) and absolute terms, i.e., terms subordinated to absolute concepts, whereby individuals are conceived absolutely (though in the case of universal concepts, indifferently), not in relation to anything else. Accordingly, on this view, Ockham is committed to holding that all connotative terms (i.e., terms subordinated to connotative concepts whereby we conceive of individuals in relation to other things, the *connotata* of the term) are semantically compositional; their semantic values are functionally determined by the semantic values of the components of their nominal definitions. Equivalently, the view holds that all connotative concepts are complex, i.e., there are no simple connotative concepts. The rationale for the view is Ockham's repeated claim that all connotative terms have nominal definitions, which coupled with the idea that nominal definitions are synonymous with their *definita* because they are subordinated to the same concept, directly yields the above theses. Panaccio's meticulous analysis of Ockham's texts, however, cast serious doubt on the assumption that nominal definitions for Ockham always have to be strictly synonymous with their *definita* on account of being subordinated to the same concept, and, correspondingly, that there are no simple connotative concepts for Ockham.

But even without trying to settle this issue of Ockham scholarship, one can note that what is problematic with a severely limited primitive vocabulary is the feasibility of providing plausible analyses in all cases when needed. However, the primitive vocabulary may be enhanced not only by means of the addition of primitive connotative terms (ones subordinated to simple connotative concepts), but also by means of enhancing the class of syncategoremata, and construe apparently primitive connotative terms as complexes of absolute terms and some "non-standard" syncategoremata. Take for instance the Latin phrase 'hominis asinus' ('donkey of a man'). This is clearly a complex term, consisting of a simple absolute term 'asinus' ('donkey') and an apparently simple connotative term, 'hominis' ('of a man'), signifying things in relation to men as their possession. However, if we look at the corresponding English term, the appearance of the simplicity of the Latin connotative term 'hominis' is dissolved in the syntactical structure of the phrase 'of a man', which seems to indicate that in the formation of the concept expressed both by 'hominis' and by 'of a man' actually several concepts are involved,

[42]See Panaccio, C. "Connotative Terms in Ockham's Mental Language", *Cahiers d'épistémologie* 9016(1990), pp. 1-22. For more recent treatments of the issue see Chalmers, D. "Is There Synonymy in Ockham's Mental Language?" in *The Cambridge Companion to Ockham,* ed. P. V. Spade, Cambridge: Cambridge University Press, 1999, pp. 76-99; and Panaccio, C. "Semantics and Mental Language," *ibid.*, pp. 53-75. Panaccio's most recent summary account of the debate can be found in his *Ockham on Concepts*.

[43]Cf., e.g., Spade, P. V. "Ockham's Distinctions between Absolute and Connotative Terms," *Vivarium* 13 (1975), pp. 55–76; Spade, P. V. "Synonymy and Equivocation in Ockham's Mental Language", *Journal of the History of Philosophy* 18(1980), pp. 9-22. For Spade's most recent discussion of the issue, see Spade, Paul Vincent, "William of Ockham", *The Stanford Encyclopedia of Philosophy (Fall 2006 Edition)*, Edward N. Zalta (ed.), `http://plato.stanford.edu/archives/fall2006/entries/ockham/`

namely, those to which the English particles 'a' and 'of', and the term 'man' are subordinated.

Thus, if the primitive vocabulary turns out to be too limited, it can be enhanced in two different ways. Either (1) by adding primitive polyadic categorematic terms, i.e., acknowledging that there are connotative terms subordinated to simple connotative concepts, which therefore cannot have strictly synonymous nominal definitions (this is going to be "Buridan's way", and this is also the way for Ockham suggested by Panaccio's interpretation), or (2) by acknowledging term-forming syncategorematic terms, subordinated to (polyadic) categorematic concepts. So, using the previously introduced restricted variables as representations of common categorematic terms having personal supposition, we might illustrate the difference in the following way:

1. $x.Dx.(y.H(y)(x.))$ — [an-x-that-is-a-donkey-that-is] [of-a-y-that-is-a-man];

2. $x.Dx.(\wp(x.)(y.Hy))$ — [an-x-that-is-a-donkey] [that-is-of] [a-y-that-is-a-man].

In (2), $\wp$ would be a distinguished logical predicate (pretty much like identity in standard quantification theory) with two arguments, the first of which is to be the possession of the second.

But no matter which way is taken, they both point to the same limitation of this approach to natural language semantics. For if whole classes of syntactically simple categorematic terms are semantically complex, in the sense that their semantic evaluation is a function of the evaluation of their nominal definition, then it would appear that the whole enterprise of Ockhamist semantics is doomed to remain programmatic until the semantic analyses of all these terms is actually carried out by providing their nominal definitions.

However, one may say that this situation need not pose a serious threat to the Ockhamist approach, especially, if one takes Buridan's (and "Panaccio's Ockham's" path). For in that case one may treat any syntactically simple term as semantically primitive as well, which is fine for most logical purposes, unless the term's meaning is relevant to a particular problem, in which case one may just provide the relevant analysis.

In any case, with this treatment of categorematic terms, the semantic construction is easy, and is obviously not committed to the vast and obscure ontology the *via antiqua* construction is apparently committed to. For on this approach any categorematic term will merely have the function of signifying individuals in the permitted categories, all of them in the same way, if the term is absolute, or some of them primarily and other secondarily, if the term is connotative, and in personal supposition will supposit for some or all of its ultimate (primary) significata.

Accordingly, the terms of a proposition in personal supposition will have the same sort of semantic function: subject and predicate will both supposit for their ultimate significata, whence the affirmative copula will have to express the identity of these supposita. For example, in the proposition 'Every man is an animal', the terms supposit for those of their ultimate significata (all men and all animals,

respectively, past, present, future and merely possible), which actually exist at the present time of the utterance of the proposition, connoted by the present tense of the copula. The copula, in turn, has to express the identity of these supposita, which is why the *via moderna* subscribes *exclusively* to the *identity theory of predication*, as opposed to the *inherence theory* of the *via moderna* (which, nevertheless, also *allowed*, non-exclusively, the identity theory).[44] Thus, the quantifier, the *signum quantitatis*, has the function of denoting how many of the subject's supposita have to be identical with the supposita of the predicate, if the proposition is to be true.

Therefore, the explicit *via moderna* reading of this sentence would be 'Every thing that is a man is identical with some animal', which is appropriately represented by the corresponding quantificational formula using restricted variables:

$$(\forall x.Mx)(\exists y.Ay)(x. = y.)$$

Accordingly, in this reconstruction, the four types of categorical propositions would be formalized as follows:

A: $(\forall x.Mx)(\exists y.Ay)(x. = y.)$

I: $(\exists x.Mx)(\exists y.Ay)(x. = y.)$

E: $\sim (\exists x.Mx)(\exists y.Ay)(x. = y.)$

O: $(\exists x.Mx) \sim (\exists y.Ay)(x. = y.)$

Indeed, given the existential import of A, these formulae would form a perfect traditional Square of Opposition.

So far, so good. But what about the significata of these propositions? Apart from stating the truth-conditions of various types of propositions in terms of his identity-theory of predication and insisting that a proposition signifies nothing over and above what its categorematic terms signify, Ockham is mostly silent on this subject, just as on many further details of his semantic theory, such as the issue of the apparent ontological commitment to non-existents (mere possibilia) in his semantics, or a general account of truth and semantic validity in a semantically closed, token-based semantics.[45] Such and similar details were, however, worked out very carefully by John Buridan.

[44] Aquinas, for instance, allows both analyses, although he regards the inherence-analysis the "more appropriate" [*magis propria*], and in the case of adjectival predicates the only acceptable one, e.g., in Super Sent., lib. 3 d. 5 q. 3 a. 3, expos.; Summa Theologiae I, q. 39 a. 6 ad 2.

[45] Most of these issues are taken up and discussed detail by Marilyn McCord Adams in her *William Ockham*, University of Notre Dame Press: Notre Dame, 1987, Vol. I, c. 11, although she also remarks on p. 394: "Ockham does not anticipate his successors' interest in refining this formulation of the correspondence theory of truth ['an expression is said to be true because it signifies things to be as they are in reality'] in the face of various objections, whether pedantic or formidable, the most notable of the latter being the semantic paradoxes".

BURIDAN'S SEMANTICS

Concepts and mental language

Buridan's semantics starts out in the same way as Ockham's. Acknowledging the subordination of written to spoken, and spoken to mental terms (i.e., concepts), establishes for him a "semantic triangle", in which concepts are natural signs of whatever we conceive by means of them, whereas the utterances and inscriptions subordinated to them are the conventional signs of the same, in virtue of their conventional subordination to concepts. Not all concepts have, however, the function of conceiving something; some concepts merely serve to determine *how* we conceive of things conceived by other concepts. This is the basis of Buridan's primary distinction between categorematic and syncategorematic concepts, and the corresponding spoken and written terms. For example, the concept of negation operating on the categorematic concept whereby we conceive indifferently of all humans (past, present, future, and merely possible ones as well) yields a complex concept whereby we conceive of all humans negatively, on account of which the concept applies to all non-humans. Again, the concept of the affirmative, assertoric, present tense copula operating on two categorematic concepts yields a complex concept, a mental proposition, whereby we conceive of all things conceived by the categorematic concepts in such a way as to conceive the identity of those significata of the categorematic concepts that are actual at the time connoted by the copula, i.e., the identity of the actual supposita of these mental terms. As these examples clearly illustrate, following Ockham's lead, Buridan wholeheartedly subscribes to the idea of a mental language, in which simple, naturally significative units of our thought, our simple concepts, are combined in their operation to yield complex representational units of our thought, our complex concepts, the representative (or "naturally significative") function of which is compositionally dependent on the functions of the simple concepts. To be sure, according to Buridan, our complex concepts are complex *only* in this sense, namely, in the compositionality of their representative function; otherwise, ontologically, they are just as simple qualities of the mind as are our simple concepts. However, since their formation and representative function is conditioned on the operation of the relevant simple concepts, it makes good sense to call them *semantically* complex.[46]

The second example also clearly illustrates another logically important feature of Buridan's distinction between categorematic and syncategorematic concepts and the corresponding terms. The concept of the copula, connoting time, does conceive of something external to the mind,[47] besides effecting the combination (*complexio*

[46]For a discussion of semantic complexity (compositionality) without syntactic or ontological complexity see my *Introduction* to John Buridan: *Summulae de Dialectica* (SD), an annotated translation with a philosophical introduction by Gyula Klima; New Haven: Yale University Press, 2001.

[47]Buridan identifies time with the motion of the sphere of the fixed stars, as conceived by means of a concept counting its revolutions. For discussion and references see Dekker, D-J. "Buridan's Concept of Time. Time, Motion and the Soul in John Buridan's Questions on Aris-

— as Buridan calls it) of categorematic concepts. Thus, Buridan classifies the copula as a "mixed" *syncategorema*, which besides its syncategorematic function also has some representative, significative function, unless we form in our minds the concept of an *atemporal* copula, which is purely syncategorematic.[48]

Buridan's mental language, therefore, contains in its "primitive vocabulary" simple concepts, some of which are syncategorematic, others are categorematic, and yet others are mixed. Purely categorematic concepts have the function of representing or naturally signifying the objects we conceive by means of them, in the way we conceive of them. Purely syncategorematic (or *complexive*) concepts do not represent anything in and of themselves, their function merely consists in modifying the representative function of other, representative concepts, by combining with them semantically complex concepts. These concepts are *semantically* complex, because their representative function is compositionally dependent on the semantic values of the concepts "making it up". Finally, mixed concepts obviously exercise both categorematic and syncategorematic functions, as we could see in the case of the tensed copula (as opposed to the purely syncategorematic, atemporal or tenseless copula). It is the semantic features of the resulting mental language that are inherited, through conventional acts of imposition, by the elements and syntactical structures of conventional spoken and written languages.

The semantics of terms

Written and spoken terms are classified as categorematic, syncategorematic or mixed, depending on the kind of concept they are subordinated to. To be sure, since imposition is conventional and as such arbitrary, this classification may change as established usage changes. And of course there are a number of *pragmatic* factors determining what counts as established usage, but if these pragmatic factors are (reasonably) fixed, then we can have a fixed interpretation of these conventional signs.

Given such an interpretation, some categorematic terms are simple, others are complex. However, some of those that are syntactically simple (i.e., they have no parts that are significative in themselves and contribute their semantic values in determining the semantic values of the whole) may still be semantically complex in virtue of being subordinated to a complex concept, the compositional character of which can be expressed by means of the syntactical structure of the appropriate nominal definition. Again, some categorematic terms are absolute and others are connotative (be they simple or complex) in virtue of being subordinated to absolute or connotative concepts (i.e., to concepts whereby conceive of things absolutely, not in relation to others, or in relation to others, respectively). Finally, some categorematic terms are singular and others are common, again depending on whether they are subordinated to singular or common concepts. Common absolute terms

totle's Physics", in: Thijssen, J. M. - Zupko, J. (eds.): *The Metaphysics and Natural Philosophy of John Buridan*, Leiden-Boston-Köln: Brill Academic Publishers, 2000, pp. 151-164.

[48] Cf. SD 4.2.3 and 4.3.4

signify all their significata indifferently in the same way, just as these significata are conceived by means of the corresponding concepts. Common connotative terms signify their significata in relation to other things, called their connotata, again, in the same way as they are conceived by means of the corresponding connotative concepts.

When categorematic terms are joined by a copula to form a proposition, they take on the function of supposition, i.e., standing for some or all of their *ultimate significata* in *personal* (or significative) supposition, or for their *immediate significata* (the token-concepts to which they are subordinated) or for *themselves* or *other tokens* of the same kind, in *material* (or non-significative) supposition. (Buridan drops simple supposition as a separate division of supposition, and simply lumps it together with material supposition under the heading of non-significative or material supposition.)

In personal supposition, and in a *non-ampliative* context,[49] *absolute* terms supposit for those of their ultimate significata that are actual at the time connoted by the present-tense assertoric copula, whereas *connotative* terms, or as Buridan more often calls them, *appellative* terms, supposit for those of their ultimate significata that are both actual at the time connoted by the present-tense assertoric copula and are actually related to the term's *appellata* in the way demanded by the term's connotation. For example, in the sentence 'A man is an animal', the absolute subject and predicate terms supposit for actual men and actual animals, as opposed to the non-actual ones still signified by these terms. In the sentence 'A man is wise, the absolute term 'man' supposits for actually existing men, whereas the connotative term 'wise' supposits for actual persons actually having wisdom, assuming the correct nominal definition of 'wise' is 'person having wisdom', where the absolute term 'wisdom' signifies individual wisdoms (given Buridan's nominalist ontology, one cannot say 'individual instances *of wisdom*'), which are the individual qualities informing the souls of wise persons. The wisdoms of wise persons are the connotata of the term 'wise', connoted by it whether they are actual or not, and *appellated* (obliquely referred to) by it in this sentence, provided they are actual. Thus, just as personal supposition is direct reference to the term's actual ultimate significata, so appellation is oblique reference to a connotative term's connotata that are actual at the time connoted by the copula of the proposition in which the term occurs.

Given that personal supposition and appellation in this way pick out the actual ultimate significata and connotata of terms that otherwise signify and connote non-actual as well as actual individuals, and that actuality is relative to time and modality, now we can make good sense of the qualification added at the beginning

[49] A context is ampliative, if it extends the range of the supposition of the term to its non-actual significata, such as tensed and modal contexts, or intentional contexts (generated, according to Buridan, by verbs, and their derivatives, signifying acts of the cognitive soul). For a reconstruction of Buridan's treatment of intentional contexts, see Klima, G. '*Debeo tibi equum*: a Reconstruction of the Theoretical Framework of Buridan's Treatment of the *sophisma*', in *Sophisms in Medieval Logic and Grammar*, ed. S. Read, Dordrecht/Boston/London: Kluwer Academic Publishers, 1993, pp. 333-347.

of the previous paragraph concerning *non-ampliative* contexts. For Buridan, *ampliation* is an extension of the range of supposition of a term from actual to past, or future, or merely possible significata of the term, demanded by its propositional context. For example, whereas in the sentence 'A man is wise' the subject term stands for actual humans, in the past tense sentence 'A man was wise' the range of supposition of the term is *ampliated* to past humans, so that the subject stands either for present or past humans. The predicate on the other hand is not held for it supposita in this disjunctive manner: it is held strictly for the time of the verb. That is to say, this sentence is true if something that *is or was* a man *was* identical with something wise. So, if Socrates was wise, this sentence is true, even if Socrates no longer exists (whence Socrates was, but is no longer, a man), and so is not identical with any presently existing wise person. Again, if the only person who used to be wise is no longer wise, although still exists, and no other person is wise at the present time, this sentence is true, even if, of course, the corresponding present-tense sentence is false. And similar considerations apply to future tense and modal sentences, where the supposition of the subject is ampliated to future or merely possible significata of the term.

One counterintuitive result of this analysis explicitly considered by Buridan is that the sentence 'An old man will/can be a boy' on this analysis will come out as true.[50] For this is equivalent to 'Something that is or will be an old man will be a boy', which is true of someone who is a boy today, and will be a boy tomorrow and will also be an old man years later. Buridan explains the problem away by pointing out that we find the result counterintuitive only because we tend to read the original not as equivalent to the sentence with the ampliated subject, but with one that is restricted to the present: 'Something that is an old man will/can be a boy', which is of course false.

However, Buridan's doctrine of ampliation, or in general, what we would call "quantification over non-existents", seems to raise more serious problems concerning Ockham's and Buridan's nominalism.[51] After all, despite all their commitment to eliminating all sorts of weird entities posited by their (moderate) realist opponents, are they not equally committed to some really weird "entities", namely, non-existents?

Quantification, existential import, and ontological commitment

Since according to Buridan's theory of predication an affirmative proposition is true only if its terms supposit for the same thing, and if at least one of the terms of an affirmative proposition supposits for nothing, then they cannot supposit for the same thing, on his theory all affirmative propositions carry existential import,

[50] Cf. SD 1.8.8 and *Sophismata* c. 4, *fourth sophism*.

[51] Ockham did not endorse the doctrine of ampliation, but he was still committed to quantifying over non-existents. Cf. Priest, G. and Read, S. "Ockham's Rejection of Ampliation", *Mind* 90(1981), pp. 274-279.

provided they are non-ampliative of their terms.[52] As has been mentioned before, this approach immediately attributes existential import to universal affirmatives (with no ampliation), which at once validates the relations of the traditional *Square of Opposition*.

In fact, Buridan would argue that even propositions with ampliated terms have analogous inferential relations, even if one might not say in their case that a universal affirmative properly speaking has *existential* import. For example, the modal universal affirmative 'Every man is possibly a philosopher' would still entail 'Some man is possibly a philosopher', still, neither of these would entail that something is actually a man. The reason is that on Buridan's theory of ampliation, these propositions would be analyzed as 'Everything/Something that is or can be a man can be a philosopher', which are true if every/some significatum of the term 'man', including everything that is or can be a man, can be a philosopher, which, however, can certainly hold even if nothing is actually a man.

Accordingly, Buridan would claim that his analysis of these propositions does not commit him to the existence of anything: on his view neither of these propositions would entail the proposition that a man exists, or even the proposition that a merely possible man exists. Indeed, in general, Buridan would perfectly agree with Quine[53] that everything exists and there are no non-existents or mere *possibilia*, for on his analysis these propositions would be true. Still, Quine would not be happy with Buridan's analysis. He would say that Buridan's analysis of propositions with ampliated terms does ontologically commit him to non-existents, by virtue of (existentially) quantifying over those non-existents.

At this point, however, we should notice that this Quinean charge against Buridan is based on our implicit acknowledgement of the Tarskian distinction between object-language and meta-language. For the point of the charge is that even if Buridan in his object language says "all the right things" according to a Quine, we can point out in his meta-language that in his analyses of propositions with ampliated terms his variables range over mere *possibilia*.

However, Buridan would flatly reject the presupposition of the objection, namely, the distinction between object-language and meta-language. He would say that the subject matter as well as the medium of his theory is one and the same language, in which he is ontologically committed to those things, and *only* those things, of which he has to say that they exist. But he is not committed to saying this of any of the *possibilia* he of course can talk about, signify, refer to, supposit for, and "quantify over", for all these semantic relations relate our expressions to objects we can conceive of, and of course we can conceive not only of things that presently exist in our narrower or broader environment.

As Buridan put it in his question-commentary on Aristotle's *De Interpretatione*:

[52] Or, if they are not interpreted according to *natural supposition*, but this issue is not relevant to our present considerations. I deal with the issue though in my "Existence and Reference in Medieval Logic".

[53] Cf. Quine, W.V.O. 'On What There Is', in Quine, W. V. O., *From a Logical Point of View*, 2^{nd} revised edition, Cambridge, MA: Harvard University Press, 1980, pp. 1-19.

> ... a name signifies what is understood by it when it is put in an expression, for to signify is to give rise to some understanding of a thing [*intellectum rei constituere*]. But by the name 'rose' we understand a rose and by the name 'roses' we understand roses. For example, [suppose] last year we, you and I, saw many red roses together. If I ask you: 'The roses we saw were red, weren't they?', then you say: 'Indeed'. And this you know to be true. But you wouldn't know this, unless you thought of those roses. Therefore, by the name 'roses', when I say 'We saw roses', you understand those things that we saw. But we saw red roses. So you think of roses. [...] the name 'rose' refers to [*supponit pro*] roses, although nothing is a rose, for according to the above-mentioned case, namely, that last year we saw many red roses, you concede the proposition 'There were many red roses last year', and you know that this is true. And since this is an affirmative [proposition], it would not be true, unless its subject, which is the name 'roses', referred to some thing or some things. But it does not refer to [any] other thing or other things, but roses. [...] we should note that we can think of things without any difference of time and think of past or future things as well as present ones. And for this reason we can also impose words to signify without any difference of time. For this is the way names signify. Therefore, by the specific concept of 'man' I conceive indifferently all men, present, past and future. And by the name 'man' all [men] are signified indifferently, present, past and future [ones alike]. So we truly say that every man who was was an animal, and every man who will be will be an animal. And for this reason it follows that the [verbs] 'think/understand' ['*intelligere*'], 'know' ['*scire*'], 'mean/signify' ['*significare*'] and the like, and the participles deriving from them, *ampliate* the terms with which they are construed to refer indifferently to present, past and future and possible [things] which perhaps neither are, nor will be, nor ever were. Therefore, even if no rose exists, I think of a rose, not one that is, but one which was, or will be, or can be. And then, when it is said: the name 'rose' signifies something, I concede this. And when you say: that [thing] is not, I concede that; but it was. If, then, you conclude: therefore, something is nothing, I deny the consequence, for in the major premise the term 'something' was ampliated to past and future [things], and in the conclusion it is restricted to present ones.[54]

[54] Buridan, J. *Questiones longe super librum Perihermeneias* (QDI), edited with an introduction by R. van der Lecq, Nijmegen: Ingenium Publishers, 1984; Ph. D. thesis, Meppel: University of Leiden, 1983, pp.12-14. Cf.: "All verbs, even in the present tense, which of their very nature can concern future, past and possible things as well as present ones such as 'think', 'know', 'mean' and the like ampliate their terms to all times, future, past and present. And what accounts for this is that a thing can be thought of without any difference of time, namely, abstracted from any place and time. And so, when a thing is thought of in this way, then a thing which was, or will be, or can be may be thought of as well as a thing which [actually] is. Therefore, if I have

So, when we say that in the proposition 'Some roses existed here' or 'Some roses could exist here' we are referring to things that existed or could exist, but do not exist. Still, according to Buridan it does not follow that therefore in these cases we are referring to (quantifying over) non-existents. For we are referring to what we are thinking of, and a non-existent or non-being cannot be thought of according to Buridan, because the proposition 'A non-being is understood' cannot be true.

Buridan considers this issue in his *Sophismata*, when he raises the question whether the *sophisma* (problem-sentence) 'A non-being is understood' is true.

First, he lays down that the proposition is affirmative with an infinite subject, that is to say, the negation preceding the term 'being' is a narrow-scope term-negation, and not a propositional negation, so the entire proposition is affirmative. Hence he argues for its truth as follows:

> [. . .] the sophism is proved: for such infinite terms are analyzed so that saying 'A non-man runs' is equivalent to saying 'What is not a man runs'. And thus saying 'A non-being is understood' is equivalent to saying 'What is not a being is understood'. But the second is true, for Antichrist, who is not a being, is understood.[55]

Next, Buridan argues for the opposite side before resolving the issue:

> O.1 The opposite is argued: for the term 'non-being' supposits for nothing, but a proposition is false if its subject supposits for nothing and it is affirmative; therefore, etc.

In his response, Buridan sides with the second position, namely, that the sophism is false, and argues for this position on the basis of his theory of ampliation.

> I respond that the sophism is false, for the term supposits for nothing. And this is clear in the following manner: for the verb 'to understand' or 'to be understood' ampliates supposition to past, and future, and even all possible things. Therefore, if I say, 'A being is understood', the term 'being' supposits indifferently for every present or past or future or possible thing. But the rule is that an infinitizing negation added to a term removes its supposition for everything for which it supposited and makes it supposit for everything for which it did not supposit, if there are any such things. Therefore, in the proposition 'A non-being is understood', the term 'non-being' does not supposit for some present, nor for some past, nor for some future, nor for some possible being;

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the common concept from which we take this name 'man', then I can think indifferently of all men, past, present and future. And this is why these verbs can concern past or future things as well as present ones." Albert of Saxony. *Perutilis Logica,* , Venice, 1518; reprint, Hildesheim-New York: Georg Olms Verlag, 1974, Tr. 2, c. 10, 8a regula. For an earlier example of the same explanation of ampliation see the selection from the *Logica Lamberti*, in *The Cambridge Translations of Medieval Philosophical Texts*, eds. N. Kretzmann and E. Stump, Cambridge: Cambridge University Press, 1988, pp. 104-163, esp. pp. 116-118.

[55] SD, p. 923ff.

> therefore, it supposits for nothing, and so the proposition is false. And I say that 'A non-being is understood' and 'What is not a being is understood' are not equivalent, for by the verb 'is' you restrict the infinity [*infinitatem*] to present things. Therefore, the supposition for past and future [and possible] things remains, and thus this has to be conceded: 'What is not [a being] is understood'. If, therefore, we are to give an equivalent analysis of 'A non-being is understood', then it will be the following: 'What neither is, nor was, nor will be, nor can be is understood', and this is false, just as the sophism was.[56]

Thus, even if we can certainly think of, refer to, and quantify over objects that do not presently exist, it does not follow, that we can think of, refer to, and quantify over non-existents: the sentence expressing this idea is just not true.

To be sure, a staunch Quinean might again try to move the objection to the level of meta-language, claiming that even if in Buridan's language, on his own analysis, the sentence 'I am referring to/quantifying over non-existents' is false, in the meta-language of his theory we can truly say this, and that is what matters. But then Buridan might again just deny the validity of that distinction, and when challenged with the semantic paradoxes and the issue of semantic closure motivating Tarski's distinction, he would just point to his own treatment of the paradoxes and his resulting, radically different conception of truth and validity, *without* any need for such a global distinction. So now we need to turn to these issues, starting with Buridan's conception of propositional signification.

The semantics of propositions: truth and validity in a semantically closed language

Buridan's semantics of propositional signification is very simple: propositions signify whatever their categorematic terms signify. Thus, the propositions 'God is God' and 'God is not God' signify the same simple thing as the term 'God' does, namely, God. Yet, the reason why these linguistic expressions are non-synonymous is that they signify different concepts in the mind, whereby the same simple thing is conceived differently. So, whereas these phrases ultimately signify the same thing *ad extra*, outside the mind, they signify different concepts immediately *apud mentem*, i.e., in the mind.

But then it is clear that these contradictory propositions cannot be verified either in terms of the existence of their immediate significata or in terms of the existence of their ultimate significata (for the existence of those would verify both, which is impossible); whence, in stark contrast to the *via antiqua* conception, their truth is not to be determined in terms of their signification at all.

Therefore, the truth of propositions is to be determined for Buridan on the basis of the supposition of their terms.[57] However, since this varies with propositional

[56] *Ibid.*
[57] See SD 9. c. 2.

context, the conditions of their truth given in terms of the supposition of their terms needs to be provided separately for different types of propositions:

> Therefore, recapitulating, we put forth the fourteenth conclusion, namely that every true particular affirmative is true because the subject and predicate supposit for the same thing or things. And every true universal affirmative is true because for whatever thing or things the subject supposits for, the predicate supposits for that thing or for those same things. And every false particular affirmative is false because the subject and the predicate do not supposit for the same thing or things. And every false universal affirmative is false because not every thing or all things which the subject supposits for are also supposited for by the predicate. And every true particular negative is true because the universal affirmative contradictory to it is false; and we have declared what the reason for this is. And every true universal negative is true because the particular affirmative contradictory to it is false; and we have declared what the reason for this is. And every false particular negative is false because the universal affirmative contradictory to it is true; and we have declared what the reason for this is. And every false universal negative is false because the particular contradictory to it is true; and we have declared what the reason for this is.[58]

These clauses, in fact, may look very much like the clauses of the satisfaction conditions of formulae in a formal semantics, serving a "definition of truth" (in a model). Indeed, we may get the same impression from what Buridan says immediately after these clauses:

> And this fourteenth conclusion, which contains eight partial conclusions, appears to be entirely true on the basis of the foregoing [points] given the principle that whatever is the cause of truth of one of the contradictories, or is required for its truth, is the cause of the falsity of the other, or is required for its falsity. But in the end we should note — since we can use names by convention, and many people commonly use this way of putting the matter — that in respect of every true proposition we say: 'It is so', and in respect of every false one we say: 'It is not so', and I do not intend to eliminate this way of speaking. But for the sake of brevity I may use it often intending by it not what it signifies on account of its primary imposition, but the diverse causes of truth and falsity assigned above for diverse propositions, as has been said.[59]

[58] *Ibid.* pp. 858-859.

[59] Buridan will often refer back to this remark, reminding us that whenever he uses his simple formula *qualitercumque propositio significat ita est* ['In whatever way the proposition signifies things to be, so things are'] or any of its stylistic variants, he uses it as a place-holder for the conditions of the truth of the several types of propositions he laid down here. Note also how this general formula is related to the colloquial phrase: *A est B, necne? — Ita est.* [literally: '*A* is

However, upon a closer look it should be clear that Buridan is not providing a "definition of truth" here whether in an Aristotelian or in a Tarskian manner. That he is not providing an Aristotelian truth-definition in terms of the existence of what a proposition as a whole signifies in the way the *via antiqua* semantics did is clear from the foregoing considerations concerning Buridan's conception of propositional signification, as well as from his remark here that whenever *he* uses the Aristotelian formula, he means it merely as a shorthand for his own clauses listed above. It is also clear that these clauses do not list satisfaction-conditions or even truth-conditions for the various types of propositions listed here (amounting to definitions of truth for these proposition types). Indeed none of these clauses are anything like Tarskian biconditionals. They are not saying 'A proposition of type T is true if and only if such and such conditions are met'. They are saying 'A true/false proposition of type T is true/false, because the supposita of their terms satisfy such and such conditions'. So, what these clauses assert is not even the necessary and sufficient conditions of the assertibility of truth or falsity of these various types of propositions; rather, they are stating the "correspondence conditions" of true or false propositions, i.e., what circumstances need to obtain in reality concerning the supposita of their terms for their truth or falsity (these circumstances being quite literally *the causes* of their truth or falsity). Finally, the same point, namely, that these clauses do not constitute Buridan's definition of truth, or strictly-speaking his statement of truth-conditions, is clear from his treatment of Liar-type paradoxes, according to which such paradox statements are simply false, despite the fact that in their case their *correspondence conditions* stated in the above clauses are perfectly met. For according to Buridan, the Liar-type paradoxes, or *insolubilia*, as they were known in medieval logic, merely show that meeting its correspondence conditions is not sufficient for the truth of a proposition.

Without going into a detailed discussion of Buridan's treatment of the Liar paradox and its cognates,[60] while focusing on its relevance to his construction of semantic theory, one can provide the gist of his solution in the following manner.

Consider the following Liar-sentence, named L:

(L) L is false

What is paradoxical about L is that apparently (1) its truth entails its falsity, whereas (2) its falsity entails its truth. Buridan's solution consists in arguing that on account of the validity of entailment (1) the Liar-sentence L is false; however, no paradox arises, because entailment (2) is not valid. Therefore, *pace* Tarski,

B, isn't it? — It is so.'] What causes the trouble in the translation is the fact that apart from such a conversational context English demands some identification of the grammatical subject of this reply, which is not demanded by Latin. This is why, in translating *qualitercumque propositio significat ita est* [literally, but not in good English: 'Howsoever the proposition signifies so it is', although T. K. Scott did use this rather awkward solution], I will use the somewhat more complicated circumlocution 'In whatever way the proposition signifies things to be, so they are', or its stylistic variants. [This is my note to this passage in my translation of Buridan's *Summulae*.]

[60] Of which he provides an astounding array in his *Summulae*.

semantic closure (i.e., the condition that a language contains its own semantic predicates and means of referring to its own elements) need not render a language inconsistent.

But how can Buridan claim that (2) is not valid? After all, if L is false, then its terms supposit for the same thing, namely, L itself, which would seem to suffice for its truth. But Buridan disagrees. First he observes that any proposition "virtually implies"[61] another proposition, stating its truth. So, L implies 'L is true'. But then, since on account of (1) L is false, 'L is true' is not true. Thus, by *modus tollens*, L is not true either, whence, by virtue of bivalence, it is false. Therefore, L cannot be true, whence its falsity cannot entail its truth, despite the fact that its *correspondence condition* is satisfied: it is false, whence its terms co-supposit for itself. Still, this is not sufficient for calling it true, since for that, besides the correspondence condition its "virtual implication condition" would also have to be met, that is to say, the "virtually implied" proposition stating its truth would also have to be true. So, what is necessary and sufficient for a proposition to be called true, according to Buridan, is not only the fulfillment of its correspondence condition, but also the fulfillment of its virtual implication condition, plus, obviously, the proposition also has to exist.

But even this formulation cannot be regarded as a Buridanian "definition of truth". After all, since the "virtual implication" condition (according to which 'L is true' implied by L must be true) contains the notion of truth, mentioning it in the right-hand side of this Buridanian equivalence would render this putative definition circular.[62] But I do not think this would be the correct interpretation of this equivalence. The right-hand side of eqtgus uivalence is rather a list of trivial, separately necessary and jointly sufficient conditions of the correct assertibility of 'true'.

In the case of non-reflexive sentences, the correspondence and virtual implication conditions are always met together, and that is why they do not give rise to paradox. What is peculiar about the Liar-type sentences, however, is that because of their meaning, their correspondence conditions can be met separately from their virtual implication condition, whence (the appearance of) a paradox arises. However, if one clearly spells out all conditions of the assertibility of 'true', it will be clear that meeting the correspondence condition without the virtual implication condition does not make a proposition true, whence the falsity of the Liar-sentence does not entail its truth; there is no genuine paradox.

But then, if Buridan's solution works for reflexive-sentences in general, then he is not compelled by considerations that motivated Tarski to introduce a global split between object language and meta-language, so his possible defense against the Quinean charges concerning ontological commitment discussed earlier may just work as well.

However, one may still raise the issue that if the list of the assertibility conditions

[61] The import of 'virtually' will be discussed soon.

[62] For this charge, see especially Read, S. L. "The Liar Paradox from John Buridan back to Thomas Bradwardine", *Vivarium* 40(2002), pp. 189-218, p. 201.

of the truth-predicate does not constitute a Buridanian definition of truth, then how can he have a working semantic definition of validity (an inference is valid if and only if the premises and the negation of the conclusion cannot be true together), which apparently needs to be based on a definition of truth?

The Buridanian answer to this question is that the notion of validity is not to be provided in terms of a definition of truth; indeed, Buridan argues that in (what we would describe as) a token-based, semantically closed system, the notion of validity cannot be based on the notion of truth. The simple reason for this is that items of our language are individual token-symbols, which are parts of Buridan's ontology just as any other really existing individual. These token-symbols come into and go out of existence, whereas their existence may be part of the conditions of their correspondence to reality and their truth. For instance, the proposition 'No proposition is negative', being a negative proposition, can never be true, for its very existence falsifies it. Still, it would not be an impossible situation in which there are no negative propositions (in fact, this was the actual situation before the appearance of humans, provided we restrict our notion of a proposition to those formed by humans whether in writing, speech or in the mind). But then, in the case of this proposition there is again a divergence between the satisfaction of truth conditions and correspondence conditions, this time on account of the existence and non-existence of the proposition itself. Therefore, if he allowed a definition of validity in terms of truth, Buridan would have to accept as valid any odd consequences that have this proposition as their antecedent.

To avoid this, Buridan proposes a different, improved definition of validity, not in terms of truth, but in terms of the correspondence conditions he laid out for different types of propositions, summarized in the (improperly interpreted) Aristotelian formula:

> The fifth conclusion is that for the validity of a consequence it does not suffice for it to be impossible for the antecedent to be true without the consequent if they are formed together [...] for this is not valid: 'No proposition is negative; therefore, no proposition is affirmative'. And this is clear because the opposite of the consequent does not entail the opposite of the antecedent. Yet, the first cannot be true without the truth of the second, for it cannot be true. Therefore, something more is required, namely, that things cannot be as the antecedent signifies without being as the consequent signifies. But in connection with this it has been determined that this is not the proper expression of the point, but we use it in the sense given above, for we cannot generally express in a single expression covering all true propositions a reason why they are true, nor concerning all false propositions a reason why they are false, as has been said elsewhere.[63]

So, by means of the re-interpreted Aristotelian formula Buridan finds a way of expressing the satisfaction of the correspondence conditions of a proposition in

[63]SD, pp. 955-956.

a given situation independently from its truth, indeed, independently from its existence in that situation. This is most obvious in Buridan's discussion leading to his final definition of logical validity, where, using the example 'No proposition is negative; therefore, some proposition is negative', he first argues against his improved definition as follows:

> Again, it is not possible for things to be as the first [proposition, i.e., the antecedent] signifies without their being as the second [the consequent] signifies; therefore, the consequence is valid. The consequence seems to be manifest from what we said a valid consequence was in the previous sophism, and you cannot otherwise express the reason why a consequence is said to be valid. But I prove the antecedent: for it follows that if things are as it signifies, then it signifies; and it follows that if it signifies, then it is; and, if it is, then things are as signified by the second.[64]

In his reply to this objection, Buridan draws a very important distinction between two possible ways of understanding his improved definition of validity:

> To the second, which seems to be troublesome, I reply that a consequence is never true or false unless it is; and thus the validity or truth of a consequence requires that its antecedent and consequent exist. And then, with this assumption, we give the rule that a consequence is valid if it is impossible for things to be as the antecedent signifies without their being as the consequent signifies. And this rule can be understood in two ways: first, that it is one proposition about impossibility in the composite sense, in the way that this is commonly used, and its sense then is that this is impossible: 'When it is formed, things are as the antecedent signifies and not as the consequent signifies'. And taken in this way the rule is not valid, for according to this rule it follows that the sophism is true. And it is according to this false rule that the argument proceeded. Taken in the other way, the rule is understood as a proposition about impossibility in the divided sense, so that its sense is: a consequence is valid if in whatever way the antecedent signifies [things to be], it is impossible for things to be in that way without their being in the way the consequent signifies [them to be]. And it is clear that this rule would not prove the sophism true, for in whatever way the proposition 'No proposition is negative' signifies, it is possible for things to be in that way, and yet for them not to be in the way in which the other signifies; for this would be that case if, while the affirmatives stayed in existence, all negatives were annihilated, and this is possible.[65]

[64] SD, pp. 956-957.
[65] SD, pp. 957-958.

So, his final definition of validity is the improved definition with the important proviso that it is to be understood in the divided sense. But in that sense it provides a clear criterion for judging the validity of a consequence, regardless of the existence of the antecedent and consequent in the possible situations in which the satisfaction of their correspondence conditions needs to be checked in order to determine the validity of the consequence in which they actually occur.

But with this notion of validity Buridan is certainly entitled in his solution of the Liar-type paradoxes to use the notion of a "virtual implication", i.e., a valid consequence whose consequent need not actually be formed. Indeed, despite his strict nominalist criteria concerning the existence of the token-sentences whose truth or falsity we assert, he can certainly talk about the validity of a *virtual* implication, because his notion of validity is not tied to the notion of truth whose assertibility would require the existence of the relevant token-sentences.

Thus, by means of the re-interpreted Aristotelian formula, as summarizing the correspondence conditions of propositions that Buridan laid down in terms of the supposition of their terms, he managed to identify the ways things are in a possible situations, regardless of whether the proposition that would signify them to be this way exists in those situations. Yet, relying on his theory of supposition, Buridan can do so without reifying the way things are as a "state of affairs", a *complexe significabile*, distinct from the ordinary things admitted in his nominalist ontology.

CONCLUSION: RECONSTRUCTING MEDIEVAL LOGIC

After this sketch of Buridan's semantics, what remains is to provide the sort of rational reconstruction I was talking about in the introduction, at least in outline, to facilitate its comparison with the competing medieval conception as reconstructed here, as well as with modern quantification theory. In fact, perhaps the best way to approach this reconstruction from "our side" is to see how the standard modern semantic construction of quantification theory would need to be modified to represent, at least in part, the contrasting features of the medieval *viae*. Next we should see what further features cannot be captured by such simple modifications, and finally what would need to be done to capture even those features for our own use.

As we have seen in connection with both medieval *viae*, much of the traditional analysis of categorical propositions, their immediate inferential relations in the Square of Opposition and traditional syllogistic can quite easily be restituted by some very simple modifications of the syntax and semantics of standard quantification theory with identity.

All we need to do is add restricted variables to the language, and evaluate them in the semantics in such a way that they pick out elements of the extension of their matrix in a model, provided this extension is not empty; otherwise they should get some evaluation that renders any affirmative predication about them false.[66]

[66]I have presented such systems in my *Ars Artium*, and "Existence and Reference in Medieval

This simple modification of quantification theory renders it capable of representing some common features of both medieval *viae*, through attributing existential import to all affirmative predications, as both *viae* did, although for different reasons. For the *via antiqua*, affirmative (non-ampliative) predications carry existential import because of the impossibility of the existence of the form signified by the predicate in a suppositum of the subject without the existence of any suppositum of the subject (barring the miraculous cases of transubstantiated bread and wine).[67] For the *via moderna*, on the other hand, affirmative (non-ampliative) predications must carry existential import because of the requirement of the co-supposition of the terms of the affirmative predication, which of course cannot take place unless both terms actually do have *supposita*.

These differences are partly representable already in the modified version of quantification theory with identity and restricted variables. In this system, the *via moderna* analysis of a universal affirmative proposition would have to be represented as a universal identity claim with two quantifiable variables:

$$(\forall x.Sx)(\exists y.Py)(x.Sx = y.Py)$$

The *via antiqua* analysis, on the other hand, would have to be represented as a universal predication where only the subject term is represented by a quantifiable variable:

$$(\forall x.Sx)(P(x.Sx)).$$

These formulae represent quite well the symmetry of the semantic functions of the two terms in the *via moderna* analysis, and the asymmetry of the same in the *via antiqua* analysis. However, it is only the *via moderna* analysis that is represented quite well by the semantic functions of the restricted variables of the first formula. The function of the predicate of the second formula (designating a subset of the domain) is nowhere near the signification function attributed to such a predicate by the *via antiqua*. Thus, in a formal semantic reconstruction of that conception, we would have to assign to the predicates of our language the sort of signification function discussed earlier. However, that will inevitably bring with it having to modify the domain of interpretation of our model, where we would have to distinguish at least actual and non-actual elements (corresponding at least to actual vs. non-actual ultimate significata of our predicates, rendering our predications true or false). But if we want to represent the finer details of the *via antiqua* conception, we will also have to introduce new terms into our language, corresponding to

Logic". A simple, non-modal system is provided here in the *Appendix*. This system I believe is about as close as one can get to Buridan's semantics by modifying quantification theory "as we know it".

[67] For the logical complications caused by this supernatural scenario, see Bakker, Paul J. J. M. "*Hoc est corpus meum*. L'analyse de la formule de consécration chez les théologiens du XIVe et XVe siècles," in Costantino Marmo (ed.), *Vestigia, Imagines, Verba: Semiotics and Logic in Medieval Theological Texts (XIIth-XIVth Century)*. Turnhout: Brepols, 1997. pp. 427-51, and Ashworth, E. Jennifer, "Medieval Theories of Singular Terms", *The Stanford Encyclopedia of Philosophy (Fall 2006 Edition)*, Edward N. Zalta (ed.), `http://plato.stanford.edu/archives/fall2006/entries/singular-terms-medieval/`, sect. 8.

abstract common terms, with the function of suppositing for the significata of the predicates representing our concrete common terms. Furthermore, at this point it will be also inevitable to introduce a copula with the function of asserting the existence of these significata. However, since these significata cannot be regarded as entities in the same sense in all cases, we would have to distinguish several senses of existence, and introduce further sub-domains into our domain of interpretation accordingly. Finally, we should also represent the function of the copula asserting the significata of entire propositions, which could then serve to provide a simple Aristotelian formula for the definition of truth-in-a-model, grounding a definition of validity. These adjustments, however, already involve major departures from the standard construction of quantification theory.

As we have seen, the *via moderna* analysis is actually much closer, at least in the first approach, to standard quantification theory. Indeed, the function attributed to predicate letters (that of denoting subsets of the domain or of its Cartesian products with itself) is not a far cry from the nominalist conception of the signification of common terms, according to which these terms signify certain individuals of the domain, or their ordered collections in the case of connotative terms. And the identity sign is clearly a good representation of the function of the copula in accordance with the *via moderna* conception.

However, again, if we need to represent the finer details of the *via moderna* conception, we need to depart considerably from the standard construction of the semantics of quantification theory. In the first place, although using restricted variables to represent common terms in personal supposition yields "the right results" concerning the square of opposition and syllogistic, nevertheless, it does so at the expense of representing simple common terms (say, F) as complex variables with an intrinsic propositional structure embedded in their matrix ('$x.Fx$', amounting to something expressible as 'thing that is an F'). But according to our *via moderna* authors it is the simple term 'man', for example, that has this referring function, and not a complex term like 'thing that is a man'. In fact Buridan would pointedly distinguish the two in various contexts. So, to represent this feature of *via moderna* semantics, we would need to devise "term-logics" along the lines proposed by Lesniewski, Lejewski, Henry, Sommers and Englebretsen.

However, even if all the finer details of *via moderna* semantics were neatly represented in a semantic theory that describes the semantic features of an object language in a distinct meta-language, the entire construction of the semantic theory, as we have seen, would be radically unfaithful to Buridan's conception. In the first place, the global split between object-language and meta-language would not do justice to his sophisticated conception of ontological commitment, offering a genuine third alternative "between" Quine's and Meinong's. In the second place, it would preclude his intriguing treatment of Liar-type paradoxes with all their implications concerning the theory of truth. Finally, it would also be incapable of representing Buridan's "relentlessly" nominalist account of logical validity in a semantically closed, token based system.

To approach these issues, therefore, one might provide a semantic construction

in which the object language is capable of "cannibalizing" its own meta-language, by introducing distinguished semantic predicates into the object-language matching those originally defined in the meta-language.

The payoff of such a project would be the ability to see exactly what it takes, in precise model-theoretical terms, to construct a nominalist semantics, facilitating its comparison both with the competing medieval conception and with a number of modern conceptions. And this, in the end, might just get us closer to a genuine understanding of the fundamental semantic relations between language, thought and reality.

APPENDIX

Language

$L := \langle C, P, V, Trm, F\rangle$ [language : constants, parameters, variables, terms, formulae]
$C := \{\ \sim, \&, =, \forall, ., (,)\}$ [constants: negation, conjunction, identity, universal quantifier, punctuation marks]
$P := Pr \cup I, Pr := \cup\{F_n^m\}, I := \cup\{a_n\}$ [parameters: predicates, individual names]
$V := X \cup X_r, X := \cup\{x_i\}$, [variables: simple variables plus restricted variables]
$X_r := \{t_n : t_n =$'$x.A$' and $A \in F\}$ [restricted variable: formula prefixed with simple variable and dot]

Trm:=I∪V [terms: individual names plus variables]

Formulae:

1. $t_1 \ldots t_m \in$ Trm, $F_n^m \in Pr \rightarrow$ '$F_n^m(t_1)\ldots(t_m)$' $\in F$
2. $t_1, t_2 \in$ Trm $\rightarrow$ '$t_1 = t_2$' $\in F$
3. $A, B \in F \rightarrow$ '$\sim (A)$', '$A\&B$' $\in F$
4. $A \in F, v \in V \rightarrow$ '$(\forall v)(A)$' $\in F$

Model

$M := \langle D, T, E, 0, 1, SGT\rangle$, where
$D \neq \emptyset, T \neq \emptyset, t \in T, E(t) \subset D, 0 \notin D, 1 \notin D, SGT : P \rightarrow D \cup P(D^m)$ [non-empty domain, set of times, times, set of existents at time t, 0 and 1: False and True, signification function: from parameters to the domain plus the power-set of the m-th Cartesian product of D with itself]

1. $SGT(a_n) \in D$
2. $SGT(F_n^m) \in P(D^m)$

Supposition (value-assignment in a model)

1. $SUP(a)(t) = SGT(a)$, if $SGT(a) \in E(t)$,
   otherwise $SUP(a)(t) = 0$

2. $SUP(x)(t) \in D$

3. $SUP(\text{`}x.A\text{'})(t) = SUP(x)(t)$, if $SUP(A)(t) = 1$,
   otherwise $SUP(\text{`}x.A\text{'})(t) = 0$

4. $SUP(\text{`}F_n^m(t_1)\ldots(t_m)\text{'})(t) = 1$, if $\langle SUP(t_1)(t), \ldots, SUP(t_m)(t)\rangle \in SGT(F_n^m) \cap E(t)^m$,
   otherwise $SUP(\text{`}F_n^m(t_1)\ldots(t_m)\text{'})(t) = 0$

5. $SUP(\text{`}t_1 = t_2\text{'})(t) = 1$, if $SUP(t_1)(t) = SUP(t_2)(t) \in E(t)$,
   otherwise $SUP(\text{`}t_1 = t_2\text{'})(t) = 0$

6. $SUP(\text{`}\sim A\text{'})(t) = 1$, if $SUP(A)(t) = 0$, otherwise $SUP(\text{`}\sim A\text{'})(t) = 0$

7. $SUP(\text{`}A\&B\text{'})(t) = 1$, if $SUP(A)(t) = SUP(B)(t) = 1$,
   otherwise $SUP(\text{`}A\&B\text{'})(t) = 0$

8. $SUP(\text{`}(\forall v)(A)\text{'})(t) = 1$, if for every $u \in RSUP(v)(t), SUP[v : u](A)(t) = 1$,
   otherwise $SUP(\text{`}(\forall v)(A)\text{'})(t) = 0$, where $RSUP(v)(t) := \{u \in E(t) : u = SUP(v)(t)\}$, if $\{u \in E(t) : u = SUP(v)(t)\} \neq \emptyset$, otherwise $RSUP(v)(t) := \{0\}$, and $SUP[v : u](w)(t) = u$, if $w = v$, otherwise $SUP[v : u](w)(t) = SUP(v)(t)$ — [$RSUP(v)(t)$ is called "the range of v at t"]

A **is true in** M iff for some $SUP, SUP(A)(t) = 1$; A **is valid** iff for every M, A is true in M.

LOGIC IN THE 14th CENTURY AFTER OCKHAM

Catarina Dutilh Novaes

This chapter is meant to complement the previous chapter on Ockham's and Buridan's respective semantic systems, and the chapters on modalities, on self-referential paradoxes and on supposition in this volume. Here, I intend to cover for as much as possible the important material from the 14th century that is not covered by these other chapters.

The 14th century was a period of intense intellectual activity in Christian Europe, in spite of the image of decline and disaster often associated with this period. By that time, the Universities of Paris and Oxford, whose birth had taken place in the previous centuries, had acquired maturity as institutions, and the different forms for intellectual investigation had been laid down. Even the Black Death in the mid-14th century did not provoke a total decline in the degree of sophistication of the knowledge being produced at the time, in spite of having taken the lives of some of its brightest masters (e.g. Bradwardine, cf. [Read, 2006b]).

Logic occupied a privileged position in the medieval curriculum; it was part of the *trivium* along with rhetoric and grammar, the three subjects a medieval student worked on at the very beginning of his career. In many senses, logic was thought to be the general method with which any student had to have a high degree of familiarity before proceeding to any other topic (cf. [Zupko, 2003, ch. 2]). So, on the one hand, at least some of the logic of that period was really meant for very young students just beginning their intellectual career; on the other hand, while it was indeed the most common for masters to move on to more 'serious' topics (especially theology) at later stages of their careers (but this was not always the case; Buridan is the most prominent but not the only example of a master having stayed at the faculty of Arts throughout his career — cf. [Courtenay, 2004]), many of them viewed logic not only as the matter to be covered by very young students. Indeed, the 14th century corpus on logic presents logical analysis of the highest quality.

But first, of course, we must clarify what was meant by 'logic' in the 14th century. That medieval logic is very different from what we call logic in 21st century is almost a truism. However, a case can be made for the non-equivocal use of the term 'logic' applied to these two radically different traditions (cf. [Dutilh Novaes, 2007, 'Conclusion']), insofar as some of the most basic traits of what is/was thought to be logic in each of these periods seem to converge in significant aspects. But this is not the place for such a conceptual, intensional analysis; rather, for

the present purposes, it is sufficient to present the **extension** of the term 'logic' in the 14th century — that is, the kinds of theories that were treated under the heading of logic in that period.

As any list of 14th century authors and texts (such as in [Spade, 1996, 329]) will show, the main logical topics treated in that period were: *insolubilia* (paradoxical propositions); modal propositions; supposition; the analysis ('proof') of propositions; obligations; and consequence. One also encounters works bearing the title '*sophismata*' (William Heytesbury, Albert of Saxony, Richard Kilvington), but *sophismata* are not theories in themselves; rather, *sophismata* are logical and/or philosophical puzzles (cf. [Pironet, 2005]). The apparatus to be used to solve a given sophisma obviously depends on the source of the problem in question, and may come from any of the familiar theoretical frameworks most used then (theories of supposition, theories of fallacies etc.).

As for the first two topics of this list, *insolubilia* and modal propositions (two of the main topics in 14th century logic), they are treated in detail elsewhere this volume, so I will not approach them here. We are thus left with the other four prominent logical genres in the 14th century: supposition, the analysis ('proof') of propositions, obligationes, and consequence. Indeed, this chapter is composed of three main parts, each of them dedicated to one of these topics – under the common heading of 'semantics', I treat supposition and, briefly, the theory of proofs of propositions. But before I move to the study of these three topics, in a preliminary section I give an overview of names, dates and places, so as to provide the reader with some of the historical background before we proceed to the conceptual analysis. The reader may also choose to turn directly to the thematic sections, if the historical aspects are not her main interest.

1 NAMES, DATES AND PLACES

While it is true that there has been growing interest in 14th century philosophy over the last decades, and that the number of publications on this subject has grown exponentially, we are still nowhere near a complete account of the historical and bibliographical events involving the authors in question. It is significant, for example, that even the exact date of death of an author as influential as John Buridan remains unknown (cf. [Zupko, 2002a, section 1]). It may be a matter of further work on the extant manuscripts and records, which remain largely unstudied in libraries, but it may also be that some of these details will never be revealed for lack of extant records.

Be that as it may, and although there is definitely a significant amount of work still to be done on manuscripts and records, scholars have managed to compile an impressive, albeit incomplete, amount of information on the philosophers of that period. Here I attempt to present the main lines of our current state of knowledge on 14th century philosophers and logicians, especially in view of the conceptual analyses to follow. Naturally, several important texts remain without definitive authorial attribution and are listed as 'Anonymous'; whenever relevant, such texts

will be mentioned in the thematic sections to follow this one.

1.1 Beyond Paris and Oxford

What is perhaps most interesting in the historical development of logic in the 14th century is the spread of logical and philosophical knowledge to places other than the two traditional centers, Paris and Oxford. Indeed, even though it would be incorrect to say that Paris and Oxford were the only centers of intellectual and academic development in Christian Europe in the centuries preceding the 14th century (the University of Bologna is generally considered to be the oldest university in Christian Europe, and there were important *studia*, i.e. schools of higher learning, in many European cities), it is undeniable that Oxford and especially Paris (which was the great center for theology from the 12th up to the 15th century) were the two great poles of development concerning the *Ars* in general (i.e. logic, rhetoric, grammar, geometry, astronomy, music and arithmetic, plus the three 'philosophies': moral philosophy, metaphysics and natural philosophy), and concerning logic in particular (cf. [De Libera, 1982], on the Paris and Oxford traditions in the 13th century). In the 14th century, however, especially in the second half of the century, this was no longer the case.

The regional element was always an important one in how academic learning was organized in the Faculty of Paris, which was divided in 'nations' in such a way that it was most common for a student to study under a master originally coming from his own home region (cf. [Courtenay, 2004]).[1] But with the creation of several new universities in different locations in Europe, by the end of the 14th century it was no longer necessary for a student to go to Paris or Oxford to obtain his degree; he could often stay within the boundaries of his own country. By the same token, the two traditional centers were no longer the only places where original and influential work in philosophy and logic was being done. It may still be useful, though, for explanatory purposes, to draw a distinction between the British and the Paris traditions in logic in the 14th century (even though there are of course multiple points of contact and mutual influence between the two traditions), and to track how each of these traditions was exported to and reworked in new centers of knowledge. Indeed, even at the time this distinction was recognized: Continental authors usually referred to the authors of the British tradition (in particular those currently referred to as the 'Mertonians') as '*Anglici*' or '*Britannici*' (cf. [Sylla, 1982, 541]).

Two interesting examples of the transmission of the British and Parisian traditions in logic are the influence of British logic in Italy and the influence of Parisian logic in Eastern Europe. Take Italy, for example: even though the country already had a great tradition of institutions of knowledge, an interesting phenomenon is

[1] The four Parisian nations were: Normandy, Picardy, France and the English/German nation. Other universities often followed Paris' example and were organized in (usually four) nations as well (but naturally, different nations other than the original Parisian ones). The University of Prague, for example, was organized in the Bohemian, Bavarian, Saxon and Polish nations (see [Ashworth, 2006, 212]).

the spread of 'British logic' in Italy in the 14th century, especially in the second half of the century. In this period, more and more Italian students were sent to Oxford[2], and often brought back to Italy the knowledge they acquired there (cf. [Courtenay, 1982, p. 17]); British masters such as Ockham and Burley also visited and worked in Italy (cf. [Courtenay, 1982, p. 17]). It is very telling that he who is perhaps the most influential logician of the first half of the 15th century was an Italian having studied in Oxford in the last decade of the 14th century, namely Paul of Venice. After obtaining his degree, he taught in Italy for most of his career (cf. [Conti, 2005b]), and this exemplifies the aforementioned phenomenon of spread of knowledge beyond the traditional centers of Oxford and Paris that took place in the 14th century.[3]

A similar phenomenon occurred in Eastern Europe with the exportation of Parisian knowledge (logic in particular), which is made even more evident by the foundation of several influential universities in the region in the second half of the 14th century — the Universities of Prague[4] in 1348, Vienna in 1365 (but to be re-founded in 1384), Erfurt (papal bull in 1379, but inaugurated in 1392), Heidelberg in 1385 and Cologne in 1388/9 are the main examples. Some of the most influential masters of the second half of the 14th century were directly related to the foundation of these universities, most notably Albert of Saxony for the University of Vienna[5] (cf. [Biard, 2004, section 1]) and Marsilius of Inghen for the University of Heidelberg (cf. [Hoenen, 2001, section 1]). These two masters are particularly representative of the spread of 'Parisian' trends (especially the 'Buridanian' approach to logic[6]) into other regions.

[2]As a result of the papal schism (1378–1417, a period during which there were two and sometimes even three 'popes', each of whom considering the other(s) to be an usurper), Italians could no longer go to Paris, since Italy and France supported opposing parties in the papal dispute.

[3]Even before Paul of Venice, Italy had already an important tradition of logicians. See for example the introduction to Blaise of Parme's logical *Questiones* [Blaise of Parme, 2001].

[4]In this respect, the University of Prague differs slightly from the other Eastern European universities at the time in that not only the teaching of Parisian masters was influential; Prague enjoyed equally close relations with Oxford. Thus, not only Buridan and Marsilius of Inghen were influential in Prague, but also Heytesbury and later Wyclif (see [Ashworth, 2006, 212]).

[5]However, Albert of Saxony stayed only for a year in Vienna (cf. [Shank, 1988, 13]). For several political and social reasons, the University of Vienna only came to existence with its re-foundation in 1384; but at that point it was again the importation of Parisian knowledge that marked its rebirth, as three of the most distinguished Germanic theologians trained in Paris (Henry of Langenstein, Henry of Oyta and Gerard of Kalkar) were recruited to be at the head of the reborn university (cf. [Shank, 1988, 17]). However, even before that, the Parisian master Thomas of Cleves was appointed chief schoolmaster at St. Stephen's Cathedral School in Vienna (the basis for the soon-to-be re-founded University of Vienna) (cf. [Read, 1991, 61]).

[6]Buridan was certainly one of the most influential logicians in the 14th century, probably more than Ockham himself (who, ironically, was more influential in Paris in the first half of the 14th century than in his own home country, England — cf. [Courtenay, 1984; 1987a]). See also [Markowski, 1984] for a detailed account of the reception of Buridan's texts in Eastern Europe.

1.2 A survey of the traditions

Establishing the relations of mutual influence between the different authors and trends in 14th century logic is definitely not a straightforward matter, and one often winds up with an oversimplification of the facts. But given this caveat, in the following sections I will attempt to present a survey of 14th century logic with respect to names, dates and places, following the thread of these two main traditions, British and Parisian logic.

1.2.1 The British tradition

Let us start with the British tradition; it is somewhat easier to follow than the Parisian/continental tradition, as it developed in a relatively more compact way. (For a detailed overview of the British tradition, [Courtenay, 1987b; Ashworth and Spade 1992] are invaluable sources of information and references to other works on the topic).

The origin of the British tradition in logic is still a matter of debate among scholars. While it seems clear that one cannot speak of a British tradition in the 12th century — a time in which activity in logic was virtually entirely concentrated in Paris — there are important British authors from the 13th century, such as William of Sherwood, Roger Bacon, Simon of Faversham and Robert Kilwardby (who nevertheless all studied and/or worked in Paris). In fact, it has been argued that, while Paris was taken over by the 'modist' fashion at the end of the 13th century, Oxford remained faithful to the 'older' terminist tradition (cf. [Ebbesen, 1985]), which was in turn re-imported into the continent at the beginning of the 14th century, and which is the matrix for the developments in the 14th century to be discussed here. But this theory also encountered opposition, to an extent that we cannot as of now speak of an entirely clear picture of these developments.

The most important British philosopher of the very beginning of the 14th century is, beyond any doubt, Walter Burley. He was extremely influential in his own time as well as in the remaining of the century (his date of death is estimated at around 1344); he exemplifies the introduction of 'new' logical and semantic tools and techniques characteristic of the late medieval period, but in his case often used to defend rather conservative views – he is viewed as the main representative of late-medieval 'realism', as opposed to the nominalism of Ockham and Buridan (for the relations of criticism but also of mutual influence between Ockham and Burley, see [Conti, 2004]).

In the early stages of his career, Burley was linked to Merton College in Oxford, the college to which most of the important British masters of the first half of the 14th century were connected (more on Merton College shortly). It is also noteworthy that he obtained his doctorate in theology in Paris, which also shows that one cannot speak of entirely independent developments in the British and Parisian traditions. Throughout his life, Burley traveled extensively in Europe for several diplomatic missions, while at the same time never stopping his scholarly work, and thus can be seen as one of the actors in the dissemination of British

logic in the continent, especially in Italy.[7]

Burley's work is also representative of the topics that were to become the logical topics par excellence throughout the 14th century: he wrote a treatise on consequences early in his career [Walter Burley, 1980] (Green-Pedersen argues that this treatise was certainly written before 1302 — cf. [Green-Pedersen, 1981], as well as a treatise on supposition, and an influential treatise on obligations [Green, 1963]. But he is perhaps most famous for his *On the Purity of the Art of Logic* [Walter Burley, 2000], of which he wrote a shorter and a longer version.[8] For the present purposes, Burley's treatises on consequences and on obligations are particularly important, as well as the parts of the *Purity* concerning consequences; they will be the starting point for the conceptual analyses of each of these topics in this chapter.

Ockham, perhaps the most famous 14th century philosopher now as well as then, was slightly younger than Burley. He led an agitated life, most notably marked by his quarrels with the Avignon popes.[9] He wrote on logic for only a very brief period of his life, before his departure to Avignon; but his work on logic, especially his *Summa Logicae* [William of Ockham, 1974] was to have a significant impact in subsequent developments. However, Ockham's writings will not be among the main objects of analysis in this chapter: his semantics is already thoroughly examined elsewhere in this volume, and his theory of obligations (described in his *Summa Logicae* III-3, chaps. 39-45) is not particularly important for the development of these theories in the 14th century. Only his writings on consequence (*Summa Logicae* III-3) will be examined in the appropriate section below.

Of the same period, Adam Wodeham is now mostly known as the secretary and assistant of Ockham in the period in which the latter was writing the *Summa Logicae* (in the 1320s); although Wodeham has made important contributions as diffusor and also critic of Ockham (especially with respect to his epistemology), his contribution to logic does not seem to have been significant. He is, though, currently thought to be the creator of the doctrine of *complexe significabile*, which was later to be defended by the Parisian theologian Gregory of Rimini (see below in the next section), and was to become an influential theory concerning the meaning and truthmakers of propositions, with applications to epistemology.

Besides the very famous Ockham and Burley, several other extremely innovative and bright masters were active in the first half of the 14th century in England (the Black Death in 1349 is usually considered as a convenient divisor between this period and the one to follow, each having quite distinct characteristics — cf. [Ashworth and Spade, 1992]). As already mentioned, most of these masters were connected to Merton College in Oxford, and are thus often referred to as the 'Mertonians' (often also as the 'Oxford Calculators' — see [Sylla, 1982]). The Merto-

[7] For brief but informative overviews of Burley's life and influence, see [Spade, 2000; Conti, 2004].

[8] On the rather awkward title of Burley's masterpiece, which might be better translated as *On the Essence of the Art of Logic*, see [Spade and Menn, 2003].

[9] For an account of Ockham's life and influence, see [Courtenay, 1999].

nians excelled not only in logic; their works on natural philosophy were probably even more influential; they are often viewed as precursors of the 'mathematical turn' in physics to take place a few centuries later (see [Sylla, 1982, 541]).

The main authors among the Mertonians of the first half of the 14th century (for our present purposes) are: Thomas Bradwardine, William Heytesbury, Richard Billingham, Roger Swyneshed (not to be confounded with the famous Richard Swyneshed, also a Mertonian and in fact known as 'The Calculator'), and Richard Kilvington (for a more detailed presentation of each of these authors, see [Ashworth and Spade, 1992]).

Until not so long ago, it was though that Thomas Bradwardine's contribution to logic was, to say the least, meager, and that his main contributions were to be found in the field of natural philosophy, mathematics and theology. But a recent interest in Bradwardine's *insolubilia* (including a new edition of the text now in preparation by Stephen Read) has arisen, showing that his work on Liar-like paradoxes was extremely innovative and sophisticated (cf. [Read, 2006b]). Since I will not be dealing with *insolubilia* in this chapter (as they are the topic of a different chapter in this volume), Bradwardine will not figure prominently in the analyses to follow. But his historical as well as philosophical importance must not be overlooked.

William Heytesbury was seemingly more prolific than Bradwardine. The list of his still extant works is rather long, and includes treatises on consequence and obligations (which are only to be found in manuscripts — cf. [Longeway, 2003, section 2]), an influential treatise on divided and composite senses, some work on *insolubilia*, among others. But he is most famous for his *Regulae solvendi sophismata* (1335)[10], a work composed of six parts, where logical, semantic and physical sophisms are dealt with. In the first part he deals with the paradoxical propositions known as *insolubilia*, and his approach to them was later to be influential, especially in the continent (e.g. Peter of Ailly's treatment of *insolubilia*). The second part is dedicated to what we now call 'reference in opaque contexts'; the third and fourth parts deal with semantic puzzles (related to the supposition of relative pronouns, in the third, and to the terms 'begins' and 'ceases' in the fourth); the last two concern physical puzzles. Thus, since the subject-matter of the first two parts is to be dealt with elsewhere in this volume, and since the last two parts do not concern logic and/or semantics directly, in the analysis to follow Heytesbury will not figure prominently. Nevertheless, it is important to notice that Heytesbury was to become very influential in Italy in the 15th century (while almost entirely forgotten in England), again exemplifying the exportation of British logic to Italy in the 14th and 15th centuries (see [Braakhuis, 1982]).

Richard Billingham, another influential Mertonian (apparently a few years younger than Bradwardine and Heytesbury), was most known in his own time for one of his works, his *Speculum puerorum* [Maierú, 1970; de Rijk, 1975; 1982], even though he also wrote on all the traditional topics in 14th century logic (obligations — also in-

[10] A transcription of the text by F. Pironet, en route for a critical edition, can be found at http://mapageweb.umontreal.ca/pironetf/Sophismata.html

fluential, cf. (Ashworth 1985) — consequences, insolubilia, supposition etc. — for complete list of his still extant works, see the bibliography in [Richard Billingham, 2003]). His *Speculum puerorum* is dedicated to what seems to be a 14$^{\text{th}}$ century invention, the theory of the 'proofs of propositions'. To 'prove' a proposition is, in a general sense, to show it to be true, but not necessarily in a rigorous, formal way (as we now understand the notion of 'proof'); the basic idea is that there are some propositions that are basic, that is, whose truth does not depend on the truth of other propositions, but that the majority of propositions are not of this nature. The task is thus to 'unfold' these propositions that are not basic into basic propositions, so that it becomes clear what the truth of these non-basic propositions depends on. For this reason, the theory of the proof of propositions is essentially a semantic theory, that is, a theory intended to explain the meaning of complex propositions in terms of more primitive ones, to which they can be reduced, and therefore will be (briefly) treated under the heading 'Semantics' below.

Billingham's treatise on the proof of propositions (*Speculum puerorum*) is not the first and probably not even the most remarkable among the treatises of the genre in the 14$^{\text{th}}$ century (see [De Rijk, 1975; Ashworth and Spade, 1992, 42]); it was, however, very influential in its time (see [De Rijk, 1976]), more than any of his other writings. Furthermore, his treatise on consequences has been given a modern edition recently [Richard Billingham, 2003], and will be mentioned in the section dedicated to consequences below.

Roger Swyneshed and Richard Kilvington are both minor figures if compared to the influential Bradwardine, Heytesbury and Billingham, but they both composed works that had considerable impact in later developments. Swyneshed is known for his treatise on *insolubilia* [Spade, 1979] — which, in spite of being heavily attacked by Heytesbury, eventually became quite popular in the 15$^{\text{th}}$ century — and for his treatise on obligations [Spade, 1977], which seemingly initiated a new trend in obligational disputations, the so-called *nova responsio* (as opposed to the *antiqua responsio*, exemplified by Burley's treatise). Indeed, Swyneshed's treatise on obligations will be one of the main objects of analysis in the section on obligations below. Kilvington will be mentioned in the same section, as parts of his quite popular *Sophismata* (cf. [Kilvington, 1990a] for the Latin text and [Kilvington, 1990b)]for the translation), which otherwise mostly deals with problems of motion and change from a logical perspective (see [Jung, 2001]), present important views on obligational disputations as well.

The next period in the history of British logic in the 14$^{\text{th}}$ century is, according to Ashworth and Spade [1992, 39] 'a period still of sophistication, even if no longer of great originality, during which earlier issues and doctrines were developed, consolidated and transmitted to the rest of Europe'. They also mention as characteristic features of this period a tendency to produce summary treatments of different logical topics, which seem to have been intended for teaching rather than to be original contributions; a strong interest in the theory of 'proofs of propositions', in the fashion of Billingham's *Speculum puerorum*, and in the problem of truth and signification of propositions. For our purposes, the main authors of this

period are: Ralph Strode, Richard Lavenham, Richard Ferrybridge, John Wyclif, and Paul of Venice (the last two being beyond any doubt the best known to us and probably the most influential ones in their time too).

Richard Lavenham was neither particularly influential nor particularly original, but his (usually short) writings in many senses illustrate exceptionally well the general themes and theories of this period. It is perhaps for this reason that many of his writings have been given modern editions (see in particular [Spade, 1974], where his treatises on consequences and on supposition are edited, and [Spade, 1978], for his treatise on obligations; see also [Spade, 1980] for a general presentation of Lavenham); this obviously means that he is a key figure for anyone wishing to understand the logic of the second half of the 14th century, given the easy access to his writings.

Ralph Strode and Richard Ferrybridge were both more influential figures than Lavenham (they were to be particularly influential in Italy — see [Maieru, 1982a] and [Del Punta, 1982]), but unfortunately most of their works have not yet been given modern editions. Ralph Strode has written a *Logica* that exemplifies perfectly the main interests of logicians in the 14th century, composed of the following treatises: two introductory chapters on the principles of logic, one on consequence, one on supposition, one on obligationes, and finally one on *insolubilia* (on the order of the treatises within the *Logica*, see [Maieru, 1982a]). His treatise on consequence has been given a modern edition [Seaton, 1973], and there is an ongoing project to edit the rest of Strode's *Logica*, but which so far has not been completed. Strode will be mentioned in the sections below dedicated to obligationes and to consequences.

Ferrybridge wrote two known works in logic, a 'Logic, or treatise on the truth of propositions' and a treatise on consequences (cf. [Ashworth and Spade, 1992, 57]). To my knowledge, neither has been given a full modern edition, but the two first chapters of the *Logica* can be found in [Del Punta, 1982], and many passages of his treatise on consequences can be found in [Pozzi, 1978]; the latter will be commented upon in the section dedicated to consequences below.

By contrast, John Wyclif and Paul of Venice are much better known to us (see [Conti, 2005a; 2005b]); their works are often easily accessible to the modern reader, including translations. Wyclif is most known for his metaphysical positions (he is the main advocate of realism in the second half of the 14th century), but his importance in the history of logic must not be underestimated. His logical doctrines are indeed usually intimately related to some metaphysical problem; for example, the issue of universals led him to reflect on the notion of predication (cf. [Conti, 2005a, section 2.3]). He did write a *Logica* and a sequel to it (ed. Dziewicki, 1893-99), where again his realist metaphysics plays a prominent role; his discussion of the notion of supposition, heavily borrowed from Burley but with important modifications, will be briefly examined below. He also wrote on the issue of the truth of propositions, on insolubilia and on the 'proof of propositions' (cf. [Ashworth and Spade, 1992]).

At first sight it may seem strange to place Paul of Venice under the heading of

'British logic'; he was after all an Italian who spent almost his entire life working in Italy. He did though spend a short period (at least three years, it would seem[11]) in Oxford in his formative years, and the logic he learned in Oxford remained his main source of influence in his subsequent writings. It makes thus good sense to place him among the 'British' logicians; moreover, in the 15th century it was mainly in Italy that 'British' logic flourished, as in Great Britain properly speaking a period of stagnation in logic occurred.

Paul's work covers an impressive array of themes, as is attested for example by the length of his *Logica Magna* (of which several parts have been recently edited and translated into English — see bibliography). His *Logica Parva* (complied around 1395, at Oxford) was one of the most influential logic textbooks in the 15th century (Paul of Venice 1984). True enough, most of Paul's career took place in the 15th century, so one might think that he should be treated elsewhere in this volume, and not in this chapter dedicated to the 14th century; but in many senses he epitomizes 14th century logic. Not only did he deal with virtually all of the important logical topics of this century (supposition, obligations, the truth of propositions etc.); he also usually summarized the logical knowledge produced in this century in his discussions, often quoting verbatim from his sources (for example, his use of Strode in his treatise on obligations — cf. [Ashworth and Spade, 1992, fn.99]), while also making original contributions to the discussions.

Another author who was of Italian origin (in fact he was Greek-born) and who adopted much of the Oxford logic framework after having studied there is Peter of Candia, later Pope Alexander V. Mostly a theologian, among his logical works are a treatise on obligationes and one on consequence (cf. [Green-Pedersen, 1985]).

Less influential figures still worth being mentioned are Henry Hopton (in particular his discussion of the truth of propositions), Robert Fland (in particular his works on consequence and on obligations, cf. [Spade, 1976; 1980c], Martinus Anglicus (in particular his works on consequence and on obligations), Johannes Venator (his *Logica* is in the spirit of Billingham's theory of 'proofs of propositions' — cf. [de Rijk, 1982] and edited in [Johannes Venator, 1999]), Robert Alington (a follower of John Wyclif) and Richard Brinkley (in particular his theory of the signification of propositions and his obligationes — [Brinkley, 1987; 1995]). John of Holland is an interesting case of an author in some senses belonging to both traditions, British and Continental. While his writings (cf. [John of Holland, 1985]) show a familiarity with British logicians, which seems to indicate that he may have studied at Oxford, he is best known for his career at the University of Prague; therefore, he will be treated in more detail in the section dedicated to the continental tradition below.

As already said, the end of the 14th century coincides with a general decline in British logic. According to Ashworth and Spade [1982, 35], it is a period of 'logic stagnation leading eventually in the sixteenth century to the rejection of the 'thorns' of scholastic logic'. In the 15th century, the most interesting and innovative contributions within the tradition of 'British' logic were to take place

[11] Cf. [Ashworth and Spade, 1992, 60].

elsewhere, in particular in Italy.

1.2.2 The Parisian/continental tradition

The continental tradition in logic in the 14th century begins with what could be described as a hiatus; according to our current state of knowledge about that period, it appears that, in the first three decades of the 14th century, no significant novelties were put forward by Parisian logicians. However, it must be said that the first half of the 14th century in Paris, and in the continent generally speaking, is as of now not as well studied as the same period in Oxford with respect to logic; but it is to be hoped that, with further research, our knowledge of this period in Paris will become more thorough in the coming years.

For as far as we can tell at present, this period in Paris was still very much market by the Modist theories, a late-13th century creation (which is treated elsewhere in this volume); indeed, what is perhaps the most important text of the Modist tradition, namely Thomas of Erfurt's *De modis significandi*, appears to have been written in the first decade of the 14th century (cf. [Zupko, 2002b]. Important Parisian Art Masters of this period (also within the general Modist trend) were Radulphus Brito and Siger of Courtrai, but one cannot speak of them as having made particularly original contributions in the domain of logic besides their influence in the development of the Modist doctrines. Another important figure of this period in Paris was Peter Auriol (see [Friedman, 2002]), who was predominantly a theologian, but whose doctrines had implications for the theory of cognition (cf. [Tachau, 1982]).

There is, however, one author of this period who is worth being mentioned in connection with the development of logic in Paris, namely Giraldus Odonis. His *Logica* [Giraldus Odonis, 1997] seems to have been written at some point in the first half of the 1320s (cf. [de Rijk, 1997, 24]); what is interesting is that it is nothing like the works in logic of the generation to follow, such as Buridan's, so apparently it was not particularly influential for subsequent developments. It was written roughly at the same time as Ockham's *Summa Logicae*, and according to de Rijk [1997, 24], 'neither in Girald's work nor in Ockham's *Summa Logicae* [...] is there any trace of acquaintance with each other's work'. The historical significance of Girald's logic is to show that there was seemingly activity within the terminist tradition in Paris at this time — contrary to the idea that the first three decades of the 14th in Paris were totally dominated by the Modist tradition, and that the terminist tradition was only practiced in Britain. Granted, Girald was familiar with Burley's work (recall that, at this period, Burley was a student of theology in Paris), so it is still possible that the terminist tradition was reintroduced in Paris as a British import, as claimed by Ebbesen [1985]. But what is very significant is how different Girald's theory of supposition is from Burley's, to the point that one wonders whether there wasn't indeed a genuine Parisian terminist tradition in this period, which however seems to have become surpassed by what can be loosely referred to as the 'British approach' in later works, such as those of Buridan.

By contrast, an Englishman[12] thought to have been working in Paris in the 1320s and 1330s, Thomas Maulevelt, illustrates the early penetration of Ockhamist logical doctrines in Paris (the history of which deserves closer attention — cf. [Courtenay, 1984]). Very little is known about him (cf. [Lorenz, 1996]), and the fact that his name is spelled in several different ways (Maulvelt, Manlevelt, Maulfield, among others), does not make the job of tracing his steps any easier. But we do know, for example, that his works in the *parva logicalia* ('textbook'-style logic) were to be very influential in Germany and Eastern Europe. Modern editions of some of his texts are now in preparation, but so far none of his works is available in print. What is in any case clear is that Thomas, while working in Paris, was already following the footsteps of Ockham in logic, and sometimes taking the Ockhamist project of ontological reduction even further than Ockham himself (cf. [Andrews, 2005], on Maulevelt's denial of the category of substance).

Be that as it may, by the end of the third decade of the 14$^{\text{th}}$ century, 'a new academic generation with different concerns was emerging [...], but its directions and importance would not become visible for almost a decade' [Courtenay, 1999a, 5]. The most famous member of this new generation is John Buridan, but other inspired masters were Nicholas of Autrecourt, Nicholas of Oresme and Gregory of Rimini. Strictly speaking, the contributions in logic of the three latter authors are not particularly significant; however, they were important figures for the general development of the *Ars* in that period. Gregory of Rimini, for example, was an Italian Augustinian who came in contact with the works of Oxford scholars in Italy (after having studied theology in Paris in the 1320s), and upon his return to Paris in the 1340s is thought to have been particularly instrumental in the spread of Oxford philosophy in general and Oxford logic in particular in Paris (cf. [Schabel, 2001, section 2]) — in the 1330s, very little attention was paid in Paris to the revolutionary works of English logicians of that period, such as Bradwardine, Heytesbury etc. (cf. [Courtenay, 1984, p. 46]).[13] Predominantly a theologian, Gregory is also known for defending the doctrine of *complexe significabile*, the doctrine according to which the object of knowledge is neither propositions nor the things in the external world signified by its terms, but rather that which is signified by the proposition (*complexe significabile*), whose ontological status was seen by some (such as Autrecourt — cf. [Thijssen, 2001, section 7]) as problematic.

John Buridan is, as already mentioned, without a doubt one of the most influential philosophers of the 14$^{\text{th}}$ century. His writings range over a wide variety of

[12] Although English, Thomas is treated in the Parisian/continental section here because he is known to have worked in Paris, and because his subsequent influence was particularly noticeable in Europe.

[13] Explicit or implicit signs of Oxford logic in Paris are always historically important, but less so the other way round. This is because one can virtually take for granted that the British logicians were always very much aware of what was going on in Paris, but the converse was not necessarily the case. For example, a manuscript on obligationes partially edited by P. V. Spade and attributed by him to a certain John of Wesel (a set of questions disputed in Paris — cf. [Spade, 1996b]) is significant insofar as it seems to show that as early as 1344 there may have been knowledge in Paris of Swyneshed's obligationes treatise and of his *nova responsio*.

topics of the *Ars* curriculum (logic, natural philosophy (physics), psychology and moral philosophy, metaphysics — he commented on all major Aristotelian texts), but, as often noted, he never moved on to 'higher' levels of intellectual activity (such as law, medicine or theology) (cf. [Zupko, 2002a]). While it was not the most usual path for a master to remain in the Arts faculty throughout his career, Buridan was not the only one to have had such a trajectory (cf. [Courtenay, 2004]). In any case, this meant that Buridan spent his entire career focusing on the subjects of the Arts curriculum, producing a large corpus of extremely sophisticated philosophical texts.

For Buridan, logic was the basic methodology permeating not only all intellectual investigation, but also a key component for the political life of a good citizen (cf. the preface to his *Summulae* [Buridan, 2001, 3]); in other words, the importance of logic for Buridan can hardly be overestimated. Of course, it must be understood that what Buridan conceived logic to be goes beyond the narrower conception that the discipline currently has (this, in fact, holds of the whole medieval tradition): for him, logic encompassed investigations that we would now consider to belong to the fields of semantics, formal epistemology, philosophy of language, metaphysics, among others. Still, Buridan produced a sophisticated and coherent system of doctrines, which has been the object of growing interest over the last couple of decades. Like Ockham, Buridan was a nominalist, that is, a defender of ontological and theoretical parsimony, but while sharing a certain common base, his doctrines differed in content and in general approach from those of Ockham's in many significant aspects. Buridan, not Ockham, is usually thought to be the pioneer of a whole new approach to logic that was to be influential for at least another century, the so-called '*via moderna*' of 'via *Buridanii*'.

For our purposes, his most important texts are his long *Summulae de Dialectica* (a heavily modified commentary of Peter of Spain's *Summulae* — available in English in [Buridan, 2001], and in Latin in several volumes, as part of an ongoing project of critically editing the whole text of the *Summulae*) and his *Treatise on Consequences* [Buridan, 1976]. In fact, since his semantics will be treated elsewhere in this volume, and since he did not write on obligations, he will be discussed in the section dedicated to consequences and, more briefly, in the analysis of the concept of supposition; however, in the section on consequence, he will indeed feature as a most prominent figure, as his treatise on consequences and his remarks on the topic in the *Summulae* are in many respects the most interesting of such medieval texts.

Albert of Saxony was once thought to have been a pupil of Buridan's, but this is now considered as highly unlikely, since they belonged to two different nations (Buridan to the Picardy nation and Albert to the English-German nation); as noted above, the most customary was for a pupil to be trained under a master of his own nation (often of his own home region). However, the influence of Buridan's doctrines over Albert is evident; in fact, Albert's work is often seen as a synthesis of Ockham's and Buridan's ideas (cf. [Biard, 2004, section 1]). This is to some extent true, but the importance of Albert as an original thinker should not be

underestimated. His most important logical work is his *Perutilis Logica* (Very useful logic — [Albert of Saxony, 1988; Kann, 1993] for the second treatise[14]), where he deals extensively with properties of terms, in particular supposition, with consequences, fallacies, *insolubilia* and obligations — in sum, the traditional logical topics in the 14$^{\text{th}}$ century. His treatise on obligations in the *Perutilis Logica* is one of the only three genuinely 'continental' treatises on obligations of this period (the others being overwhelmingly British or written under British influence — cf. [Braakhuis, 1993]). He also wrote a *Sophismata* and several question commentaries in logic (cf. [Biard, 2004, section 1]) (one of such sets of questions has received a modern edition — [Albert of Saxony, 2002]). We shall be interested in particular in his treatment of supposition, and, to some extent, his treatment of consequence.

William Buser may have been a pupil of Albert of Saxony (they were members of the same English-German nation in Paris), and in turn Thomas of Cleves and Marsilius of Inghen (who will be discussed below) were later pupils of William (cf. [Read, 1991, 71]). Besides these interesting relations of 'intellectual hereditariness', the importance of William Buser for the present purposes is mainly that he is the author of one of the only three continental treatises on obligations. Other than his treatise on obligations, no other logical text by him is known (in fact the only other written record by him still extant is his last will — cf. [Kneepkens, 1993, 343]).

Thomas of Cleves is another still obscure Parisian figure of the mid-14$^{\text{th}}$ century (for his biography, see [Bos and Read, 2001, 15–18]). He was a pupil of William Buser, and appears to have become a full master of arts in 1365, in Paris. One interesting aspect of his biography is the fact that he became the schoolmaster of the St. Stephen's Cathedral School in Vienna, which (as already mentioned) was the foundation for the University of Vienna to be re-founded in 1384, exemplifying thus the spread of the 'Buridanian' tradition in logic in Eastern Europe. As far as his writings are concerned, we have now recent editions of his treatise on concepts [Bos and Read, 2001] and a reconstruction of his *Logica* (in [Bos, 2004]). For the present purposes, his position with respect the fourth mode of personal supposition, i.e. collective supposition, will be particularly important in the section on supposition theory below.

Marsilius of Inghen, who was a few years younger than Albert of Saxony (born around 1340) and also a pupil of William Buser, had a decisive role in the establishment of the *via Buridanii* as one of the two main approaches to logic in the late 14$^{\text{th}}$ and 15$^{\text{th}}$ century. In particular, as already mentioned, he was one of the founders and many times the rector of the University of Heidelberg, again exemplifying the spread of Parisian logic in Eastern Europe. At a later stage of his life he eventually obtained his degree in theology, but for most of his career he was writing predominantly on logic, natural philosophy and metaphysics (see [Hoenen, 2001, section 1]). Noteworthy are his treatises on the properties of terms: on supposition, ampliation, appellation, restriction; and his treatises on obligations,

[14] Selections from Albert's masterpiece are available in English translation by T. Parsons *et al.* at `http://www.humnet.ucla.edu/humnet/phil/faculty/tparsons/download/AlbertSL.pdf`

insolubles, and consequences. Unfortunately, so far only his treatises on the properties of terms have received a modern edition [Bos, 1983]. Marsilius will be a central figure in our discussion on supposition below.

John of Holland is another interesting case of an author somehow belonging to both the Continental and the British traditions. While there is no conclusive evidence to the effect that he did study at Oxford, his familiarity with the works of some Oxford logicians such as Heytesbury and Bradwardine is an indication that this might have been the case (cf. [Bos, 1985, *14*]). In fact, little is known about him, but we do know that he studied in Prague[15], and was later to become Dean of the faculty of arts in Prague in 1369. His treatises on supposition, fallacies, obligation and *insolubilia* have received modern editions [John of Holland, 1985].

Peter of Ailly was a Parisian master who wrote his best known work in 1372, his *Concepts and Insolubles* [Ailly, 1980]. This work deals with mental language and in particular the signification of mental and spoken terms, and, as the title says, with *insolubles*. His definition of signification was to be very influential in the 15th century. Although influential for subsequent developments, Ailly will not be treated in any of the analyses to follow, since his main contribution to logic concerns *insolubilia* and semantic notions such as that of signification, which for reasons of space will not be dealt with here.

Besides France, Britain, Central Europe and Italy, there was also vivid intellectual activity in Spain; indeed, a handful of universities were founded in Spain in the 14th century. But contrasting with the 15th and 16th centuries, when Spain was to become one of the main centers for original work within the Scholastic tradition, we know of no Spanish logicians in the 14th century having had the same influence and importance as later authors such as Domingo de Soto. Worth noticing, however, is that the (otherwise) famous St. Vincent Ferrer composed an interesting treatise on supposition around 1372 (edited in [Trentman, 1977]). Particularly significant is the fact that St. Vincent Ferrer went through his whole student career in his native Spain; so while he seemed to be acquainted with most of the important logical texts of the 14th century, he was in practice outside the circle of influence of the main centers, and claimed that his main source of inspiration was St. Thomas Aquinas. St. Vincent will be briefly mentioned in the section on supposition below.

By contrast, an author such as Blaise of Parme indicates that, in Italy, the 'usual' 14th century authors were indeed very influential, such as Ockham and Buridan. The logic taught at Italian universities then was referred to (and dismissed) by humanists such as Petrarca as 'Ockhamist logic', and Blaise is perhaps the most prominent example thereof. His only surviving logical text is a set of questions on Peter of Spain's *Tractatus* (edited recently by J. Biard and G. Federici Vescovini — [Blaise of Parme, 2001]), which is in many ways idiosyncratic for a 14th century logical work in that it does not treat of supposition and other typical 14th century topics. It does have a section on consequence, but it does not

[15]But remember that, unlike other Eastern European universities, Oxford logic was quite influential in Prague, so John's knowledge of British logic may have been acquired in Prague.

consider for example the crucial material vs. formal distinction with respect to consequence. Blaise of Parme is, in any case, an essential author for the understanding of the development of logic in Italy.

John Dorp, working at the very end of the 14$^{\text{th}}$ century, is most famous for his commentary on Buridan's logic (John Dorp 1499), which in 1393 became required reading material for a student to obtain the degree of Bachelor of Arts in Paris (cf. [Lorenz, 1996, 148]). Dorp's commentary consisted of the main text from Buridan's *Summulae* (the parts supposedly taken from Peter of Spain, but with significant modifications), and Buridan's own commentary was replaced by Dorp's more concise commentary (this was indeed the form of the early printed editions of Buridan's *Summulae*, that is, in fact with Dorp's commentary — cf. [Klima, 2001, xxxii]). Dorp deals with and even solves many of the tensions that could be felt in Buridan's logical doctrines, for example with respect to the effect of the negation over the personal supposition of terms (cf. [Karger, 1993]) — his ingenious solution to this problem will be discussed in the section on supposition below.

1.3 Conclusion

From the foregoing considerations, the picture that emerges of the 14$^{\text{th}}$ century is of an extremely active period of intellectual and academic activity, in particular with respect to logic. Many were the authors involved in these activities, and many were their contributions to the field. We will now see that the result of all these activities was exceptionally sophisticated logical analysis.

2 SEMANTICS

2.1 Supposition

While at the beginning of the 14$^{\text{th}}$ century the concept of supposition and the doctrines built upon it were already respectable and mature elements of the terminist logic tradition, one can surely speak of a further development of these doctrines in the 14$^{\text{th}}$ century. The concept of supposition was one of the most important conceptual tools used in fields as wide-ranging as natural philosophy and theology; another typical place to capture the development of the concept of supposition and its uses is in the *sophismata* literature. But here, I will focus on treatises on supposition properly speaking.

Moreover, for reasons of space, the discussion to follow is not intended to be comprehensive in a historical sense: it is impossible to mention the doctrines held by every single significant author, or to discuss all the important aspects concerning the concept of supposition of this period. Rather, the discussion here is thematic in that it focuses on a few interesting conceptual developments concerning the notion of supposition, and particular authors are mentioned only insofar as they are representative of a given position.

In the post-Ockham and post-Buridan period, the development of supposition theories can be summarized as their successors essentially dealing with the 'loose ends' left by these earlier authors in their theories. While it is undeniable that Ockham as well as Buridan had constructed impressive (and mutually different) semantic systems where the notion of supposition was central, a few aspects and conclusions that could be drawn from their systems had not been discussed by them as thoroughly as one might wish. So several of the masters in the generations to follow saw it fit to draw and discuss some of these conclusions and to offer solutions to the difficulties emerging from Ockham's and Buridan's systems, as we shall see. (For a comprehensive and systematic approach to supposition theory, including the definition of its main concepts, I refer to T. Parson's piece in this volume.)

2.1.1 Simple supposition: yes or no?

One of the recurring debates concerning the concept of supposition in the 14th century regarded simple supposition, more explicitly whether it should be included among the main kinds of supposition. Traditionally, there are three main kinds of supposition: personal, simple and material (such as in Peter of Spain and William of Sherwood). Personal supposition occurs when a term in a proposition stands for thing(s) that fall(s) under it. For example, if the term 'man' in a given proposition would stand for actual men, then it would have personal supposition. Material supposition occurs when a term in a proposition stands for a word (in particular for itself). Finally, simple supposition occurs when a term in a proposition stands for the corresponding universal, that is, in the case of 'man', the abstract universal 'manhood'.

Ockham notoriously denied existence to those 'universals' for which terms in simple supposition were said to stand, and contended that there is no such thing as a common nature that Plato and Socrates shared insofar as they are both men. The only thing common to Socrates and Plato in this respect is that the (mental, written or spoken) term 'man' can be correctly predicated of each of them. So, for Ockham, if simple supposition were to be the supposition for universals, it would at best be a completely idle concept, since there are no such things in his ontology. At this point, two options were open to Ockham: (i) either to exclude simple supposition from his theory of supposition altogether, (ii) or else to keep it but with a significant reformulation. Ockham opted for (ii): for him, simple supposition becomes the supposition for a mental term (a concept) (in chapter 68 of Ockham's *Summa Logicae*). One may wonder why he chose to maintain and reformulate simple supposition instead of simply getting rid of it (as Buridan later did), and a good guess would be that he did so for the sake of conservativeness and respect for the tradition.

By contrast, Buridan, who shared Ockham's denial of universals, opted for (i). For Buridan, it makes no sense to distinguish the supposition for mental terms (simple supposition for Ockham) from the supposition for written and/or spoken terms (material supposition for Ockham); as far as Buridan is concerned, they

are both supposition for terms, and thus both material supposition (cf. chapter 4.3.2 of Buridan's *Summulae*). He added that, being the conventionalist about language that he was, he did not really care whether some people prefer to call the supposition for mental terms simple supposition [Buridan, 2001, 253]. But from the point of view of the theoretical simplicity advocated by Ockham himself, it would seem more reasonable not to multiply concepts and terms unnecessarily; if the concept of simple supposition becomes theoretically superfluous, one may as well get rid of it. So it would seem that Buridan's position was overall more coherent.

Another related point is that, while Ockham ascribed different kinds of supposition to terms in mental language as well (yielding a few counterintuitive results related to equivocation in mental language[16]), Buridan only allowed for personal supposition in mental language [Buridan, 2001, 522], whereas in written and spoken language both personal and material supposition could occur. In this respect too Buridan's position was more coherent, as argued in the literature (cf. [Spade, 1980b]).

However, this was not the end of the story. Authors of the following generations kept on debating whether simple supposition was indeed required or in fact superfluous for a theory of supposition. Most nominalists, i.e. those who followed Ockham and Buridan in their denial of the existence of universals, ultimately opted for simplicity and followed Buridan in his exclusion of simple supposition. This was in particular the case of Marsilius of Inghen. Moreover, in the second half of the 14$^{\text{th}}$ century, with the revival of realism about universals with Wyclif and his followers, simple supposition became again an important theoretical tool within this trend.[17] But interestingly, there was an author who followed Ockham both in his ontology (denial of universals) and in his inclusion of simple supposition in the theory of supposition — a position that, as already argued, is not entirely straightforward. This author boldly holding Ockham's banner was Albert of Saxony (cf. [Berger, 1991]).

Moreover, Albert's position with respect to simple supposition and to the supposition of mental terms is also rather idiosyncratic: he accepted personal, simple and material supposition in spoken and written language, but only personal and material supposition in mental language.[18] This is rather surprising, since one

[16] Cf. [Spade, 1980b; Normore, 1997]. The problem is essentially the following: ambiguity in spoken/written language (equivocation/amphiboly) is accounted for by Ockham in terms of a one-many mapping between spoken/written expressions and mental expressions (an ambiguous expression is one corresponding to more than one mental expression). However, how can he account for ambiguity in mental language if there is no super-mental level to play the role that the mental level plays for the spoken/written level?

[17] The realist tradition in 14$^{\text{th}}$ century logic is still largely understudied. Its first exponent was, as already mentioned, Burley, who for example maintained the traditional definition of simple supposition as the supposition for a universal. In the second half of the century, realism underwent a revival, mainly with Wyclif and his followers (Robert Alyington, Johannes Sharpe — see the entries on these authors in the *Stanford Encyclopedia of Philosophy*, and a whole issue of *Vivarium* 43(1) (2005) dedicated to the realists).

[18] "[...] a mental term cannot have simple supposition, but only material or personal supposi-

could expect him either to side with Ockham in maintaining that all the different kinds of supposition that occur in spoken and written language also occur in mental language, or to side with Buridan in maintaining that there is only personal supposition in mental language. However, there is a striking coherence in his position, as I shall argue.

The first element to be taken into account is that his formulations of these three modes of supposition differ slightly but significantly from those of Ockham. Personal supposition is defined roughly in the same way as by Ockham, as the supposition for the things that the term signifies; material supposition, by contrast, is defined as the supposition for **terms** but not necessarily spoken or written terms, as it is by Ockham. According to Albert's definition, a term having material supposition can supposit for itself or for a similar term (be they written, spoken or mental). It is clear thus that, in mental language, if a given mental term supposits for itself or for another term (mental or otherwise), Albert's definition allows for it to be a case of material supposition, whereas for Ockham it had to be a case of simple supposition, since his definition of material supposition necessitated that the supposition be for a spoken or written term. Albert, however, with the definitions of personal and material suppositions thus stated, could already account for all the different kinds of supposition in mental language, making thus simple supposition in the mental realm superfluous.

He was still left with the cases of terms in spoken or written language suppositing for mental terms, which were presumably excluded from his definition of material supposition. Following Ockham and against Buridan, he prefers to keep such cases in a category of their own — perhaps in order to maintain a more fine-grained and discriminating taxonomy — a category in which the supposition of a term for itself could not occur. From this point of view, what characterizes and gives unity to the concept of material supposition is above all the possibility of supposition of a term **for itself** (a spoken term for a spoken term, a written term for a written term, and a mental term for a mental term), whereas simple supposition would deal with the 'left-over' cases where this could not occur, namely the cases where a spoken or written term was explicitly meant to supposit for a concept.

As for the risk of provoking equivocation in mental language by the ascription of different kinds of supposition to mental terms, such as in Ockham, Albert avoids this problem with another feature of his logical system: unlike most of his immediate predecessors such as Ockham and Buridan and his own contemporaries, Albert denies that some propositions must be distinguished, that is, that they are ambiguous.[19] Now, if there is no equivocation with respect to propositions in written and spoken language, where there are three kinds of supposition, then a fortiori there will not be this kind of equivocation in mental language either.

In sum, Albert's preservation of simple supposition seems to be better motivated

tion." (Translation by Parsons *et al*, p. 12, on the website mentioned above).

[19]See [Ashworth, 1991, 156]. Albert contends that the propositions that must be distinguished according to others in fact correspond to the conjunction or disjunction (depending on the case) of each of their possible readings.

than Ockham's, and that on purely logical/semantic grounds — that is, it is not motivated by ontological considerations (i.e. realism about universals) as it is in the case of Wyclif and Paul of Venice (see chap. 2 of Paul's treatise on supposition in his *Logica Magna* — [Paul of Venice, 1971]).[20] Moreover, thanks to the position he holds with respect to propositions in general, i.e. that they should not be distinguished, he escapes the risk of introducing equivocation in mental language even though he accepts different kinds of supposition in mental language.

However, the simplicity of Buridan's doctrines, namely the exclusion of simple supposition from spoken and written language and of all kinds of supposition except for personal supposition from mental language, remained very appealing; it is not by chance that the vast majority of nominalists sided with Buridan and not with Albert, and that virtually all other upholders of simple supposition were essentially motivated by ontological considerations.

2.1.2 A fourth mode of personal supposition?

As shown in T. Parson's contribution to this volume, in the 14$^{\text{th}}$ century the modes of personal supposition were virtually always associated to relations of ascent and descent between propositions and the corresponding singular propositions. The *descensii ad inferiora* are 'certain types of inferences in which the common terms of which the mode of supposition is being characterized is replaced by singular terms falling under it, appearing in either nominal or propositional conjunctions or disjunctions.' [Klima and Sandu, 1990, 177]. Singular terms are proper names or, as most frequent in the case of descents, demonstrative pronouns (usually accompanied by the appropriate common term). For example, in the case of 'Every man is an animal', if it is the supposition of the term 'man' that is at stake, then the singular propositions in question would be of the form 'This man is an animal', 'That man is an animal' etc. (pointing at each individual falling under the term 'man', i.e. each man), and the question is then how the descent to these singular propositions can be made, i.e. either nominally or propositionally, and either conjunctively or disjunctively.

Let the basic form of categorical propositions be represented as "ΦA is ΦB",[21] where A and B are terms and Φ stands for any syncategorematic expression (such as 'Every', 'Some', 'No' etc.) or the absence thereof. The different kinds of propositional descent can then be characterized as:

> Propositional conjunctive descent for $A \Leftrightarrow$ from 'ΦA is ΦB' one can descent to 'This A is ΦB and that A is ΦB and ...'

[20] Notice though that in his *Logica Parva* Paul of Venice only recognizes personal and material supposition.

[21] In fact, most often than not there was no quantifying expression preceding the predicate. However, in a few cases there was such an expression (such as in an example to be discussed below, 'Socrates differs from every man'), and therefore for the sake of generality I introduce a place-holder for a quantifying expression also in front of the predicate. See also [Karger, 1993] for more of such examples.

> Propositional disjunctive descent for $A \Leftrightarrow$ from 'ΦA is ΦB' one can descent to 'This A is ΦB or that A is ΦB or ...'
>
> Nominal disjunctive descent for $A \Leftrightarrow$ from 'ΦA is ΦB' one can descent to 'This A or that A or ... is ΦB.'
>
> Nominal conjunctive descent for $A \Leftrightarrow$ from 'ΦA is ΦB' one can descent to 'This A and that A and ... is ΦB.'

These definitions apply, *mutatis mutandi*, to the predicates of a proposition as well.

With respect to the main propositional forms (A, E, I, O), it is evident that at least three types of descent are required to account for the supposition of their terms:

(A) Every S is $P \Leftrightarrow$ Propositional conjunctive descent is possible for the subject and nominal disjunctive descent is possible for the predicate: 'This S is P and that S is P and ...' and 'Every S is this P or that P or etc.'

(E) No S is $P \Leftrightarrow$ Propositional conjunctive descent is possible for both subject and predicate: 'This S is not P and that S is not P and etc.' and 'No S is this P and no S is that P and ...'

(i) Some S is $P \Leftrightarrow$ Propositional disjunctive descent is possible for both subject and predicate: 'This S is P or that S is P or etc.' and 'Some S is this P or some S is that P or ...'

(O) Some S is not $P \Leftrightarrow$ Propositional disjunctive descent is possible for the subject and propositional conjunctive descent is possible for the predicate: 'This S is not P or that S is not P or etc.' and 'Some S is not this P and some S is not that P and ...'

Notice that, whenever (conjunctive or disjunctive) propositional descent is possible, so is the corresponding nominal descent (since propositional descent corresponds to wider scope and thus to a stronger reading of the proposition), and whenever conjunctive descent is possible, so is disjunctive descent (due to the logical properties of conjunctions and disjunctions).[22] We thus have:

Propositional conjunctive descent is possible	$\Rightarrow$	Propositional disjunctive descent is possible
		Nominal conjunctive descent is possible
		Nominal disjunctive descent is possible
Propositional disjunctive descent is possible	$\Rightarrow$	Nominal disjunctive descent is possible
Nominal conjunctive descent is possible	$\Rightarrow$	Nominal disjunctive descent is possible

In the early 14th century, three of these patterns of descent were associated to modes of personal supposition:

[22] Naturally, these two claims ought to receive a formal proof, but I take them to be sufficiently intuitive so that these proofs are not necessary in the present context.

	Conjunctive	Disjunctive
Nominal	XXXXXXX	Merely confused
Propositional	Confused and distributive	Determinate

Naturally, since nominal disjunctive descent is always possible, merely confused supposition is not defined as the supposition of a term whenever nominal disjunctive descent is possible (otherwise all terms would have merely confused supposition); rather, it is (usually) defined as the supposition of a term when **only** nominal disjunctive supposition is possible. Similarly, determinate supposition is not defined as the supposition of a term whenever propositional disjunctive descent is possible; rather, it is defined as the supposition of a term when propositional disjunctive descent is possible but not propositional conjunctive descent (otherwise the categories of confused and distributive, and determinate supposition would overlap — cf. [Read, 1991b, 74]).

It is easy to see why at first no interest was paid to nominal conjunctive descent: it does not correspond to any of the terms in the four traditional categorical propositional forms. Accordingly, since it was tacitly assumed that all other propositions could be in some way or another reduced to one of these four forms, it was thought that only those three kinds of personal supposition were required to account for the supposition of terms in propositions.

But even at its early stages, the suppositional framework was also applied to cases other that these four traditional propositional forms. For example, in the case of exceptive propositions of the form 'Only S is P', it was recognized that S has merely confused supposition (cf. [Marsilius of Inghen, 1983, 59] — see below). In other cases, it was necessary to bend the concepts of the modes of personal to such an extent (for instance, by rephrasing the original sentence so that it would fit into one of the recognized propositional forms, but with rather implausible results) that one cannot help but wonder whether supposition theory with only these three modes of personal supposition was complete in the sense of being able to account for all, or at least most, propositions. But, since in most formulations of the modes of personal supposition merely confused supposition was in practice a 'catch-all' category (cf. [Read, 1991b, 75]) (since nominal disjunctive descent is always possible, as noted above), in the end no semantic phenomenon was excluded from the taxonomy with these three modes of personal supposition.

But at some point in the first half of the 14^{th} century, some authors were led to acknowledge at least the logical possibility of a fourth mode of descent, namely nominal conjunctive descent. According to [Read, 1991b, 74], the first mention to nominal conjunctive descent that we know of is to be found in Thomas Maulevelt's *De Suppositionibus*: there, Maulevelt says that when merely confused supposition occurs, nominal disjunctive **as well** as nominal conjunctive descent are possible. Maulevelt's example of the supposition of a term which is best accounted for by nominal conjunctive descent instead of nominal disjunctive descent is 'Socrates

differs from every man'. According to Maulevelt, the descent allowed for the term 'man' giving the intended meaning of the proposition is 'Socrates differs from this man and that man and...', and not 'Socrates differs from this man or that man or...' (although in principle, the second descent should also be allowed, since nominal conjunction should imply the corresponding nominal disjunction). In other words, according to Maulevelt, nominal conjunctive descent is not only a logical possibility; it is also the actual descent required by some real cases.

Albert of Saxony, writing after Maulevelt, explicitly rejects his analysis of 'Socrates differs from every man' as requiring nominal conjunctive descent; according to him, 'man' in this case should have determinate and not merely confused supposition (cf. [Read, 1991b, 80]). Moreover, Albert criticizes the inclusion of the clause for nominal conjunctive descent in the definition of merely confused supposition, thus implicitly defending the idea that nominal conjunctive descent is not a phenomenon that needs to be taken into account in the definition of modes of personal supposition (being at best only a logical possibility). Obviously, for those who reject nominal conjunctive descent as a relevant phenomenon, such as Albert of Saxony, three modes of personal supposition provide a complete picture of the (personal) supposition of all terms not only in the sense that all cases were taken into account (which happens anyway if merely confused supposition is defined as a catch-all clause), but also in the sense that these three categories are sufficiently fine-grained and discriminating so as to give a coherent grouping of these semantic phenomena, since they correspond to the three relevant kinds of descent.

Among those who recognize nominal conjunctive descent as an important phenomenon, two positions are possible; either to associate nominal conjunctive descent to merely confused supposition, together with nominal disjunctive descent (as did Maulevelt and later Paul of Venice — cf. [Read, 1991a, 53]), yielding thus a rather heterogeneous notion of merely confused supposition as a 'miscellaneous' category; or to associate nominal conjunctive descent to a fourth mode of personal supposition altogether. According to our current state of knowledge, while the notion of a *descensus copulatim* (nominal conjunctive supposition) seems to have been familiar in Paris and Oxford in the 1350s and 1360s, the first to associate a fourth mode of supposition to nominal conjunctive descent seems to have been Thomas of Cleves (cf. [Read, 1991a; 1991b]) in his *Suppositiones* written in the first half of the 1370s in Paris[23], before he moved on to be the rector of St. Stephen's cathedral school in Vienna This fourth mode of personal supposition became known as *collective supposition*.

Some of the examples usually associated with nominal conjunctive descent (from [Read, 1991a]) were: 'You are not every man', 'No animal is every man', and 'Some penny will be seen by every man' (in all three cases with respect to 'man'). But in such cases, the opponents of *descensus copulatim* usually proceeded by showing that, if this kind of descent was possible at all, so were other kinds of descent,

[23] At this point, no surviving manuscript of Thomas of Cleves' *Suppositiones* has been located; but a reconstruction of it has been made recently (in [Bos, 2004]) on the basis of references made to this work in other texts.

and therefore the supposition of such terms could be classified among the three usual modes of personal supposition (since, presumably, the category of collective supposition would be defined as the cases where **only** nominal conjunctive descent would be possible).[24] Their usual strategy consisted of an appeal to Ockham's Razor, to the effect that if the job of accounting for the different modes of personal supposition could be done with only three categories, then there was no need to posit a fourth one.[25]

But a fourth type of examples poses more serious difficulties. It is epitomized by the proposition 'All the apostles of God are twelve'. The nominal conjunctive descent under 'the apostles of God' is a very natural one indeed, either with demonstrative pronouns or even with proper names: 'Peter and James and John and Judas etc. are twelve'. But nominal disjunctive descent seems not to be allowed, since it is not of each of them that the predicate 'twelve' can be predicated, but rather of all of them taken collectively.[26] In other words, only nominal conjunctive descent seems to be allowed, and if this kind of descent is not accounted for in the definitions of the (three) modes of personal supposition, then no mode of personal supposition can be assigned to 'the apostles of God'.

There were different replies from those who rejected the notion of a fourth mode of personal supposition and nominal conjunctive descent, with various degrees of plausibility. Some proposed to treat 'all the apostles of God' as a singular term used to refer to the collection of apostles of God, having thus discrete supposition; others implausibly rephrased the proposition as 'All all of the apostles of God are twelve' and attributed confused and distributive supposition to the subject (cf. Read 1991a, 80).

It is worth noticing that, in his 1372 treatise on supposition (thus roughly at the same time as Thomas of Cleves was presumably writing his treatise on supposition), Vincent Ferrer advocates a rather idiosyncratic position with respect to the modes of personal supposition: he also recognizes three modes of common personal supposition, but not the three traditional ones. According to him, common personal supposition is subdivided into determinate, distributive confused, and collective confused (Trentman 1977, 134); in other words, he does recognize

[24] That poses a logical problem since, according to the usual laws for conjunction and disjunction, whenever nominal conjunctive descent is possible, so is nominal disjunctive descent, as indeed nominal disjunctive descent is always possible. So arguing that, in such cases what we have are cases of merely confused supposition because nominal disjunctive descent is possible is in some sense fallacious, since this holds of the other modes of personal supposition as well. In this sense, collective supposition should be defined as the cases where nominal conjunctive and nominal disjunctive descents are possible, but no propositional descent is possible. However, as we shall see shortly, there are cases where nominal conjunctive descent seems to be possible but **not** nominal disjunctive descent, which would violate the usual rules for conjunction and disjunction.

[25] An anonymous author of a commentary on Marsilius' Parva Logicalia says: "everything can be explained without positing collective supposition", and "one should not multiply entities without necessity" (cf. [Read, 1991a, 79]).

[26] Indeed, this seems to indicate that the logical behavior of nominal conjunctive and disjunctive descents, or in any case the semantics of collective nouns, is more complicated than what the mere truth-functional properties of conjunction and disjunction can account for.

Thomas of Cleves's 'fourth' mode of personal supposition, but to him this is not a fourth but rather a third mode, because he does not recognize merely confused personal supposition. While this may seem an awkward position at first sight, it is not altogether implausible considering that, for Vincent Ferrer, and following very early (12th and 13th century) notions of supposition, it is only the **subject** of a categorical proposition that has supposition, not the predicate (cf. [Trentman, 1977, 89-92]). Given that, usually, merely confused personal supposition concerns the supposition of the predicate (in particular in universal affirmative propositions), if one does not attribute supposition to the predicate, then there may seem to be no need to recognize merely confused personal supposition.

But the fact that Ferrer recognized collective supposition poses a historiographic problem. While there is a clear line of continuity between the other authors mentioned so far (Maulfelt, Albert of Saxony, Thomas of Cleves, all roughly belonging to the same Parisian nominalist tradition), Vincent Ferrer was, as already mentioned, educated in Spain, and saw himself as belonging to a Thomist tradition, thus completely out of the circle of influence of the nominalist tradition. Read [1991a, 74] argues that Ferrer's treatise 'does not read like that of an author inventing an original theory'; that is, presumably collective supposition was already a recognized mode of supposition in the tradition within which he was schooled. We could thus have two independent 'inventions' of the concept of collective supposition. But our knowledge of the 14th century Spanish logical tradition is as of yet still insufficient in order to tell us whether the concept of collective supposition was indeed independently developed within this tradition (which seems unlikely, given the coincidence in terminology), or whether there were earlier points of contact between the two traditions in such a way that there might have been influence in one direction or in the other (or both).

In sum, the debate concerning nominal conjunctive descent and a fourth mode of supposition was a heated one in the second half of the 14th century. While it is fair to say that the majority of the authors preferred to maintain the traditional scheme with three modes of common personal supposition (see [Ashworth, 1978]), for the sake of theoretical parsimony and probably also out of respect for the tradition, many authors nevertheless recognized the fourth kind of descent. But all in all, considered from a systematic point of view, it seems that supposition theory containing four modes of personal supposition is indeed a more sophisticated version of the traditional theory: it respects the logical symmetry of nominal and propositional descent, and it allows for a very intuitive account of the semantics of some terms (even if some of them can also be accounted for with only the three traditional kinds of descent).

2.1.3 Different modes of supposition also for material supposition

In the traditional formulations of theories of supposition, in particular with the 13th century summulists (William of Sherwood, Peter of Spain), but also with authors of the first half of the 14th century such as Ockham and Buridan, there

is a remarkable asymmetry between personal supposition on the one hand and simple/material supposition on the other hand. While personal supposition is further divided into the different modes of personal supposition, as just discussed, the same does not occur with simple and material supposition.

The different modes of personal supposition concern the fact that, in a given proposition, we do not always wish to talk about every single object that falls under a given term; for example, when using the term 'man', we sometimes wish to talk about all men, but sometimes only about some of them. In anachronistic terms, the modes of personal supposition are the medieval counterpart of theories of quantification, and, roughly speaking, allow us to determine how many individuals are being 'talked about' with a given proposition.

At first sight, it may seem natural that personal supposition should have the privilege of being further subdivided, while simple and material supposition should not: given that simple and material supposition are not the significative kinds of supposition, why should we quantify over things that do not fall under a given term? What is the need to quantify over the word that a term stands for if it has material supposition, or, even more awkwardly, over the universal that it stands for if it has simple supposition? For example, if the term 'man' has simple supposition, then there is only one thing it can stand for, namely the unique universal 'manhood', so there is no point in discussing how many universals the term 'man' stands for if it has simple supposition.

But further reflection quickly shows that there may be indeed a point in further distinguishing the kinds of supposition other than personal supposition. In the 14^{th} century, material supposition is usually not defined exclusively by the fact that the term in question supposits for itself; it may also supposit for other terms, either its equiform occurrences or even non-equiform terms (for example, a term in a given case — nominative, accusative etc. — may supposit for the same term but in a different case, thus generally not being equiform). So it is clear that, just as much as with personal supposition, there is a rather wide range of objects (different occurrences of different words) that a given term can stand for if it has material supposition (and that has led some to conclude that material supposition is such a wide-ranging concept that it winds up becoming ineffective — cf. [Read, 1999]).

As for simple supposition, for as long as it was exclusively the supposition for the unique universal corresponding to a given term, there was indeed not much point in further distinguishing different kinds of simple supposition with respect to quantification — notice that Burley, for example, distinguished simple supposition into absolute and compared simple supposition [Burley, 2000, 92], but this subdivision concerned different ways in which a term had simple supposition, and not the number of things being talked about. However, with Ockham's reformulation of the notion of simple supposition as the (non-significative) supposition for concepts, this issue acquired an entirely different shape. For Ockham (in his mature theory) concepts are simply the very acts of conceiving (things) by a given intellect; that is, they are (temporary) attributes of the intellect. In other words,

concepts, just as much as spoken words, have different temporal occurrences: the concept *man* does not correspond to one single concept that perdures in the intellect, but rather to each and every occurrence of this concept in the intellect every time it conceives of men, just as much as the spoken word 'man' corresponds to each of its temporal occurrences.

Hence, if both simple supposition and material supposition concern a wide range of objects for which a term can supposit (different occurrences of a given concept or of a given word), just as much as personal supposition, then it seems counterintuitive that only personal supposition should receive further distinctions. In fact, Ockham's and Buridan's nominalism seems almost to require these distinctions; for both of them, all that exists are the different actual occurrences of written, spoken and mental terms – that is, what we now call 'tokens', but not the corresponding types. For this reason, also in simple and material supposition it seems necessary to consider the number of entities (concepts, words) being talked about with a given proposition, just as much as with personal supposition.

Buridan seemed to be well aware of the fact that one can quantify over the different occurrences of a term. Take this passage, from the first treatise of the *Summulae*:

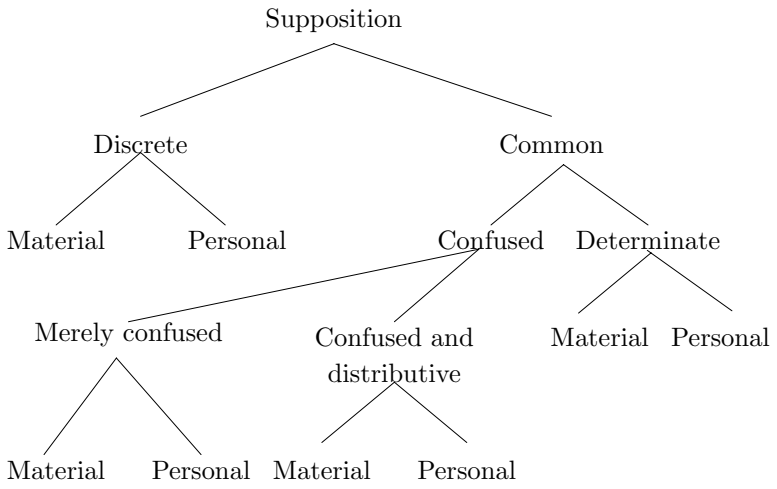
> Next, we should say that if the subjects in the aforementioned propositions supposit materially, then the proposition "[A] man is a species" is [...] indefinite, for the term 'man' is not to be understood as supposing only for itself, but indifferently for other similar terms as well [...]. And in this way "Every man is a species" would be universal, and "Some man is a species", i.e. "Some term man is a species" would be particular, and "This man is a species", i.e. "This term man is a species", would be singular. [Buridan, 2001, 92][27]

But Buridan does not go as far as actually applying the different modes of personal supposition to material supposition, even though from saying that a proposition is universal to acknowledging that its subject has confused and distributive supposition is but a small step (as it was widely recognized that the subject of an affirmative universal proposition has confused and distributive supposition, and similarly that the subject of a particular or indefinite supposition has determinate supposition and the subject of a singular proposition has discrete supposition).

To my knowledge, the first author to have made the small but significant step of introducing the distinctions originally pertaining only to personal supposition also to other kinds of supposition is Buridan's follower, Marsilius of Inghen. In his treatise on supposition [Marsilius of Inghen, 1983, pp. 52-97], Marsilius presents a compelling and elegant way of structuring the different kinds of supposition and their subdivisions. What is most remarkable about his doctrine is that, unlike his predecessors, who began by dividing proper supposition into personal, simple and material, Marsilius actually begins by the division of what were traditionally the different modes of personal supposition. This is a sensible move, considering that

[27] I am indebted to Gyula Klima for having drawn my attention to this passage.

the subdivisions of the different modes of supposition are considerably more complicated than the division between personal and material supposition (recall that Marsilius, following Buridan, does not recognize simple supposition as a class of its own and views the supposition for mental terms as a kind of material supposition). Here is a tree representing his divisions:



We thus have four kinds of material supposition, just as much as four kinds of personal supposition. Thereby, Marsilius is able to present a more fine-grained account of the phenomenon of terms standing for other terms. Indeed, a term may stand for:

- one specific non-ultimate significate[28] only – discrete material supposition, for example if I say 'This 'man' is written in red' pointing at a specific occurrence of the word 'man' (cf. [Marsilius of Inghen, 1983, 55]);

- any non-ultimate sigificate in a disjunctive way — determinate material supposition, for example if I say 'Man is written on this page' meaning that there is at least one occurrence of the word 'man' in a given page (cf. [Marsilius of Inghen, 1983, 57]);

- any non-ultimate significate with disjunction of the term — merely confused material supposition, for example with 'Only man is a monosyllabic word', from which follows 'Only this [occurrence of] man or that [occurrence of] man etc. is a monosyllabic word' (cf. [Marsilius of Inghen, 1983, 59]);

[28]For Marsilius, following Buridan, the ultimate significates of terms are the things that fall under them, such as men for 'man', concepts for 'concept' etc., and the non-ultimate sigificates of terms are the things that they also signify — the corresponding mental, spoken and written terms — but not ultimately.

- every non-ultimate significate in a conjunctive way — confused distributive material supposition, for example with 'Every man is a monosyllabic word', meaning that every single occurrence of 'man' is monosyllabic (cf. [Marsilius of Inghen, 1983, 59]).

Indeed, in these divisions we see the recognition of individual concepts, inscriptions and utterances as legitimate members of the ontology (something that was already crucial for Buridan), and the possibility of talking about them in a much more refined way. With these divisions, we can attribute a certain property to one specific occurrence of a term, to some or even to all its occurrences; in other words, here we certainly have a conceptual predecessor of the important token-type distinction, which was to be fully developed only in the 20th century.

Marsilius's reformulation (or, perhaps better put, improvement) of supposition theory so as to attribute the so-called modes of personal supposition to material supposition as well seems to have become the standard practice in the 15th century. It is symptomatic that Paul of Venice, in his very influential *Logica Parva* [Paul of Venice, 1984, 147], also adopted these distinctions of modes of supposition for material supposition.

2.1.4 Problems and solutions for the negation

Both in Buridan's treatise on supposition and in the part of Ockham's *Summa Logicae* dedicated to supposition (the last chapters of Part I), one of the main topics are the syntactic rules determining which mode of (personal) supposition a term has on the basis of the syncategorematic terms present in a proposition (quantifying terms such as 'some', 'every', negating terms etc.) and word order. We learn for example that an affirmative sign of universality ('every') causes the term immediately following it to have confused and distributive supposition, and that it causes a term mediately (i.e. not immediately) following it to have merely confused supposition (so in 'Every man is an animal', the subject, which follows 'every' immediately, has confused and distributive supposition, while the predicate, which follows 'every' mediately, has merely confused supposition). We also learn that a negative sign of universality ('no') causes all terms following it (immediately and mediately) to have confused and distributive supposition, and that when the negation is placed relative to the copula (as in 'A man is not a stone'), it causes the predicate to have confused and distributive supposition. As for determinate supposition, it occurs when a term immediately follows a sign of particularity ('some') or when it does not follow any syncategorematic term (as in 'a man is an animal').

But matters become significantly more complicated once one departs from the four traditional propositional forms ('Every A is B', 'Some A is B', 'No A is B' and 'Some A is not B'), especially when iteration of syncategorematic terms occurs. As argued by Parsons in [Parsons, 1997] and in his piece in this volume, in the 14th century it became current to classify the modes of personal supposition globally rather than locally, as had been done in the 13th century — that is, taking into

account the whole propositional context, and not only the syncategorematic term immediately preceding a given term. For this reason, the occurrence of several syncategorematic terms all having the same categorematic terms under their scope posed the problem of the effect of those embedded syncategorematic terms over each other with respect to the (personal[29]) supposition of the categorematic terms in question.

In particular, the treatment of the negation requires a great deal of ingenuity. The basic problem is: what is the effect of a negating term over the supposition of a term which, if the negating term was removed, would have such-and-such personal supposition in a given proposition? In other words, rules determining the kind of supposition that a term would have if a negation is added to the proposition where it stands are required for all three cases, namely if the term in the original proposition had determinate, confused and distributive or merely confused supposition. And this is where the issue arises.

Buridan, for example, offers an explicit rule concerning the effect of the negation over a term that, without the negation, would have determinate or merely confused supposition: *A negating negation distributes every common term following it that without it would not be distributive and does not distribute anything that precedes it.* [Buridan, 2001, 269]

That means that, if in a proposition $\mathbf{P}$, a term A has determinate supposition, and if a negation is added to $\mathbf{P}$ (yielding $\mathbf{P}^*$) in such a way that A follows the negation (immediate or mediately), then A will have confused and distributive supposition. For the purposes of clarity in the exposition, let me introduce a few notations in order to express this rule more precisely:

$\mathrm{Det}(A)_P \Leftrightarrow$ The term A has determinate supposition in proposition P.
$\mathrm{Dist}(A)_P \Leftrightarrow$ The term A has confused and distributive supposition in proposition P.
$\mathrm{Conf}(A)_P \Leftrightarrow$ The term A has merely confused supposition in proposition P.
$\langle\sim, A\rangle_P \Leftrightarrow$ The negation is followed by term A in proposition P.

Buridan's rule can then be formulated as follows:

Rule 1 $\quad \mathrm{Det}(A)_P \& \langle\sim, A\rangle_{P^*} \rightarrow \mathrm{Dist}(A)_{P^*}$
Rule 2 $\quad \mathrm{Conf}(A)_P \& \langle\sim, A\rangle_{P^*} \rightarrow \mathrm{Dist}(A)_{P^*}$

There is, however, a serious problem concerning the effect of a negating sign upon a term that, without the negation, would have confused and distributive supposition. Given the structure of the theory, it seems at first sight impossible to provide a general rule for the negation and for confused and distributive supposition, for the following reason. Consider the four traditional kinds of categorical propositions:

[29] Or material supposition, if one follows Marsilius of Inghen in applying the traditional modes of personal supposition also to material supposition.

Some A is B (1) —is the contradictory of→ No A is B (2) (contradiction 1)

Every A is B (3) —is the contradictory of→ Some A is not B (4) (contradiction 2)

'Some A is B' (1) should be equivalent to 'Not: No A is B' ($2'$) (the contradictory of (2)) and 'Every A is B' (3) should be equivalent to 'Not: Some A is not B' ($4'$) (the contradictory of (4)). If these equivalences hold, then the supposition of the terms in (1) and ($2'$) should be the same: A and B have determinate supposition in (1), so they should have the same kind of supposition in ($2'$).

For this to happen, the effect of the negation in ($2'$) should be to turn the confused and distributive supposition of A and B in 'No A is B' into determinate supposition. This is indeed the rule proposed by Ockham: *Nevertheless, it should be noted that the aforementioned rules hold only in the case where the term in question would not stand confusedly and distributively if the negation sign or the relevant verb or name were taken away. For if the term were to stand confusedly and distributively when one of these expressions [negation sign] were taken away, then with the addition of such an expression it would stand determinately.* [Ockham, 1998, 214]

Ockham's rule can be formulated as follows:

Rule 3o $\quad \mathrm{Dist}(A)_P \& \langle \sim, A \rangle_{P^*} \rightarrow \mathrm{Det}(A)_{P^*}$

But what about the equivalence between (3) and ($4'$)? In (3) A has confused and distributive supposition and B has merely confused supposition. So the same should occur in ($4'$). However, in 'Some A is not B', A has determinate supposition and B has confused and distributive supposition. According to rule 1, the negation would make A have confused and distributive supposition in ($4'$), that is, the same supposition of A in (3) (so far, so good). But what about B? According to the rule proposed by Ockham, since it has confused and distributive supposition in 'Some A is not B', it would have **determinate** supposition in ($4'$), under the effect of the negation. But in fact it ought to have **merely confused** supposition, because of the equivalence between (3) and ($4'$). So the rule stated by Ockham does not safeguard this equivalence.

Buridan, on the other hand, presents a rule that does safeguard the equivalence between (3) and ($4'$): *A common term is confused nondistributively by two distributive [parts of speech] preceding it, either of which would distribute it without the other.* [Buridan, 2001, 275]

Buridan's rule can be formulated as follows:

Rule 3b $\quad \mathrm{Dist}(A)_P \& \langle \sim, A \rangle_{P^*} \rightarrow \mathrm{Conf}(A)_{P^*}$

That is, under the effect of two negations, B in ($4'$) would have merely confused supposition, which is the desired result. But then the equivalence between (1) and

(2′) would no longer be preserved: B would have merely confused supposition in (2′), whereas it ought to have determinate supposition, as in (1).

Thus, neither of these rules is able to preserve both equivalences: thus formulated, a rule concerning the effect of the negation upon a term originally with confused and distributive supposition either preserves the equivalence between (1) and (2′) (Ockham's rule) or it preserves the equivalence between (3) and (4′) (Buridan's rule). This is due to the following asymmetry: in the case of contradiction 1, the opposition in the supposition of B in each proposition is between determinate supposition and confused and distributive supposition, whereas in contradiction 2 the same opposition is between merely confused supposition and confused and distributive supposition. Therefore, it would seem impossible to provide a homogeneous account of the effect of the negation (or other distributive term) upon terms with confused and distributive supposition (for a systematic approach to this problem, see part 8 of Parson's contribution to this volume).

However (and fortunately for the general robustness of supposition theory as a semantic framework), later masters were well aware of this difficulty and proposed ways to deal with it. Already at the end of the 14$^{\text{th}}$ century, in his popular commentary to Buridan's *Summulae*, John Dorp [1499] proposed a method to determine the supposition of terms following a negation based on the idea that the proposition should be rephrased in such a way that the negation would be all the way at the end of the proposition, in which case the usual (i.e. positive) rules for the determination of the personal supposition of a term could be applied. On this approach, '[t]he problem of assigning the mode of supposition to a term following a negation becomes that of determining a procedure for bringing negative sentences into a non-ordinary form [one where the negation only precedes the verb] such that the mode of supposition of each term remains unchanged.' [Karger, 1993, 419]

On the basis of Dorp's examples, Karger proposes a reconstruction of what this procedure would be like, essentially based in the idea of bringing the negation towards the end of the proposition, immediately before the verb, and introducing universally quantifying signs (to recover the distributive effect of the negation) where there was none, and deleting such universally quantifying terms previously present. Here is an example (cf. [Karger, 1993, 419], and [Read, 1991a, fn. 8] for Dorp's text):

> *Nullum animal omnis homo est* (No animal is every man) is rephrased as
> *Omne animal homo non est* (Every animal a man is not).

Nullum is replaced by *Omne*, *omnis* preceding *homo* in the original proposition is deleted, and the negation comes to be followed only by the verb *est*. Now, it is clear that *animal* has confused and distributive supposition (which was clear already in the original proposition), but moreover it becomes apparent that *homo* has merely confused supposition (as it follows mediately the universal sign), whereas in the original proposition it would have been unclear (from 3o and 3b alone) whether it

had determinate or merely confused supposition.

A different approach to the same issue can be found for example in the writings of the early 17th century philosopher and theologian John of St. Thomas (admittedly a few centuries off our period);[30] there, one finds a precise account of the effect of distributive terms (the negation in particular) upon terms already having confused and distributive supposition. For this purpose, one has to consider the whole propositional context, i.e. the supposition of the other term in the proposition. Here is how John formulates it: *If two universal signs simultaneously affect the same term, then you must see how it remains after the first negation or universal sign is removed; and if it remains distributive with reference to a term having determinate supposition, then it originally had confused supposition; if however the term remains distributive with reference to a term having confused supposition, it originally was determinate.* [John of St. Thomas, 1955, 69]

Here are his own examples: 'For example, if I said, *No man is not an animal*, then when the first negative, i.e. the *no*, is taken away, *animal* becomes distributive with reference to *man*, which is determinate. Thus originally *animal* had confused supposition. However, if I said, *Not every man is an animal*, then when I take the *not* away, *man* becomes distributive with reference to *animal* which is confused. And thus *man* originally had determinate supposition.' [John of St. Thomas, 1955, 69]

Making use of the symbolism introduced here, these rules can be formulated as:

Rule 3o′ $\mathrm{Dist}(A)_P\&\ \mathrm{Conf}(B)_P\&\langle\sim, A\rangle_{P^*} \rightarrow \mathrm{Det}(A)_{P^*}$

Rule 3b′ $\mathrm{Dist}(A)_P\&\ \mathrm{Det}(B)_P\&\langle\sim, A\rangle_{P^*} \rightarrow \mathrm{Conf}(A)_{P^*}$

If a uniform account of the effect of distributive terms upon terms already having distributive and confused supposition could not be provided, this would have been a serious drawback for theories of supposition as a whole. Seemingly, at the time of Ockham and Buridan a solution for this issue had not yet been found; however, later authors such as John Dorp and John of St. Thomas were clearly aware of the problem, and succeeded in finding appropriate rules to deal with it. Clearly, many other cases may seem problematic and appear to be, at first sight, unaccountable for within supposition theory; but the reformulation of the rules for confused and distributive supposition above shows that the supposition framework is more resourceful than one might expect at first sight, allowing for constant refinement.[31]

[30] But this approach was already known in the 15th and 16th centuries, see [Ashworth, 1978, 600].

[31] See (Klima and Sandu 1991) for the use of supposition theory to account for complex quantificational cases.

2.2 Other important developments

2.2.1 The doctrine of proof of terms/propositions

Besides the 'older' semantic tradition based on the concept of supposition, another semantic tradition became very influential in the 14th century.[32] This tradition was known as the doctrine of the proof of terms (*probationes terminorum*) or propositions, and its most influential text was Billingham's *Speculum Puerorum*. Billingham's text was not the first in this tradition (for example, de Rijk dates Martin of Alnwick's text also edited in [De Rijk, 1982] as earlier than Billigham's), but it seems to have been the main source for the popularity of this genre in the second part of the 14th century.

'Proof' here is not to be understood in its mathematical/logical sense, as demonstration; in this sense, to prove a proposition is to show what its truth depends on, in particular which simpler propositions must be true in order for a given proposition to be true. It is essentially an analytic procedure, in which the meaning of a 'difficult' proposition is decomposed in terms of simpler propositions, known as immediate propositions, on the basis of the analysis of the term(s) causing the proposition to be a 'difficult' one. Immediate propositions are those, according to Billingham, which cannot be proved (verified) except by a direct appeal to the senses or the understanding. These would be primarily propositions with directly referential pronouns or adverbs such as 'I', 'this', 'that', 'here', 'now' etc (cf. Billigham, *Probationes Terminorum*, in [De Rijk, 1982, 49]).

There are three basic techniques to 'prove' a proposition, according to whether the proposition is an exponible, a resoluble or an officiable proposition. An exponible proposition is one that corresponds to several propositions taken in conjunction, such as 'Socrates begins to be white', which corresponds to 'Socrates was not white and Socrates is now white'. Propositions containing comparative and superlative terms, or the verbs 'begins', 'ceases' and 'differs from' are analyzed in this fashion. A resoluble proposition is one that involves the descent from a general term to discrete terms, such as 'A man is running', which is proved by an appeal to sense experience codified by the propositions 'This is a man' and 'This is running'. Finally, an officiable proposition is one containing the nominalization of a proposition with the accusative-plus-infinitive construction, and corresponds roughly to what is now known as 'opaque contexts' (modalities and verbs related to propositional attitudes such as 'think', 'believe' etc.).

Although we dispose of several texts presenting the doctrine of the proof of propositions/terms, it is still in fact quite understudied. Indeed, it is fair to say that we still do not really understand the purpose and the mechanisms defining it (see [Spade, 2000, part IV]). For this reason, it is to be hoped that scholars will at some point take up the challenge of analyzing this doctrine systematically, as

[32] This approach was sometimes used instead of supposition theories (cf. Johannes Venator's *Logica* which uses the doctrine of the proof of propositions exclusively), but at other times both theories co-existed together, for example as distinct chapters of the same work (such as Paul of Venice's *Logica Parva*).

it is still by and large murky terrain.

2.2.2 Other developments worth mentioning

For reasons of space, it is impossible here to treat the totality of influential 14th century semantic theories; but before we move on to the next section, a few more of these should be at least mentioned, so that the interested reader may further pursue his/her investigations. One of them is the doctrine of the supposition of relative pronouns, a vivid topic already in Ockham's *Summa Logicae* (Part I, chap. 76), which is the medieval counterpart of modern theories of anaphora (cf. [King, 2005]).

Another important aspect which, for reasons of space, could not be addressed here are the fascinating discussions on the semantics of propositions. In the same way that the medieval authors were interested in what single terms stood (supposited) for, many 14th century authors raised similar questions concerning phrases and propositions (see for example Chapter 6 of [Spade, 1996]). One of such questions was what, in the extra-mental physical realm, the accusative-plus-infinitive constructions then known as *dicta* (which are in fact nominalizations of indicative propositions) corresponded to. Another important question concerned what in the physical world, if anything, makes a true proposition true. Some of the authors who addressed such issues are Burley (cf. [Cesalli, 2001]), Wyclif (cf. [Conti, 2005a, 2.1; Cesalli, 2005]), Henry Hopton (cf. [Ashworth and Spade, 1992, 51]), [Richard Brinkley, 1987]; Paul of Venice (part II, fascicule 6 of his *Logica Magna*), among others, often in treatises bearing the conspicuous titles of *De significato propositionis* and *De veritate et falsitate propositionis*, or similar ones.

In sum, while 14th century semantics has been a popular topic of research in the last decades, this tradition itself is so rich that much of it still remains to be further studied and better understood. Therefore, the foregoing analyses should not be seen as an exhaustive account of this tradition but rather as a starting point for further research.

3 CONSEQUENCES

Theories of consequences are considered to be genuine medieval inventions. Of course, investigations on the nature of logical and inferential relations between propositions have existed ever since logic has existed; but medieval theories of consequence present a characteristic approach to the issue and a level of systematization that is arguably not to be found in previous investigations. Some (see [Moody, 1953]) see in them the forerunners of the 'propositional turn' in logic that took place to its full extent only in the 19th century with Frege, after millennia of predominance of term logic — that is, of logical systems whose basic units were terms, such as in traditional Aristotelian logic, and not propositions.[33]

[33] With the notable exception of Stoic logic, which is usually recognized as the first propositional system of logic in the history of logic (see [Mates, 1973]).

This picture, however, is not entirely accurate, as the logic of terms also occupies a prominent place in 14th century theories of consequence, as we shall see. But 14th century logicians were probably the first to attempt a systematization of the propositional rules of inference that we now take as fundamental, such as contraposition, *ex falso*, the behavior of conjunctions, disjunctions etc. Medieval logicians sought not only to establish the validity of such basic rules; they also made inquiries on the very nature of logical consequence and inference. In this sense, their investigations overlap not only with modern 'proof theory', but also with modern philosophy of logic (as exemplified by modern discussions on the nature of logical consequence such as [Etchemendy, 1990]).

In this section, I begin with a brief historical overview of this logical genre, and then move on to the three main subjects that the medievals discussed with respect to consequence: a general definition/criterion of what is to count as consequence; the distinctions of different kinds of consequence; and the most widely accepted rules of consequence. Finally, I show that at that time too there was no absolute consensus as to what rules of inference should be accepted, and that a few authors questioned some of the rules that were otherwise widely accepted.

3.1 Historical development

3.1.1 Origins

The precise historical origin of 14th century theories of consequence is still controversial among specialists. It is still something of a mystery why and how, all of a sudden, at the beginning of the 14th century, treatises bearing the title *De consequentiis* or the like began to appear. Why then, and not before? Naturally, the subject itself, that is, the logical and inferential relations between propositions, was very often discussed by earlier authors; the very term '*consequentia*' was in constant usage (in the same sense) since at least the 12th century, and dates as far back as Boethius in the 5-6th century (cf. [Boh, 1982, 302]). But no treatises or chapters were specifically dedicated to the topic or bore such titles before the 14th century.

According to an influential hypothesis concerning the origin of theories of consequences, they stemmed essentially from the tradition on the Topics (cf. [Bird, 1961; Stump, 1982]). The *Topics* was the fifth book of Aristotle's *Organon*, a book that can be described as a rather loose collection of rules for the conduction of non-demonstrative reasoning and argumentation. At first sight, this hypothesis makes good sense: in the tradition of Aristotelian logic, the role of the Topics was often that of accounting for the patterns of (correct) inference and reasoning that did not fit into the syllogistic system presented in the *Prior Analytics*. While it is a wonder of systematicity and formality, syllogistic is not a very wide-ranging theory in that it accounts for only a small portion of the patterns of reasoning that we are prepared to accept as valid. The Topics, even though not as rock-solid as syllogistic patterns, provided an account of many more of such patterns of reasoning. So, conceptually, it would seem quite natural that the tradition on the Topics

would be at the origin of theories of consequences, as these are essentially theories about the relations between propositions that go beyond the patterns recognized by syllogistic. Moreover, some earlier investigations on the notion of consequence were made explicitly within the context of an analysis of the Topics; Abelard, in the 12th century, developed a sophisticated theory of the logical relations between propositions precisely in the part of his *Dialectica* [Abelard, 1956] dedicated to the topics.

However, this hypothesis did not receive the historical confirmation that one could have expected. It has been argued [Green-Pedersen, 1984, 270] that the late 13th century literature on the Topics, that is, the period immediately preceding the emergence of treatises on consequences, gives absolutely no clue of what was to come; that is, there is no significant similarity between the contents of these 13th century treatises on the topics and 14th century treatises on consequences. Therefore, it has been concluded that the Topics could not have been the main source for 14th century theories of consequences.[34]

Although our current state of knowledge on the matter still does not allow for a conclusive account of these developments, the picture that at this point seems more plausible is that different strands of traditional Aristotelian logic converged in order to give rise to the 14th century theories of consequences. It seems that at least three other traditions contributed to the development of theories of consequence: treatises on *syncategoremata*, especially in connection with the *syncategorema* '*si*' (corresponding to the 'if ... then' structure in English); the analysis of hypothetical syllogisms, a concept absent from Aristotle's logic and introduced by Boethius in the 6th century AD (his treatise *De hypotheticis syllogismis* is referred to 6 times in Burley's *De puritate artis logicae* — cf. [Green-Pedersen, 1984]); and commentaries on the *Prior Analytics* — indeed, it is in the *Prior Analytics* that Aristotle explicitly states a formulation of the notion of 'following' that is arguably the (remote) source for the most fundamental definition of consequence in the 14th century.[35]

Be that as it may, the importance of the Topics for the development of 14th century theories of consequences should not be altogether dismissed. It is worth noticing that two of the first authors having written explicitly on consequence, Ockham and Burley, are both in some way or another influenced by the Topics. Burley explicitly says that all valid consequences are based on dialectical Topics [Burley, 2000, 158]. By contrast, the relation of Ockham's theory of consequence to the Topics is more convoluted; Green-Pedersen argued convincingly that Bird's reconstruction of Ockham's theory within the framework of the Topics [Bird, 1961] is not satisfactory [Green-Pedersen, 1984, 268], but he also confirms that Ockham's 'intrinsic' and 'extrinsic' middles, crucial concepts for his theory of consequence, are concepts essentially taken (albeit heavily modified) from the topical framework.

[34] Chapter E of [Green-Pedersen, 1984], on topics and the theory of consequence, is the most comprehensive survey of these developments that I am aware of.

[35] "A deduction is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so." *Prior Analytics* 24^b19-20.

In sum, while we are not yet able to reconstruct a complete history of the development of these theories, at this point it seems that the most plausible hypothesis is that at least these four traditions — topics, *syncategoremata*, hypothetical syllogisms and *Prior Analytics* — must be taken into account to explain the rise of theories of consequences in the 14$^{\text{th}}$ century. Different aspects of each of these traditions contributed to the development of different aspects of the theories of consequence.[36] Green-Pedersen [1984, 295] argues for example that the late 13$^{\text{th}}$ century treatises that most resemble early 14$^{\text{th}}$ century treatises on consequences are 'the treatises on syncategorematic words and a number of sophism-collections arranged after syncategoremes.' Some of these connections will be commented upon in what follows, when specific aspects of theories of consequence are discussed.

3.1.2 Development in the 14$^{\text{th}}$ century

The 14$^{\text{th}}$ century treatises on consequences can be divided in roughly four groups:[37]

1. The treatises on consequences from the very beginning of the 14$^{\text{th}}$ century (Burley's *De consequentiis* and two anonymous treatises of roughly the same time — cf. [Green-Pedersen, 1981]) are in fact rather unsystematic collections of rules of consequence/inference. It seems as though their purpose was solely to provide 'rules of thumb' to deal with *sophismata* related to some syncategorematic terms; no conceptual or systematic discussion of the nature of consequences is presented therein.

2. The second stage of their development is represented by Burley's *De Puritate* [Burley, 2000], the chapters on consequence in Ockham's *Summa* (III-3), a few Pseudo-Ockham treatises and the *Liber consequentiarium* edited in [Schupp, 1988]. I group these texts together because in them the concept of (intrinsic and extrinsic) middles plays a crucial role and thus the presence of the (reworked) topical framework is more clearly perceived. They display a much deeper interest in the very nature of consequences than the previous group, presenting general definitions and criteria of what is to count as a consequence as well as divisions of kinds of consequence.

3. The third stage is represented by Buridan's treatise (Buridan 1976) and the treatises inspired by it, most notably Albert of Saxony's (a section of his *Perutilis logica*), the commentary on the *Prior Analytics* formerly attributed to Scotus (Pseudo-Scotus 2001)[38], and Marsilius of Inghen's (which is still only to be found in manuscripts). In these treatises the doctrine of intrinsic and

[36] For several of these tentative connections, see chapter E of [Green-Pedersen, 1984].

[37] My division is inspired by the division presented in [King, 2001, fn. 1], but with a few modifications. Here I refer mostly to treatises that can be found in print. For a list of these printed treatises, see [King, 2001]; for a list of British treatises only available in manuscripts, see [Green-Pedersen, 1985].

[38] On the identity of the author of this text, see [Read, 1993, fn. 10].

extrinsic middles has disappeared completely, and they present sophisticated analyses of the nature of consequences. What characterizes them as a group is the definition of formal consequence based on the substitutivity criterion (more on it below). This tradition can be referred to as the continental tradition on consequences.

4. The fourth group of treatises is predominantly British, and significantly more represented in number of treatises than group (3). It is represented by the treatises by Robert Fland, John of Holland, Richard Billingham, Richard Lavenham, Ralph Strode, among others. What characterizes this group as such is the definition of formal consequence in terms of containment of the consequent in the antecedent; it is a distinctly epistemic definition of formal consequence, when compared to that of group (3).[39]

In sum, the development of theories of consequence in the 14th century is characterized by an early and rather 'primitive' stage, then by a stage of further development but with emphasis on the idea that consequences need something extrinsic to validate them (most generally quasi-Topics), and then by two further traditions that run more or less parallel, the British tradition and the continental tradition.[40]

3.2 General definition of consequence

Before we inspect the different general definitions of consequence, we must first address the question of the relation between consequences and conditionals. For explanatory purposes, it may be convenient to consider three related but distinct concepts:

- A conditional proposition, a proposition that relates logically two embedded phrases, which are themselves not assertions properly speaking.
- A consequence, which is a logical relation between assertions or propositions.
- An inference, which is the action performed by somebody of inferring a conclusion from a (set of) (asserted) premise(s).

The medieval authors were well aware of these distinctions, or at least of the distinction between conditional sentences and consequences/inferences, but they often treated these different notions simultaneously, causing some confusion among modern interpreters.[41] Conditional sentences were generally treated by the medievals under the general heading of 'hypothetical propositions', along with conjunctions, disjunctions and others (cf. for example Chapter 1.7.3 of Buridan's

[39] For a compelling account of how this epistemic notion of formal consequence may have been the background for the emergence of the Cartesian notion of inference, see [Normore, 1993].

[40] This is in any case the general picture, but of course I do not claim that there are no exceptions to it — that is, there may very well be treatises that do not fit this description.

[41] For example: if the medieval notion of *consequentia* is not to be assimilated to that of a conditional sentence or implication, as I will argue, then the discussion on whether Ockham knew of material implication based on his theory of consequence seems rather misguided.

Summulae — [Buridan, 2001]); however, many of them recognized that a true conditional corresponds to a valid consequence/inference[42] (cf. Ockham, *Summa Logicae*, Part II, chapter 31, where he says that he will not discuss conditionals extensively since their logical properties correspond to the logical properties of consequences). Moreover, if the historical hypothesis of the influence of theories on the syncategoremata and on hypothetical syllogisms for the development of theories of consequences is correct, it becomes then patent that the two notions of conditionals and consequences were intimately related for our authors.

Nevertheless, as argued in [King, 2001], there are various reasons to conclude that the medievals were not only aware of these distinctions, but also that they viewed their *consequentiae* as primarily corresponding to arguments and inferences, i.e. to relations between statements, and not to conditional statements. Perhaps the most conspicuous of these reasons, taken from the structure of Ockham's *Summa*, is that consequences are treated there in Part III, dedicated to arguments, and not in part II, dedicated to propositions. Buridan explicitly addresses this distinction between conditionals and arguments in 7.4.5 of his *Summulae de Dialectica*, and while he says that *consequentia* is a twofold concept (covering conditionals and arguments), in most cases the logical properties of conditionals are treated under the heading 'conditionals', and what is dealt with in sections on consequence is predominantly logical properties of arguments. King [2001, 123] also discusses how Burley presents rules of consequences involving conditional propositions, explicitly contrasting 'the conditional sentences that enter into such reasoning with the consequences made out of them'. Hence, for the purposes of the present analysis, medieval consequences are not to be understood as conditional sentences but rather as inferences/arguments.

Now, as for the general definition of consequence, most authors of the 14^{th} century accept at least as a necessary condition for a (valid) consequence that the antecedent cannot be true while the consequent is false; many accept this as a sufficient condition as well. This is, of course, the very familiar modal definition of consequence, present in Aristotle and also widely accepted in current (philosophy of) logic (at least as a necessary condition). Another formulation of the same idea is that a consequence is valid if from the contradictory of the consequent the contradictory of the antecedent follows [Ockham, 1974, 728][43], or similarly that the contradictory of the consequent is incompatible with (the truth of) the antecedent [Burley, 2000, 149]. In fact both Ockham and Burley give several equivalent formulations of this core idea, but the key point is obvious: for our authors, the most fundamental characteristic of a consequence is that the truth of the antecedent is incompatible with the falsity of the consequent.

Buridan's formulation of the fundamental criterion of what is to count as a consequence follows the same idea, but it is more convoluted because he has to take

[42] We shall see in due course that absolute consequences correspond to necessarily true conditionals, while as-of-now consequences correspond to contingently true propositions.

[43] Naturally, this should not be seen as a definition, as it is obviously circular, but rather as a rule of thumb for the recognition of putative consequences.

into account a few extreme counterexamples related to his commitment to actually formed propositions as the bearers of truth value (for the details of Buridan's discussion, see [Dutilh Novaes, 2005c]). His final formulation is 'a proposition is antecedent to another when it is related to it [the other] in such a way that it is impossible for things to be in whatever way the first signifies them to be without their being in whatever way the other signifies them to be, when these propositions are put forward together.' [Buridan, 1976, 22]. Pseudo-Scotus [2001] also considers this formulation, but he notices that it is not immune to a form of what is now known as Curry paradox[44], so he adds a clause to the effect that the definition does not hold in such extreme cases.

In the British tradition, variations of the same basic definition of consequence can be found, as for example in Billingham [Billingham, 2003, 80], Strode (in [Pozzi, 1978, 237]) and Paul of Venice [Paul of Venice, 1983, 167]; in fact, these authors present this definition without much discussion or analysis, as opposed to what one can find in Buridan's treatise for example. That seems to indicate that at later stages the definition was seen as unproblematic. It must also be noted that, while earlier authors such as Ockham and Buridan consider to be consequences only those pairs of propositions that satisfy the criterion, later authors such as Billingham and Paul of Venice (cf. same as above) recognized invalid consequences — that is, those that do not satisfy this criterion — as consequences nevertheless, making thus the distinction between valid and invalid consequences (while for Buridan and others, an invalid consequence was simply not a consequence).

3.3 Types of consequence

Medieval logicians recognized that the class of pairs (or triplets etc.) satisfying the general modal definition of consequence is rather heterogeneous. Indeed, one of the main focuses of treatises on consequences is the distinction of different kinds of consequences. There are three main kinds of distinctions: natural vs. accidental consequences, absolute vs. as-of-now consequences and formal vs. material consequences. While the same terms are generally used by different medieval authors, they often mean different things and have dissimilar criteria differentiating one kind from another. However, the natural vs. accidental distinction, while very important in early theories of consequence, is to be found only in Burley in the 14th century (cf. [Green-Pedersen, 1984, 286; Pozzi, 1978, 58], so we shall not spend any time on it here. It is really the *ut nunc* vs. simple distinction and, even

[44]The paradoxical case put forward by Pseudo-Scotus is the following: 'God exists, hence this argument is invalid'. If this consequence is valid, then it has a necessary antecedent and a false consequent (since the consequent says that it is invalid). But then it is invalid. In sum, if it is valid, it is invalid, thus by *reductio ad absurdum* it is invalid. But if it is invalid, it is necessarily so, since the premise is a necessary proposition; therefore, we have a consequence with a necessary consequent, thus satisfying the modal criterion, but which is bluntly invalid, thus violating the modal criterion. See [Read, 2001] for an account of Pseudo-Scotus 'paradox', where it is also shown that this so-called paradox did not obtain the desired effect of violating the modal criterion.

more so, the formal vs. material distinction that are crucial for the understanding of 14th century theories of consequence. Besides these, there is also a distinction that is sometimes found explicitly stated but sometimes not, and which sometimes overlaps with the formal vs. material distinction but sometimes does not, namely the distinction *bona de forma* vs. *bona de materia*.

It is important to understand that these divisions are not necessarily meant to be sub-divisions of each another; they are also often understood as alternative, overlapping ways of dividing consequences. Ockham, for example, presents several such distinctions but does not present them as sub-divisions of one another (*Summa* III-3, Chapter 1). So let us now take a closer look at each of them.

Formal vs. material consequences. This distinction first appears in Ockham (cf. [Green-Pedersen, 1984, 287]), but afterwards it is to be found in virtually all treatises on consequence in the 14th century. However, what distinguished formal from material consequences varies per author.

For Ockham, this distinction is related to his doctrine of intrinsic and extrinsic middles. Ockham says that a consequence is formal when there is such middle, intrinsic or extrinsic, validating the consequence; otherwise, when there isn't such middle and the consequence holds only in virtue of its very terms (*Summa Logicae*, p. 589), it is a material consequence. As already mentioned, Ockham's extrinsic and intrinsic middles are reminiscent of the topical framework, but they are extremely modified versions of topical concepts, as shown by Green-Pedersen [1984, chapter E].

But what are intrinsic and extrinsic middles? An extrinsic middle is a proposition not containing the terms that form the antecedent and the consequent of the putative consequence, but which is a general rule describing the fact (ontological, logical or other) that warrants the passage from the antecedent to the consequent. An intrinsic middle, by contrast, is formed by the very terms that form the antecedent and the consequent of the putative consequence. Ockham says that some formal consequences hold only in virtue of an extrinsic middle, while others need an intrinsic as well as an extrinsic middle to hold. Syllogisms, for example, hold only in virtue of extrinsic middles; a consequence such as 'Only a man is a donkey, therefore every donkey is a man' holds in virtue of the following extrinsic middle: 'an exclusive and a universal with transposed terms signify the same and are convertible'. But a consequence such as 'Socrates is not running, therefore a man is not running' requires that the intrinsic middle 'Socrates is a man' be true in order to hold (and it still requires an extrinsic middle to validate it mediately). (Ockham, *Summa Logicae*, p. 588).

What exactly a material consequence is for Ockham is still a matter of controversy among scholars, as the passage where this notion is explained is known to be corrupted (*Summa Logicae*, p. 589).[45] Ockham gives two examples of material consequences, one of the *ex impossibili* kind (from an impossible proposition anything follows) and one of the *ad necessarium* kind (a necessary proposition follows

[45] See [Schupp, 1993] on the corruption of the text.

from anything), and that has led many commentators to believe that these are the only kinds of material consequences that Ockham recognizes. This matter cannot be settled at this point, but some have also raised the hypothesis that Ockham's notion of material consequence would go beyond such specific cases (see [Schupp, 1993; Kaufmann, 1993]).

Buridan, by contrast, is crystal clear concerning his distinction between formal and material consequence. His very terms are: 'Formal consequence means that [the consequence] holds for all terms, retaining the form common to all. Or [...] a formal consequence is that which, for every proposition similar in form which might be formed, it would be a good consequence.' [Buridan, 1976, 22-23]. A material consequence is one that does not satisfy this criterion but only the modal criterion.

While it is not immediately obvious why Ockham chose this nomenclature for his distinction[46], in the case of Buridan the terminology espouses perfectly the traditional Aristotelian notions of form and matter: the matter of a proposition is defined by its categorematic terms, while its form is defined by its syncategorematic terms. Thus, a formal consequence is one that holds in virtue of its form (the meaning of its syncategorematic terms); a material consequence is one that does not hold in virtue of its form alone but also in virtue of its matter (the meaning of its categorematic terms). Another way of describing Buridan's criterion is with the notion of substitutivity: a formal consequence is one that holds in all substitutional instances of its categorematic terms.

Buridan's use of the substitutional criterion is remarkable in that it is immune to much of the recent criticism against this criterion, most notably in [Etchemendy, 1990]. This is so because the substitutional criterion is applied only to consequences which already satisfy the modal criterion of incompatibility between the truth of the premise and the falsity of the conclusion. Under purely substitutional accounts of (logical) consequence, a clearly invalid logical consequence such as 'Bill Clinton was a president of the USA, thus Bill Clinton was male' comes out as valid if 'Bill Clinton' is seen as the only non-logical term of the consequence (as it happens to be so that, thus far, all presidents of the USA have been male, and thus all substitutional instances for 'Bill Clinton' will also validate the consequent). But in Buridan's account, this putative consequence would not be considered as a valid consequence in the first place, since it is not incompatible for somebody to be the president of the USA and not be a male (in fact this may happen even in the foreseeable future); therefore, it cannot be a formal consequence because it is not a consequence to start with. Buridan's account is best seen as what Shapiro [1998] has coined the 'hybrid' notion of formal (logical) consequence (which he presents as the most accurate conceptual characterization of logical consequence), that is,

[46] Ockham's choice of the term 'formal consequence' seems to be related to John Duns Scotus' notion of 'formal distinction'. Moreover, references to *consequentia formalis* can be found in Scotus' writings, for example in his *Quaestiones super libros Elenchorum* (p. 77 of the Vivès edition of *Opera Omnia*). On the relation between Scotus and Ockham on this matter, see [Martin, 2004].

the notion according to which a formal (logical) consequence must satisfy both criteria, the modal one and the substitutional one (see [Dutilh Novaes, 2005b].

This criterion for differentiating consequences had been proposed before by Abelard, who distinguished perfect from imperfect consequences [Abelard, 1956, 253-4]: perfect consequences received their warrant from their structure (*complexio*) alone, while imperfect inferences needed external warrant (for which Abelard turned to the Topics — cf. [Abelard, 1956, 256-7]). The same criterion was later rediscovered by Bolzano [1973] and further developed by Tarski [2002].

Intuitive though as it may seem to the modern reader, the definition of formal consequence based on the substitutional criterion was not widely accepted in the 14$^{\text{th}}$ century. As already mentioned, while the treatises on consequence influenced by Buridan did maintain this criterion (Pseudo-Scotus, Albert of Saxony, Marsilius of Inghen), the majority of treatises followed a different notion of formal consequence. This alternative notion of formal consequence is what we could call the containment notion: a consequence is formal iff the consequent is contained in the antecedent, in such a way that whoever understands the antecedent necessarily understands the consequent.[47]

Here is Lavenham's formulation (as quoted in [King, 2001, 133]): 'A consequence is formal when the consequent necessary belongs to the understanding of the antecedent, as it is in the case of syllogistic consequence, and in many enthymematic consequences'. Strode's similar formulation is: 'A consequence said to be formally valid is one of which if it is understood to be as is adequately signified through the antecedent then it is understood to be just as is adequately signified through the consequent. For if someone understands you to be a man then he understands you to be an animal.' (as quoted in [Normore, 1993, 449]). Many other authors held similar definitions, such as Billingham [2003, 80] and Fland [1976].

Besides the fact that this is notably an epistemic notion of formal consequence (as opposed to Buridan's substitutional notion), for the authors adopting the containment notion, the extension of the concept of formal consequence is usually wider than that of those adopting the substitutional notion of formal consequence. Lavenham explicitly says that some enthymematic consequences are formal consequences — that is, consequences that, with the addition of an extra premise, acquire a syllogistic form, such as 'Socrates is a man, thus Socrates is an animal', which acquires a syllogistically valid form with the addition of 'Every man is an animal'. For Buridan, however, enthymematic consequences are not formal consequences because they do not satisfy the substitutional criterion, e.g. this particular example is not valid for all substitutional instances of 'man' and 'animal'. They do become formal consequences with the addition of the missing premise, but before that occurs they are merely material consequences (cf. [Buridan, 1976, 23]).

Another different criterion for formal consequences is presented at the very end of the 14$^{\text{th}}$ century by Paul of Venice; he characterizes formal consequences as those

[47] The containment notion of (formal) consequence is not a 14$^{\text{th}}$ century invention. For Abelard, it was a necessary condition for all valid consequences; in the 13$^{\text{th}}$ century, it was held by authors such as Faversham and Kilwardby.

'in which the opposite of the consequent is repugnant formally to the antecedent', by which he means that the antecedent and the opposite of the consequent cannot even be conceived together. His example of a material consequence is 'God is not, therefore no man is': even though it is (or so he thinks) metaphysically impossible for men to be without God being, it is not inconceivable.

Consequences *bona de materia* vs. *bona de forma*. A criterion differentiating consequences that is intimately related to but not always identical to the formal vs. material distinction is the distinction *bona de forma* vs. *bona de materia*. For those who adopt the substitutional criterion to define formal consequences, this distinction is usually equivalent to the formal vs. material distinction; but for those who follow different criteria for formal consequences, there is indeed a point in recognizing that some consequences are valid no matter what their categorematic terms are, if their syncategorematic terms are retained. Ockham, for example, while using the rather idiosyncratic notion of 'middles' to define his formal consequences, also recognizes that some consequences are valid '*de forma*' (cf. Summa III-1, cap. 13, 32-35). Similarly for Paul of Venice; next to his 'conceivability' distinction between formal and material consequences, he adds the distinction between consequences '*bona de forma*' and '*bona de materia*' [Paul of Venice, 1984, 168]. Confusingly, though, he says that 'man runs, therefore animal runs' is a consequence '*bona de forma*', while it clearly does not satisfy the substitutional criterion that is usually associated with the notion of a consequence '*bona de forma*'.

Absolute vs. as-of-now consequences. The other important distinction of consequences for 14th century authors is the one between absolute vs. as-of-now (*simplex vs. ut nunc*) consequences. Intuitively, the idea is that absolute consequences hold always and necessarily, while as-of-now consequences hold at a specific time or under specific assumptions (in particular at the moment indicated by the verbs in the consequence).

A clear account of this distinction is to be found in Pseudo-Scotus [2001]: for him, the absolute vs. as-of-now distinction applies only to material consequences (recall that for Ps.-Scotus a formal consequence is one that satisfies the substitutional criterion), and amounts to the modal value of the missing premise that can be added in order to turn the (enthymematic) consequence into a formal one. That is, if the missing premise is a necessary proposition, then the consequence is an absolute one (it always holds, since the condition for it to hold, namely the truth of the 'missing premise', always obtains); if the missing premise is a contingent truth (it has to be true with respect to the time indicated by the verbs of the consequence, otherwise the original material consequence does not hold), then the original material consequence holds only in some situations, namely the situations in which the contingent proposition happens to be true, and is thus an as-of-now consequence.

Although there may be slightly different formulations of the absolute vs. as-of-now distinction, these differences are in fact conceptually immaterial if com-

pared to the very different formulations of the distinction formal vs. material consequence. An important difference, however, is that, while for some authors (Pseudo-Scotus, Buridan) the absolute vs. as-of-now distinction is posterior to the formal vs. material distinction and applies only to material consequences, there are also authors (e.g. Peter of Mantua — cf. [Pozzi, 1978, 61]) who present the absolute vs. as-of-now distinction as primary, and who see the formal vs. material distinction as applying only to absolute consequences (as-of-now consequences are always material consequences). In the latter case, clearly the distinction can no longer be cast in terms of the modal value of the missing premise, since at least in some cases of absolute formal consequences (e.g. valid syllogisms) there is in fact no missing premise. Alternatively, for Peter of Mantua, an as-of-now consequence is a consequence in which the contradictory of the consequent can indeed be true at the same time as the antecedent, but not at the time indicated by the copula or verb in question (the present if the verb is present-tense, the past if it is past-tense etc.).

For a while, modern commentators have been particularly interested in as-of-now consequences (in particular with respect to Ockham — cf. [Mullik 1971]), as some of them (cf. [Bohener, 1951]) saw these consequences as possible forerunners of modern material implication. The first problem with this association, as already argued, is that it seems unfitting to view medieval consequences as conditional sentences/implications (material or otherwise). Moreover, as-of-now consequences are contingent only insofar as the truth of the 'missing premise' is contingent. For the rest, the logical relation of necessity between the propositions involved is just as tight as with other consequences, provided that the missing premise is true — if the missing premise is true, then the truth of the antecedent is incompatible with the falsity of the consequent. In other words, as-of-now consequences display a stronger logical relation than mere truth-functional material implication.

The modern reader may be wondering: what is the point in distinguishing these different kinds of consequences? Is it yet one of those futile exercises that logicians of all times tend to be fond of, but with no practical application? Not so; the reason for such distinctions is in fact very practical: the different rules of consequences (to be presented in the next section) apply to specific kinds of consequences. That is, some rules apply only to formal or absolute consequences; others apply to material or as-of-now consequences. And such rules are extremely useful for the purposes of construing an argumentation; in fact, it seems that construing valid arguments is really the ultimate purpose of theories of consequences. But for a sound application of such rules, it is essential to identify the kind of a consequence in question, whence the importance of these criteria being as clearly formulated and effective as possible.

3.4 Rules of inference recognized by the medieval authors

Let us now look at the logical rules of consequence recognized by our 14th century authors. Some of them had already been identified by earlier authors[48] (for example the rules of opposition, equipollence and conversion for categorical propositions described in Aristotle's *De Interpretatione*); but what is remarkable in at least some of the medieval treatises is how they attempt at a systematization of these rules, such that from primary rules secondary rules are derived (see for example the first chapters of Burley's *De Puritate*, The Shorter Treatise — [Burley, 2000, 3-26]). Granted, other treatises are really no more than rather unsophisticated lists of rules, with no attempt to link the logical properties of each of them together. But quite a few of them present what we could view as the first stages of a proof theory.

Here, I present some of these rules making use of a notation inspired by Gentzen-style sequent calculus. The list of rules presented here is not exhaustive in that not all the rules studied and recognized by the medievals will be presented; the purpose here is to give the reader an idea of the level of logical sophistication attained by medieval treatises on consequences. A more thorough and extremely useful listing of the rules recognized by the medieval authors can be found in [Pozzi, 1978, 69-73].[49]

Burley, for example, lists ten main rules and several other rules that follow from these main rules. The first four rules are indeed easily rendered within the conceptual framework of propositional calculus, while the other six rely heavily on the properties of terms as well (and this is why it is inaccurate to say that medieval theories of consequences are purely propositional in nature; logical properties of terms still play a prominent role). Boh [1982, 312-314] presents a neat reconstruction of main rules 1 to 4, plus their derived rules, but the problem with his reconstruction is that it implies the view that consequences are conditionals, a view that, as already said, is rejected here.

Burley's rule 2, for example, states that 'whatever follows from a consequent follows from the antecedent', or alternatively, 'whatever is antecedent to the antecedent is antecedent to the consequent' [Burley, 2000, 4]. This is basically a formulation of the Cut-rule in sequent calculus (with the difference that no mention is made to the contextual propositional variables that are included in sequent calculus for the sake of generality).

$$\text{Rule 2} \quad \frac{A \Rightarrow B \quad B \Rightarrow C}{A \Rightarrow C}$$

[48] It is also often said that the Stoics are the genuine pioneers of propositional logic; however, there is as of yet no evidence of direct or even of indirect influence from Stoic logic on the development of medieval theories of consequences. That is, even if many of such rules of consequence had already been recognized by the Stoics, it all seems to indicate that the medievals re-discovered them independently.

[49] Pozzi's study is based on the treatises on consequences of the following authors: Ockham, Burley, Pseudo-Scotus, Buridan, Albert of Saxony, Ralph Strode, Peter of Mantua, and Richard Ferrybridge.

From this he derives a few other rules, among which: (2′) 'whatever follows from a consequent and from its antecedent follows from the antecedent by itself' [Burley, 2000, 6] and (2″) 'whatever follows from a consequent with something added follows from the antecedent with the same thing added' [Burley, 2000, 7].

$$\text{Rule } 2' \quad \frac{A \Rightarrow B \quad A, B \Rightarrow C}{A \Rightarrow C}$$

$$\text{Rule } 2'' \quad \frac{A \Rightarrow B \quad B, C \Rightarrow D}{A, C \Rightarrow D}$$

Burley derives (2′) from Rule 2 plus what he takes to be a logical fact, which the modern reader may recognize as a special case of right-weakening: 'every proposition implies itself together with its consequent'; similarly, he derives (2″) from Rule 2 plus simultaneous applications of special cases of right-weakening and left-weakening: 'an antecedent together with something added implies the consequent with the same thing added'. His arguments can be reconstructed as follows:

$$(2') \qquad \frac{\dfrac{A \Rightarrow B}{A \Rightarrow A, B}\,\text{WR} \qquad A, B \Rightarrow C(\text{hyp.})}{A \Rightarrow C}\,\text{Rule 2}$$

$$(2'') \qquad \frac{\dfrac{A \Rightarrow B}{A, C \Rightarrow B, C}\,\text{WR, WL} \qquad B, C \Rightarrow D(\text{hyp.})}{A, C \Rightarrow D}\,\text{Rule 2}$$

After having shown that the medievals did know the procedure of deriving rules from primitive rules, I now present a few other rules recognized by them (taken from Pozzi's very useful list, unless otherwise stated), not paying specific attention to the deductive structure between these rules:

'From the impossible anything follows.'

$$\frac{}{\bot \Rightarrow A}$$

In the case of material as-of-now consequences the requirement is weaker: from a false proposition anything follows.

'The necessary follows from anything.'

$$\frac{}{A \Rightarrow T}$$

Similarly, in the case of material as-of-now consequences the requirement is weaker: a true proposition follows from anything (these two weaker formulations

for as-of-now consequences are to be found in Buridan and Pseudo-Scotus (cf. [Pozzi, 1978, 69]).

'If the antecedent must be conceded, so must be the consequent.'

$$\frac{\Rightarrow A \quad A \Rightarrow B}{\Rightarrow B}$$

'If the consequent must be denied, so must be the antecedent.'

$$\frac{\Rightarrow \neg B \quad A \Rightarrow B}{\Rightarrow \neg A}$$

'From the contradictory of the consequent the contradictory of the antecedent follows': contraposition [Ockham, 1974, 728].

$$\frac{A \Rightarrow B}{\neg B \Rightarrow \neg A}$$

'Whatever follows from the contradictory of the antecedent follows from the contradictory of the consequent.'

$$\frac{A \Rightarrow B \quad \neg A \Rightarrow C}{\neg B \Rightarrow C}$$

'Whatever is antecedent to the contradictory of the consequent is antecedent to the contradictory of the antecedent.'

$$\frac{A \Rightarrow B \quad C \Rightarrow \neg B}{C \Rightarrow \neg A}$$

'From a conjunction to one of its parts constitutes a valid consequence.'

$$\frac{}{A \& B \Rightarrow A}$$

'From one of its parts to the whole disjunction constitutes a valid consequence.'

$$\frac{}{A \Rightarrow A \vee B}$$

'From a conditional with its antecedent to its consequent constitutes a valid consequence.'[50]

$$\frac{}{A \rightarrow B, A \Rightarrow B}$$

Now something perhaps slightly surprising for the modern reader: rules for consequences that do not obtain (which I represent by '$\not\Rightarrow$'). In modern systems

[50]Here again it is clear that our authors were very much aware of the distinction between conditionals and consequences.

this is not necessary, as the enumeration of the valid rules is supposed to be exhaustive, and all patterns not falling within the patters of these valid rules are immediately false. The medievals, however, did not have the ambition of presenting exhaustive lists of valid rules, and therefore it was also useful for them to know how to identify when a consequence did not hold.[51]

'Whatever does not follow from the antecedent, does not follow from the consequent.'

$$\frac{A \Rightarrow B \quad C \not\Rightarrow B}{C \not\Rightarrow A}$$

'Of that from which the consequent does not follow, the antecedent does not follow from it either'

$$\frac{A \Rightarrow B \quad C \not\Rightarrow B}{C \not\Rightarrow A}$$

They also discussed rules involving modalities, such as 'from the necessary the contingent does not follow' or 'from the possible does not follow the impossible'.[52]

The rules presented here are those that can be easily formulated within a purely propositional framework. However, as already noted, treatises on consequences contained many more rules which, by contrast, were based on properties of terms; these rules were just as significant, and the only reason why they are not treated here is because their formulation presupposes concepts from elsewhere (such as supposition theory — for reasons of space, it is not possible to go into such details here). But just to illustrate the point, here is an example, Burley's Rule 7 in the shorter version of *De Puritate*: 'an inference holds from a distributed superior to an inferior taken either with distribution or without distribution. But an inference does not hold from an inferior to a superior with distribution. For it follows: 'Every animal runs, therefore, every man runs, and a man runs', but not conversely.' [Burley, 2000, 16] 'Animal' is the superior of 'man', and from a proposition where 'animal' is distributed follow both the proposition where its inferior 'man' is distributed and the one where it is not.

3.5 Dissident voices

So far I have treated medieval theories of consequences as if they were homogeneous with respect to the rules of consequence accepted by their authors. While not all authors stated all these rules explicitly, they are in any case all compatible with one another. However, a small minority rejected two of the very basic rules of the notion of consequence presented so far, namely the *ex impossibili* and the *ad necessarium* rules. These rules follow naturally from the modal definition of

[51] In the same manner that theories of fallacies — that is, theories of apparently sound but in fact unsound reasoning — were crucial for medieval logicians.

[52] More on modal inferences can be found in the chapter on modalities in the present volume.

consequence (if it is seen as a sufficient condition), such that those who accept this definition as a sufficient condition for a consequence must admit the validity of these two principles. This is so because the definition states that B is a consequence of A if it is impossible for A to be true while B is false. If it is impossible for A to be true *tout court*, the definition is satisfied *a fortiori* for any B whatsoever, and hence *ex impossibili* must hold. Similarly, if *ex impossibili* holds, by contraposition *ad necessarium* must hold.

But in several periods of the history of logic, some have seen these two principles as highly counterintuitive[53] given that, according to them, propositions that are otherwise not related by meaning or logic in any way whatsoever are in a relation of consequence with one another: 'It is raining and it is not raining, therefore I am God'; 'The cat is on the mat, therefore it is raining or it is not raining' would be examples of such counterintuitive inferences/consequences. On a semantic level, to deny the validity of these rules amounts to denying the modal definition of consequence as a sufficient criterion; on a syntactic level, modifications of the usual rules are required, since the derivation of any proposition from a contradiction can be obtained very easily from the usual rules of deduction (as shown by the famous Lewis argument, which may have been known already in the 12th century — cf. [Martin, 1986]): from P and not-P follows P; from P follows P or Q; but from P and not-P not-P also follows, and hence by disjunctive syllogism Q follows from P or Q and not-P. To block this derivation, at least one of these otherwise very natural rules must be discarded.

Prior to the 14th century, illustrious logicians such as Abelard and Kilwardby had already restricted their notion of consequence, not accepting the modal definition as a sufficient condition. Abelard, for example, required that the consequent be contained in the antecedent for a consequence to hold [Abelard, 1956, 253]. (As we have seen, 14th century logicians also made use of this criterion, but to define a sub-class of the valid consequences and not as a necessary and sufficient condition for all consequences.) With this move, *ex impossibili* and *ad necessarium* no longer hold.

In the 14th century we know of at least a few dissident voices. A certain Nicolaus Drukken of Dacia, writing in Paris in the 1340s, proposes a revision of the sufficient criterion of a valid consequence such that the 'total significate of the consequent be signified by the antecedent' [Read, 1993, 241][54] in his commentary to the *Prior Analytics* (edited by [Green-Pedersen, 1981]). Richard Ferrybridge also rejects *ex impossibili* and *ad necessarium* if the impossible and necessary propositions in question are impertinent to the other proposition in a putative consequence, precisely because he requires there to be a relation of relevance between antecedent and consequent (cf. [Pozzi, 1978, 60]. In practice, what such authors seem to be proposing is that the criterion of containment of the consequent in the antecedent be used as a necessary and sufficient criterion for all valid consequences, and not

[53] Recent examples of such dissident voices are paraconsistent and relevant logicians.

[54] Read [1993, 251] mentions that the early 16th century philosopher Domingo de Soto was another dissident voice with respect to *ex impossibili*.

only for the formal ones as in the case of Billinham, Strode *et al.*

3.6 Conclusion

14th century theories of consequences are without a doubt among the most important and most interesting developments in the logic of this century. Given the considerable length of the literature on the topic, and for reasons of space, here I had to focus on its main lines of development and disregard some of the secondary points and details that are nonetheless very interesting (such as Buridan's reformulation of the modal criterion in order to accommodate his token-commitment, Pseudo-Scotus analysis of a Curryian paradox, among others). Indeed, my purpose here was to give the reader a hint of the richness of this material, and encourage him/her to go look further.

4 OBLIGATIONS

4.1 Introduction

Obligationes were a regimented form of oral disputation. It consisted of two participants, Opponent and Respondent; Opponent would put forward several propositions, and Respondent was expected to accept, deny or doubt them on the basis of specific rules (the discussion of which will constitute the core of this chapter). It is without a doubt one of the main logical genres of the 14th century: virtually every important author of this period wrote on *obligationes*. However, contrary to what is sometimes thought, *obligationes* are not a 14th century invention; interesting 13th century Parisian treatises[55] indicate that the genre was already quite developed at that time and place.[56] Even though the contents of these treatises fall out of the scope of the present investigation, it may be added that the theory of *obligationes* presented in them is very much in the spirit of the earliest of such treatises in the 14th century, namely Burley, indicating thus that Burley was most probably inspired by this early tradition.

With *obligationes* we have a phenomenon that falls in the context of the OXINAT theory (cf. [Ebbesen, 1985]) previously mentioned, according to which the typical logical topics of the 14th century were re-introduced into Paris as British import, after a period of modistic predominance. Indeed, after these 13th century Parisian treatises on *obligationes*, few continental treatises on the topic were written in the 14th century; the fact is that in the 14th century, *obligationes* was an overwhelmingly British genre.

The 14th century British tradition on obligations begins, as already said, with Burley; his treatise seems to have been written in the very first years of the century (cf. [Braakhuis, 1993, 323]). Then, for at least two decades, nothing much seems

[55] Such as those edited in [Braakhuis, 1998] and [de Rijk, 1974; 1975; 1976].

[56] See also [Martin, 2001] for very early (12th century) developments in the genre.

to have been written on *obligationes* (in any case, we have not been able to unearth anything so far) until Kilvington's *Sophismata* [Kretzmann and Kretzmann, 1990], written before 1325), which contains interesting remarks on the genre (without actually presenting a full-fledged theory of *obligationes* — see [Spade, 1982]). Kilvington's remarks seem to have sparked renewed interest in the genre, especially among the Oxford Calculators, who then began to write prolifically on *obligationes*. The first of such treatises is Roger Swyneshed's treatise (≅ 1330-1335) [Spade, 1977], which indeed inaugurated a new trend within the genre, later to be named '*nova responsio*' by Robert Fland in his treatise (≅ 1350) [Spade, 1980c], as opposed to the '*antiqua responsio*' represented by the Burley-style form of *obligationes*. As for what exactly differentiates the *antiqua* from the *nova responsio*, this will be the core of the conceptual discussion to follow.[57]

Other British authors having written on *obligationes* in the decades following Swyneshed's treatises are (for the dates, I follow [Braakhuis, 1993]): Billingham (≅ 1350s) (cf. [Ashworth, 1985]), Martin of Alnwick (≅ 1350s), Richard Brinkley (≅ 1350s) [Spade, 1995], Ralph Strode (≅ 1360s) (cf. [Ashworth, 1993; Dutilh Novaes, 2006b]), an anonymous Mertonian [Kretzmann and Stump, 1985], and Richard Lavenham (later 14th century) [Spade, 1978]. Paul of Venice, who for our present purposes is counted among 'British' authors, has a long treatise on *obligationes* in the *Logica Magna* [Paul of Venice, 1988], which is heavily inspired by Strode's treatise, and a short chapter on *obligationes* in the *Logica Parva*. Among these, Martin of Alnwick, Richard Lavenham and, to some extent, Robert Fland, follow Swyneshed's *nova responsio* style of *obligationes*; the others remain by and large faithful to the *antiqua responsio*.

As for continental authors, currently we only know of six continental authors who wrote *obligationes* treatises: Albert of Saxony (≅ 1350) (cf. Braakhuis 1993), John of Wesel (≅ 1350) (cf. [Spade, 1996b]), William Buser (≅ 1355) (cf. [Keepkens, 1982; 1993; Pozzi, 1990] for the edited text), Marsilius of Inghen (just before 1360) (cf. [Keepkens, 1982, 159-160]), John of Holland (just after 1360) (cf. John of Holland 1985) and Peter of Candia (very end of 14th century) (cf. [Keepkens, 1982, 154]). The last two are thought to have studied in England, and therefore their exposure to the British *obligationes* literature can be taken for granted. As for the others: it has been argued convincingly that Albert of Saxony would have drawn significantly from Billingham's treatise, or in any case from the chapter on *obligationes* of the general manual *Logica oxoniensis*, which in turn is basically Billingham's text (see [Ashworth, 1985; Braakhuis, 1993]). As for William Buser and Marsilius of Inghen, it is certain that they would have had direct contact with Albert and his *obligationes*; Buser's text resembles Albert's in many aspects, and Marsilius's text in turn is visibly inspired by his master Buser's text (cf. [Keep-

[57] Notice though that these terms, '*antiqui*' and '*moderni*', are not consistently used by our authors; given the natural flow of generations, those that are referred to as '*moderni*' often end up being referred to as '*antiqui*' in subsequent generations (cf. [Spade, 1980, 42; Pozzi, 1990, 17, fn. 25]). In any case, in this section I will use the term '*nova responsio*' to refer to Swyneshed's style and '*antiqua responsio*' to refer to Burley's style of *obligationes*.

kens, 1982]). Concerning John of Wesel (in Paris in 1344-1353), of whom little is known, Spade argues that his text shows 'a close familiarity with the writings of Oxford logicians from the first half of the fourteenth century, in particular with those of Roger Swyneshed' [Spade, 1996b, 3]). Therefore, we may conclude that all these continental *obligationes* treatises of the 14$^{\text{th}}$ century were directly or indirectly under the influence of the British literature on the topic.

And as for the *antiqua responsio* vs. *nova responsio* dichotomy with respect to the continental authors, it seems that, while all of them were aware of the innovations introduced by Swyneshed, they were mostly critical of them. John of Wesel seems to be sympathetic to some of Swyneshed's views (cf. [Spade, 1996b]); as for Peter of Candia, in spite of the attribution by some commentators of the qualification of *modernus* to him (cf. [Keepkens, 1982, 154]), from the description of his text in [Pozzi, 1990, 55] it becomes apparent that he was not a *modernus* with respect to relevance, which is the main aspect considered here.[58] All the other authors clearly side with the *antiqui*.

In what follows, I concentrate on Burley's and Swyneshed's treatises, as they are the most famous treatises of each of these trends (it is in fact a condensed version of [Dutilh Novaes, 2005a; 2006a]). Moreover, Burley's treatise has received a partial translation into English [Burley, 1988], and it seems reasonable to focus on a text that can be consulted also by those who do not read Latin.

In his treatise, Burley describes six kinds of *obligationes*: *petitio, sit verum, institutio, positio, depositio* and *dubitatio*. Swyneshed, by contrast, recognizes only three kinds: *positio*, *depositio* and *impositio* (Burley's *institutio*). We shall be focusing on *positio*, as it is arguably the most representative form of *obligationes* (even though *impositio* in particular also offers the opportunity for very interesting semantic analysis).

4.2 Burley's treatise: antiqua responsio

The disputation has two participants, Opponent and Respondent. In the case of *positio*, the game starts with Opponent putting forward a proposition, called *positum*, which Respondent must accept as true for the sake of the disputation, unless it is self-contradictory. Opponent then puts forward propositions (the *proposita*), one at a time, which Respondent must either concede, deny or doubt, on the basis of inferential relations with the previously accepted or denied propositions, or, in case there is none (and these are called irrelevant or impertinent propositions[59]), on the basis of the common knowledge shared by those who are present. Respondent loses the disputation if he concedes a contradictory set of propositions. At

[58] Unfortunately, this text is still only available in the form of manuscripts (S. Brown is reportedly preparing and edition of the text), and I have not had the occasion of examining the manuscripts myself.

[59] Throughout the text, I use the terms 'relevant' and 'pertinent' as synonymous, as much as 'irrelevant' and 'impertinent'. The terms in Latin are '*pertinens*' and '*impertinens*', but they are often translated as 'relevant' and 'irrelevant', for example in the translation of Burley's treatise.

the end, Opponent and possibly a jury determine whether Respondent responded well, in particular if he was able to keep consistency

I have argued elsewhere that Burley's *obligationes* is best seen as a logical game of consistency maintenance (cf. [Dutilh Novaes, 2005a]): here, I will follow this basic idea.[60]

4.2.1 Reconstruction

DEFINITION 1 (The obligational game (Burley)).

$$\boldsymbol{Ob} = \langle K_C, \Phi, \Gamma, R(\phi_n)\rangle$$

$\mathbf{K}_C$ is the common state of knowledge of those present at the disputation complemented by the *casus*. The *casus* was usually a proposition to be assumed as true, often to make it explicit that the *positum* was false. An example from Burley: "Suppose Socrates is black, and suppose it is posited that Socrates is white" [Burley, 1988, 378]. That Socrates is black is the *casus*, a proposition which all the participants are to assume to be true, and 'Socrates is white' is the *positum*.[61]

$\mathbf{K}_C$ is an incomplete model, since some propositions do not receive a truth-value in it: for some propositions, it is not known whether they are true or false, although it may be known that they are true-or-false (these must be doubted — cf. [Burley, 1988, 381]). So, the state of common knowledge is a state of imperfect information: it includes all information that is considered common sense (that the pope is in Rome, all religious dogmas etc.), plus information circumstantially available, due to the pragmatics of the disputational situation, plus the *casus*.

Φ is an ordered set of propositions. It is the set of propositions actually put forward by O(pponent) during an obligation. Each element of Φ is denoted by 'ϕ_n', where n is a natural number, denoting the place of $\phi_n$in the ordering. The order corresponds to the order in which the propositions are put forward by O, starting with ϕ_0 (the *positum*).

Γ is an ordered set of sets of propositions, which are formed by R(espondent)'s responses to the various ϕ_n. How each Γ_n is formed will be explained below. The ordering is such that Γ_n is contained in Γ_{n+1}.

$\mathbf{R}(\phi_n)$ is a function from propositions to the values 1, 0, and ?. This function corresponds to the rules R must apply to respond to each proposition ϕ_n. 1 corresponds to his accepting ϕ_n, 0 to his denying ϕ_n and ? to his doubting ϕ_n (cf. [Burley, 1988, 381]).

The **procedural rules** of the game are quite simple: O first puts forward a proposition. If R accepts it (according to $\mathrm{R}(\phi_0)$ defined below), then the game

[60] In fact, even though the game-interpretation is to my mind the one that best explains the data, how to interpret obligationes in terms of modern concepts is still an on-going issue. Other proposals have been put forward, such as *obligationes* as a logic of counterfactuals, *obligationes* as thought-experiments, *obligationes* as belief-revision, among others. I have discussed each of these proposals in [Dutilh Novaes, 2007, 3.2].

[61] For more on the notion of *casus*, see [Yrjönsuuri, 1993].

begins. Then O puts forward a further proposition, R responds to it according to $R(\phi_n)$, and this procedure is repeated until the end of the game.

The **logical rules** of the game are defined by $R(\phi_n)$, in the following way:

DEFINITION 2 (Rules for *positum*).

$$R(\phi_0) = 0 \text{ iff } \phi_0 \Vdash {}^{62}\bot$$
$$R(\phi_0) = 1 \text{ iff } \phi_0 \nVdash \bot$$

The rule defining the response that R should give to ϕ_0 (the *positum* — [Burley, 1988, 378]) has interesting consequences for the idea that *obligationes* are games of consistency maintenance. If R is obliged to accept at the beginning a proposition that entails a contradiction — for example, any paradoxical proposition such as Liar sentences and the like — then there is no possible winning strategy for R. He cannot maintain the consistency of a set of propositions that, from the outset, contains a contradictory paradoxical proposition. So the rules of the game stipulate that there always be a winning strategy for R, starting from this restriction upon the *positum*. Burley expresses this clause by saying that it must be in the Respondent's power to satisfy the requirement (of not falling in contradiction) (cf. [Burley, 1988, 376]).

DEFINITION 3 (Rules for propositа).

$$R(\varphi_n) = 1 \text{ iff } \left\} \begin{array}{l} \Gamma_{n-1} \Vdash \phi_n, \text{ or} \\ \Gamma_{n-1} \nVdash \phi_n, \Gamma_{n-1} \nVdash \neg\phi_n \text{ and } {K_c}^{63} \Vdash \phi_n \end{array} \right.$$

$$R(\phi_n) = 0 \text{ iff } \left\} \begin{array}{l} \Gamma_{n-1} \Vdash \neg\phi_n, \text{ or} \\ \Gamma_{n-1} \nVdash \phi_n, \Gamma_{n-1} \nVdash \neg\phi_n \text{ and } K_C \Vdash \neg\phi_n \end{array} \right.$$

$$R(\phi)n) =? \text{ iff } \Gamma_{n-1} \nVdash \phi_n, \Gamma_{n-1} \nVdash \neg\phi_n, K_C \nVdash \phi_n, K_C \nVdash \neg\phi_n$$

That is, if Respondent fails to recognize inferential relations and if he does not respond to a proposition according to its truth-value within common knowledge, then he responds badly (cf. [Burley, 1988, 381]).

Formation of Γ_n. The different sets of propositions accepted by R (i.e., the propositions to which R has committed himself in the disputation) are formed in the following way:

DEFINITION 4 (The sets Γ_n).

If $R(\phi_n) = 1$, then $\Gamma_n = \Gamma_{n-1} \cup \{\phi_n\}$

[62] I use the forcing turnstyle $\Vdash$ throughout to express the relation of semantic implication between propositions. That is, within obligationes the relation of 'following' is not defined syntactically or proof-theoretically, but rather semantically.

[63] In case of K_C, it is not so much that K_C semantically implies a proposition ϕ_n, but rather that ϕ_n is contained in K_C (therefore a fortiori K_C also implies ϕ_n). For the sake of simplicity, I use only the forcing turnstyle.

If $R(\phi_n) = 0$, then $\Gamma_n = \Gamma_{n-1} \cup \{\neg\phi_n\}$
If $R(\phi_n) =?$, then $\Gamma_n = \Gamma_{n-1}$
In particular, if $\mathrm{R}(\phi_0) = 1$, then $\Gamma_0 = \{\phi_0\}$. If $\mathrm{R}(\phi_0) = 0$ or $\mathrm{R}(\phi_0) = ?$, then there is no disputation.

These rules mirror closely the clauses of Lindenbaüm's lemma, the main idea being that propositions are gradually added to a set of propositions (which starts with one single element, the *positum*), while consistency is also maintained. There is a significant difference, though, in that, in the construction of a maximal consistent set according to this lemma, if the set $\Gamma_n = \Gamma_{n-1} \cup \{\phi_n\}$ formed is inconsistent, then the construction simply continues with Γ_{n-1}, i.e. the so far largest consistent set built. In the *obligationes* framework, however, if an inconsistent set is constructed, the procedure comes to a halt, as respondent has responded badly and thus lost the game.

Outcome. O wins the game if it is recognized that $\Gamma_n \Vdash^{\perp}$; that is, if R has conceded a contradictory set of propositions. R wins the game if, when the disputation is declared to be over, it is recognized that $\Gamma_n \nVdash^{\perp}$. The clause about the stipulated time concerns the feasibility of the game: the construction of maximal-consistent sets of propositions is not feasible within human time, therefore respondent is expected to keep consistency only during a certain time.

4.2.2 Can respondent always win?

The rules of the obligational game as defined guarantee that there always be a winning strategy for R. This is due to two facts: one is a stipulated rule of the game and the other is a general logical fact.[64] The relevant rule of the game is: a paradoxical *positum* should not be accepted. As stated by Burley himself, the point of this clause is exactly to guarantee that R stands a chance to win. Therefore, R always starts out with a consistent set of propositions.

It is a general principle of logic (and also the backbone of Lindenbaüm's lemma) that any consistent set of propositions can always be consistently expanded with one of the propositions of a contradictory pair $\phi_n$and $\neg\phi_n$.[65] R starts with a consistent set of propositions (the set composed of the *positum*); so at each move, there is in theory at least one 'correct' way of answering, i.e. either accepting or denying ϕ_n, which maintains the set of accepted and denied propositions consistent.

4.2.3 Why does R not always win?

But why does the game remain hard? It is a fact that R often makes wrong choices and is not able to keep consistency, even though to keep consistency is

[64]This fact has already been noticed by J. Ashworth: 'a certain kind of consistency was guaranteed for any correctly-handled disputation' (Ashworth 1981, 177).

[65]**Proof**: Assume that Γ is consistent. Assume that $\Gamma \cup \{\phi\}$ is inconsistent. Thus $\Gamma \Vdash \neg\phi$ (1). Moreover, assume that $\Gamma \cup \{\neg\phi\}$ is inconsistent. Thus $\Gamma \Vdash \phi$ (2). From (1) and (2), it follows that $\Gamma \Vdash \phi\&\neg\phi$, that is, that Γ is inconsistent, which contradicts the original assumption. The principle to be proven follows by contraposition.

always logically possible. If it were easy, then it would not fulfill its pedagogical and theoretical purposes. It seems that *obligationes* remains a difficult kind of disputation for Respondent for two basic reasons: Opponent makes use of intricacies (for example, the phenomena of synonymy and equivocation) of the language being used in the game to set up 'traps' for Respondent; and the game is essentially dynamic.

4.2.3.1 Intricacies of the language To have a glimpse of the kind of trap Opponent may set up, take a look at the hypothetical disputation represented in the scheme below (it is not an example taken from Burley's text, but it is very much in the spirit of the examples he proposes).

Proposition	Calculation	Verdict	Outcome
ϕ_0: **You are in Rome or you are the Pope.**	Possible	Conceded	$\Gamma_0 = \{\phi_0\}$
ϕ_1: **You are in Rome.**	$\Gamma_0 \nVdash \phi_1$, $\Gamma_0 \nVdash \neg\phi_1$, $\mathrm{K}_C \Vdash \neg\phi_1$	Denied	$\Gamma_1 = \{\phi_0, \neg\phi_1\}$
ϕ_2: **The pope is in Rome.**	$\Gamma_1 \nVdash \phi_2$, $\Gamma_1 \nVdash \neg\phi_2$, $K_C \Vdash \phi_2$	Conceded	$\Gamma_2 = \{\phi_0, \neg\phi_1, \phi_2\}$
ϕ_3: **You are the pope.**	$\Gamma_2 \Vdash \phi_3$ (from ϕ_0 and $\neg\phi_1$)	Conceded	$\Gamma_3 = \{\phi_0, \neg\phi_1, \phi_2, \phi_3\}$
ϕ_4: **You are in Rome.**	$\Gamma_3 \Vdash \phi_4$ (from ϕ_2 and ϕ_3)	Conceded	$\Gamma_4 = \{\phi_0, \neg\phi_1, \phi_2, \phi_3, \phi_4\}$

But $\phi_1 = \phi_4$. So $\Gamma_4 \Vdash^{\perp}$.

Where did it go wrong? Why was Respondent forced both to accept and to deny ϕ_4 in the last round? Could he have avoided the trap? A closer inspection of the propositions shows that ϕ_2 is not irrelevant (i.e., with no inferential connections to the previously accepted or denied propositions), as it might seem at first. Actually, from ϕ_0 and $\neg\phi_1, \neg\phi_2$ follows: by negating the first disjunct, Respondent has already (logically) committed himself to the second disjunct, that he is the Pope. So if he is not in Rome and he is the Pope, then the Pope cannot be in Rome. So Respondent should deny ϕ_2, instead of accepting it as a proposition irrelevant and true according to K_C, even though he has not explicitly granted ϕ_3 yet.

4.2.3.2 The game is dynamic Another source of difficulty in this game is its dynamic character. This is related to the inclusion of irrelevant propositions, accepted or denied according to K_C, in the set of propositions that will be used to respond to each *propositum* still to come. Burley himself attracts the reader's attention to this point: "One must pay special attention to the order [of the propositions]." [Burley, 1988, 385]

This means that, during a disputation, it may occur that (1) $\phi_0, \phi_1 \Vdash \phi_2$ but $\phi_0, \phi_2 \nVdash \phi_1$, or else that (2) $\phi_0, \phi_1 \Vdash \phi_2$ but $\phi_0 \nVdash \phi_2$. (1) is related to the obvious asymmetric character of implication, and (2) to the dynamic nature of the game, what I shall call the 'expansion of the informational base Γ_n'. This can be best seen if we examine what happens in terms of models during an obligational disputation. For that, here are some definitions:

DEFINITION 5. Γ_n= Informational base, i.e. a set of propositions.

DEFINITION 6. $\boldsymbol{UM}_n$ = The class of models that satisfy informational base Γ_n.

DEFINITION 7. $\boldsymbol{UM}\phi_n$ = The class of models that satisfy ϕ_n.

DEFINITION 8. $\boldsymbol{UM}_n \Vdash \Gamma_n$ iff $\boldsymbol{UM}_n \Vdash$ P for all P in Γ_n.

A model that satisfies a set of propositions satisfies each of them (i.e. they are all true in this model). It is clear that, if $\Gamma_k = \{\phi_n\} \cup \{\phi_m\}$, then $\boldsymbol{UM}_k = \boldsymbol{UM}\phi_n \cap \boldsymbol{UM}\phi_m$. So, the set of models that satisfy Γ_k is the intersection of all the models that satisfy each of the elements of Γ_k. Similarly, if $\Gamma_{n+1} = \Gamma_n \cup \{\phi_{n+1}\}$, then $\boldsymbol{UM}_{n+1} = \boldsymbol{UM}_n \cap \boldsymbol{UM}\phi_{n+1}$.

THEOREM 9. *If* $\Gamma_n \Vdash \phi_{n+1}$ *and* ϕ_{n+1} *is accepted, then* $\boldsymbol{UM}_n = \boldsymbol{UM}_{n+1}$.

Assume that, at a given state of the game, $\Gamma_n \Vdash \phi_{n+1}$. According to $\mathrm{R}(\phi_n), \phi_{n+1}$ must be accepted, forming $\Gamma_{n+1} = \Gamma_n \cup \{\phi_{n+1}\}$. Now take $\boldsymbol{UM}_n$, that is, all the models that satisfy Γ_n. According to the model-theoretic definition of implication (i.e. P $\Vdash Q$ iff Q is true in all models where P is true, that is, if $\boldsymbol{UM}_P \Vdash Q$), if $\boldsymbol{UM}_n \Vdash \Gamma_n$ and $\Gamma_n \Vdash \phi_{n+1}$, then $\boldsymbol{UM}_n \Vdash \phi_{n+1}$. Since $\Gamma_{n+1} = \Gamma_n \cup \{\phi_{n+1}\}$, $\boldsymbol{UM}_n \Vdash \Gamma_n$ and $\boldsymbol{UM}_n \Vdash \phi_{n+1}$, then $\boldsymbol{UM}_n \Vdash \Gamma_{n+1}$. It is defined that $\boldsymbol{UM}_{n+1} \Vdash \Gamma_{n+1}$, so $\boldsymbol{UM}_n = \boldsymbol{UM}_{n+1}$.

Thus, all the models that satisfy Γ_n also satisfy Γ_{n+1}.

THEOREM 10. *If* $\Gamma_n \nVdash \phi_{n+1}$ *and* ϕ_{n+1} *is accepted, then* $\boldsymbol{UM}_{n+1} \subset \boldsymbol{UM}_n$.

Assume that, at a given state of the game, $\Gamma_n \nVdash \phi_{n+1}$ and $\mathrm{K}_C \Vdash \phi_{n+1}$. According to $\mathrm{R}(\phi_n)$, ϕ_{n+1} must be accepted, forming $\Gamma_{n+1} = \Gamma_n \cup \{\phi_{n+1}\}$. $\boldsymbol{UM}_{n+1}$ is the intersection of $\boldsymbol{UM}_n$ and $\boldsymbol{UM}\phi_{n+1}$ ($\boldsymbol{UM}_{n+1} = \boldsymbol{UM}_n \cap \boldsymbol{UM}\phi_{n+1}$). But because $\Gamma_n \nVdash \phi_{n+1}$, $\boldsymbol{UM}_n \nVdash \phi_{n+1}$. So not all models that satisfy $\Gamma_n$also satisfy ϕ_{n+1}. Since $\Gamma_{n+1} = \Gamma_n \cup \{\phi_{n+1}\}$, not all models that satisfy Γ_n also satisfy Γ_{n+1}. Thus $\boldsymbol{UM}_{n+1} \neq \boldsymbol{UM}_n$. But Γ_n is contained in Γ_{n+1}, so all models that satisfy Γ_{n+1} also satisfy Γ_n – $\boldsymbol{UM}_{n+1} \Vdash \Gamma_n$. So $\boldsymbol{UM}_{n+1} \subset \boldsymbol{UM}_n$.

Thus, all the models that satisfy Γ_{n+1} are contained in the set of models that satisfy Γ_n.

Summing up; in an obligational game, $\boldsymbol{UM}_{n+1} \subseteq \boldsymbol{UM}_n$. If $\Gamma_n \Vdash \phi_{n+1}$, $\Gamma_n \Vdash \neg\phi_{n+1}$ or $\mathrm{R}(\phi_{n+1}) = ?$, then $\boldsymbol{UM}_n = \boldsymbol{UM}_{n+1}$, otherwise $\boldsymbol{UM}_{n+1} \subset \boldsymbol{UM}_n$. That is, the larger the informational base, the fewer models will satisfy it, and greater the constraints on the choice between $\neg\phi_n$ and ϕ_n will be (a model-theoretic way to see why a larger base implies that more propositions will have inferential relations with Γ_n). Clearly, the base is expanded (and therefore the range of models that satisfy it is reduced) only by inclusion of 'irrelevant' propositions.

4.3 Swyneshed's treatise: nova responsio

The main modification introduced by Swyneshed concerns the notion of pertinent propositions: while for Burley whether a *propositum* was pertinent was determined on the basis of all the previously accepted or denied propositions (*positum* and *proposita*), for Swyneshed being pertinent is a property of a *propositum* only with respect to the *positum*.[66] There were other aspects that were thought to characterize the *nova responsio* (such as the treatment of pragmatically inconsistent *posita* — cf. [Braakhuis, 1993, 334]), but here I focus on the redefinition of the notion of pertinent proposition and its corollaries (such as the behavior of conjunctions and disjunctions). As a result, while the main purpose of Burley's *obligationes* seems to be consistency maintenance, Swyneshed's *obligationes* seems to have as its main purpose that of inference recognition.[67]

4.3.1 Reconstruction

In Swyneshed's version, an obligation corresponds to the following quadruple:

DEFINITION 11 (The obligational game (Swyneshed)).

$$\boldsymbol{Ob} = \langle \Sigma, \Phi, I, R(\phi_n) \rangle$$

Σ is an ordered set of states of knowledge S_n. This is the first significant difference with respect to Burley's theory. In the latter, all irrelevant propositions were supposed to be answered to according to the static state of common knowledge K_C. Changes in things during the time of the disputation were not supposed to affect the response to (irrelevant) propositions, all the more since, once proposed and accepted or denied, these were included in the 'informational base' of the disputation. So, in Burley's theory, if, at a certain point, 'You are seated' is proposed to Respondent, and Respondent is indeed seated, he should accept the proposition. Subsequently, if Respondent stands up, and Opponent proposes 'You are not seated', Respondent should deny it, because it contradicts the set of previously accepted/denied propositions, and this logical relation has priority over reality.

In Swyneshed's theory, since irrelevant accepted or denied propositions are not included in the informational base of the disputation, as we shall see, the state of knowledge is not required to be static. So the response to irrelevant propositions, according to Swyneshed's theory, should take into account the changes in things during the time of the disputation; therefore, what we have is a series of states of knowledge S_n, ordered according to their index n, which is a natural number and corresponds to the stage of the disputation in which the state of common

[66] This fact has been acknowledged by virtually all studies on medieval *obligationes*, including [Stump, 1981; Ashworth, 1981; 1993; Spade, 1982b; Keffer, 2001] etc.

[67] In fact, from this point of view, the disagreement between Burley and Swyneshed may be viewed as a disagreement concerning the very purpose of *obligationes*, in terms of a student's training.

knowledge must come into play.[68]

Φ is an ordered set of propositions ϕ_n. (No difference here with respect to Burley's theory.)

I is an ordered set of responses $\iota_n = [\phi_n; \gamma]$. Responses are ordered pairs of propositions and one of the replies 1, 0 or ?, corresponding to Respondent's response to proposition ϕ_n.[69] Note that the index of the response need not be (but usually is) the same as the index of the proposition, in case the same proposition is proposed twice, in different moments of the disputation (in which case, for convenience, it is referred to by the index it received in the first time it was proposed).

$\mathbf{R}(\phi)$ is a function from propositions to the values 1, 0, and ?. This definition is identical to the definition of $\mathbf{R}(\phi)$ in the reconstruction of Burley's theory, but the **function** corresponding to the rules of Swyneshed's theory is different from the function of Burley's theory, since the rules are different.

4.3.2 Rules of the game

Swyneshed's **procedural rules** are quite simple (cf. [Spade, 1978, §72][70]), and identical to the procedural rules in Burley's theory. By contrast, the logical rules are quite different from Burley's.

Swyneshed's analysis of the requirements for a proposition to be accepted as *obligatum* (that is, the first proposition proposed, named *positum* in the specific case of *positio*) is less extensive than Burley's. Since an inconsistent *positum* gives no chance of success for Respondent, Burley clearly says that the *positum* mustn't be inconsistent. Swyneshed does not follow the same line of argumentation; rather, he requires that a proposition be contingent to be a *positum* (§ 73). This excludes impossible propositions — always false — and necessary propositions – always true –, and that is a necessary requirement in view of the *ex impossibili sequitur quodlibet* rule: if Swyneshed's rules of *obligationes* are indeed meant to test Respondent's abilities to recognize inferential relations, an impossible *obligatum* would make the game trivial (any proposition would follow).[71] Moreover, from a necessary proposition only necessary propositions follow, so if the *obligatum* is a necessary proposition, then the game becomes that of recognizing necessary

[68] But why use states of knowledge, and not simply states of affairs? Because (both in Burley's and Swyneshed's theories) proposed propositions whose truth-value is unknown to the participants of the disputation — for example, 'The Pope is sitting now' — should be accordingly doubted. We are dealing here with imperfect states of information.

[69] In my reconstruction of Burley's theory, responses were not primitive constituents of the game. But to express some of the interesting properties of Swyneshed's theory, the notion of responses is crucial.

[70] All references are to Spade's edition of the text. For the relevant passages and translations, see [Dutilh Novaes, 2006a].

[71] Notice that Swyneshed's reason for excluding impossible propositions is different from Burley's: trivialization of the game *versus* absence of a winning strategy for Respondent. Keffer [2001] has also remarked that impossible (and true) *posita* have a *Trivialisierungseffekt* on both kinds of responses, but for different reasons (pp. 158-164).

propositions, i.e. a deviation from its (presumed) original purpose.

So the rule for accepting the *positum* could be formulated as:

DEFINITION 12 (Rules for *positum*).

$R(\phi_0) = 0$ iff, for all moments n and m, and for one reply γ, $\iota_n = [\phi_0; \gamma]$ and $\iota_m = [\phi_0; \gamma]$.

$R(\phi_0) = 1$ iff, for some moments n and m, for two replies γ and κ, $\gamma \neq \kappa$, $\iota_n = [\phi_0; \gamma]$ and $\iota_m = [\phi_0; \kappa]$

Moreover, Swyneshed also gives instructions as to how to respond to the *positum* if it is **posited again** during the disputation (§§ 62-64). A *positum* which is re-proposed must be accepted, except in the cases of a *positum* which is inconsistent with the very act of positing, admitting and responding in an obligational context. The paradigmatic example is 'Nothing is posited to you': it should be accepted as a *positum*, according to the rules above, but if it is again proposed during the same disputation, it should be responded to as if it were an irrelevant proposition. In this case, it would be denied, even though it had been accepted as *positum*. Burley, by contrast, would probably not accept such pragmatically inconsistent propositions as *posita* in the first place (cf. [Braakhuis, 1993, 330]).

In effect, from the start, the set of all propositions (not only those put forward during the disputation, which constitute Φ) is divided in two sub-sets, namely the set of propositions that are **pertinent** with respect to the *positum* ϕ_0(§4, §7) — denoted $\Delta_{\phi 0}$- and the set of those that are **impertinent** with respect to the *positum* ϕ_0(§8) – denoted $\Pi_{\phi 0}$. The sets are defined as follows:

DEFINITION 13 (Pertinent and impertinent propositions).

$$\Delta_{\phi 0} = \{\phi_n \varepsilon \Delta_{\phi 0} : \phi_0 \Vdash \phi_n \text{ or } \phi_0 \Vdash \neg\phi_n\}$$
$$\Pi_{\phi 0} = \{\phi_n \varepsilon \Pi_{\phi 0} : \phi_0 \nVdash \phi_n \text{ and } \phi_0 \nVdash \neg\phi_n\}$$

Assuming that any proposition implies itself, the *positum* ϕ_0 belongs to $\Delta_{\phi 0}$.[72] E. Stump [1981, 167] mentions the possibility of allowing for a second *positum* at any given moment of the disputation. In this case, obviously the two sets defined above must be revised, and the set of pertinent propositions is defined by the conjunction of the two (or more) *posita*.

4.3.2.1 Proposita The rules for responding to proposed propositions other than the *positum* are better formulated in two steps, first for the pertinent, then for the impertinent propositions, as this division is in fact the decisive aspect of the game in Swyneshed's version.

DEFINITION 14 (Rules for *proposita*).

[72] Except for the *posita* that are (pragmatically) repugnant to the act of positing (cf. §64); according to Swyneshed, those should be answered to as if they were impertinent, thus belonging to $\Pi_{\phi 0}$.

Pertinent propositions ($\phi_n \neq \quad \phi_0$, $\phi_n \; \varepsilon \quad \Delta_{\phi 0}$) ((§24, §67, §68).

$$\mathbf{R}(\phi_n) = 1 \text{ iff } \phi_0 \Vdash \phi_n \; ^{73}\mathbf{R}(\phi_n) = 0 \text{ iff } \phi_0 \Vdash \neg\phi_n$$

Impertinent propositions ($\phi_n \; \varepsilon \Pi_{\phi 0}$) (§ 26, §69)

$$\mathbf{R}(\phi_n) = 1 \text{ iff } S_n \Vdash \phi_n$$
$$\mathbf{R}(\phi_n) = 0 \text{ iff } S_n \Vdash \neg\phi_n$$
$$\mathbf{R}(\phi_n) =? \text{ iff } S_n \nVdash \phi_n \text{ and } S_n \nVdash \neg\phi_n$$

The game ends when Opponent says '*Cedat tempus obligationis'*. From Swyneshed's text, it seems that Opponent can say it at any time; he will say it when Respondent has made a bad move, and thus has lost the game (§98), but he may say it when he is satisfied with the performance of Respondent, who until then has not made any bad move, and therefore has 'won' the game.

4.3.3 Logical properties of nova responsio

Now I discuss some of the noteworthy logical properties of *nova responsio*, in particular those related to the reformulation of the notion of impertinent proposition.

4.3.3.1 The game is fully determined It is clear that, in Swyneshed's *obligationes*, only one answer to each proposition is correct at a given point. That this is the case is seen from the fact that R(ϕ) really is a function, assigning exactly one value to each argument of its domain (the class of propositions). Swyneshed's rules divide the class of propositions in two sets and in five sub-sets: pertinent propositions — 1. repugnant to or 2. following from the *positum* — and impertinent propositions — 3. which are known to be true; 4. which are known to be false; 5. which are not known to be true and are not known to be false. These five subsets exhaust the class of propositions, and for each of them there is a defined correct answer. In other words, at each stage of the disputation, Respondent's moves are totally determined by the rules of the game.

4.3.3.2 The game is not dynamic The game played according to the *antiqua responsio* is, as we have seen, dynamic in that the player must take into account all previous moves of the game in their corresponding order. By contrast, the game played according to the *nova responsio* is 'static': the response to a proposition is entirely independent of the order in which it occurs during the disputation, as it is entirely independent of all previous moves except for the first one, relative to the *positum*. In effect, for any proposition ϕ_n, at any round n of the disputation, the reply to ϕ_n is always the same.

[73] Clearly, if the introduction of extra *posita* occurs, then this definition holds for the set of *posita*, instead of for the first *positum* only.

Indeed, the great difference with respect to Burley's theory is that, in Swyneshed's version, the game is totally determined already once the *positum* has been posited, from the start, and not only at each move. All Respondent has to do is to determine correctly the two sets of pertinent and impertinent propositions from the outset. Opponent can do nothing to interfere with Respondent's winning strategy, as it simply consists of assessing correctly the presence or absence of relations of inference between the *positum* and the proposed propositions. Once more, the fact that the game is totally determined from the moment the *positum* is posited means that the order of presentation of the *proposita* does not matter, and that Opponent cannot do much to make the game harder for Respondent. Moreover, it also means that, during a disputation, only one response is the right one for a given proposition, independent of when it is proposed. In Burley's game, it can happen that a proposition is first doubted (as impertinent and unknown) and then accepted or denied (it has become pertinent in the meantime, given the expansion of the informational base). This cannot occur in Swyneshed's game.

There is one exception to this rule: impertinent propositions whose truth-values change during the course of the disputation. Swyneshed says that these propositions should be responded to according to the state of knowledge of that moment, and therefore the response depends on the moment in which they are proposed — but not on the moment **within the disputation** in which they are proposed (their relative position with respect to other propositions). Similarly, if such propositions are proposed twice during the same disputation, they may receive different answers, as a consequence of a change in things.

4.3.3.3 Two disputations with the same *positum* will prompt the same answers, except for variations in things. This is perhaps the main motivation for the changes introduced by Swyneshed to the obligational game. In many passages, he emphasizes that the response to impertinent propositions must vary only in virtue of changes in things, and not in virtue of other previously accepted/denied propositions. Indeed, the crucial element of a winning strategy for Swyneshed's game is the accurate definition of the two sets of propositions relative to a *positum* (the set of pertinent propositions and the set of impertinent ones).

So, if the game is defined once the *positum* is posited, then any two disputations with the same *positum* have **the same winning strategy**, that is, the establishment of the same two sets of pertinent and impertinent propositions. Since the propositions proposed by Opponent may vary, two disputations with the same *positum* will not necessarily be identical. But any given proposition proposed in both disputations will belong to the same set of propositions — either pertinent or impertinent — in both cases.

Again, the dissimilarity with Burley's theory is striking. In Burley's version of the game, the *positum* was merely one of the propositions constituting the set according to which a proposed proposition was to be evaluated as pertinent or impertinent (the others being the previously accepted/denied propositions). So in two disputations having in common only the *positum*, a given proposition proposed

in each of them was most likely bound to receive different responses.

4.3.3.4 Responses do not follow the usual properties of the connectives

One of the most discussed aspects of the *nova responsio*, not only among medieval authors but also among modern commentators, is the non-observance of the usual behavior of some sentential connectives, in particular conjunction and disjunction. This is a corollary of the basic rules of the *nova responsio*, and it was thought to be one of its distinctive traits.[74]

Apparently, these two corollaries have struck some of Swyneshed's contemporaries as very odd, and were for them sufficient reason to reject the *nova responsio* as a whole. However, in careful inspection, it is only in appearance that two of the most fundamental laws of logic — the truth-conditions of conjunction and disjunction – are being challenged. As Yrjönsuuri suggested [Yrjönsuuri, 1993, 317], it is as if the bookkeeping of a Swyneshed-style obligational disputation featured two columns of responses, one for pertinent propositions and one for impertinent propositions. **Within each column**, the laws for conjunction and disjunction are in effect observed. So, if in one of the columns two propositions have been correctly granted, then their conjunction will also be granted (disregarding changes in things); similarly, if in one of the columns a disjunction has been correctly granted, then at least one of the disjuncts will have to be granted too.[75] This fact only emphasizes the idea that the crucial aspect of playing a Swyneshed-style game of *obligationes* is the correct division between pertinent and impertinent propositions.[76]

4.3.3.5 The set of accepted/denied propositions can be inconsistent

Perhaps the most surprising feature of Swyneshed's *obligationes* is the little importance attributed to consistency maintenance. That is, if one takes the set of all propositions granted and the contradictories of all propositions denied during a disputation, this set is very likely to be **inconsistent**, and this feature struck many medieval authors as very odd (cf. [Keffer, 2001, pp. 164-166]). There are two main sources of inconsistency in Swyneshed's game: the most obvious one is the case of impertinent propositions which receive two different responses in different times of the disputation (in particular if they are first denied and then accepted or vice-versa), in virtue of changes that occurred in things during the time of the disputation. The second source of inconsistency for this set is the behavior of conjunctions and disjunctions explained above (§101).

But again, the bookkeeping metaphor implies that this corollary is not as awkward as it seems. Since the set of propositions that follow from a proposition is

[74] Cf. [Stump, 1981, 139]. For formal proofs of these properties, see [Dutilh Novaes, 2006a; Keffer, 2001, pp.176-178].

[75] For simplicity, I am disregarding impertinent propositions whose truth-value may change during the disputation.

[76] For a discussion of the apparent conflict arising in cases in which a conjunction or disjunction is formed by propositions taken from both columns, see [Dutilh Novaes, 2006a].

always consistent, the column for pertinent propositions will always be consistent, — for each contradictory pair of propositions $(A, \neg A)$, a given proposition B implies either one of them $(B \rightarrow A)$, or the other $(B \rightarrow \neg A)$, or none, but never both contradictory propositions. Therefore, it will never be the case that a *positum* forces the granting of a proposition A and its contradictory $\neg A$. By contrast, the column for impertinent propositions can very well be inconsistent, in the case of impertinent propositions whose truth-value changes during the disputation and which are in fact proposed twice (and receive different responses).

These considerations indicate thus that Swyneshed has no interest whatsoever in the set formed by all granted and denied propositions during a disputation, and that he is perfectly willing to accept its inconsistency. For Burley, on the contrary, the ultimate goal of the *obligationes* game is to keep this very set consistent. So the differences between the two versions of the game do not only regard the rules governing them, but seemingly the very motivations for playing the game.

4.4 Conclusion

This section is not intended to give a comprehensive picture of the *obligationes* genre; the topic is much more complex than what can be covered in just a few pages. My goal was just to outline some of the interesting logical properties of this genre so as to inspire the reader to go look for further literature on the topic. Some significant aspects of *obligationes* had to be left out for reasons of space; in particular, the sophisms (i.e. logical puzzles) treated in the *obligationes* literature are particularly interesting, especially those related to self-reference and to pragmatic inconsistencies — see for example [Ashworth, 1993; Pironet, 2001]

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MEDIEVAL MODAL THEORIES AND MODAL LOGIC

Simo Knuuttila

The main ancient sources of early medieval logic were Boethius's translations of Aristotle's *Categories* and *De interpretatione* and Porphyrius's introduction to the *Categories* (*Isagoge*), Boethius's commentaries on these works and on Cicero's *Topica*, as well as a number of other logical treatises by Boethius, including *De syllogismis categoricis*, *De hypotheticis syllogismis*, *De topicis differentiis*, and *De divisione*.[1] Some historians, following medieval usage, call early medieval logic based on these sources the old logic, as distinct from the new logic the textual basis of which was extended by twelfth-century recovery of the rest of Aristotle's *Organon* (*Prior Analytics*, *Topics* and *On Sophistic Refutations*, translated by Boethius, and *Posterior Analytics*, translated by James of Venice in the second quarter of the twelfth century).[2] The elements of distinctly medieval logic and semantics had been introduced since the twelfth century in what was called

[1]The translation of the *Categories* is edited by L. Minio-Paluello in *Aristoteles Latinus* I.1-5 (Bruges-Paris: Desclée de Brouwer, 1961) and the translation of the *De interpretatione* by L. Minio-Paluello in *Aristoteles Latinus* II.1-2 (Bruges-Paris: Desclée de Brouwer, 1965). The translation of Porphyry's *Isagoge* is edited by L. Minio-Paluello and B.G. Dod in *Aristoteles Latinus* I.6-7 (Bruges-Paris: Desclée de Brouwer, 1966). Boethius's commentary on Aristotle's *Categories* is found in *Patrologia Latina* (ed. J.-P. Migne), 64, 159-294, *De syllogismis categoricis* in *Patrologia Latina* 64, 793-832, *In Ciceronis Topica* in *Patrologia Latina* 64, 1039-1169; an English translation with notes and an introduction by E. Stump, and *De differentiis topicis* in *Patrologia Latina* 64, 1174-1216; an English translation with notes and essays by E. Stump, *Boethius's* De topicis differentiis (Ithaca and London: Cornell University Press, 1978), *Boethius's* In Ciceronis Topica (Ithaca and London: Cornell University Press, 1988). Boethius's two commentaries on *De interpretatione* are edited by C. Meiser (Leipzig: Teubner, 1877-1880), his two commentaries on Porphyry's *Isagoge* by S. Brandt, Corpus Scriptorum Ecclesiasticorum Latinorum 48 (Leipzig: G. Freytag, Vienna: F.Tempsky, 1906), *De hypotheticis syllogismis* by L. Obertello (Brescia: Paideia, 1969) and *De divisione* by J. Magee, with a translation and commentary (Leiden: Brill, 1998). Some other ancient works were also used, such as Apuleius's *Peri hermeneias* (ed. C. Moreschini in *Opera* III (Stuttgart: Teubner, 1991), Augustine's *Dialectica*, trans. with notes by B.D. Jackson from the text edited by J. Pinborg, Synthese Historical Library 16 (Dordrecht: Reidel, 1975), and pseudo-Augustinean *De decem categoriis*, ed. L. Minio-Paluello, in *Aristoteles Latinus* I.1-5, 133-75. For the sources of eleventh and twelfth century logic, see J. Marenbon, 'Medieval Latin Commentaries and Glosses on Aristotelian Logical Texts, Before c. 1150 AD' in Ch. Burnett (ed.), *Glosses and Commentaries on Aristotelian Logical Texts* (Warburg Institute Surveys and Texts 23), (London: Warburg Institute, 1993), 77-127, republished and revised in J. Marenbon, *Aristotelian Logic, Platonism, and the Context of Early Medieval Philosophy in the West*, Variorum Collected Studies Series (Aldershot: Ashgate, 2000).

[2]*Analytica priora*, ed. L. Minio-Paluello, *Aristoteles Latinus* III.1-4 (Bruges-Paris: Desclée de Brouwer, 1962), *Analytica posteriora*, ed. L. Minio-Paluello and B.G. Dod, *Aristoteles Latinus* IV.1-4 (Bruges-Paris: Desclée de Brouwer, 1968), *Topica*, ed. L. Minio-Paluello and B.G.

modern logic (*logica moderna*) and some historians call terminist logic because of its concentration on an analysis of the properties of terms, such as signification, supposition, appellation, and ampliation. This trend was also associated with the rise of some new genres in logic: the treatises on sophisms, which were influenced by Aristotle's *On sophistic refutations*, insoluble propositions, syncategorematic terms, and the logic of 'obligational' disputations.[3] The subjects of the old and new logic were combined with those of terminist logic in many early and mid-thirteenth century introductions and compendia, the most influential of which proved to be Peter of Spain's *Tractatus*.[4]

Ancient modal theories were extensively dealt with in Boethius's two commentaries on *De interpretatione*, early medieval authors being acquainted with Aristotelian and other ancient modal paradigms through these works before the later Aristotle reception. The modal themes which were brought into the discussion by the new logic were mainly the modal syllogistics and the conversion of modal propositions which formed part of it. Early medieval theories of modality were also influenced by Augustine's ideas about divine power and freedom which deviated from the philosophical assumptions of ancient views of possibility. There were analogous discussions of ancient philosophical theories and their relationship to divine modalities in Arabic philosophy. Arabic modal theories influenced Latin discussions mainly through the translations of the works of Averroes.[5]

Dod, *Aristoteles Latinus* V.1-3 (Bruges-Paris: Desclée de Brouwer, 1969), *De sophisticis elenchis*, ed. B.G. Dod, *Aristoteles Latinus* VI.1-3 (Bruges-Paris: Desclée de Brouwer and Leiden: Brill, 1975). See also S. Ebbesen, 'Echoes of the *Posterior Analytics* in the Twelfth Century' in M. Lutz-Bachmann, A. Fidora, P. Antolic (eds.), *Erkenntnis und Wissenschaft. Probleme der Epistemologie in der Philosophie des Mittelalters* (Berlin: Akademie Verlag, 2004), 69-92; id., *Commentators and Commentaries on Aristotle's Sophistici Elenchi*, Corpus Latinum Commentariorum in Aristotelem Graecorum VII.1-3 (Leiden: Brill, 1981); id., 'Medieval Latin Glosses and Commentaries on Aristotelian Logical texts of the Twelfth and Thirteenth Centuries' in Ch. Burnett (ed., 1993), 129-77; N.J. Green-Pedersen, *The Tradition of the Topics in the Middle Ages* (Munich: Philosophia Verlag, 1984).

[3] See L.M. de Rijk, *Logica Modernorum. A Contribution to the History of Early Terminist Logic I: On the Twelfth Century Theories of Fallacy, II.1-2: The Origin and Early Development of the Theory of Supposition*, Wijsgerige teksten en studies 6, 16 (Assen: van Gorgum, 1962, 1967); H.A.G. Braakhuis, *De 13de eeuwse tractaten over syncategorematische termen. Inleidende studie en uitgave van Nicolaas van Parijs' Sincategoreumata*, 2 vols (Meppel: Krips Repro, 1979); S. Read (ed.), *Sophisms in Medieval Logic and Grammar*, Nijhoff International Philosophy Series 48 (Dordrecht: Kluwer, 1993); M. Yrjönsuuri (ed.), *Medieval Formal Logic: Obligations, Insolubles and Consequences*, The New Synthese Historical Library 49 (Dordrecht: Kluwer, 2001), P. Spade, *The Mediaeval Liar: A Catalogue of the Insolubilia-Literature* (Toronto: The Pontifical Institute of Mediaeval Studies, 1975); id., *Thoughts, Words and Things: An Introduction to Late Mediaeval Logic and Semantic Theory*, `http://pvspade.com.Logic/`.

[4] *Tractatus called afterwards Summule logicales*, ed. L.M. de Rijk, Wijsgerige teksten en studies 22 (Assen: Van Gorcum, 1972).

[5] For medieval Arabic Logic, see T. Street, 'Arabic Logic' in J. Woods and D.M. Gabbay (eds.), *Handbook of the History of Logic* I (Amsterdam: Elsevier, 2004), 471-556; for modalities in Arabic philosophical theology, see T. Kukkonen, 'Possible Worlds in the *Tahâfut al-falâsifa.* Al-Ghâzalî on Creation and Contingency', *Journal of the History of Philosophy* 38 (2000), 479-502; A. Bäck, 'Avicenna and Averroes: Modality and Theology' in T. Buchheim, C.H. Kneepkens and K. Lorenz (eds.), *Potentialität und Possibilität. Modalaussagen in der Geschichte der Metaphysik* (Stuttgart and Bad Canstatt: Frommann-Holzbook, 2001), 125-45.

In Section (1) I shall deal with some influential medieval interpretations of modal concepts from Boethius to Aquinas. I do not mean that these were shared by everybody before Aquinas and by nobody after him. Since the eleventh century recovery of philosophical modal theories, Boethian formulations of central ancient conceptions had often been considered congenial, but were also qualified by theological considerations. These discussions were accompanied by some attempts to redefine modal concepts using the idea of alternativeness. While all these trends influenced mid-thirteenth century discussions, many thinkers were particularly interested in interpreting modal terms in the light of Aristotelian essentialism. This approach is also found in Robert Kilwardby's commentary on Aristotle's *Prior Analytics* (c. 1240) which became the standard thirteenth-century textbook for modal syllogistics. Many thirteenth-century paradigms lost their significance in early fourteenth-century discussions of modal theory, as will be shown in section (3). Before this, however, I shall discuss the details of earlier logical modal theories including modal syllogistics in section (2). Section (3) is about fourteenth-century developments and section (4) about medieval theories of applied modalities.

1 GENERAL SEMANTIC PARADIGMS FROM BOETHIUS TO THOMAS AQUINAS

1.1 Extensional Interpretation of Modality

In the introductory remarks of his commentary on Chapter 9 of Aristotle's *Peri hermeneias*, Thomas Aquinas classifies various types of propositions on the basis of their 'matter':

> In necessary matter all affirmative propositions are determinately true; this holds for propositions in the future tense as well as in the past and present tenses; and negative ones are false. In impossible matter the contrary is the case. In contingent matter, however, universal propositions are false and particular propositions are true. This is the case in future tense propositions as well as those in the past and present tenses. In indefinite ones, both are at once true in the future tense propositions as well as those in the past and present tenses. (*In Peri herm.* I.13, 168 [5], trans. Oesterle, with changes)[6]

The matter of a proposition is associated with the habitude of a predicate to a subject and is explained as follows:

> If the predicate is *per se* in the subject, it will be said to be a proposition in necessary or natural matter, for example 'Man is an animal'

[6] *In Aristotelis libros Peri Hermeneias et Posteriorum analyticorum expositio*, ed. R. Spiazzi (Turin: Marietti, 1964); *Aristotle: On Interpretation. Commentary by St. Thomas and Cajetan*, J.T. Oesterle, Mediaeval Philosophical Texts in Translation 11 (Milwaukee, Wisc.: Marquette University Press, 1962).

> and 'Man is risible'. If the predicate is *per se* repugnant to the subject, as in a way excluding the notion of it, it is said to be a proposition in impossible or remote matter, for example 'Man is an ass'. If the predicate is related to the subject in a way midway between these two, being neither *per se* repugnant to the subject nor *per se* in it, the proposition is said to be in possible or contingent matter. (*In Peri herm.* I.13, 166 [3], trans. Oesterle, with changes)

Aquinas employs the terms *enunciatio* and *propositio* as synonyms and takes them to mean statement making sentences. For reasons of simplicity, I shall use the term 'proposition' for these and related terms in medieval authors.[7]

The ancient theory of the matter of propositions was often associated with the rules of contraries, subcontraries and contradictories in the traditional square of opposition. While these rules defined how the members of various opposed pairs were related to truth and falsity, it was thought that they could be further specified by classifying propositions on the basis of their matter. An interesting feature in Aquinas's account of the contingent matter is that universal affirmative and negative propositions are false and particular affirmative and negative propositions are true. Comparing this with what is said about propositions in other matters, modal differences can be characterized as corresponding to a descending order in the frequency of true cases: the predicate is not truly said of any subject in impossible matter, it is truly said of some subjects in contingent matter and of all subjects in necessary matter. Aquinas's formulations are possibly influenced by Ammonius's commentary on Aristotle's *Peri hermenias*, translated into Latin by William of Moerbeke in 1268, but the ancient theory of the matter of propositions was also known through the works of Boethius and found in many Latin authors before the translation of Ammonius's work.[8]

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[7]For medieval terminology, see G. Nuchelmans, *Theories of the Proposition. Ancient and Medieval Conceptions of the Bearers of Truth and Falsity* (Amsterdam: North-Holland, 1973).

[8]Ammonius, *Commentaire sur le Peri hermeneias d'Aristote. Traduction de Guillaume de Moerbeke*, ed. G. Verbeke, Corpus Latinum Commentariorum in Aristotelem Graecorum (Louvain: Publications Universitaires, Paris: Béatrice-Nauwelaerts, 1961), 168.41-169.57; 175.69-176.95; 178.27-32; 179.38-43; 180.61-181.82; 199.23-201.63; the Greek text is edited by A. Busse in Commentaria in Aristotelem Graeca 4.5 (Berlin, 1897). Boethius was acquainted with the same doctrine which Ammonius explains, though he does not use the same terminology. Ammonius's work was not known to Boethius; see R. Sorabji, 'The Tree Deterministic Arguments Opposed by Ammonius', in Ammonius, *On Aristotle:* On Interpretation 9, trans. D. Blank, with Boethius, *On Aristotle:* On Interpretation 9, first and second Commentaries, trans. N. Kretzmann, with Essays by R. Sorabji, N. Kretzmann and M. Mignucci (London: Duckworth 1998), 3-15. In dealing with the opposition between the pairs of universal and particular propositions, Boethius distinguishes between predications which are natural (necessary), impossible and neither natural nor impossible. While universal affirmative and negative contraries of the third case are both false, the corresponding particular propositions are both true. A universal affirmative proposition is true in the first case and a universal negative proposition in the second case (*In Periherm.* II, 177.18-178.8; see also 303.15-306.13; 325.8-15.) Garland the Computist explains the same classification, distinguishing between propositions with natural, remote and impossible matter (*Dialectica*, ed. L. M. de Rijk, Wijsgerige teksten en studies 3 (Assen: van Gorcum, 1959), 54.21-30; 82.25-30.) In a late eleventh century logical treatise attributed to William of Cham-

Some of those who discussed the matter of propositions did not maintain that both of the particular sub-contraries are true in contingent matter, apparently thinking that it was somewhat strange to exclude the possibility, say, that at a certain time no human being is sitting. Those who did not find this problematic could think that universal and particular propositions should basically be understood as making assertions without temporal restrictions (*simpliciter*) and not how things are at the moment of utterance (*ut nunc*).[9] In this approach universal and particular propositions are regarded as statements about the generic features of the order of being, such as natural kinds and the necessary or contingent properties of their members. It is possible that a true predication in contingent matter is universal *ut nunc* though it is particular *simpliciter.*

The Thomist version of the doctrine of the matter of propositions is in agreement with what some historians call the statistical or frequency model for modality in ancient philosophy. This refers to the habit of associating the notion of necessity with omnitemporal actuality or actuality in all members of a species, contingency with actuality at some times or in some members of a species, and impossibility with the lack of actuality in these respects. Possibilities are dealt with from the point of view of their actuality in history without an idea of alternative domains.[10] This way of thinking, which was known to medieval thinkers through Boethius, was later employed by many authors, although it never was the only modal paradigm.

Aristotle made use of modal terms in this manner, particularly in the contexts in which he discussed eternal beings, the natures of things, the types of events, or generic statements about such things. (See, e.g., *Met.* IX.8, 1050b6-34; IX.10, 1051b9-30; XII.6, 1071b18-22; XIV.2, 1088b14-25; *De caelo* I.12, 282a4-25; *Phys.* III.4, 203b29-30; *De generatione et corruptione* II.11, 338a1-3). According to the temporal version of this paradigm as it is found in Boethius, what always is, is by necessity, and what never is, is impossible. Possibility is interpreted as expressing what is at least sometimes actual (*In Periherm.* I, 200.20-201.3, II,

peaux by its editor, propositions are divided on the basis of their having natural, contingent or remote matter; universal opposite propositions are false in contingent matter and particular ones are true; see Y. Iwakuma, 'The *Introductiones dialecticae secundum Wilgelmum* and *secundum G. Paganellum*', *Cahiers de l'Institut du moyen-âge grec et latin, Université de Copenhague* 63 (1993), I.3.2 (60); II.1.2-3 (66).

[9]Aquinas writes in his commentary on Aristotle's *Posterior Analytics* that predications can be taken to be 'as of now', as dialecticians sometimes do, or *simpliciter* with respect to all times, as is done in demonstrations; see *In libros Posteriorum Analyticorum expositio*, ed. R.M. Spiazzi (Turin: Marietti, 1964), I, 9.79. Treating a temporally indefinite proposition as true 'as of now' is to take it as true at the moment of utterance. See note 11 below.

[10]The term 'statistical interpretation of modality' was introduced into the modern discussion by Oscar Becker in *Untersuchungen über den Modalkalkul* (Meisenheim am Glan: Anton Hain, 1952) and it is sometimes applied in historical studies, particularly by Jaakko Hintikka in *Time and Necessity: Studies in Aristotle's Theory of Modality* (Oxford: Clarendon Press, 1973). The assumption that all genuine possibilities will be actualized was called the principle of plenitude by Arthur O. Lovejoy who traced its various forms in the history of Western thought in *The Great Chain of Being: A Study of the History of an Idea* (Cambridge, MA: Harvard University Press, 1936). For this modal conception in medieval authors, see S. Knuuttila, *Modalities in Medieval Philosophy* (London: Routledge, 1993), 51-4, 101-3, 113-14, 119-21, 130-3.

237.1-5). Correspondingly, a property which belongs to all members of a group is a necessary property. If it is not actual at all in that group, it is impossible; and if it is exemplified at least in one member, it is possible (*In Periherm.* I, 120.24-121.16).

Aristotle sometimes treats affirmative sentences as token reflexive type sentences which express how things are at the moment of their utterance and consequently have changing or unchanging truth-values (e.g., *Cat.* 5, 4a23-b2). In *Metaphysics* IX.10 this is associated with the frequency paradigm as follows:

> Regarding contingent things, then, the same opinion or the same proposition comes to be false and true, and it is possible at one time to have truth and at another time to be in error, but regarding things that cannot be otherwise opinions are not at one time true and at another false, the same opinions being always true or always false. (1051b13-17)

When contingency is equated with the change in things and truth-values, it follows that unchangingly true sentences are necessary and unchangingly false ones are impossible.[11] This seems to be the background of some *prima facie* obscure formulations in Boethius:

> For it is only those things that can both be and not be that not always are and not always are not. For if they always were, their status could not change, and so they would of necessity be; but if they always were not, it would be necessary that they not be. For of course just as the very nature of the things, events or states of affairs coming about is various, so also does the one or the other part of the contradiction have variable truth. And, indeed, it is always true or false – not, however, one definitely, in such a way that this one is determinately true, or that one – but in either of two ways. And so just as the very status of things, events or states of affairs is mutable, so also is the truth or falsity of the propositions dubitable. And indeed it comes about that as regards some things the one is more often but not always true and the other is more rarely true although it is not necessary that it be false. (*In Periherm.* I, 124.30-125.14; trans. Kretzmann)[12]

This text is part of Boethius's discussion of the truth of singular future contingent propositions. He does not argue that these as such have a changing truth-value, but assumes that as their objects belong to changeable things which make present

[11] See also Thomas Aquinas, *In duodecim libros Metaphysicorum Aristotelis expositio*, ed. M.-R. Cathala and R.M. Spiazzi (Turin: Marietti, 1964), IX.11 (1900): 'Regarding to those things which can be compounded and divided one and the same proposition is clearly sometimes true and sometimes false, for example 'Socrates is sitting' is true when he sits and false when he stands up. And the same holds true of an opinion. But regarding the things which cannot be otherwise, namely those which are always compounded or always divided, it is not possible that a same proposition or opinion is sometimes true and sometimes false, but that which is true is always true and which is false is always false.'

[12] For the translation, see note 8 above.

tense propositions about their prevalence sometimes true and sometimes false, it is illuminating to characterize future propositions about the objects of this kind as variably true or false. Since 'Socrates is sitting' is sometimes true and sometimes false, depending on when it is uttered, the proposition as such is not simply true or false. It is this indeterminacy which Boethius tries to express in calling temporally determinate future contingent propositions variably true or false.[13]

There is a similar shift from temporally definite cases to temporally indefinite ones in Boethius's discussion of Aristotle's remark that what is necessarily is when it is (*De int.* 19a23). Boethius draws a distinction between simple necessity and temporal or conditional necessity (*In Periherm.* I, 121.20-122.15; II, 241.1-242.15). By temporally conditional necessity he means the immutability of a state of affairs as long as a condition is actual. One example of unconditional (simple) necessity is the motion of the sun, of which he says that, as distinct from temporal necessity, 'it is not because the sun is moved now, but because it will always be moved, that there is necessity in its motion' (II, 241.5-7). Conditional necessities are of two sorts: the condition may be the actuality of what is said to be necessary, e.g. Socrates is necessarily seated when he happens to be seated, or the actuality of the subject, e.g. fire is necessarily hot as long as it exists (II, 187.24-188.2; 243.1-6). Boethius is particularly interested in the first group of conditional necessities. He thinks that this necessity expresses the necessity of the present and stresses that it does not imply any other necessity which has deterministic implications.

The Boethian view of the necessity of the present shows how it was understood in ancient modal thought which did not involve the idea of synchronic alternatives. Boethius typically maintains: 'For since the same person cannot be sitting and not sitting at the same time, whoever is sitting cannot be not sitting at the time when he is sitting; therefore, it is necessary that he be sitting' (II, 241.10-13). It is assumed here that

(1) $-M(p_t\& - p_t)$

implies

(2) $-(p_t\& M_t - p_t)$

which is equivalent to

(3) $p_t \rightarrow L_t p_t$.

Boethius thought that the temporal necessity of the present is qualified by the possibility to be otherwise at another time. Referring to the consequent in (3), he states that 'if we separate from the proposition ... the temporal condition, then truth vanishes from the whole proposition' (242.1-3).[14] One might wonder how the alleged necessity of Socrates's sitting at a certain moment of time is qualified

[13] See also *In Periherm.* I, 106.11-23; 123.16-24.

[14] In his similar account Ammonius writes: 'That you are necessarily seated or walking, as long as one of these holds of you, is true, but not absolutely, for we are not always walking or sitting' (153.24-6).

by what he does at other times. According to Boethius, this shows that sitting as such does not inhere in Socrates by necessity (II.242.22-243.2). Many twelfth and thirteenth century authors applied this analysis in discussing the sentence 'A standing man can sit', adding to the true divided (or *de re*) reading 'at another time' or something similar.[15] Another Boethian strategy was to refer to antecedent contingency; I shall return to this in 1.3 below.

Even though there are many examples of the frequential use of modal terms in Western authors from Boethius to Thomas Aquinas, they did not define modal terms through the ideas of various degrees of generality or changing and unchanging truth.[16] These aspects of modalities were regarded as concomitant rather than definitional. Modifying Boethius's classification of ancient theories from this point of view, Thomas Aquinas writes that:

> Some who distinguished them by results – for example Diodorus – said

[15] See, for example, the anonymous texts in De Rijk 1962, 210.10-17; 311.8-15; 316.1-7; 613.23-8; De Rijk 1967 (II.2), 687.28-688.1; William of Sherwood, *Introductiones in logicam*, ed. M. Grabmann in *Sitzungsberichte der Bayerischen Akademie der Wissenschaften*, Phil.-hist. Abteilung 1937, 10 (Munich: Verlag der Bayerischen Akademie der Wissenschaften, 1937), 90.11-24, trans. with an introduction and notes by N. Kretzmann in *William of Sherwood's Introduction to Logic* (Minneapolis: University of Minnesota Press, 1966), 141-2; Peter of Spain, *Tractatus* VII.68, 70-1; Lambert of Auxerre, *Logica*, ed. F. Alassio (Florence: La Nuova Italia, 1971), 158.

[16] For an equation of modal and temporal notions (necessarily-always, possibly-sometimes), see Averroes's commentary of Aristotle's *De caelo* in *Aristotelis opera cum Averrois commentariis* (Venice 1562-74), vol. V, 84A-D, 87H-M, and Avicenna's treatise *al-Masa'il*, quoted in T. Street, 'Fahraddīn Ar-Rāzī's Critique of Avicennan Logic' in D. Perler and U. Rudolph (eds.) *Logik und Theologie. Das Organon im arabischen und lateinischen Mittelalter*, Studien und Texte zur Geistesgeschichte des Mittelalters 84 (Leiden: Brill, 2005), 104. Further typical examples are the principles that if something can be destroyed, it will be destroyed (Moses Maimonides, *The Guide of the Perplexed*, trans. with introduction and notes by Sh. Pines (Chicago: University of Chicago Press, 1974), II, 247, 249; Thomas Aquinas, *Summa theologiae*, ed. P. Caramello (Turin: Marietti, 1948-50) I.2.3; *In Post. an.* I.13, 117; cf. Aristotle, *Metaphysics* XII.6), what is possible with respect to a species will be actualized (Moses Maimonides in a letter to Samuel ibn Tibbon, quoted in Ch.H. Manekin, 'Problems of 'Plenitude' in Maimonides and Gersonides' in R. Link-Salinger, R. Long and Ch.H. Manekin (eds.), *A Straight Path: Studies in Medieval Philosophy and Culture. Essays in Honor of Arthur Hyman* (Washington: Catholic University of America Press, 1988), 187, and that universal propositions are false and particular propositions are true in contingent matter (De Rijk 1967, II.2, 115.5-12; 138.24-6; Albert the Great, *Commentarius in Perihermenias*, in *Opera omnia*, ed. A. Borgnet, vol. I (Paris: Vivès, 1890), 5.6, 422, and the texts mentioned in note 8 above). Gersonides also assumed that all genuine possibilities will be actualized; see *The Book of Correct Syllogism* I.1.14 in *The Logic of Gersonides. A Translation of* Sefer ha-Heqqesh ha-Yashar (The Book of the Correct Syllogism), with introduction, commentary, and analytical glossary by Ch.H. Manekin, The New Synthese Historical Library 40 (Dordrecht: Kluwer, 1992). One of distinctions between modalities *per se* and *per accidens*, which were employed in logical treatises, was based on the idea that the modal status of a temporally indefinite tensed proposition may be changeable. According to the early thirteenth-century *Logica 'Ut dicit'*, true future tense singular propositions referring to non-existent future things, such as 'The Antichrist will be existent', are necessary *per accidens*, for they cannot have been false in the past nor false now, though they will be false in the future (de Rijk 1967, II.2, 390.21-2);'necessarily' is said to mean the same as 'in all times' in this treatise (411.8-9). It was more usual to associate accidental necessities and impossibilities with past tense singular propositions; see note 111 below. Peter of Spain wrote that 'Antichrist has not been' will be accidentally impossible, though it is true now (*Tractatus* IX.4).

> that the impossible is that which never will be, the necessary that which always will be, and the possible that which sometimes will be, sometimes not. The Stoics distinguished them according to exterior restraints. They said the necessity was that which could not be prevented from being true, the impossible that which is always prevented from being true, and the possible that which can be prevented or not. However, the distinctions in both of those cases seem to be inadequate. The first distinctions are a posteriori, since something is not necessary because it always will be, but will always be because it is necessary; this holds for the possible as well as the impossible. The second designation is taken from what is external and accidental, since something is not necessary because it does not have an impediment, but does not have an impediment because it is necessary. (*In Peri herm.* I.14, 183 [8], trans. Oesterle, with changes)

Aquinas criticized Diodorus for regarding the effects of necessity or possibility as their definitions and the Stoics for similarly considering something external as the basis of definitions. He then describes the true model based on the nature of the things:

> Others distinguished these better by basing their distinctions on the nature of things. They said that the necessary is that which in its nature is determined only to being, the impossible that which is determined only to non-being, and the possible that which is not altogether determined to either. (Ibid.)

It may be noticed that when the characterizations of modalities are turned in the way suggested by Aquinas, one gets the assumption typical of the frequency model that what is possible is sometimes actual. Combining frequential and essentialist ideas was not unusual among Western thinkers before Aquinas, but this is also found in Moses Maimonides, Avicenna and Averroes. I shall return to early medieval qualifications of this model because of divine possibilities and some other considerations.

1.2 Possibility as a Potency

Another Aristotelian modal paradigm used by Boethius was that of possibility as potency. In *Met.* V.12 and IX.1 potency is said to be the principle of motion or change either as an activator (in Latin *potentia activa*) or as a receptor of a relevant influence (in Latin *potentia passiva*). One group of possibilities is defined as those which are based on potency. The types of potency-based possibility belonging to a species are recognized as possibilities because of their sometime actualization – no natural potency type can remain eternally frustrated (*De caelo* I.12). According to Boethius, some potencies are never unrealized, their nature being such that they are always actual and as such necessarily so. When potencies

are not actualized, their ends are said to exist potentially (*In Periherm.* II, 453.10-455.19). Necessarily actual potencies leave no room for the potencies of their contraries. There are no contrary potencies in these cases, Boethius says, because they would remain unrealized forever and the constitution of nature cannot include elements which are in vain (II, 236.11-18). It is implied here that all natural types of potency must show their reality through actualization. The potencies of non-necessary features of being do not exclude contrary potencies. They are not always and universally actualized, but as potency-types even these are taken to fulfil the criterion of genuineness mentioned (II, 237.1-5). This is in agreement with the 'statistical' model of modality.

The theory of active and passive potencies was originally meant to explain how and why a singular change takes place.[17] Possibilities as active and passive potencies are the dynamic aspects of an actual change. This background to the potency paradigm made it a cumbersome model for singular possibilities. While it allowed Aristotle and his medieval followers to speak about unrealized possibilities in the sense of partial possibilities, i.e., as the correlates of active or passive potencies, full singular possibilities were actualized when they could be actualized. Natural passive potencies could not be actualized without an active power and they were necessarily actualized when an active power activated them and there was no external hindrance (*Met.* IX.5, 1048a5-21).

According to Thomas Aquinas, the generic natural potentialities, divided into passive propensities (*potentia passiva*) and related activating principles (*potentia activa*), are necessary features of things, determined by the nature of the subjects in which they occur. These necessary structures cannot be violated, and therefore miraculous events, which take place against the common course of nature, must occur through a special supernatural causation. Corresponding to the supernatural active power, there is a passive *potentia obedientiae* by which creatures may receive exceptional influences from the divine cause.[18]

Boethius believed that the passive and active conditions of a great number of events were embedded in the antecedent causes of these events, but in criticizing the Stoic causal determinism he also taught that according to an Aristotelian view there were indeterminate events based on free choice, chance and *ad utrumlibet* contingency which were not determined by preceding causes.[19] As for the efficient causes, he distinguished between necessary causes which always produced their effects and lower causes which may be prevented because of an indeterminacy factor in the causal nexus of nature.[20] Related frequential ideas became popular

[17] For agent and patient in Aristotle's natural philosophy in general, see S. Waterlow, *Nature, Change and Agency in Aristotle's Physics* (Oxford: Clarendon Press, 1982), 159-203.

[18] *Summa theologiae* I.82.1; *In Metaph.* V.6, 832-4; IX.1, 1782; for obedient powers, see *De potentia*, ed. P.M. Pession in *Quaestiones disputatae*, vol. II (Turin: Marietti 1965), 1.3, ad 1; 6.1, ad 18.

[19] See N. Kretzmann, '*"Nos ipsi principia sumus":* Boethius and the Basis of Contingency' in T. Rudavsky (ed.), *Divine Omniscience in Medieval Philosophy*, Synthese Historical Library 25 (Dordrecht: Reidel, 1985) 28-9; Knuuttila 1993, 47-51.

[20] *In Periherm.* II.197.10-198.3; *In Ciceronis* Topica, *Patrologia Latina* 64, 1148, 1152, trans-

through Averroes's works in the thirteenth century.[21] Following the Averroistic view, Thomas Aquinas and Siger Brabant, his contemporary in Paris, defined necessary natural causes as causes which, when actual as cause, always bring about their effect. Contingent causes are divided into those which bring about the effect in most cases (*ut in pluribus*) and are in a few cases prevented by chance and those which are not associated with a natural tendency to a definite result (*ad utrumlibet*). A particular cause is here considered necessary or contingent depending on how causes of the same type usually behave. Similarly, the effect which is necessary with respect to its actual causes can be called contingent by referring to what happens in other similar cases.[22] In asking how things can be contingent if they are eternally known by divine omniscience and included in the immutable providential plan, Aquinas sometimes states that things are contingent, if their proximate causes are not necessary, i.e., causes which always bring about their effect.[23]

While the conceptions of power and potentiality were widely regarded as essential elements in understanding modality, Anselm of Canterbury attempted to base the whole of modal semantics on these notions. Putting forward a detailed analysis of direct (proper) and indirect (improper) modes of agency, he suggested that the

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lated in Stump 1988, 162-3.

[21]For Averroes's view of causation, see A. Maier *Die Vorläufer Galileis im 14 Jahrhundert* (Rome: Edizioni di Storia e Letteratura, 1949), 219-50.

[22]See Siger of Brabant, *Quaestiones in Metaphysicam: Edition revue de la Reportation de Munich, texte inédit de la Reportation de Vienne*, ed. W. Dunphy, Philosophes médiévaux 24 (Louvain-la-Neuve: Editions de l'Institut Supérieur de Philosophie, 1981), 369.51-370.81; 378.58-380.26; *De necessitate et contingentia causarum* in J. Duin, *La doctrine de la providence dans les écrits de Siger de Brabant*, Philosophes médiévaux 3 (Louvain: Editions de l'Institut Supérieur de Philosophie, 1954) , 28.10-30.49; 32.1-34.34; Thomas Aquinas, *Scriptum super libros Sententiarum*, ed. P. Mandonnet and M.F. Moos, 5 vols (Paris: Lethielleux, 1929-56), I.38.1.4-5; *Summa contra gentiles*, ed. C. Pera, P. Marc, P. Caramello (Turin: Marietti, 1961-7), I.67.3; *De malo*, ed. P. Bazzi and P. M. Pession in *Quaestiones disputatae*, vol. II (Turin: Marietti, 1965), 16.7; *In Peri herm.* I.13, 172; 14, 194, 197; *In octos libros Physicorum Aristotelis expositio*, ed. M. Maggiòlo (Turin: Marietti, 1954), II.8.210.

[23]*Scriptum super libros Sententiarum* I.38.1.4-5. While repeating this view in his later works (*Summa contra gentiles* I.85.4; III.72.2; *Summa theologiae* I.14.13, ad 1), Aquinas qualified it stating that he did not mean that things could be considered necessary as such. He stressed that God as the first cause provided things which naturally necessary or contingent causes so that contingent events, particularly the acts of free will, were not reducible to a necessitating ultimate cause, though dependent on God's irresistible will which transcended the natural orders of necessity and contingency (*Quaestiones disputatae I: De veritate*, ed. R.S. Spiazzi (Turin: Marietti, 1964), 23.5; *In Peri herm.* I.14, 197; see also H. Goris, *Free Creatures of an Eternal God: Thomas Aquinas on God's Infallible Foreknowledge and Irresistible Will* (Utrecht: Thomas Instituut, Leuven: Peeters, 1996), 293-302). Siger of Brabant refers to some Parisian masters who argued that associating the modality of an effect with that of its proximate cause was compatible with metaphysical determinism (*De necessitate et contingentia causarum* 26.67-79). In order to avoid this, he stated that the first cause left room for individual coincidences the results of which were not known even by divine foreknowledge (*De necessitate et contingentia causarum* 26.80-28.9; 31.78-32.00; 42.10-25). When accused of heresy, he became more cautious when speaking about indeterminate events in nature (*Quaest. in Met.* (Vienna), 370.1-387.89; R. Hissette, *Enquête sur les 219 articles condamnés à Paris le 7 Mars 1277*, Philosophes médiévaux 22 (Louvain: Publications Universitaires, Paris: Vander-Oyez, 1977), 42-3.

concepts of potency and impotency should be treated in the same way. According to Anselm, 'A is necessarily B' properly ascribes a constraint to A and 'A is possibly B' properly ascribes a capacity to A. Modal terms proper refer to properties of things, such as the power to bring something about, or constraints caused by other things. If modal terms are used in another way, they are used improperly. Since Anselm regards the notion of capacity as the basic modal notion, something is necessary in an improper sense with respect to causes which cannot prevent it, and something is similarly impossible with respect to causes which cannot bring it about. There are no constraining necessities proper in God, and divine impossibilities are similarly analysed by referring to external impossibilities. In trying to solve problems associated with the notions of freedom, sin and grace, Anselm also employed the distinction between full and partial potencies. The shortcomings of these analyses and some artificial constructions of the meaning of modal statements based on the distinction between proper and improper capacity show that the notion of potency is too narrow a basis for modal semantics.[24]

1.3 Temporal modalities

A third ancient modal paradigm in Boethius meant for discussing singular possibilities could be called the diachronic model of modality. In chapter 9 of *Peri hermeneias,* Aristotle tries to qualify the necessity of the present — a corollary of the lack of simultaneous alternatives in his theory — stating that not everything which is actual is necessary *simpliciter*, without qualification (19a23-4). If necessity without qualification means 'necessary even without the temporal qualification', as Boethius reads it in his first commentary, Aristotle's point would be that the temporal necessity of actual events does not imply that such events are necessary in themselves in the sense that they always take place in similar circumstances.[25] As already mentioned, this is a problematic attempt to qualify the necessity of a definite singular event with the help of frequential considerations. Some medieval and modern interpreters have opted for another interpretation: Aristotle wanted to show that the necessity of an event at a certain time does not imply that it would have been antecedently necessary. There are some places in which Aristotle speaks about genuine singular possibilities with respect to definite future points of time which may be realized or remain unrealized (*De int.* 19a13-17, *EN* III.5, 1114a17-19, *Met.* VI.3).[26] This idea of diachronic modalities was considered important in the later Peripatetic tradition. It was argued against Stoic determinism that there are genuine future alternatives which remain open

[24]See E. Serene, 'Anselm's Modal Conceptions' in S. Knuuttila (ed.), *Reforging the Great Chain of Being*, Synthese Historical Library 20 (Dordrecht: Reidel, 1981), 117-62; S. Knuuttila, 'Anselm on Modality' in B. Davies and B. Leftow (eds.), *The Cambridge Companion to Anselm* (Cambridge: Cambridge University Press, 2004), 111-31.

[25]See Hintikka 1973, 147-175; Boethius, *In Periherm.* I.121.25-122.4.

[26]This is a central theme in S. Waterlow, *Passage and Possibility: A Study of Aristotle's Modal Concepts* (Oxford: Clarendon Press, 1982); see also G.H. von Wright, *Truth, Knowledge, and Modality* (Oxford: Blackwell, 1984), 72-8.

until the moment of time to which they refer. Even the Stoics spoke about alternative prospective possibilities which are not yet fixed at the level of known causes, but they also regarded fate as an active potency which ultimately necessitates everything. In the Peripatetic theory of diachronic modalities it is assumed that there are transient individual alternative possibilities, but those which will not be realized disappear.[27] When Boethius refers to chance, free choice, and possibility, which restrict the domain of causal necessity, his examples include temporalized modal notions which refer to diachronic prospective possibilities at a given moment of time.[28] A temporally determinate prospective possibility may not be realized at the time to which it refers, in which case it ceases to be a possibility. Boethius did not develop a theory of simultaneous synchronic possibilities which remain intact even when diachronic possibilities have vanished, insisting that only what is actual at a certain time is possible at that time with respect to that time.

The model of diachronic modalities provided Boethius with a more satisfactory tool for qualifying the necessity of the present than the frequency interpretation did. Instead of arguing that when the condition of the conditionally necessary proposition is removed, the proposition itself is contingent, he could also remark that when the antecedent conditions of a temporally necessary state of affairs are considered, it may be realized that it was not necessary before it was actualized.[29] (See also 2.3 below.)

1.4 Divine Modalities

While Boethius's works formed the main source of knowledge about philosophical modal theories for early medieval thinkers, Augustine's doctrine of creation made them aware of a theological discussion of possibility and necessity which was based on different ideas. Augustine argued that God simultaneously created the first things and the seminal reasons for later things out of nothing. The creation was based on an eternal free act of God's perfectly good will, and took place through his omnipotence. In Augustine's Trinitarian view, the Son is a perfect image of the Father and, as the Word, the seat of the ideas of finite beings which in a less perfect manner can imitate the highest being. The ideas refer to possible actualization in the domain of mutability. In this sense the possibilities have an ontological foundation in God's essence.[30] This became the dominating conception of theological modal metaphysics until Duns Scotus departed from it.[31]

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[27] See R.W. Sharples, *Alexander of Aphrodisias on Fate*, text, translation and commentary (London: Duckworth, 1983); S. Bobzien, *Determinism and Freedom in Stoic Philosophy* (Oxford: Clarendon Press, 1998).

[28] See, e.g., *In Periherm.* I, 106.11-14; 120.9-16; II, 190.14-191.2; 197.20-198.3; 203.2-11; 207.18-25.

[29] *In Periherm.* II, 245.4-246.19. See also Thomas Aquinas, *In Peri herm.* I.15, 201; Albert the Great, *Comm. in Periherm.* I.5.6, 421; Siger of Brabant, *De necessitate et contingentia causarum* 32.4-18.

[30] S. Knuuttila, 'Time and Creation in Augustine' in E. Stump and N. Kretzmann (eds.), *The Cambridge Companion to Augustine* (Cambridge: Cambridge University Press, 2001), 103-115.

[31] Knuuttila 1993, 135-6.

Augustine thought that the world was not necessary and that many possibilities remained unrealized. In book 12 of *The City of God*, he criticizes the ancient doctrines which claimed that the only permissible notion of infinity is that of potential infinity, arguing that an infinite series of numbers actually exists in God's mind, and that God could create an infinite number of individuals and know each of them simultaneously.[32] Augustine regards God's omnipotence as an executive power with respect to his free choice, which is conceptually preceded by knowledge about alternative possibilities. God could have done other things, but did not want to.[33] Augustine's remarks on various unrealized possibilities remained sketchy, but the basic idea is pretty clear and was often repeated in the early medieval doctrine of God as acting by choice between alternatives. This involved an intuitive idea of modality based on synchronic alternativeness.[34]

The discrepancy between the Catholic doctrine of God's freedom and power and the philosophical modal conceptions was stressed in Peter Damian's *On Divine Omnipotence* and formulated in a less ambiguous manner by Anselm of Canterbury.[35] Like Peter Damian, Anselm also avoided speaking about the restrictions of divine omnipotence and preferred formulations according to which all necessities and impossibilities are subject to God's will. He did not mean that all necessary truths were dependent on God's will, for example those about God's existence or properties or the principles of logic. His point was that the natural order was created. Events which take place in accordance with the common course of nature or against it are naturally necessary or impossible with respect to the laws of the created order, but God as the Lord of this order can bring about effects which are naturally impossible. Miraculous divine interventions do not strictly speaking violate natural patterns, since they are meant to be subordinate.[36] These ideas were codified in the commonly employed distinction between possibilities *secundum inferiorem causam* as possibilities *secundum cursum naturae* and possibilities *secundum superiorem causam* as God's possibilities.[37] Some theologians

[32]Augustine, *De civitate Dei*, ed. B. Dombart and A. Kalb, Corpus Christianorum Series Latina 47-8 (Turnhout: Brepols, 1955), 12.9

[33]'Potuit sed noluit'; *Contra Faustum*, ed. I. Zycha, Corpus Scriptorum Ecclesiasticorum Latinorum 25 (Vienna: F. Tempsky, Leipzig: G. Freytag, 1891), I.30.

[34]Plotinus also thought that the various levels of reality outside the One, the first principle, are caused and thus different from Aristotle's conception of higher spheres. However, the power of being eternally proceeds from the One and does not leave any genuine possibility of being unrealized. See H.Buchner, *Plotins Möglichkeitslehre*, Epimeleia 16 (Munich and Salzburg: Anton Pustet, 1970); for the principle of plenitude in Proclus, see T. Kukkonen, 'Proclus on Plenitude', *Dionysius* 18 (2000), 103-28.

[35]For Peter Damian, see *Lettre sur la Toute-Puissance Divine* (*De divina omnipotentia*), introduction, critical text, translation and notes by A. Cantin, Sources Chrétiennes 191(Paris: Cerf, 1972), 603a-604b, 610d-615b, Knuuttila 2004, 115-18.

[36]See, for example, *Cur Deus homo* 2.17 in F. Schmitt (ed.), *Opera omnia* (Edinburgh: Nelson, 1946-61), vol. II, 122.25-30; *Meditatio redemptionis humanae* (*Opera omnia* III, 86.60-2); *De conceptu virginali et de originali peccato* (*Opera omnia* II, 153.7-9, 154.4-15); for non-created modalities, see, for example, *Proslogion* 3-5 (*Opera omnia* I, 102-4); *De grammatico* 1-2 (*Opera omnia* I, 145-6).

[37]Peter Lombard, *Sententiae in IV libris distinctae* ed. PP. Collegium S. Bonaventurae

asked whether one should apply the notion of possibility without qualification to divine possibilities and the notion of qualified possibility, such as natural possibility, to those spoken of in accordance with the lower causes, or vice versa.[38] This is a sign of an increasing awareness of the fact that the conception of omnipotence, which included the idea of eternal alternatives, was very different from philosophical modalities, and there were some twelfth-century attempts to explicate the idea of synchronic alternatives more systematically.

In addition to the doctrines of omnipotence and eternal choice between providential options, divine modalities were discussed in treating the compatibility of divine omniscience and the contingency of events in the created world. In his longer commentary on *De interpretatione*, Boethius says of God that 'he knows future events as happening contingently and not necessarily so that he does not ignore the possibility that something else might take place.'[39] In his later work *Consolation of Philosophy*, Boethius argues that God is atemporal and has timeless knowledge of everything. God's timelessness involves his having the whole history present to him simultaneously. God's knowledge is not foreknowledge, since it is not temporally located, but the predictions of future contingents are true or false from the point of view of God's eternal knowledge of the things referred to. It is necessary that if God knows that p, then p. This 'conditional necessity' does not imply the 'simple necessity' of p.[40] This approach was very influential and was further developed in Aquinas's theory of God who grasps all combinations of things at particular times by one eternal vision. God has an immediate knowledge of all things and their relative temporal order, though none of them is past or future with respect to His cognition. The objects of divine omniscience are necessary by supposition (i.e., with respect to God's knowledge and providential plan), but many of them are contingent with respect to their proximate causes.[41] God's eternal and immutable vision includes the knowledge of things which are future contingents in the temporal order, and he can supranaturally inform lower intellects about these things – otherwise there could not be true prophetic predic-

(Grottaferrata: Collegium S. Bonaventurae ad Claras Aquas, 1971-81), II.18.5-6; Simon of Tournai, *Disputationes*, ed. J. Warichez, Spicilegium Sacrum Lovaniense, Études et Documents 12 (Louvain: Spicilegium Sacrum Lovaniense,1932), 35.2 (104.32-105.29); Peter of Poitiers, *Sententiae*, I.7 (65.444-66.452, 67.498-68.502); Alan of Lille, *Regulae caelestis iuris*, ed. N.M. Häring, *Archives d'histoire doctrinale et littéraire du moyen âge* 78 (1981), 164-5. This was the background of the scholastic doctrine of God's absolute and ordained power. For similar discussions in medieval Arabic thought, see Kukkonen 2000.

[38] William of Auxerre, *Summa aurea*, ed. J. Ribaillier, Spicilegium Bonaventurianum 16-20 (Paris: Editions du CNRS and Rome: Collegium S. Bonaventurae ad Claras Aquas, 1980-86), I.11.2 (206-9).

[39] *In Periherm.* II, 226.9-12.

[40] *Philosophiae consolatio*, ed. L. Bieler, Corpus Christianorum Series Latina 94 (Turnhout: Brepols, 1957), V.6, 25-32; cf. Augustine, *De civitate Dei* XI.21. Boethius's conception of God's atemporal knowledge is meant to explain how future contingent events retain their indeterminateness while being certainly known as atemporally present. See also J. Marenbon, *Boethius* (New York: Oxford University Press, 2003), 123-45.

[41] *Scriptum super libros Sententiarum*, I.38.1.4-5; *Summa contra gentiles*, I.66-7; *Quaestiones disputatae I: De veritate*, 2.12; *In Peri herm.* I.14, 194-6; *Summa theologiae* I.14.13.

tions.[42] While prophetic statements are formally future contingent propositions, their being true is based on a revelation of atemporal knowledge. This is how Aquinas tries to combine the doctrine of divine omniscience and the lack of definite truth and falsity in future contingent propositions.[43] (See also 1.6 below.)

An influential formulation of divine foreknowledge was put forward by Peter Abelard. William of Champeaux, one of Abelard's teachers, dealt with an argument against the compatibility between contingency and divine omniscience which was discussed in Augustine's *City of God* (V.9). In response to Cicero's *De fato* and *De divinatione*, Augustine refuted the claim that the possibility of events having happened otherwise implies the possibility of error in God. William stated that the antecedent of the argument is true but the consequent is false and therefore the consequence is not valid. Peter Abelard, in discussing the same example in his *Dialectica* and *Logica 'Ingrdientibus'*, applied a systematic division which he elsewhere drew between modal statements *de sensu* or in the compound sense and modal statements *de re* or in the divided sense.[44] Abelard's analysis of Cicero's argument was often repeated in medieval theology, since it was presented in slightly modified form in Peter Lombard's *Sententiae*, which became the standard medieval introduction to theology. Abelard states that when the proposition 'A thing can be otherwise than God knows it to be' is taken to mean that it is possible that a thing is otherwise than God knows, which corresponds to what he elsewhere calls the *de sensu* reading, the antecedent is false. When the antecedent is taken to say that a thing may be otherwise than God knows it to be, which corresponds to what Abelard elsewhere calls *de re* possibility, the antecedent is true, but the consequent is false, since if things were otherwise, God would possess different knowledge of them. This shows that the consequence is not valid.[45] Following Peter Abelard, Peter Lombard formulated the same view in stating that 'Things cannot be other than as God foreknows them' is true in the compound sense and false in the divided sense. The truth of the compound sense saves God's infallibility and the falsity of the divided sense expresses God's freedom and the

[42] *Summa theologiae* II-2.171.3.

[43] For various interpretations of divine knowledge and future contingents in Aquinas, J.F. Wippel, 'Divine Knowledge, Divine Power and Human Freedom in Thomas Aquinas and Henry of Ghent' in T. Rudavsky (ed.), *Divine Omniscience and Omnipotence in Medieval Philosophy*, Synthese Historical Library 25 (Dordrecht: Reidel, 1985), 213-41; Goris 1996, 213-54; W.L. Craig, *The Problem of Divine Foreknowledge and Future Contingents from Aristotle to Suarez* (Leiden: Brill, 1988), 103-26. See also J. Marenbon, *Le tempts, l'éternité et la prescience de Boèce à Thomas d'Aquin* (Paris: Vrin, 2005).

[44] For this terminology, see *Super Periermenias XII-XIV*, ed. L. Minio-Paluello in *Twelfth Century Logic: Texts and Studies II: Abaelardiana inedita* (Rome: Edizioni di Storia e Leteratura, 1958), 3-47. Later authors often used the expression *de dicto* instead of *de sensu.* See below, section 2. For the historical background of Abelard's argument, see J. Marenbon, *The Philosophy of Peter Abelard* (Cambridge: Cambridge University Press, 1997), 226-8.

[45] Peter Abelard, *Philosophische Schriften I. Die Logica 'Ingredientibus'*, ed. B. Geyer, Beiträge zur Geschichte der Philosophie und Theologie des Mittelalters XXI, 1-3 (Münster: Aschendorff, 1919-27), 429.26-430.36; *Dialectica*, ed. L.M. de Rijk, Wijsgerige teksten en studies 1 (Assen: van Gorcum, 1956), 217.27-219.24.

metaphysical contingency of the future. It is assumed that when something is, it is possible that it is not at that very instant of time at which it is actual.[46]

1.5 Modality as Alternativeness

Augustine's doctrine of God's eternal choice involved an intuitive idea of modality as alternativeness which influenced early medieval theological discussions of divine omnipotence and omniscience. Some authors regarded this as a special theological matter which did not affect the use of traditional ideas in other disciplines, an attitude supported by the general Aristotle reception in the thirteenth century, but there were twelfth-century thinkers who realized the philosophical significance of this new modal conception.

According to Abelard, the possibilities and necessities belonging to the individuals of a species are determined by their shared nature. What nature demands or allows and what is repugnant to it is the same for all members of a species, and what is possible is seen in what has taken place in the representatives of the species.[47] One could regard these essentialist possibilities as abstract Philonian possibilities, many of which never become or even can become actual in an individual.[48] This is one aspect of Abelard's modal views, but there are others. He assumes that what is actual is temporally necessary at a certain point of time as no longer avoidable, but he also argues that unrealized alternatives are possible at the same time in the sense that they could have happened at that time. Some of the alternatives of a singular being are real counterfactual alternatives. These are unrealizable because of some previous changes in the conditions of the subject. There are also merely imaginable alternatives, such as Socrates' being a bishop, which never had a real basis in things.[49] (See also 2.3 below.)

Gilbert of Poitiers stresses the idea that natural regularities which are called natural necessities are not absolute, since they are chosen by God and can be overridden by divine power. This had become a widespread theological view in the twelfth century. Gilbert explained it in the light of the Augustinian view of God's acting by divine will, which chooses between alternative providential plans, and divine omnipotence as an executive power. It has been sometimes thought

[46] *Sententiae* I.38.2.

[47] *Dialectica* 193.36-194.3; 385.1-8.

[48] Philo's definitions of modal concepts are described in Boethius's commentary on *De interpretatione* (II, 234.10-21). One of his examples was that it is possible for a piece of wood at the bottom of the sea to be burnt, in virtue of the fitness of the subject. See also Alexander of Aphrodisias, *In Aristotelis Analyticorum priorum librum I commentarium*, ed. M. Wallies, Commentaria in Aristotelem Graeca 2.1 (Berlin: Reimer, 1883), 184.6-12; John Philoponus, *In Aristotelis Analytica Priora commentaria*, ed. M. Wallies, Commentaria in Aristotelem Graeca 13.2 (Berlin: Reimer, 1905), 169.19-20.

[49] See the discussion of *Logica 'Ingredientibus'*, 272.39-273.19, and some other texts in C.J. Martin, 'An Amputee is Biped. The Role of the Categories in the Development of Abelard's Theory of Possibility' in J. Biard and I. Catach-Rosier (eds.), *La Tradition médiévale des Categories (XIIe-XIVe siècles)*, Philosophes médiévaux 45 (Louvain-la-Neuve: Éditions de l'Institut Supérieur de Philosophie, Louvain and Paris: Peeters, 2003), 225-42.

that Gilbert wanted to deny the necessity of the past in this context, but the texts interpreted in this way can be read as meaning that temporal necessities are based on God's free choice and do not imply any lack of Divine power. There is an interesting formulation of Plato's 'Platonitas' in Gilbert. He says that this includes all what Plato was, is and will be as well as what he could be but never is. Even though Gilbert does not explain why there is a modal element in the individual concept, it was probably needed in order to speak about Plato in alternative possible histories or, as Abelard did, about Socrates as a bishop. Gilbert seems to have been the first to formulate an individual concept in this way.[50]

A third context of the systematic interest in synchronic alternatives was the new twelfth-century theory that declarative singular propositions should be primarily treated as temporally definite and as having an unchanging truth-value. This approach was developed by twelfth-cantury authors who were later called *nominales.* One of their theses was that 'What is once true is always true.'[51] This was applied in discussing of the question of how the beliefs of Abraham and others who lived before the coming of Christ and believed various things about him were the same as the beliefs of those who live after His coming – the previous beliefs were formulated in future tense statements and the latter in present or past tense statements. According to the *nominales*, one could regard as basic a temporally definite propositional content which is expressed by various tensed expressions depending on when they are uttered. While tensed statements about temporally definite singular events have a changing truth-value, the corresponding non-tensed propositions are unchangingly true or false.[52]

Peter of Poitiers, one of the twelfth-century authors taking this approach, argued that propositions pertaining to contingent things have a truth-value on the basis of God's eternal choice. These truth-values would be otherwise, if the providential design of the world were different in relevant respects. The contingent truth-values of future contingent propositions do not prevent future events from being indeterminate. They may begin to be temporally unavoidable, but they do not begin to be metaphysically contingent nor do the propositions pertaining to them begin to be true or false.[53] This is in accordance with the immutability of God's knowledge. God does not know contingent things through tensed propositions,

[50]See Knuuttila 1993, 75-82.

[51]See Y. Iwakuma and S. Ebbesen, 'Logico-Theological Schools from the Second Half of the 12^{th} Century: A List of Sources', *Vivarium* 30 (1992), 196, n. 46; 199, n. 50a, b; 200, n. 51a; 201, n. 52d; 205, n. 62d; 206, n. 64a. For the history of the principle, see also J. Marenbon, 'Vocalism, Nominalism and the Commentaries on the *Categories* from the Earlier Twelfth Century', *Vivarium* 30 (1992), 58-61 and S. Ebbesen, 'What Must One Have an Opinion About', *Vivarium* (30), 1992, 73-4. For the history of twelfth-century *nominales*, see W. Courtenay, 'Nominales and Nominalism in the Twelfth Century' in J. Jolivet, Z. Kaluza, A. de Libera (eds.), *Lectionum varietates. Hommage à Paul Vignaux (1904-1987)* (Paris: Vrin 1991), 11-48.

[52]Nuchelmans 1973, 177-89; for some later examples of distinguishing between temporally definite and temporally indefinite declarative sentences, see H. Goris, 'Tense Logic in 13^{th}-Century Theology', *Vivarium* 39 (2001), 161-84.

[53]Peter of Poitiers, *Sententiae* I, ed. P.S. Moore and M. Dulong, Publications in Medieval Studies 7 (Notre Dame: The University of Notre Dame Press, 1961), I.7.133-43; I.12.164-82; I.14.328-53.

since their truth-value is changeable. If God's knowledge is described by using tensed propositions, analogously to the articles of faith before and after the coming of Christ, one should read them so that they signify the same. This became a well-known position, since it was also employed in Peter Lombard's *Sententiae*.[54]

The formulations by Peter Abelard, Gilbert of Poitiers, Peter Lombard and Peter of Poitiers discussed above exemplify twelfth-century deviations from the Aristotelian thesis 'What is necessarily is when it is.' This was traditionally understood as implying the principle of the necessity of the present, which was not questioned in ancient modal theories. Since God's knowledge about contingent things was regarded as unchangeable, the contingency of this knowledge also implied the denial of the Aristotelian equation of immutability with necessity, a denial regarded as an explicit doctrine of the *nominales*.[55]

The idea of modal alternativeness was also discussed by Robert Grosseteste in early thirteenth century. Grosseteste taught that the opposites of actualized contingent things are no longer realizable possibilities, though they are possible in the sense that they could have been included in God's eternal providential choice. Actual history is an explication of one of the divine alternatives with respect to which things are primarily called necessary, possible or impossible. Modalities at this basic level are called modalities 'from eternity and without beginning'. Mathematical truths are necessary in this way. In addition to these 'simple' necessities, there are necessities and impossibilities which have a beginning and which are eternal contingencies in the sense that God could have chosen their opposites.[56] The contingency of the divine acts of knowledge and will is based on an atemporal causal priority between the power and its acts.[57]

1.6 Future Contingents

Some Stoics took Aristotle to deny that future contingent propositions are true or false, as Boethius reports in his second commentary on Aristotle's *De interpretatione* (208.1-4). Future contingent propositions were regarded as true or false in Stoic logic, the Stoics taking the universally valid principle of bivalence to imply the predetermination of all future events. (See Cicero, *De fato*, 20-1.)[58] Boethius regarded the Stoic view of future contingent propositions as well as the Stoic characterization of Aristotle's position as false, his interpretation being based on

[54] *Sententiae* I.12.193-223; I.13.192-220; cf. Peter Lombard, *Sententiae*.1.39.1; 41.3.

[55] Ebbesen and Iwakuma 1992, 194

[56] Robert Grosseteste, *De libero arbitrio* in *Die philosophischen Werke des Robert Grosseteste*, ed. L. Baur, Beiträge zur Geschichte der Philosophie des Mittelalters 9 (Münster: Aschendorff, 1912), 168.26-170.33, 178.28-9; N. Lewis, 'The First Recension of Robert Grosseteste's *De libero arbitrio*, *Mediaeval Studies* 53 (1991), 1-88.

[57] Ibid. 178.24-6.

[58] On the basis of a note in Simplicius's commentary on Aristotle's *Categories* (407.6-13), it is argued that some Aristotelians also qualified the principle of bivalence; for this and other ancient deviations from the principle, see R. Sorabji, *Necessity, Cause, and Blame: Perspectives on Aristotle's Theory* (Ithaca, NY: Cornell University Press, 1980), 92-3. For the Stoic views, see also Bobzien 1998, 59-86.

the distinction between definite and indefinite division of truth and falsity in the contradictory pairs of propositions. This terminology was also used in Ammonius's commentary. Since this work was not known to Boethius, both authors apparently based their commentaries on earlier Greek discussions in which these qualifications were introduced.[59] According to Boethius, Aristotle argues that if all pairs of contradictory propositions definitely divide truth and falsity and all propositions are definitely true or definitely false, then everything necessarily occurs as it does.[60] The fact that contingent future things, events and states of affairs are not determined refutes the thesis that all affirmations or negations are definitely true or definitely false.[61] The disjunctive pairs of contradictory future contingent propositions can be said to divide truth and falsity only in a very special sense; 'the whole body of the contradiction does indeed separate into truth and falsity, but this truth and falsity is undifferentiated and alterable.'[62] Boethius interprets Aristotle's final solution as follows:

> For it is necessary as regards future and contingent contradictions that the whole contradiction have one part true and the other false. For example, if someone affirms that there is going to be a sea battle tomorrow ... and if someone denies it ... the whole contradiction will indeed have one part true, the other false; but there will not be one of them definitely true and the other definitely false. (*In Periherm.* I, 122.26-123.10, trans. Kretzmann)[63]

The majority interpretation of contemporary commentators is that Ammonius and Boethius ascribe to Aristotle the view that the predictions of future contingent things and their denials differ from other contradictory pairs, because truth and falsity are not definitely distributed between these propositions, which are consequently neither definitely true nor definitely false. This is taken to mean that they are not true or false. In answering the Stoic criticism, Boethius might have thought that future contingent propositions have the disjunctive property of being true-or-false, which would mean something other than simply lacking a truth value.[64]

[59] Sorabji 1998. Ammonius's commentary was translated by William of Moerbeke in 1268 and was used by Thomas Aquinas. See note 8.

[60] Boethius thinks that the term 'definitely' can be added to Aristotle's argument as part of what Aristotle means (*In Periherm.* I, 108.18-26; 125.20; II, 204.8-25; see also I, 109.9-17; 110.28-112.4; 114.8-24; II, 208.7-23; 211.26-213.4).

[61] *In Periherm.* II, 219.5-17.

[62] *In Periherm.* I, 108.2-5; cf. I, 106.30-107.6; II, 208.7-18.

[63] This contradiction is said to be indefinitely both true and false (I, 124.28); according to the longer commentary, 'one part of the contradiction is true and the other false only indefinitely' (II, 246.12-13).

[64] Craig 1988, 79-98; R. Gaskin, *The Sea Battle and the Master Argument*, Quellen und Studien zur Philosophie 40 (Berlin: de Gruyter, 1995), 146-84; N. Kretzmann, 'Boethius and the Truth about Tomorrow's Sea Battle' in Ammonius, *On Aristotle:* On Interpretation 9, 24-52. D. Frede argues argues that Boethius gives up the principle of bivalence and does not extend it to include the disjunctive formulation; see 'The Sea-Battle Reconsidered : A Defense of the Traditional Interpretation, *Oxford Studies in Ancient Philosophy*, vol. III (Oxford: Clarendon Press, 1985), 44-5.

Another interpretation of Boethius and Ammonius holds that future contingents are not definitely true or false, because their truth-makers are not yet determined, but are true or false in an indeterminate way. No qualification of the principle of bivalence is involved. True statements are either determinately true or simply (indeterminately) true.[65] While Ammonius and Boethius assumed that Aristotle denied the definite truth of predictions which they took to imply determinism, it is less clear how they understood the indefinite truth of these. Boethius's formulations often suggest that future contingent propositions are true-or-false without being simply true or false, but perhaps he was not quite sure about this.[66]

The past and the present are necessary in Boethius. Prospective contingent alternatives with respect to a future event remain open until the relevant causes are settled or the event takes place and the alternative options vanish. Correspondingly he seems to take the truth of future propositions to mean that things cannot be otherwise, for the antecedently assumed actuality of future truth-makers implies that alternative prospective possibilities refer to things which are rendered temporally impossible by the actualized alternatives.

In his early commentary on *De interpretatione*, Abelard follows Boethius's analysis of future contingent propositions. He understands Boethius's comments in the way most contemporary commentators do and accepts the idea that future contingent propositions are merely true-or-false. Contradictory present tense propositions are determinately true and determinately false and also disjunctively determinately/necessarily true or false (*etiam sub disjunctione*), whereas contradictory future contingent propositions are merely disjunctively true or false (*tantum sub disjunctione*).[67]

Abelard changed his view in the *Dialectica* and his longer commentary on *De interpretatione* in *Logica 'Ingredientibus'*. The historical order of these two works is not quite clear. Many scholars have argued that the *Dialectica* is earlier[68], but it is also possible that the texts contain parts written at different times. While the main structure of Aristotle's argument in *De interpretatione* 9 is understood in the same way in the *Dialectica* and the early commentary on *De interpretatione*, Abelard now argues that future contingent propositions are true or false, although not determinately or necessarily so, and takes this to be Aristotle's view as well. The difference between future contingent propositions and other propositions has nothing to do with bivalence; it concerns the determinateness or indeterminateness of the truth of propositions and what is signified by propositions. Misguided opinions are based on the mistaken idea that the necessity or determinateness of

[65] M. Mignucci, 'Truth and Modality in Late Antiquity: Boethius on Future Contingent Propositions' in G. Corsi, C. Mangione and M. Mugnai (eds.), *Atti del convegno Internazionale di Storia della Logica. Le Teorie delle Modalità* (Bologna: CLUEB, 1989), 47-78, and 'Ammonius' Sea Battle' in Ammonius, *On Aristotle:* On Interpretation 9, 53-86.

[66] See also Sorabji 1998.

[67] *Editio super Aristotelem De interpretatione*, ed. M. Dal Pra in Pietro Abelardo, *Scritti di logica* (Florence: La Nuova Italia, 1969), 100.13-19; 112.7-113.3.

[68] See Marenbon 1997, 40-8.

a disjunction implies that the disjuncts are necessary or determinate.[69] This is close to what Mignucci regards as Boethius's position.

The commentary on *De interpretatione* 9 in *Logica 'Ingredientibus'* involves Abelard's most detailed analysis of the structure of Aristotle's argument. Chapter 9 is described as a dialectical discussion of the thesis that in all contradictories one part is necessarily true and the other false. The correct reading of this is that of all contradictory pairs necessarily one part is true and the other false. This does not imply that one is necessarily true and the other necessarily false, although one is true and the other false.[70]

Abelard pays attention to some propositions about past and present states of affairs the truth and falsity of which depend on future, such as 'Socrates is the name of a man going to eat tomorrow' or 'He has spoken falsely', when this is said of a man who said yesterday that Socrates will eat tomorrow. The truth and falsity of propositions of this kind are not knowable without knowledge of future contingent states of affairs which are indeterminate and epistemically inaccessible to human beings. Hence the propositions have an indeterminate truth and falsity and do not differ from future contingent propositions in this respect. In discussing these examples, Abelard states that 'Aristotle calls propositions determinately or indeterminately true with respect to the determinate or indeterminate states of affairs they propose'. This terminology is part of Abelard's rational reconstruction of Aristotle's discussion; being probably aware that Aristotle did not use these terms at all, Abelard suggests that Aristotle would consider propositions indeterminately true or false if their truth depended on indeterminate future contingent states of affairs. Propositions which propose the actual inherence of truth in a future contingent proposition are of the form: "Socrates will eat tomorrow' is true'. As far as determinateness is understood in accordance with what Abelard calls Aristotle's view, this is no more determinate than 'Socrates will eat tomorrow'. As for the truth as a present fact, Abelard repeats the remark from the *Dialectica* that one could call this present state of affairs determinate and, correspondingly, the propositions determinately true or false. In this sense all true propositions are determinately true, but this has nothing to do with causal determination. Abelard also remarks that God knows whether any proposition is true and false, and all true propositions are determinately true with respect to this omniscience. The determinateness in the sense of divine knowability pertains to truth-values and truth-makers, but this supranatural knowability does not make things necessary.[71] While the truth-values of these propositions are in principle knowable, human beings cannot know them without supranatural illumination. The present truth of a future contingent proposition implies that what is predicted will obtain but not that it is determinate, since future contingent states of affairs are inde-

[69] *Dialectica* 211.28-32; 212.36-213.7; 221.15-24.

[70] *Logica 'Ingredientibus'* 431.13-432.9; 445.22-446.29.

[71] *Logica 'Ingredientibus'* 421.28-422.30. For truth and determinateness in Abelard, see also C. Normore, 'Future Contingents' in N. Kretzmann, A. Kenny, J. Pinborg (eds.), *The Cambridge History of Later Medieval Philosophy* (Cambridge: Cambridge University Press, 1981), 362-3; N.T. Lewis, 'Determinate Truth in Abelard', *Vivarium* 25 (1987), 81-109.

terminate.[72] In discussing the necessity of the present, Abelard follows Boethius in stating that what is actual at a certain point of time is necessary in the sense that it can no longer be avoided,[73] but he also argues that unrealized alternatives may be possible at the same time in the sense that they could have happened at that time.[74] The actuality of a contingent state of affairs at a specified future time does not exclude the non-temporal possibility of simultaneous alternatives and the truth of a proposition about this state of affairs does not make it necessary. This seems to be the background of Abelard's deviation from Boethius's approach to the truth and falsity of future contingent propositions.

According to Albert the Great and Thomas Aquinas, Aristotle argues in *De interpretatione* 9 that future contingent propositions differ from other assertoric propositions in not being determinately true or determinately false. Their general view of the structure of Chapter 9 is similar to that of Boethius. Instead of the Boethian definite-indefinite distinction, Albert and Aquinas employed the terms 'determinate' and 'indeterminate', as most medieval commentators did.[75] Assertoric propositions are related to truth or falsity in the same way as their correlates are related to being or non-being. When future things are indeterminate with respect to being and non-being, the contradictory propositions about them must also be indeterminate with respect to truth and falsity. A future contingent proposition and its denial form a disjunction which is necessarily true. The members of this disjunction are disjunctively true or false. The indeterminate truth or falsity of a member of the disjunction does not imply that it is true or that it is false. It is merely true-or-false. Because of the prospective indeterminateness of a sea-battle, the propositions pertaining to it 'must be true or false under disjunction, being related to either, not to this or that determinately.'[76] According to Aquinas, truth is not altogether lacking in a pair of contradictory singular future contingent propositions. It not true to say that both of these are false, for if one of these is false, the other is true, and *vice versa*.[77]

Following Boethius, Aquinas thinks that contingency pertains to the future. Past and present things are necessary. The absence of the idea of synchronic alternatives in Boethius's approach makes a true proposition about a future contingent event determinately and necessarily true. Aquinas describes this assumption as follows:

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[72] *Logica 'Ingredientibus'* 422. 33-40.

[73] *Logica 'Ingredientibus'* 437.37-438.17.

[74] See note 49 above.

[75] Albert the Great, *Comm. in Periherm.* I.5.4-5, 418-21; Thomas Aquinas, *In Peri herm.* I.13, 170-1.

[76] Thomas Aquinas, *In Peri herm.* I.15, 202-3; Albert the Great, *Comm. in Periherm.* I.5.6, 422; see also Averroes, *De interpretatione* in *Aristotlis Opera cum Averrois commentariis* I.T (Venice, 1562–71), 82va and Peter of Ireland, *Expositio et quaestiones in librum Aristotelis Peryermenias seu De interpretatione*, ed. M. Dunne, Philosophes médiévaux 34 (Louvain-la-Neuve: Éditions de l'Institut Supérieur de Philosophie, Louvain and Paris: Peeters, 1996), I.17, 132.164-133.173; I.19, 143.47-56.

[77] *In Peri herm.* I.13, 175.

> If it is always true to say of the present that it is, or the future, that it will be, it is not possible that this not be or that it will not be. The reason for this consequence is evident, for these two are incompossible, that something truly be said to be, and that it not be, for this is included in the signification of the true that what is said to be is. If therefore that which is said concerning the present or the future is posited to be true, it is not possible that this not be in the present or future. (*In Peri herm.* I.13, 173 [10]; trans. Oesterle, with changes)[78]

While the necessity of the disjunction of future contingent contradictory propositions does not imply that one of these is necessarily true, the truth of one part implies that it is necessarily true, because it excludes the truth of the other as impossible. In commenting on Aristotle's thesis that everything is necessarily when it is, though not simply necessary, Aquinas writes:

> This necessity is founded on the principle that it is impossible at once to be and not be, for if something is, it is impossible that it not be at the same time; therefore it is necessary that it be at that time ... Similarly, if something is not, it is impossible that it be at the same time. This is not absolute necessity, but necessity by supposition. (I.15, 201 [2], trans. Oesterle, with changes)

Necessity by supposition is the impossibility of being otherwise when one is actual. This necessity of the present does not imply the antecedent necessity of what is actual. Similarly the determinate division between truth and falsity in a contradictory pair of present propositions does not imply that even beforehand one or the other is determinately true.[79] Following Boethius, Aquinas assumes, as distinct from Abelard, that the disjunctive necessity of a contradictory pair of future contingent propositions implies that if one part were true, the other would be impossible, because the truth implies the actuality of the predicted state of affairs, and it is necessarily actual when it is actual, since antecedently possible alternatives are excluded.[80]

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[78] For incompossible possibilities, see also *In Aristotelis libros De caelo et mundo expositio*, ed. R.M. Spiazzi (Turin: Marietti, 1952), I.29, 280; cf. *Summa theologiae* I.24.4, ad 2. See also the discussion in F. Mondadori, 'Leibniz on Compossibility: Some Scholastic Sources' in R.L. Friedman and L.O. Nielsen (eds.), *The Medieval Heritage in Early Modern Metaphysics and Modal Theory, 1400-1700*, The New Synthese Historical Library 53 (Dordrecht: Kluwer, 2003), 324-6.

[79] *In Periherm.* I.15, 201.

[80] See also 1.4 above. After Aquinas, many authors endorsed the interpretation that Aristotle did not regard future contingent propositions as true or false. John Buridan was one of the few who read Aristotle like Abelard. All assertoric statements are true or false though those about future contingents are not determinately true or false; *Questiones longe super librum* Perihermeneias, ed. R. van der Lecq, Artistarium 4 (Nijmegen: Ingenium Publishers, 1983), I.10. Since most theologians thought that divine omniscience presupposed bivalence, the discussion of future contingents was divided into historical constructions of Aristotle's view and the systematic discussions in theology which usually followed the Abelardian lines. Peter Auriol argued that since future contingent propositions lack a truth value, even God is aware of the future in a way

1.7 Essentialist Assumptions

In his treatise *On Hypothetical Syllogisms* Boethius speaks about two kinds of conditionals which express a necessary consequence between the antecedent and the consequent. The consequence is accidentally necessary when the antecedent and consequent are immutably true but have no internal link between them, for example 'If fire is hot, the heavens are spherical'. In a non-accidental consequence, which Boethius calls natural, there is a conceptual connection between the parts; for example, 'If something is human, it is an animal'. Abelard also teaches that a genuine conditional expresses a necessary consequence in which the antecedent of itself requires the consequent. These were taken to express immutable laws of nature derivable from the nature of things.[81] A related distinction between *per se* and *per accidens* necessary propositions was employed in mid-thirteenth century discussions of modal conversion and modal syllogistic. Robert Kilwardby states that some necessary connections between terms are merely accidentally necessary in the sense that the things signified are inseparable. These necessities are not dealt with in modal syllogistic the necessity propositions of which express *per se* necessities explained in *Posterior Analytics* I.4. The first type is said to occur when the definition of the subject includes the predicate and the second type when the definition of the predicate includes the subject. Typical *per se* necessary propositions were those expressing the properties determined by the substantial form of a subject or, as in the second class, other features based on the genus-species structure. Terms themselves were necessary if they stood necessarily for what they signified, for example 'horse'. Other terms were accidental, for example 'white' or 'walking'. (See also 2.4 below.)

Necessary propositions which were not *per se* necessary were often exemplified by propositions about inseparable accidents. In the *Isagoge*, Porphyry defines the inseparable accident as something which cannot actually be removed from its subject though the subject can be conceived of without it (3.5-6). The idea of the degrees of necessity and impossibility was also developed in late ancient discussions of indirect proofs and impossible hypotheses. In order to defend Aristotle's indirect proofs with impossible premises, Alexander of Aphrodisias argued that Aristotle had in mind impossibilities which were not nonsensical.[82] Some late ancient authors were interested in impossible hypotheses as tools for conceptual

which does not imply that future contingent statements are true or false. This was an exceptional view. See C. Schabel, *Theology at Paris, 1316-1345: Peter Auriol and the Problem of Divine Foreknowledge and Future Contingents* (Aldershot: Ashgate, 2001).

[81] Boethius, *De hypotheticis syllogismis*, I.3.7; Abelard, *Dialectica*, 253.28-30; 279.12-14; 280,12-18; 283.37-284.17; see also Garland the Computist, *Dialectica*, 141.7-22. Some later twelfth-century masters regarded the principle that the antecedent is not true without the consequent as a sufficient condition for the truth of a conditional and accepted the so-called paradoxes of implication. See C.J. Martin, 'Logic' in J. E. Brower and K. Guilfoy (eds.), *The Cambridge Companion to Abelard* (Cambridge: Cambridge University Press, 2004), 164-5, 179-81.

[82] See the report in Simplicius, *In Aristotelis Physicorum libros quattuor posteriores commentaria*, ed. H. Diels, Commentaria in Aristotelem Graeca 10 (Berlin: Reimer, 1895), 1039.13-14, 26-7; translated by C. Hagen in Simplicius, *On Aristotle's Physics 7* (London: Duckworth, 1994), 105.

analysis. In the arguments which were called Eudemian procedures something impossible was assumed in order to see what followed. The impossibilities discussed in this way by Philoponus and Boethius show similarities with Porphyry's characterization of inseparable accidents as something which cannot occur separately but can be separated in thought. This analysis, which was counterpossible rather than counterfactual, influenced the branch of medieval obligations logic which was called *positio impossibilis.* The rules of this logical theory were often applied to theological problems – impossible hypotheses were regarded as conceivable but doctrinally impossible.[83]

In thirteenth-century philosophy, Aristotelian impossible hypotheses were also explained with the help of special abstract possibilities referring to states of affairs which could not exist. It was thought that the possibilities of a thing are determined by its genus, species, and matter. Something which is possible for a subject as a member of a genus may be impossible for it as a member of a species, and the same holds of its being a member of a species and a singular being individuated by matter. This was an essentialist counterfactual analysis without the idea of alternative possibilities.[84] In explaining why Aristotle says that the elements can be continuous with each other and with the celestial bodies, Aquinas writes as follows:

> But it must be said that the contingent and the impossible are taken in different ways when something is demonstrated of a genus and when something is demonstrated of a species. For when a species is treated, that must be taken as impossible which is repugnant to either the genus or the specific difference from which the nature of the species is constituted. But when a genus is treated, everything which is not repugnant to the nature of the genus is taken as contingent, even though it may be repugnant to the differentia which constitutes the species. For example, if I am speaking of animal, I can say that every animal being winged is contingent. But if I descend to the consideration of man, it would be impossible for this animal to be winged. Now Aristotle is speaking here about movers and mobile objects in general ... Therefore, he states as a contingency that all mobile objects are continuous with each other.

[83] C.J. Martin, 'Thinking the Impossible: Non-Reductive Arguments from Impossible Hypotheses in Boethius and Philoponus', *Oxford Studies in Ancient Philosophy* 19 (1999), 279-302, id., 'Impossible *positio* as the Foundation of Metaphysics or, Logic on the Scotist Plan', in C. Marmo (ed.), *Vestigia, imagines, verba. Semiotics and Logic in Medieval Theological Texts* (Turnhout: Brepols, 1997), 255-76; id., 'Obligations and Liars' in S. Read (ed.), *Sophisms in Medieval Logic and Grammar* (Dordrecht: Kluwer, 1993), 357-81. An example analysed in twelfth century Christological discussion was that a person is both man and donkey. This was regarded as naturally impossible and supernaturally possible since Jesus Christ was a human being and a divine being simultaneously. See S. Knuuttila, '*Positio impossibilis* in Medieval Discussions of the Trinity' in Marmo (ed., 1997), 277-88;

[84] See Averroes, *Commentary on Aristotle's Physics* in *Aristotelis Opera cum Averrois Commentariis*, vol. 5 (Venice 1562-1574), VI.15, 255vb; VII.2, 307vb; Thomas Aquinas, *In Phys.*IV.12, 538; VI.3, 774; VII.2, 896; VIII.1, 1009; S. Knuuttila, 'On the History of Theory of Modality as Alternativeness' in T. Buchheim *et al.* (ed., 2001), 233-5.

> This, however, is impossible if mobile objects are considered according to their determinate natures. (*In Phys.* VII.2.896)[85]

Aquinas's formulations illustrate the nature of medieval essentialism, also showing some baroque aspects of the theory.

2 MODAL SEMANTICS AND MODAL LOGIC IN TWELFTH AND THIRTEENTH CENTURIES

2.1 Equipollences and Oppositions

In the Latin translation of Aristotle's *Peri hermeneias* by Marius Victorinus, Aristotle's two terms for possible, *dunaton* and *endechomenon*, were apparently rendered by *possibile* and *contingens*.[86] Boethius states that these words may be taken as meaning the same, although they are not used in quite the same way: while the privation of possibility is expressed by *inpossibile*, the corresponding word *incontingens* is not used.[87] Many twelfth-and thirteenth-century authors thought that the terms *possibile* and *contingens* are used as synonyms or, if not, the term *possibile* is used for what is not impossible (possibility proper) and *contingens* for what is neither impossible nor necessary (two-edged possibility).[88] Using the term 'possible' for the basic modality which includes necessity and contingency became

[85]The translation is quoted from Thomas Aquinas, *Commentary on Aristotle's Physics*, translated by R.J. Blackwell, R.J. Spath, and W.E. Thirlkel (London: Routledge and Kegan Paul, 1963).

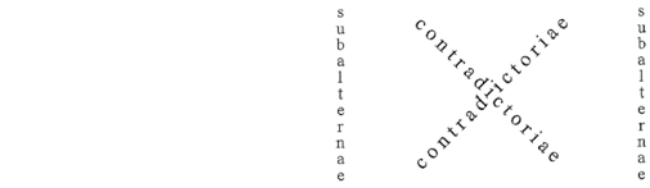
[86]A. Becker-Freysing, *Die Vorgeschichte des philosophischen Terminus 'contingens': Die Bedeutungen von 'contingere' bei Boethius und ihr Verhältnis zu den Aristotelischen Möglichkeitsbegriffen*, Quellen und Studien zur Geschichte und Kultur des Altertums und des Mittelalters D 7 (Heidelberg: Bilabel, 1938), 20-4.

[87]*In Periherm.* II, 382.17-22; 384.6-7; 392.17-393.12.

[88]For the synonymous use, see Peter Abelard, *Dialectica* 193.31; *Super Periermenias XII-XIV*, 8, 37; William of Sherwood, *Introductio in logicam*, 44-5; trans. Kretzmann (1966), 47, 50; Peter of Spain, *Tractatus*, I.24. In an anonymous twelfth-century commentary on Aristotle's *Prior Analytics* (Ms. Orléans BM 283) *possibile* refers to what is not impossible and *contingens* to this or to what is neither impossible nor necessary; f. 181a. (For this work, see S. Ebbesen, 'Analyzing Syllogisms or *Anonymus Aurelianensis III* — The (Pesumably) Earliest Latin Commentary on the *Prior Analytics*, and Its Greek Model, *Cahiers de l'Institut du Moyen-Âge Grec et Latin, Université de Copenhague* 37 (1981), 1-20.) In the *Dialectica Monacensis* the term *possibile* is characterized as expressing something like a genus of two species of modality, namely the necessity and the contingent (De Rijk II-2, 481.9-13, 20-21). Lambert of Auxerre and Robert Kilwardby suggest that while the common term may be 'contingent' or 'possible', 'contingent' is used for two-edged possibilities (Lambert of Auxerre, *Logica*, 40.1-42.2; Robert Kilwardby, *In libros Priorum Analyticorum expositio* (Venice, 1516, reprinted Frankfurt am Main: Minerva 1968), 7ra-b); see also Albert the Great, *Commentarius in librum I Priorum Analyticorum*, ed. A. Borgnet in *Opera omnia* I (Paris: Vivès, 1890), IV.4, 546. Roger Bacon and some others stated that *contingens* is used of what is true or actual but can be false or un-actual, while *possibile* refers to what is false or un-actual but can be true or actual; Roger Bacon, *Summulae dialectices, I: De termino, II: De enuntiatione*, ed. A. de Libera, *Archives d'histoire doctrinale et littéraire du moyen âge* 61 (1986), II.1.6, 247, 260-1, 395-6, 408-10; de Rijk 1967, II.1, 467; II-2, 391.19-19.

more common in late medieval logic. The contingency which was separated from necessary and impossible was often divided into natural (*naturale* or *natum*) and indefinite (*infinitum*) depending whether the contingency was founded on a natural tendency which was realized in most cases or was without such a tendency.[89] In Boethius's frequential classification, the contingency was divided into what takes place in most cases, in few cases, and indeterminately, all these being cases of *utrumlibet*.[90] While some medieval authors called non-necessary contingency *ad utrumlibet*,[91] it became more usual to equate *ad utrumlibet* with the indefinite contingency.[92]

Modifying Boethius's systematization of Aristotle's remarks in *De interpretatione* 12 and 13, the authors of this period often presented the equipollences between modal terms and opposed relations between modal propositions with the help of the following diagram:



The square could be taken to refer to modals *de dicto* or singular modals *de re*.[93]

[89] The background of this division was Aristotle's *Prior Analytics* I.13, 32b4-14; see *In Pr. an.* (Orléans) 186b; Lambert of Auxerre, *Logica*, 42.5-25; Robert Kilwardby, *In Pr. an.* 7vb- 8rb; Albert the Great, *In Pr. an.* I.12, 477a.

[90] *In Periherm.* II, 240.6-21; 248.11-26.

[91] *In Pr.an.* (Orléans) 181a; *Dialectica Monacensis*, 481.18-21; Lambert of Auxerre, *Logica*, 42.4-5.

[92] Robert Kilwardby, *In Pr. an.* 7vb, 8rb; Roger Bacon, *Summulae dialectics III: De argumentatione*, ed. A. de Libera, *Archives d'histoire doctrinale et littéraire du moyen âge* 62 (1987), III.1.2, 62. Both uses are found in Albert the Great, *In Pr. an.* I.12, 476a-477a; 1.14, 481b; see also K. Jacobi, *Die Modalbegriffe in den logischen Schriften des Wilhelm von Sherwood und in anderen Kompendien des 12. und 13. Jahrhunderts: Funktionsbestimmung und Gebrauch in der logischen Analyse* (Cologne: Brill, 1980), 92-4; H. Lagerlund, *Modal Syllogistics in the Middle Ages*, Studien und Texte zur Geistesgeschichte des Mittelalters 70 (Leiden: Brill, 2000), 23-5.

[93] de Rijk 1967, II-1, 469-70; II-2, 393.6-394.5; 431.19-26; 483.1-484.5; William of Sherwood, *Introductiones in logicam*, 45.11; Peter of Spain, *Tractatus* I.25, 16.12-13; Roger Bacon, *Summulae dialectics* II.1.6, 354; Thomas Aquinas, *De propositionibus modalibus*, ed. H.-F. Dondaine, *Opera omnia* 43 (Rome: San Tommaso, 1976), 422. Peter of Abelard explained the relations without a diagram (*Super Periermenias* 22.4-25.5).

Peter Abelard tried to define the opposed relations between quantified *de re* modals as well. He thought that these were the same as those between singular modal propositions, which is completely wrong. This question was not much discussed before its satisfactory solution in the early fourteenth century. (See p. 554 below.)

In discussing the imperfect first figure assertoric-contingency syllogisms, Aristotle formulated the principle 'If when A is, B must be, then when A is possible, B must be possible'. On the basis of this, he describes syllogisms as follows: 'If someone were to put the premises as A and the conclusion as B, it would not only follow that B is necessary, if A is necessary, but also that B is possible, if A is possible' (*Prior Analytics* I.15, 34a5-7, 22-4). These are the rules of inference

(4) $p \rightarrow q \vDash Lp \rightarrow Lq$

and

(5) $p \rightarrow q \vDash Mp \rightarrow Mq$.

In *Metaphysics* IX.9, 1047b14-20, Aristotle argues for (5) by applying the characterization of potentiality in *Metaphysics* IX.3, 1047a24-6, which is based on the definition in *Prior Analytics* I.13, 32a18-19: 'I use the expressions 'to be possible' and 'what is possible' in application to something if it is not necessary but nothing impossible will result if it is put as being the case.' If this is applied to possibility proper, it could be formulated as

(6) $Mp \;\&\; (p \rightarrow q) \vDash Mq$.

Aristotle did not develop his general remarks on propositional modal logic, but these principles were dealt with in later ancient discussions of the logic of conditionals and related issues.[94] Similar rules were often put forward in early medieval logical treatises.[95]

2.2 Modalities de dicto *and* de re

In dealing with modal propositions in his later commentary on *De interpretatione*, Peter Abelard argues that modal terms in the proper grammatical sense of the word are adverbs which modify the inherence between a subject and a predicate. Terms like 'necessarily', 'well' or 'rapidly' are used in this manner; adverbs like 'possibly' or 'falsely' are analogously called modals, although they do not modify actual inferences. Nominal modal terms occur in propositions like 'Necesse est

[94] See W. and M. Kneale, *The Development of Logic* (Oxford: Clarendon Press, 1962), 128-38; J. Vuillemin, *Nécessité ou contingence: l'aporie de Diodore et les systèmes philosophiques* (Paris: Les Editions de Minuit, 1984); S. Bobzien, 'Chrysippus's Modal Logic and Its relation to Philo and Diodorus' in K. Döring and T. Ebert (eds.), *Dialektiker und Stoiker* (Stuttgart: Franz Steiner, 1993), 63-84.

[95] See, e.g., Peter Abelard, *Dialectica*, 202.6-8; 219.5; *Logica 'Ingredientibus'* 429.36-7; 430.16-17.

Socratem currere'. When this is understood in accordance with its grammatical construction, it is not a genuine modal proposition, the mode itself being said of the grammatical subject which is the accusative and infinitive phrase. The nominal mode can also be understood in an adverbial manner, however, in which case the proposition expresses a modalized predication.[96] In Abelard's view, these and some further ambiguities demanded a systematic investigation of the properties of various propositions including modal terms. In his approach, grammatical distinctions were transformed into a logical distinction between *de sensu* modes and *de re* modes. Abelard states that in the present treatise he uses the terms *de sensu* and *de re* in the same way as Aristotle uses the expressions 'in a compound sense' (*per compositionem* or *coniunctionem*) and 'in a divided sense' (*per divisionem*) in *Sophistici elenchi* 4, 166a23-30, and that the distinction, understood in this way, maintains an inference from an affirmative possibility proposition *de sensu* to a corresponding affirmative possibility proposition *de re*. When possibility propositions *de sensu* are interpreted in a compound sense, both *de sensu* readings and *de re* readings are in fact *de re* readings which express modalized predications about actual things. The difference between these is that a modal proposition *de sensu* in a compound sense asserts that a subject which is said to have a predicate possibly, necessarily, or impossibly has it together with all descriptions of the subject mentioned in the proposition. A modal proposition *de re* does not imply this combination.[97] The compound *de sensu* reading of 'An F is necessarily/possibly/impossibly G' is:

(7) An x which is F is necessarily/possibly/impossibly G while remaining F

and the *de re* reading is

(8) An x which is F is necessarily/possibly/impossibly G.

In addition to this analysis, Abelard mentions that *de sensu* modalities can also be understood in an impersonal manner in which case they do not refer to capacities or incapacities embedded in actual beings but instead to what can or cannot be. A *de sensu* possibility proposition in this strict sense does not imply a *de re* possibility proposition with an existential import. Abelard's examples are 'Every substance is a spirit' and 'My son is living' said by a person not having a son. These are not true in the Aristotelian sense, but possible in the sense of what is not repugnant to nature.[98] The differences between Abelard's strict *de sensu* — reading and (7) and (8) could be expressed in standard modern notation as follows:

(9) $\Diamond Ex(Fx \ \& \ Gx)$

[96] *Super Periermenias* 3.7-11.16.

[97] *Super Periermenias* 13.15-14.4; 30.25-31.10.

[98] *Super Perihermenias* 29.13-30.25; cf. 20.13-21.15. For impersonal modal constructions in Abelard, see K. Jacobi, 'Diskussionen über unpersönliche Aussagen in Peter Abaelards Kommentar zu *Peri hermeneias*' in E.P. Bos (ed.), *Medieval Semantics and Metaphysics*, Artistarium Supplementa 2 (Nijmegen: Ingenium Publishers, 1985), 30-40.

(10) $Ex(Fx \ \& \ \Diamond(Fx \ \& \ Gx))$

(11) $Ex(Fx \ \& \ \Diamond(Gx)$.

As already mentioned, Abelard applied the distinction between compound and divided meanings, without using these terms, in his influential analysis of the compatibility between contingency and divine omniscience. After Abelard the sections about modal propositions in logical treatises often begin with remarks on adverbial and nominal modes. According to the *Dialectica Monacensis*, the modal adverb qualifies the composition between the subject and the predicate. Modal propositions with nominal modes can be taken to mean the same as corresponding adverbial modal propositions or can be taken to mean that what is expressed by the accusative and infinitive element is necessary, possible, or impossible. The adverbial modes are said to be dealt with in Aristotle's *Prior Analytics* and the non-adverbial nominal modes in his *De interpretatione.*[99] The structure of adverbial modal propositions without negation is:

(12) quantity/subject/mode, copula/predicate.

The question of the quality (affirmative, negative) is associated with the problem that the sign of negation can be located in two different places:

(13) quantity/subject/negation, mode, copula/predicate

or

(14) quantity/subject/mode, negation, copula/ predicate.

If modal propositions with a negation are read in accordance with (13), then the mode is denied; if they are read in accordance with (14), the modal adverb qualifies a negated predication. The quality and quantity of modal propositions are determined by those of the corresponding non-modal propositions. Thus (14) is a negative modal proposition and (13) an affirmative proposition with a negated mode. Propositions with non-adverbial nominal modes are always singular, their form being:

(15) subject/copula/mode.

In the *Dialectica Monacensis*, modal propositions with the structure of (12)–(14) are also called *de re* or divided modalities and those with the structure of (15) *de dicto* or compound modalities.[100] This terminology was commonly employed in later medieval treatises.[101] In the twelfth century some logicians began to apply the grammatical distinction between categorematic and syncategorematic terms to

[99]de Rijk 1967, II-2, 479.35-480.26.
[100]de Rijk 1967, II-2, 570.22-9.
[101]See Jacobi 1980, ch. 4.

modal terms. These were taken to have a categorematic meaning as *de re* modes and a syncategorematic meaning as *de dicto* modes.[102]

The distinction between various interpretations of possibility propositions was extensively dealt with in twelfth- and thirteenth-century discussion of Aristotle's examples of fallacies based on shortcomings in distinguishing between compound and divided meanings. The compound interpretation of 'A standing man can sit' was usually taken to be 'It is possible that a man sits and stands at the same time'. Many authors formulated the corresponding divided interpretation as involving a reference to a later or earlier time.[103] The reference to another time was based on the assumption that the necessity of the present prevents the acceptance of

(16) p (now) & $\Diamond - p$ (now).

The authors who regarded (16) as false did not operate with counterfactual alternatives. They thought that if an unactualized present possibility is assumed to be actualized, something impossible follows, as is seen in the rule of thirteenth-century obligations logic, according to which one should deny that the 'now' of a disputation is the present 'now', if it is assumed for the case of argument that a contingent proposition which is false now is true.[104]

In discussing the proposition 'That which is standing can sit' the author of the *Dialectica Monacensis* states that the *de re* possibility can be predicated of the subject abstracting from the subject's actual qualifications; this may imply a counterfactual interpretation of the modal proposition. Another example of the idea of modal alternativeness in this and some other works is found in the theory that the word 'can' may ampliate the subject term so that it stands for actual and merely possible beings.[105]

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[102]For some examples, see *Syncategoremata Monacensia* in Braakhuis 1979, 99-100, translated in N. Kretzmann and E. Stump (eds.), *The Cambridge Translations of Medieval Philosophical texts I: Logic and the Philosophy of Language* (Cambridge: Cambridge University Press, 1988), 168-9; William of Sherwood, *Syncategoremata*, ed. J.R. O'Donnell, *Mediaeval Studies* 3 (1941), 72-3, trans. with an introduction and notes by N. Kretzmann in *William of Sherwood's Treatise on Syncategorematic Words* (Minneapolis: University of Minnesota Press, 1968), 100-101; Jacobi 1980, 86-7, 207-9, 339-41.

[103]See notes 15 and 29 above.

[104]In *De obligationibus* attributed to Willam of Sherwood, the author writes: 'When a false contingent proposition concerning the actual instant of time has been posited one must deny that it is the actual instant. This is proved as follows. Let A be the name of the actual instant; it is discrete name and not a common name. When it is false that you are in Rome, it is impossible that it is true then or in A. It can become true only through a motion or through a change, but it cannot become true trough a motion in A, because there is no motion in an instant. And it cannot become true through a change, since if there were a change to truth in A, the truth would be in A, for whenever there is a change, there is the term of that change. So it is impossible that this false proposition becomes true in A. Therefore, if it is true, A is not actual.' The text is edited in R. Green, *The Logical Treatise 'De obligationibus': An Introduction with Critical Texts of William of Sherwood (?) and Walter Burley* (PhD diss., University of Louvain, 1963).

[105]de Rijk 1967, II-2, 570.18-571.9; 623.33-624.26. Cf. 1.5 above.

2.3 Modalities with Temporal Determinations

Avicenna and Abelard showed particular interest in alethic modalities with determinations. Like Boethius, Abelard divided propositions with a *dum* -phrase ('so long as') into two groups, depending on whether the determination was intrinsic or extrinsic, i.e., whether it is the same as the modalized predicate or not. Both of these can be understood in two ways. The word *dum* can be taken to indicate time and to form a compound predicate. In this 'modal' reading the intrinsic or extrinsic *dum* phrase is included as a restriction in what is predicated with a modal qualification. Or they may be considered as complex propositions which have as parts a modal proposition and a non-modal *dum* proposition and assert that something is necessary or possible or impossible and things are as the *dum* phrase says they are.[106] There were similar discussions in Avicenna. In the part on logic of his *Pointers and Reminders*, Avicenna distinguishes between absolute necessity propositions (like 'God exists), which are always true and refer to eternal things, and various conditional necessity propositions. These are true while the non-eternal substances referred to by the subject terms exist (like 'Man is necessarily a rational body'), some descriptions are attached the substances (like 'All that is moving is necessarily changing'), what is predicated is actual or a definite or indefinite time is actual.[107] Propositions which are necessary on the condition that the subject is qualified by a description show similarities to Abelard's *de re* compound modal propositions.

Apart from the distinction between two basic readings Abelard's remarks on the propositions which contain a modal term and a *dum* phrase are not always easily understood. If the temporal *dum* phrase is included in the modal proposition, one may wonder how to understand the difference between simple modal propositions and modal propositions with temporal determinants. Abelard states that a determined possibility proposition implies a corresponding simple possibility proposition, but not *vice versa*, and a simple necessity proposition implies a determinate necessity proposition, but not *vice versa*.[108] These principles could be derived from the standard idea that simple possibilities are temporally indefinite and may be prevented from being actualized as temporally quanified, but Abelard also refers to a distinction between treating the determination as pertaining to a modal term and treating it as pertaining to the scope which is modalized.[109]

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[106] *Dialectica* 206.14-37; *Super Periermenias* 36.22-37.23.

[107] Avicenna, *Pointers and Reminders*, part one, *Logic,* trans. by S.C. Inati (under the title *Remarks and Admonitions*, Toronto: Pontifical Institute of Mediaeval Studies, 1984), 92-4: see also T. Street, 'An Outline of Avicenna's Syllogistic', *Archiv für Geschichte der Philosophie* 84 (2002), 133-4; P. Thom, *Medieval Modal Systems: Problems and Concepts*, Ashgate Studies in Medieval Philosophy (Aldershot: Ashgate, 2003), 67-8, 74-6. For temporal and modal concepts in medieval Arabic logic, see also N. Rescher, *Temporal Modalities in Arabic Logic*, Foundations of Language Supplementary Series 2 (Dordecht: Reidel, 1967).

[108] *Super Perihermenias* 36.11-21.

[109] *Super Perihermenias* 41.23-42.6. See also the discussion in R. Pinzani, *The Logical Grammar of Abelard*, The New Synthese Historical Library 51 (Dordrecht: Kluwer Academic Publishers, 2003), 111-16.

'Socrates necessarily sits when he sits' is true in the first case but false in the second case. Perhaps he means that while the modal status of a proposition pertaining to a temporally definite event is basically independent of temporal changes, it can be changeable when the modal qualification is evaluated from a temporal point of view. Things which are not necessary now may be so to-morrow. This is in agreement with Abelard's view about unchangeable metaphysical necessities and possibilities and temporal modalities which are associated with real potencies.[110]

After Abelard, an often discussed theory about time and modality was associated with the distinction between temporal modalities *per se* and *per accidens*. *Per se* necessary propositions were said to be true whenever they were uttered and *per accidens* necessary propositions, true past tense singular propositions referring to a definite event, were unchangeably true after having begun to be true. The same analysis was applied to impossible propositions as well.[111]

2.4 Modal Conversion

While the conversion of assertoric propositions was frequently discussed in early medieval logic, the modal conversions were not often dealt with before the *Prior Analytics* began to be used in logic teaching in the thirteenth century. One earlier example is Peter Abelard's remark that modal propositions in the divided sense are only convertible into assertoric propositions, not into modal propositions; for example, 'For every man it is possible to run' converts into 'Something for which it is possible to run is a man'.[112] This was found problematic when it was realized that conversion of modals into modals played an important role in Aristotle's modal syllogistics which seemed to concentrate on modalities in the divided sense.[113] According to Aristotle (*An. pr.* I.3), necessity propositions are converted in the same way as the corresponding assertoric propositions: a universal affirmative predication (AaB) implies a converted particular predication (BiA), a particular affirmative predication (AiB) is equivalent with a converted predication (BiA), and a universal negative predication (AeB) is converted into universal negative predication (BeA). The first conversion was called accidental and the others simple. While these rules are not problematic with respect to modals in the compound sense, Aristotle employed them in proving modal syllogisms some of which seem to be acceptable only when the premises are modal propositions in

[110] Abelard often thinks of possibility in terms of potency attaching to things of a particular species; see J. Marenbon, 'Abelard's Concept of Possibility' in B. Mojsisch and O. Pluta (eds.), *Historia philosophiae medii aevi: Studien zur Geschichte der Philosophie des Mittelalters* (Amsterdam and Philadelphia: Gruener, 1991), 595-609, reprinted in Marenbon 2000; for the idea of metaphysical alternatives, see C.J. Martin, 'Abelard on Modality: Some Posibilities and Some Puzzles', T. Buchheim *et al.* (eds. 2001), 97-124; Martin 2003.

[111] See de Rijk II-1, 371; II-2; 429.1-10; 481.22-482.14; Roger Bacon, *Summulae dialectices* 2.1.6, 366-72; William of Sherwood, *Introductiones in logicam* 41.8-16, trans. Kretzmann 1966, 41, and note 16 above.

[112] *Super Perihermenias* 11.17-12.20.

[113] For this view of modal syllogistics, see *Summe Metenses* in de Rijk 1967, II-1, 468; *Dialectica Monacensis*, de Rijk 1967, II-2, 480.10-16; Lambert of Auxerre, *Logica* 30.17-23.

the divided sense. One of the problems of his modal syllogistics is that reading these conversion rules in the divided sense leads to obvious difficulties: the actuality of subjects changes into necessity and possibility into actuality. Contrary to what one might expect, the relationship between compound and divided modalities was not dealt with in mid-thirteenth-century discussions of Aristotle's rules, these being associated with various philosophical ideas of the nature of necessity and contingency. Many authors tried to redefine syllogistic necessity propositions in a way which would match the conversion rules. This took place in treating some often repeated counter-examples, such as

(17) Everything literate is necessarily a human being

(18) Everything healthy (or awake) is necessarily an animal

or

(19) Everything white is necessarily a body.

It was considered an obvious fact that no animal is necessarily healthy or that 'some animal is healthy' is not a necessary truth. Some authors argued that in examples like (17)–(19) the denominative subject terms refer to things which are qualified by non-essential forms and that these terms should be treated in the same way when they are predicates.[114] 'An animal is necessarily healthy' is taken to mean that an animal is necessarily that which is healthy. Accordingly, (17) could be read: 'Every literate being is necessarily a member of the class of human beings', but even then the conversion 'Some human beings are necessarily members of the class of literate beings' is false, if this is not understood, as Lambert of Auxerre suggests, to mean that some human beings are necessarily those who are literate, that is, human beings.

Robert Kilwardby, after mentioning this pretty artificial idea, moves to a 'more probable' interpretation which is based on the view that convertible necessity premises in modal syllogistic are necessity propositions *per se* and not *per accidens*, like (17)–(19), which are not convertible. In affirmative necessity propositions *per se*, the subject is *per se* connected to the predicate. In negative necessity propositions *per se*, the subject is apparently *per se* incompatible with the predicate. In accidental necessity propositions, the connection is not based on a *per se* inherence or repugnance.[115] In explaining the notion of *per se* later in his work,

[114] See Lambert of Auxerre, *Logica* 39.1-41; Robert Kilwardby, *In Pr. an.* 7ra; Roger Bacon, *Summulae dialectices* III.1.2, 58, 60; cf. *In Pr. an.* (Orléans) f. 180b.

[115] *In Pr. an.* 7ra-b; cf. 45rb for essential negative propositions. Kilwardby's concludes in 7rb: 'Therefore, when Aristotle teaches that necessity propositions are convertible, he means that only *per se* necessity propositions are convertible'. Aristotle did not say this; Kilwardby followed the medieval commentator habit of maintaining that what commentators found reasonable was what Aristotle said or meant. Kilwardby seems to assume that syllogistic necessity propositions should be taken in the divided sense or at least not in the composite sense (27va). For *per se* necessity in Kilwardby, see also Lagerlund 2000, 25-42; Thom 2003, 93-6; P. Thom, *Logic and Ontology in the Syllogistic of Robert Kilwardby*, Studien und Texte zur Geistesgeschichte des Mittelalters 92 (Leiden: Brill, 2007), 19-28.

Kilwardby refers to Aristotle's discussion of different types of essential predication in *Posterior Analytics* I.4. He does not give a detailed explanation of why the *per se* inherences or incompatibilities make propositions convertible, but he does associate convertible propositions with terms signifying their subjects invariably and in this sense necessarily.[116] This excluded the conversion of *per se* necessary predications involving accidental terms, such as 'All walking is necessarily moving.'

There were similar discussions among Arabic philosophers. Avicenna did not accept the conversion of affirmative necessary propositions, although he considered the conversion of universal negative necessity propositions as valid.[117] Averroes restricted the syllogistic necessity premises to necessity propositions *per se.* In these propositions the terms, such as 'animal', are essential, invariably signifying necessary constituents of things. Accidental terms, such as 'walking' or 'white', signify accidental things. The background to this distinction is *Post. An.* I.4.[118] The conversion of these propositions is regarded as unproblematic. The idea that the terms of syllogistic necessity premises are themselves essential also occurs in Kilwardby, although he does not use quite the same terminology.

Averroes, Robert Kilwardby, Albert the Great, and their followers regarded Aristotle's modal syllogistics as a correct theory of modalities, the explication of which often demanded metaphysical considerations. Restricting the modal conversion of necessity propositions to those involving essential terms is an example of this attempt to discern the unity and coherence of Aristotle's theory. The same orientation dominated the discussion of the conversion of contingent propositions.

According to Aristotle, a two-edged negative contingency proposition (neither necessary nor impossible) implies an affirmative contingency proposition of the same quantity, and all contingency propositions are converted by the conversion of terms into particular contingency propositions (*An. pr.* I.3, 25a37-b19; 13,

[116] *In Pr. an.* 8va; 22ra; 25rb; Roger Bacon, (*Summulae*, III.1.2, 59) refers to Aristotle's first and second mode of *per se* predication (*An. post.* I.4, 73a34-b5). Cf. Albert the Great, *In Pr. an. IV.10,559a.*

[117] Avicenna's discussion of the conversion differs from other medieval approaches because of his special view that the subject terms and predicate terms of assertoric propositions stand for all possible applications. The instantiations of the subject term of a universal affirmative proposition are asserted to have the predicate at least once (*Pointers and Reminders* 2.5, trans. in Street 2002, 134). Other classes of proposition are treated in an analogous way. Avicenna took this to mean that universal negative assertoric propositions are not convertible (3.3; Street 2002, 143). The terms of modal propositions are ampliated in the same way as the terms of assertoric propositions. Universal negative necessity propositions are simply convertible and other necessity propositions are not convertible. Universal negative possibility propositions do not convert. Affirmative possibility proper propositions convert like assertoric propositions (3.5; Street 2002, 144-5; Thom 2003, 65-6, 69).

[118] Averroes's detailed discussion of the structure of modal propositions is included in his separate essay on modal syllogistics, translated into Latin in the *Quaesita octo in librum Priorum Analyticorum Aristotelis* in *Aristotelis Opera cum Averrois Commentariis* I.2b (Venice 1562-1574); see IV.3, 83-4. In his commentary on the *Prior Analytics* (*Media expositio in libros Priorum Resolutoriorum* in *Aristotelis Opera cum Averrois Commentariis* (Venice, 1562-74), 1I.1) Averroes simply says that necessary predications express essential connections; see 45va. See also Thom 2003, 81-5.

32a29-b1). This was taken for granted in mid-thirteenth-century logical treatises. It was noted that while the converted propositions of indefinite contingency propositions are of the same type of contingency, the conversion of natural contingency propositions or of those of the minor part contingency, whether with respect to quality or terms, result in different modal propositions.[119] There were extensive discussions of the kinds of contingency in the commentaries by Robert Kilwardby and Albert the Great and in other treatises by their contemporaries based on various philosophical ideas of contingency.[120] Following Aristotle's remark in *An. pr.* I.13, 32b23-32, according to which 'A contingently belongs to B' may mean either 'to that to which B belongs' or 'to that to which B contingently belongs', Kilwardby argues that the major premise in uniform first figure contingency syllogisms is read in the second way, having the form 'Everything/something that is contingently B is contingently A'.[121] Contingency premises are ampliated, if syllogistic relations do not demand restrictions.[122] In explaining the difference between necessity propositions and contingency propositions in this respect, Kilwardby teaches that syllogistic terms are substantial or accidental. Substantial terms necessarily stand for the things they signify, while accidental terms contingently stand for them. Since the terms in *per se* necessity propositions are substantial, 'Every A is necessarily B' and 'Whatever is necessarily A is necessarily B' mean the same.[123]

Averroes did not have much to add to what Aristotle says about the convertibility of contingent propositions, except that the subject terms of syllogistic contingency propositions are always meant to be read with the prefix 'Everything/something which is or is contingently'.[124] As distinct from the mid-

[119] *In Pr. an.* (Orléans), 186b; Lambert of Auxerre, *Logica*, 42.26-49.43; Robert Kilwardby, *In Pr. An.* 8rb-va; Peter of Spain, *Syncategoreumata*, ed. with an introduction by L.M. de Rijk, with an English translation by J. Spruyt, Studien und Texte zur Geistesgeschichte des Mittelalters 30 (Leiden: Brill, 1992), VII.6, 284-7: 17-19, 294-7; Albert the Great, *Pr. an.* I.13, 477a-480b; 14, 482b; Roger Bacon, *Summulae* III.1.2, 61-6; Jacobi 1980, 83-4, 382. For the kinds of contingency, see also Averroes, *Media expsitio in libros Priorun Resolutoriorum* 35vb-36ra.

[120] According to Aristotle, universal negative possibility proper propositions are simply convertible into universal negative possibility propositions (*An. pr.* 1.3, 25b3-14). It was concluded from this that a universal negative contingency proposition is convertible into a universal negative possibility proposition; see Lambert of Auxerre, *Logica*, 47.14-48.7; Robert Kilwardby, *In Pr. an.* 9ra; Roger Bacon, *Summulae*, III.1.2, 64; albert the Great, *In Pr.an.* I.14, 480a-482b.

[121] Kilwardby, *In Pr. an.* .

[122] *In Pr. an.* 19vb; 21ra-b; Kilwardby says later that the subject term of the minor premise of a universal first-figure syllogism may also be actual (22rb).

[123] *In In Pr. an.* 21ra-b; 22ra-b. See also Thom 2007, 22-3. These considerations did not influence conversion rules. Kilwardby did not find it problematic that the contingency premises in various mixed third-figure moods, which he did not treat as ampliated, were converted in Aristotle's modal syllogistic.

[124] *Media expositio in libros Priorum Resolutoriorum*, 38rb; 44ra-b. According to Averroes, Alfarabi mistakenly argued that the subject-term ampliates to the possible in all syllogistic premises. In fact this is true only of contingency propositions (28vb). Alfarabi's lost commentary on the *Prior Analytics* may be the source of Avicenna's view of ampliation (note 117 above). See also T. Street, "'The Eminent Later Scholar" in Avicenna's *Book of the Syllogism*', *Arabic Science and Philosophy* 11 (2001), 205-18.

thirteenth century Latin commentators, Averroes taught that dealing with the various types of non-necessary contingent propositions is not relevant in logic.[125]

Aristotle's *Prior Analytics* continued to be part of university logic teaching in the thirteenth century. Kilwardby's commentary was used in this context as is shown by the fact that many of the questions dealt with by him were considered in later thirteenth-century treatises on the *Prior Analytics.* These have not yet been edited. The next edited work is Richard Campsall's *Questions on Prior Analytics*, written in Oxford *c.* 1305.[126] Campsall often refers to various views which show that he was familiar with modal ideas different from those in Kilwardby. Fourteen of twenty questions are about modal propositions and two are about past and future tense propositions. Campsall almost always begins his discussions of modal questions by referring to the distinction between compound and divided modal propositions, apparently thinking that one should deal with the logic of both readings, as he does to some extent. This became a standard procedure after Campsall. Analysing the fine structure of singular modal propositions and reducing the role of metaphysical principles were further new features of Campsall's work in comparison to the mid-thirteenth century logic of modalities. He wanted to show that Aristotle's theory was consistent, although it was associated with various problems. In trying to resolve these, Campsall was obliged to formulate some rules which were not used by Aristotle.

Campsall treats the conversions of necessity and contingency propositions in the compound sense as the conversions of their assertoric contents (*dicta*). These conversions are based on the rules that when assertoric propositions are qualified by the notions of necessity or contingency (neither necessary nor impossible), the converted propositions are qualified by necessity or contingency, with the exception that the conversion of universal affirmative contingency propositions does not always result in the same kind of contingency.[127] Divided necessity propositions are also converted in the same way as assertoric propositions, and the same holds true of affirmative contingency propositions.[128] Negative contingency propositions are regarded as equivalent to affirmative contingency propositions of the same quantity.[129] Campsall regards these as Aristotelian modal principles and makes use of some additional rules in order to defend their questionable validity.

Campsall describes the notion of necessity as follows:

> An affirmative necessity proposition in the divided sense is true only

[125] *Media expositio in libros Priorum Resolutoriorum* 36rb.

[126] *Quaestiones super librum Priorum Analyticorum* in *The Works of Richard Campsall* I, ed. E.A. Synan (Totonto: Pontifical Institute of Mediaeval Studies, 1968); there are a lot of mistakes in this is edition. For unedited works, see Ch.H. Lohr, 'Medieval Latin Aristotle Commentaries', *Traditio* 23-30 (1967-74).

[127] 5.39 (111); 6.21 (121); 7.32 (135). When 'contingens' and 'possibile' are not treated as synonyms, 'contingens' refers to what is neither necessary nor impossible and 'possibile' to what is not impossible (as in 7.32). Campsall does not discus the distinctions between the kinds of contingency on which earlier authors spent much time.

[128] 5.40 (111); 6.22-4 (121-2); 7.33 (135-6).

[129] 16.21 (250).

> if something contained under the predicate term is unchangeable with respect to something contained under the subject term, as long as this exists. This holds when the terms are common. When the terms are singular, it is required that what is signified by one is not changeable with respect to what is signified by the other, as long as this exists.[130]

Unchangeable features signified by common terms may be transcendental properties, such as 'something', substantial forms, properties based on the species nature of things, such as 'mortal', 'living', or other inseparable attributes.[131] Divided affirmative necessity propositions with common or singular terms express that things under a necessarily predicated term are in various ways the same as actual things under a subject term, as long as both exist.[132] Campsall believes that this reading of divided necessity propositions explains why they are regulated by the Aristotelian rules of conversion. The strict restriction to actual things is not separately explained since Campsall probably thought that Aristotelian necessity premises should be treated in this way and that only actual things can have necessary properties. Much attention is paid to the example 'A pale Socrates is necessarily Socrates'. This is said to be false for the reason that what the accidental subject term signifies is not an invariable characterization of Socrates.[133] Campsall does not deal with the traditional universal or particular counter-examples, thinking that they can be analysed into the conjunctions or disjunctions of singular propositions with terms standing for accidental combinations.

The equation of necessity with unchanging relations pushes Campsall's theory toward the temporal frequency approaches of necessity. It is also characterized by some versions of the principles that the present is necessary and that possibility implies actuality. Let us take a look at his proofs of the conversion of divided necessity propositions. According to Campsall, 'Every/some B is necessarily A' is converted into 'some A is necessarily B', because the negation of the converted proposition 'Every A is possibly not B' is not compatible with the proposition to be converted. This is allegedly shown by the fact that were the negation true, a particular being, say c, which is B and necessarily A would be possibly not B. This is said to be impossible, apparently because c is necessarily B *qua* being unchangingly A.[134] Campsall's proof of the convertibility of universal negative divided necessity proposition is similar — this time he states that 'this consequence is necessary: c can be one of those which are now contained under B; therefore it

[130] 6.25 (122-3).

[131] 5.43-5 (112-3); 9.18 (157).

[132] For the actuality condition, see 5.40 (111); divided negative propositions express that the things under the terms are necessarily separated (5.38 (110)).

[133] 6.25-31 (122-5).

[134] 6.22 (121-2). According to Lagerlund, Campsall assumes that if c is possibly not B now, it is not B now (2001, 68). Campsall explicitly denies this principle later (19.21 (297)) and the argument can be understood as indicated above. Campsall maintains, however, that if c is not B now, it is necessarily not B now (5.50 (114)). If all negative propositions about actual things are necessary, it is clear that there are no simultaneous *de re* alternatives. In this sense actuality implies necessity. It seems that Campsall was not fully aware of the problems embedded in his formulations.

is one of those which are now contained under B'.[135] Possibility implies actuality in the sense that an affirmative possibility proposition about actual connections between things implies the corresponding affirmative proposition.

Campsall states that Aristotle's rules for the conversion of divided contingency propositions can be applied to predications the subject terms of which stand for things which are under them or which are contingently under them. As far as the predicate term stands for actual things, the distinction pertains to contingent attributes of things which are necessarily (*per se*) or contingently (*per accidens*) under the subject term.[136] Contingency means in this context that things signified by the terms are not invariable with respect to each other. Applying the standard supposition analysis, Campsall states that a universal affirmative contingency proposition in the divided sense means that A_1 is contingently B_1 or B_2 and so on and A_2 is contingently B_1 or B_2 and so on. A particular proposition is a disjunction of these. 'This A is contingently this B' is said to express a conjunction of two possibility proper propositions one of which is affirmative and the other is negative, 'This A is possibly this B and this same A is possibly not this same B'. The converted form can be neither necessary nor impossible, because the coversions of these would be incompatible with the original contingency proposition.[137] If the terms stand for actual things, the negative possibility proposition denies that the attribute which possibly and therefore actually belongs to the subject does so necessarily, i.e., A and B as the attributes of this c are not invariable with respect to each other.

2.5 Modal Syllogistics

Avicenna wrote a brief summary of Aristotle's modal syllogistics, but his own theory was different, concentrating on necessities and proper possibilities understood in the same way as divided modal propositions in Latin discussions and being based on ampliated terms and conversion rules mentioned above. He also sketched some ideas about modal syllogistics for propositions with temporal determinants. Insofar as Avicenna treated the terms of assertoric propositions as modally amplified and equated the truth-conditions of purely assertoric propositions and possibility propositions, syllogisms with assertoric and necessity premises are similar to syllogisms with possibility and necessity premises and syllogisms with assertoric and possibility premises or with assertoric premises to uniform possibility syllogisms.[138]

While Averroes's commentary on the *Prior Analytics* follows the main lines of Aristotle's text, his separate treatise on modality involves new systematic ideas, mainly the theory of accidental and *per se* necessary essential terms. According to Averroes, necessary syllogistic premises are *per se* necessary propositions with

[135] 5.40 (111).
[136] 7.33 (135), 17.56 (270).
[137] 7.34-6 (135-7). See also Lagerlund 2000, 69-72.
[138] See note 117 above; Street 2002; Thom 2003, 67-80. For later Arabic criticism of Avicenna's temporal frequency approach to modality and modal syllogistic, see also Street 2005.

per se necessary essential terms. Since modal premises are of the divided type, assertoric premises in Aristotelian mixed necessary-assertoric-syllogisms must have a predicate term which in fact is necessary. Universal and particular propositions of this kind are called assertoric *per se* and necessary *per accidens*, since the predicate always belongs to the subject when the subject is actual. Propositions with a necessary subject term and a non-necessary predicate term are said to be temporally assertoric, since the predicate belongs to the subject merely at some times. The first premise in mixed assertoric-necessary syllogisms must be of this kind (or a proposition of necessary matter). The conclusion in the first case is necessary *per accidens* and in the second case temporally assertoric.[139] This is a speculative explanation of Aristotle's asymmetric treatment of mixed necessary-assertoric syllogisms and mixed assertoric-necessary syllogisms.[140] Analogous essentialist ideas were developed in thirteenth-century Latin discussions.[141] According to Averroes, all syllogistic contingency propositions are ampliated with respect to potential subjects.[142] Assertoric premises in mixed contingency-assertoric syllogisms are based on universal predications which hold for all times for all subjects or at least for most subjects.[143]

The first known Latin commentary on *Prior Analytics* (Ms. Orléans BM 283) involves fairly detailed explications of Aristotle's modal moods and discussions of many problems dealt with in ancient commentaries. A concise summary of Aristotle's modal syllogistic is also provided in the late twelfth-century *Dialectica Monacensis*, and the elements of modal syllogistics seem to have been discussed in logic courses in the first part of the thirteenth century.[144] In the 1240's, Robert Kilwardby's commentary became an authoritative work on Aristotle's *Prior Analytics*. The remarks on modal syllogistics in Albert the Great's commentary were largely derived from Kilwardby.

According to Kilwardby, the premises and the conclusion in uniform necessity syllogisms should be necessary *per se*, as is clear from his interpretation of the conversion rules of necessary propositions.[145] In mixed first-figure syllogisms with a major necessity premise and a minor assertoric premise, the non-modalized premise should be *simpliciter* assertoric rather than merely *ut nunc* (as-of-now). A simply assertoric premise of this mixture includes a predication which is *per se*

[139] *Quaesita octo in librum Priorum Analyticorum*, IV.3, 84.

[140] See also the discussion in Thom 2003, 81-5.

[141] Gersonides tries to develop further Averroes's remarks in the modal part of his *Book of the Correct Syllogism*; see Manekin, 1992, 309.

[142] See note 124 above.

[143] *Media expositio in libros Priorum Resolutoriorum*, 45va-b.

[144] See, e.g., Braakhuis 1979, 243-4; de Rijk 1967, II-1, 468; Peter of Spain, *Syncategoreumata*, 304; *Compendium examinatorium Parisiense*, ed. C. Lafleur and J. Carrier (Québec: Faculté de Philosophie, Université Laval, 1992), 176-94.

[145] While Kilwardby does not analyse the structure of necessity premises in discussing the uniform necessity syllogism (15ra-16ra), he explains the nature of *per se* necessity premises elsewhere (16va; 21ra; 22ra; 25rb). In treating the exposition proofs of Baroco and Bocardo, Kilwardby states that either particular premises are changed into universal premises in expository syllogisms or expository syllogisms are formed by taking one singular example and having two singular premises (16ra).

in the first or second way (cf. Aristotle, *An. post.* I.4) and consequently necessary *per se* and not merely *per accidens.* Similarly Kilwardby states that in mixed first-figure syllogisms with contingent and assertoric premises the assertoric premise must be simply assertoric, but now not necessary. It is simply assertoric if the predicate belongs to the subject essentially, invariably or by natural contingency.[146] Kilwardby states that even though the conclusions in mixed first-figure necessary-assertoric moods and mixed assertoric-necessary moods with simply assertoric premises are necessary, they are syllogistically derived as necessary only in the first case. The modal character of the predicate in the conclusion of perfect first-figure syllogisms follows that of the first premise which involves the whole syllogism in accordance with the *dici de omni et nullo.*[147]

In explaining the varieties in reading assertoric premises in mixed syllogisms, Kilwardby states that a first-figure major necessity premises 'appropriates' to itself a minor which is unrestrictedly assertoric, i.e., necessary *per se.* No such appropriation occurs in first-figure mixed assertoric-necessity syllogisms, the first premise being either necessary or true as-of-now. In the second figure, while the universal negative necessity premise appropriates to itself an unrestricted assertoric premise, a particular necessity premise cannot appropriate a universal to itself, nor an affirmative a negative. There are similar considerations about third-figure moods.[148]

In uniform contingency syllogisms, the subject term of the major premise is read 'Everything/something that is contingently A' but this ampliation does not concern mixed contingency syllogisms.[149] This major contingency premise is always false when the subject is an accidental term and the predicate a substantial term. If the same applies to contingent minor premises, it seems that a major premise can be true with a minor only when the middle term is accidental, taking for granted that both terms cannot be substantial in a true contingency proposition. Kilwardby explains, however, that if the subject term of the minor premise is not ampliated, it may be true even when the predicate is a substantial term.[150]

Kilwardby's considerations show similarities to Averroes's classification of modal propositions with essential and accidental terms. Both authors assume that propositions of the same form can have different interpretations depending on how they are related to other propositions in a syllogism. These interpretation rules are based on various metaphysical assumptions. Kilwardby's *ad hoc* remarks on contingency premises read in the sense of natural contingency in certain moods show the same approach.[151]

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[146]Kilwardby, *In In Pr. an.* 16va; 25rb; see also Albert the Great, *In In Pr. an.* IV.10, 558b-559b.

[147]Kilwardby, *op. cit.* 16vb-17ra; 21ra-b; Albert the Great, *op. cit.* IV.4, 526a-b. For the application of the *dici de omni et nullo* in this context, see also Lagerlund 2001, 41-2.

[148]16vb; 17va-b; 18vb; 24vb. See also Thom 2007, 148, 160-1, 165-6, 172-4ch. 5.

[149]Kilwardby, *op. cit.* 19vb, 21ra-b; Albert the Great, *op. cit.* IV.2, 540b-541a.

[150]Kilwardby, *op. cit.* 22ra; Albert the Great, *op. cit.* IV.4, 545a-b.

[151]In a mixed first-figure necessity-contingency mood the first negative premise 'appropriates'

As distinct from his forerunners, Campsall comments separately on Aristotelian modal syllogisms with premises in the compound and divided senses. Uniform necessity syllogisms are valid on both readings. Syllogisms in the compound sense are reduced to the principle

(20) If the premises are necessary, the consequent is necessary.

Syllogisms in the divided sense are valid because they are regulated by the same principles as the assertoric syllogisms, i.e., *dici de omni et nullo*.[152] Mixed necessity-assertoric moods are valid in the compound sense on the basis of (20), when the assertoric premise (*de inesse* proposition) is of the *de inesse simpliciter* type, i.e., necessarily true. These consequences are not syllogistic, however, and the same holds of mixed assertoric-necessity moods with a necessary conclusion. Mixed necessity-assertoric syllogisms in the divided sense are valid when the assertoric premise is *de inesse simpliciter* or *ut nunc primo modo*, i.e., a proposition which express a necessary relation between actual things. Propositions *de inesse secundo modo* express accidental relations and assertoric minor premises of this kind do not make valid syllogisms. Campsall describes the validity of the perfect necessity-assertoric syllogisms by referring, like Kilwardby, to the principle that the whole syllogism is implicitly included in the first premise. The middle term is said to be sufficiently the same in both premises only when the minor premise is in fact necessary.[153]

If syllogistic premises are contingent in the compound sense, it does not follow that the consequent is contingent in the compound sense. Aristotelian uniform contingency syllogisms are valid for divided modals when the middle term is ampliated with respect to contingency.[154] Aristotelian first-figure mixed contingency-assertoric syllogisms are said to be valid for divided modals provided that the contingency premise is not ampliated and the assertoric premise is assertoric *simpliciter* or *ut nunc primo modo*.[155] Because affirmative and negative contingency propositions with terms standing for actual things imply affirmative assertoric propositions and the contingent premises in mixed necessary and contingent syllogisms are of this type, the conclusions are assertoric. Aristotle seemingly denied this, but in fact he wanted to say that the assertoric conclusion was evident in negative moods and less apparent in affirmative moods.[156] The conclusions are

to itself a natural contingency minor; *op. cit.* 27va-b. For Kilwardby's discussion of contingency syllogisms, see also Thom 2007, ch. 6.

[152]Campsall 11.58-9 (194); see also Lagerlund 2000, 74.

[153]12.33-5 (205). Campsall states that mixed assertoric-necessity Baroco and mixed necessity-assertoric Bocardo with necessary conclusions are syllogistically valid. When Aristotle did not accept these, he did not mean that they were not acceptable — it was only because the indirect proof involved a possibility proper syllogism which was presented later in the *Prior Analytics.* The corresponding mixed necessity-assertoric Baroco and mixed assertoric-necessity Bocardo are valid consequences, but not syllogistic (14.16-19 (221-2)).

[154]17.48-9, 17.70 (268-9, 274).

[155]18.36-8 (283-4).

[156]20.14-17 (304). Campsall does not deal with Aristotelian mixed contingency-necessity moods, apparently regarding them immediately obvious when the contingency premises are not ampli-

assertoric also in mixed assertoric-contingency syllogisms, for the predicate of the contingency premise must stand for actual things and consequently be equivalent to an assertoric premise, but this time Campsall says that the assertoric conclusions do not follow syllogistically.[157]

As far as the middle premises of contingent syllogisms stand for actual things, as is the case with mixed moods with contingent minor premises, there is no difference in the logic of affirmative premises and contingency premises. A negative assertoric premise is said to imply the corresponding necessity premise. Why is this step associated with negative propositions and not with affirmative propositions? Campsall seems to think that negative premises express actual incompatibilities which are analogous to essentialist *de re* necessities. The possible denial of 'Socrates is pale' does not refer to the actual time at which Socrates is white; Campsall says that as long as Socrates exists, being pale is variable with respect to him and 'sometimes when he is 'Socrates is a pale Socrates' is true and sometimes it is false'.[158] Even though a *de re* possibility of not being does not imply non-being now, it seems to imply non-being later.

It has been argued that the asymmetry between affirmative and negative *de re* possibilities is based on the insight that if a contingency proposition with actual terms implies two possibility propositions, one affirmative and one negative, and the affirmative implies actuality, the negative cannot imply actuality. The notion of contingency would otherwise be self-contradictory. Campsall was allegedly familiar with the Scotist denial of the necessity of the present and applied it to affirmative *ut nunc* propositions, although he continued to think that true negative *ut nunc* propositions were necessarily true.[159] However, if Campsall made use of the Scotist idea of simultaneous alternatives, why did he think that what can be actual now is actual now and that actual things are contingent only if they change later? Scotus's idea was that contingently actual things can be otherwise at the very moment of their actuality. Campsall cannot say this, because all propositions expressing present alternatives are necessarily false.[160] While there are no alternative possibilities with respect to the present, actual things are not simply necessary if they can change later. This is one of the traditional ways to qualify the necessity of the present.

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ated.

[157]18.71-2 (291-2). Campsall states that when the first premise of a first figure mood is affirmative *simpliciter* or *ut nunc primo modo*, the syllogistic conclusions are contingent and not merely non-impossible, as in negative moods. Aristotle says that all conclusions are possible, and Campsall states that this follows when the major premise may also be *ut nunc secundo modo* (18.54-7 (287-9)). This is meant to explain Aristotle's formulations. Campsall's earlier remark that Aristotle did not operate with assertoric premises of this kind in mixed syllogisms apparently pertains to minor assertoric premises (12.32, (205)).

[158]12.31 (204), 19.21 (297).

[159]Lagerlund 2000, 87-90. For Scotus's view, see 3.1.

[160]See note 134 above.

3 LATER MEDIAEVAL DEVELOPMENTS

3.1 Modalities in Philosophy and Theology

It has been assumed that the increasing interest in modal syllogistic and modal logic in general in early fourteenth century was associated with certain philosophical and theological developments which added to the interest in modal theories. These often involved the idea of simultaneous alternatives which did not play a significant role in mid-thirteenth-century essentialist theory.

Following the twelfth-century model, Henry of Ghent (d. 1293) applied obligations logic to Trinitarian doctrine by assuming doctrinally impossible positions in order to see what followed from them and what did not.[161] This was an influential idea. Several early fourteenth-century authors found an obligational analysis as a useful tool for analysing conceptual connections between theological concepts. The most popular version of this logic was called *positio.* It deals with how an increasing set of true and false propositions might remain coherent in a disputation in which an opponent puts forward a contingent and false initial proposition and a respondent accepts this and accepts or denies other propositions in a logically consistent way. Irrelevant propositions are granted, denied or doubted according to the best knowledge of the respondent. Relevant propositions should be treated in a consistent way. These either follow from the initial position and/or what has been granted and/or the opposites of what has been correctly denied or are incompatible with them. Thirteenth-century *positio* rules denied that a now false but possible position could refer to the time of an actual obligations discussion. This was in agreement with the traditional doctrine of the necessity of the present.[162] As part of his new interpretation of modal concepts, John Duns Scotus dropped this rule, a revision which made it possible to understand obligational answers as partial descriptions of how things could be instead of regarding them as a internally consistent set of propositions without a sensible interpretation.[163]

Scotus's revision of obligations rules was in agreement with his modal meta-

[161] Henry of Ghent, *Summae quaestionum ordinarium* I.56 (Paris 1520, reprinted St. Bonaventure, N.Y.: The Franciscan Institute; Louvain: Nauwelaerts; Padenborn: Schöningh, 1953), vol. 2, 92r-v. One twelfth-century predecessor is *Tractatus Emmeranus de impossibili positione*, edited by L.M. de Rijk in 'Some Thirteenth Century Tracts on the Game of Obligation', *Vivarium* 12 (1974), 94-123; an English translation by M. Yrjönsuuri in M. Yrjönsuuri (ed.), *Medieval Formal Logic*, The New Synthese Historical Library 49 (Dordrecht: Kluwer, 2001), 217-23.

[162] See note 104 above.

[163] For Scotus' revision, see *Lectura* I.39.1-5, 56, 59 in in *Opera omnia*, ed. C. Balić *et al.* (Vatican Ciry: Typis Polyglottis Vaticanis, 1950-), vol. 17; an English translation in John Duns Scotus, *Contingency and Freedom: Lectura* I.39, introduction, translation and commentary by A. Vos Jaczn., H. Veldhuis, A.H. Looman-Graaskamp, E. Dekker and N.W. den Bok, The New Synthese Historical Library 42 (Dordrecht: Kluwer Academic Publishers, 1994). For obligations logic, see M. Yrjönsuuri, *Obligationes: 14^{th} Century Logic of Disputational Duties*, Acta Philosophica Fennica 55 (Helsinki: Societas Philosophica Fennica, 1994) and note 83 above; for obligations logic in fourteenth-century theology, see H. Gelber, *It Could Have Been Otherwise. Contingency and Necessity in Dominican Theology at Oxford 1300-350*, Studien und Texte zur Geistesgeschichte des Mittelalters 81 (Leiden: Brill, 2004).

physics. In arguing for the contingency of the created order Scotus writes:

> I do not call something contingent because it is not always or necessarily the case, but because the opposite of it could be actual at the very moment when it occurs.[164]

This is a denial of the traditional thesis of the necessity of the present and the temporal frequency characterization of contingency. In the Scotist definition, the meaning of the notion of contingency is spelt out by considering simultaneous alternatives. What is actual is contingent only if, instead of being actual, it could be not actual. This conception of simultaneous contingent alternatives is part of an argument that the first cause does not act necessarily. According to Scotus, the eternal creative act of divine will is free only if it is a choice between alternatives and could be other than it is in a real sense.[165]

That God acts by choice between alternatives had been a common theological view since Augustine, but Scotus formulated its conceptual foundation in a new way, which had consequences for how modal terms in general were understood. In the Augustinian tradition, it was thought that metaphysical possibilities are ultimately based on the divine essence and represent the ways in which it could be imitated by created things. Scotus was the first to deviate from this metaphysical tradition in which possibilities are founded on divine being. According to Scotus, when God as an omniscient being knows all possibilities, he does not know them by turning first to his essence. Possibilities can be known in themselves.[166] In fact they would be what they are even if there were no God. Scotus states that if it is assumed that, *per impossibile,* neither God nor the world exists and that the proposition 'The world is possible' then existed, this proposition would be true. The actual world is possible as it is, and this possibility and the possibilities of unrealized things are primary metaphysical facts which are not dependent on anything else.[167]

Scotus calls the propositional formulations of pure possibilities logical possibilities (*possibile logicum*). These express things and states of affairs to which it is not repugnant to be, which means that their descriptions do not involve a contradiction. Possibilities as such have no kind of existence of their own, but are real in the sense that they form the precondition for everything that is or can be.[168] A great deal of Scotus's discussion of metaphysical themes concentrates on the

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[164] *Ordinatio* I.2.1.1-2, 86 (*Opera omnia*, vol. 2); *De primo principio*, ed. and translated by W. Kluxen (Darmstadt: Wissenschaftliche Buchgesellschaft, 1974), IV.4.

[165] *Lectura* I.39.1-5, 58; see also L. Honnefelder, *Duns Scotus* ((Munich: C.H. Beck, 2005), 85-6.

[166] *Ord.* I.35, 32 (*Opera omnia*, vol. 6); Knuuttila 1993, 103-4; S. Knuuttila, 'Duns Scotus and the Foundations of Logical Modalities' in L. Honnefelder, R. Wood, M. Dreyer (eds.), *John Duns Scotus: Metaphysics and Ethics*, Studien und Texte zur Geistesgeschichte des Mittelalters 53 (Leiden: Brill, 1996), 135-36.

[167] *Ord.* I.7.1, 27 (*Opera omnia*, vol. 4); ; *Lect.*I.7, 32 (*Opera omnia*, vol. 16); *Lect.*; I.39.1-5, 49; *Quaestiones super libros Metaphysicorum Aristotelis libri VI-IX*, ed. R. Andrews *et al.* (St. Bonaventure: The Franciscan Institute, St. Bonaventure University, 1997), 9.1-2, 18.

[168] Knuuttila 1996, 137-41.

modal explication of being and the disjunctive transcendental notions of necessity and contingency.[169]

God's omniscience involves all possibilities and as objects of God's knowledge they receive an intelligible or objective being. Some of these are included in God's providential plan of creation and will receive an actual being. The description of a possible world at a certain moment consists of compossible possibilities. Though possibilities necessarily are what they are, the actualizations of non-necessary possibilities are not necessary but contingent. All things which are contingently actual at a certain moment could be not actual at that very moment. Since all finite beings are contingently actual, the contingent alternative possibilities are possible with respect to the same time, though they are not compossible with what is actual. According to Scotus, impossibilities are incompossibilities between possible ingredients, such as Socrates's sitting at a certain time and Socrates's not sitting at that same time. God could have chosen a world in which the first happens by Socrates's free will or a world in which the second happens by Socrates's free will. Since these possibilities are real possibilities, though not compossible, they are Socrates's possibilities in alternative histories.[170]

Scotus's new modal metaphysics influenced early fourteenth-century philosophy and theology in many ways. From the point if view of logic and its applications to natural philosophy, the most significant ideas were the distinction between logical and real modalities and the association of logical modalities with alternative imaginative domains. Thirteenth-century essentialist assumptions were largely dropped from modal logic based on new modal semantics.

3.2 Early Fourteenth-Century Modal Logic

I shall sketch the main lines of early fourteenth-century modal logic by concentrating on three works. The first is William Ockham's *Summa logicae* (*OSL*). In respect to the variety of the combinations of types of premises in modal syllogisms, this includes the most extensive fourteenth-century discussion of the topic. The

[169] For modalities in Scotus's metaphysics, see L. Honnefelder, *Scientia transcendens. Die formale Bestimmung der* S*eiendheit und Realität in der Metaphysik des Mittelalters und der Neuzeit (Duns Scotus – Suarez – Wolff – Kant – Peirce)* (Hamburg: Meiner, 1990), 3-108.

[170] *Ord.* I.43, 14; *Lect.* I.39.1-5, 62-5; *Ord.* I.35, 32, 49-51; I.38, 10. For Scotus' modal theory, see also Knuuttila 1993, 139-46; C. Normore, 'Scotus, Modality, Instants of Nature, and the Contingency of the Present' in Honnefelder, Wood and Dreyer (eds., 1996), 161-74; Normore, 'Duns Scotus's Modal Theory' in T. Williams (ed.), *The Cambridge Companion to Duns Scotus* (Cambridge: Cambridge University Press, 2003), 129-60; S.P. Marrone, 'Revisiting Duns Scotus and Henry of Ghent on Modality' in Honnefelder, Wood and Dreyer (eds., 1996), 175-89. Many scholars have paid attention to similarities between Scotus and Leibniz and regarded both as predecessors of the ideas codified in possible worlds semantics. Scotus, as distinct from Leibniz, did not use the term 'possible world', but it received a theoretical role in modal discussions before Leibniz; see S.K. Knebel, 'Necessitas moralis ad optimum. Zum historischen Hintergrund der Wahl der Besten aller möglichen Welten', *Studia Leibnitiana* 23 (1991), 3-24; J. Schmutz, 'Qui a inventé les mondes possibles' in J.-C. Bardout and V. Jullien (eds.), *Les mondes possibles*, Cahiers de philosophie de l'Universite de Caen (Caen: Presses universitaires de Caen, 2006), 9-45.

second is John Buridan's *Treatise on Consequences* (*BC*). According to the editors, Ockham's *Summa logicae* was written in 1323 and Buridan's *Tractatus* in 1335.The third is an anonymous *Questions on Prior Analytics* (*PS*). Its anonymous author is known as Pseudo-Scotus since it was included in the seventeenth-century edition of Duns Scotus's works. I refer to this work as *PS*. Buridan discussed modal logic and modal syllogistic also in his *Summulae de Dialectica* and in *Quaestions on Prior Analytics*. Some modal questions are dealt with in his *Questions on Peri hermeneias*.[171] I refer mainly to the *Treatise on consequences* because it contains a carefully considered summary of his modal theory. It seems that the treatise by Pseudo-Scotus is later than the works of Ockham and Buridan, many of the questions being similar to those in Buridan's *Questions on Prior Analytics*.[172]

Buridan presents modal logic as part of his general theory of consequences, which includes sections on the equipollences between various modes in combination with negations and on the relations of the standard square of opposition between them, the nature and conversions of compound and divided (*de dicto* and *de re*) modal propositions, the mutual relationship between these two types of modal propositions and their relations to assertoric propositions, further modal consequences, and modal syllogisms.[173] The same themes occur in all works mentioned. Fourteenth-century modal logic strove for generality, which had effects on the attitudes towards Aristotle's modal syllogistics. For one thing, it was thought

[171]William Ockham, *Summa logicae*, ed. Ph. Boehner, G. Gál, S. Brown, Guillelmi de Ockham Opera philosophica, vol. III (St. Bonaventure, N.Y.: St. Bonaventure University, 1974); John Buridan, *Tractatus de consequentiis*, ed. H. Hubien, Philosophes médiévaux 16 (Louvain: Publications Universitaires, Paris: Vander-Oyez, 1976); id., *Questiones longe super librum Peri hermeneias*, ed. R. van der Lecq, Artistarium 4 (Nijmegen: Ingenium Publishers, 1983); id., *Quaestiones in Analytica priora*, ed. H. Hubien (not published); idem, *Summulae de Dialectica*, an annotated translation with a philosophical introduction by G. Klima (New Haven: Yale University Press, 2001); Pseudo-Scotus, *In librum primum Priorum Analyticorum Aristotelis quaestiones* (John Duns Scotus, *Opera omnia*, ed. L. Wadding (Lyons, 1640), vol. I, 273-341. For the date and author of this treatise, see S. Read, 'Self-Reference and Validity Revisited' in Yrjönsuuri (ed., 2001), 184. Buridan's *Treatise on Consequences* is translated in King, 1985.

[172]For late medieval modal logic and modal theories, see also G.E. Hughes, 'The Modal Logic in John Buridan' in G. Corsi, C. Mangione and M. Mugnai (eds.), *Atti del convegno Internazionale di Storia della Logica. Le Teorie della Modalità* (Bologna: CLUEB, 1989), 93-111; Knuuttila 1993, 149-96; Spade 1996, ch. 10; Lagerlund 2000; Thom 2003; Normore 2003; S. Knuuttila, 'Medieval Theories of Modality', *Stanford Encyclopedia of Philosophy*, `http://plato.stanford/edu/entries/modality-medieval/` (2006).

[173]In the first half of the fourteenth century, many logicians wrote detailed studies on consequences, either in comprehensive works on logic, such as Walter Burley's *De puritate artis logicae* or William Ockham's *Summa logicae* or in independent treatises, such as Burley's *De consequentiis* and Buridan's *Tractatus de consequentiis*. For recent works on late medieval discussions of consequences, see P. King, 'Consequences as Inference: Mediaeval Proof Theory' in M. Yrjönsuuri (ed., 2001), 117-45; I. Boh, 'Consequence and Rules of Consequences in the Post-Ockham Period', in M. Yrjönsuuri (ed., 2001), 147-81; C. Dutilh Novaes, 'Buridan's *consequentia*: Consequence and Inference within a Token-based Semantics', *History and Philosophy of Logic* 26 (2005), 277-97. While Buridan and Pseudo-Scotus based their discussions on modal syllogistic on the theory of modal consequences, Ockham did not deal with modal syllogistic in the section on consequences of the *Summa logicae*.

that the notion of possibility proper (Mp = -L-p) must be added to modal syllogistics as the basic notion. Furthermore, it was considered imperative to distinguish between modal premises in the compound (*de dicto*) and the divided (*de re*) senses and to divide *de re* modals into two groups depending on whether the subject terms refer to actual things or possible things. Aristotle's modal syllogistics was regarded as a fragmentary theory in which the distinctions between different types of fine structure were not explicated.

The truth conditions of categorical propositions in the traditional square of opposition were given in terms of supposition theory as follows: provided that no semantic paradoxes are involved in the propositions, a universal affirmative proposition is true iff the predicate term stands for everything for which the subject term actually stands. A particular affirmative proposition is true iff the predicate terms stands for some of those for which the subject term actually stands. Negative propositions did not have existential import, and negatives with empty subject-terms were considered true. Aristotelian assertoric conversion rules, viz., the simple conversions between universal negative propositions and affirmative particular propositions and the so-called accidental conversion from affirmative universal proposition to particular affirmative proposition are valid in this interpretation.[174] These rules were in agreement with the identity theory of predication which was also employed by Campsall.[175]

As far as these authors treated propositions as tensed, the above remarks were taken to pertain to propositions the copulas of which were understood as present tense verbs. Since the twelfth century, there had been various attempts to define the truth conditions of past and future tense propositions. This discussion continues in early fourteenth-century logic. According to Buridan, the supposition of the subject term of the past tense proposition 'A was B' is ampliated so that it stands for past and present As while the predicate term stands for past Bs; 'A was B' is read as 'What was or is A was B'. Similarly, in 'A will be B' the supposition of the subject term is ampliated so that it stands for present and future As. It is read as 'What is or will be A will be B'. The restrictive phrase 'which is' (*quod est*) prevented ampliation when it was added to the subject term.[176] Some early thirteenth-century logicians applied the theory of ampliation to possibility propositions so that their subject terms associated with the verb 'can' were taken to stand for actual and merely possible beings, which is how the subjects in divided possibility propositions without the restriction (*quod est*) were understood by Ockham, Buridan and Pseudo-Scotus.[177] Following the traditional doctrine of the matter of propositions, fourteenth-century logicians stated that assertoric

[174] *OSL* II.2-4, 249-66; II.21, 318-21; *BC* I.5, 25-6; I.8, 44-5; *PS* I.12-15, 290-96.

[175] See Lagerlund 2000, 86.

[176] See *BC* I.6, 26-30; I.8.14, 45-7; *Summulae de Dialectica* IV.6.2, 299-300; cf. *OSL* II.7, 269-72; II.22, 321-5; *PS* I.17, 297-9. In question 11 of his treatise on the *Prior Analytics*, Campsall discussed syllogisms with tensed premises, and this syllogistic tense logic was developed further in Ockham's *Summa logicae* III-1.17-19, 406-11.

[177] *OSL* I.72, 216; II.25, 331-2; *BC* I.6, 27; *PS* I.3, 277; I.26, 311. See also Lagerlund 2000, 108-12, 138-40, 171-6.

propositions are necessary, possible, or impossible depending on whether they can or cannot be true or false. Only those propositions were counted as modal, however, which included modal terms connected to the copula (divided modals) or connected by the copula to propositions or to *dicta* (compound modals).[178]

The conversions of compound necessity and possibility propositions with respect to the *dictum* were not considered problematic. As in Campsall, these were said to hold by the rules that if the antecedent of a valid consequence is possible then the consequent is possible, and if the antecedent is of necessity then the consequent is of necessity or, as with simple conversions, if one of the convertibles is possible (necessary) then the other is possible (necessary). Of compound contingency propositions only those with simply convertible *dicta* are converted; the rule that if the antecedent is contingent then the consequent is contingent is not valid.[179] The whole logic of compound modal propositions was in fact based on the Aristotelian principles for propositional modal logic which, as noted above, were widely known in early medieval logic as well. When modal syllogisms were regarded as syllogisms with respect to the *dicta*, connecting the mode 'necessary' with the premises and conclusions of valid assertoric syllogisms yielded valid modal syllogisms. This was based on (4) and the rule that if the conjuncts are necessary, the conjunction is necessary. Uniform syllogisms consisting of compound contingency or possibility modals were not considered valid, because the compossibility of two possible premises was not assured. Ockham and Pseudo-Scotus remark that because *de dicto* necessities are compossible with any de *dicto* possibilities or contingencies, mixed compound necessity and possibility or contingency syllogisms with possible or contingent conclusions are valid. Buridan did not mention this.[180]

The main object of the fourteenth-century modal logic was the theory of divided modals. Some treatises include discussions in which the logical relations between various divided modal propositions were codified in the same way as the relations between the types of assertoric propositions in the square of opposition. Buridan taught that there were two types of copula, the affirmative 'is' and the negative 'is-not', and that modality was part of the copula in divided modal propositions. Combining the equivalences between quantifying words with negations with equivalent modalities, Buridan arranged divided modals into eight groups of nine equivalent formulae. In the *Summulae*, these groups are presented in a diagram showing the relations of contradiction, contrariety, sub-contrariety, and sub-alternation between them.[181]

[178] *BC* II.1, 56; *PS* I.25, 309; William Ockham., *Expositio in librum Perihermenias Aristotelis*, ed. A. Gambatese and S. Brown, Opera philosophica II (St. Bonaventure: St. Bonaventure University, 1978), 459-66.

[179] *OSL* II.24-5, 27, 327-8, 330-31, 334; *BC* II.7.12-14, 72-4; *PS* I.25, 310; I.30, 319.

[180] *PS* I.27, 313; I.33, 323; *OSL* III-1.20, 412-13; III-1.23, 419; III-1.44, 474; III-1.47, 479; *BC* IV.1.1, 113.

[181] For this octagon of opposition, see *Summulae de dialectica* I.8.4-7 and the diagram in *Questiones longe super librum Perihermeneias* II.9, 87; see also Hughes 1989, 109-10; E. Karger, 'Buridan's Theory of the Logical Relations between General Modal Formulae' in H.A.G. Braakhuis and C.H. Kneepkens (eds.), *Aristotle's* Peri hermeneias *in the Latin Middle Ages*, Artistarium supplementa 10 (Groningen – Haren: Ingenium Publishers, 2003), 429-44.

According to Buridan and Pseudo-Scotus, the subject terms of all divided modals are ampliated to stand for actual and possible beings which fall under those terms. The phrase 'what is' (*quod est*) attached to subject terms restrict them to standing for actual beings only.[182] Divided necessity modals with restricted subject terms are not converted simply or accidentally, and the same holds true of the conversions of divided necessity propositions with non-restricted subject terms, with the exception that universal negative propositions are convertible simply.[183] Unlike Buridan and Pseudo-Scotus, Ockham did not accept any conversions of terms of divided necessity propositions.[184] In fact he did not treat divided necessity propositions with unrestricted subject terms at all in his modal logic.[185]

As to unrestricted divided possibility propositions, Ockham, Buridan and Pseudo-Scotus state that affirmative modals are converted in the same way as assertoric propositions, those with restricted subject terms not being convertible.[186] According to Buridan and Pseudo-Scotus, an unrestricted divided proposition *de contingenti ad utrumlibet* can be converted into one of the opposite quality, but no conversions of the terms are valid.[187] They treated these propositions as ampliated with respect to possibility. Ockham states that if the subject terms are ampliated with respect to contingency, unrestricted universal contingency propositions convert into particular contingency propositions.[188]

In discussing the logical properties of the unrestricted divided modal propositions, Buridan and Pseudo-Scotus made some comments on the question whether such propositions should be treated as categorical propositions with a disjunctive subject (what is or can be ...), as they did, or whether they should be taken as complex propositions.[189] Pseudo-Scotus claimed that they could be read as conjunctions or disjunctions as follows: 'Every A is-possibly(-not) B' is a conjunction of 'Everything which is A is-possibly(-not) B' and 'Everything which can be A, etc.' and 'Some A is-possibly(-not) B' is the disjunction: 'Something which is A is-possibly(-not) B' or 'Something which can be A, etc.'[190]

It has been suggested that one could supply a Kripke-style possible worlds semantics for Buridan's modal system as an axiomatic basis for it.[191] I think

[182] *BC* II.4, 58; II.6, 61, 63; IV.1, 111; *Summulae de dialectica* IV.6.2, 299; *PS* I.26, 312-13.
[183] *BC* II.6.6, 67; *PS* I.26, 312-13.
[184] *OSL* II.24, 329-30; III-1.21, 416.
[185] See also Lagerlund 2000, 112-15.
[186] *OSL* II.25, 331-2; III-1.24, 423-4; *BC* II.6.5, 66-7; *PS* I.26, 312.
[187] *BC* II.6.7, 68; *PS* I.30, 320.
[188] *OSL* II.27, 338; III-1.27-8, 430-33.
[189] *BC* II.4, 58-60; *PS* I.26, 310-11.
[190] Pseudo-Scotus's remarks on the conversion of these readings are sketchy and problematic. It is not clear why he thinks that possibility propositions would be convertible in the same was as assertoric propositions, the simple conversion of universal negative included. Necessity propositions are said to be non-convertible except when they are about the necessary characteristics of a necessary being. The author does not mention that a conversion from universal negative to particular negative necessity propositions would be acceptable in this approach. See also Lagerlund 2000, 172-6.
[191] Hughes 1989.

that the general ideas of Buridan and Pseudo-Scotus can be described with the help of some models similar to those on which the possible worlds semantics is based, although many basic theoretical questions of modern formal semantics were beyond the purview of medieval logicians. Constructing the details of the intuitive model which the authors possibly had in mind is a cumbersome task. Ockham and Buridan state that the truth of 'A white thing can be black' demands the truth of 'This can be black' and that 'This can be black' and '"This is black" is possible' mean the same.[192] It is reasonable to suppose that the possible truth of 'This is black' means that it is true in at least one of the possible states of affairs in which the possible being referred to by 'this' occurs and that the necessary truth of 'This is black' means that it is true in all possible states of affairs in which the thing referred to by 'this' occurs. Buridan and Pseudo-Scotus assume that he subject terms of quantified divided modal propositions stand for possible beings. The truth of these propositions demand the truth of all or some relevant singular propositions of the type just mentioned. Compound possibility propositions maintain that assuming their *dicta* as true does not imply any contradictions. Buridan typically did not show any understanding of the idea of abstract unrealizable possibilities in Averroes and Aquinas – he took it for granted that if a counterfactual state of affairs is possible, it can be coherently imagined as actual.[193]

One might wonder why, instead of regarding the reading with actual or possible subjects as basic for all modals, Ockham discussed the necessity propositions with restricted subject terms. This makes his modal syllogistics less systematic than those of Buridan and Pseudo-Scotus who argued that if divided possibility propositions are ampliated, the equipollences between various modals requires that necessity propositions are ampliated.[194] It seems that the differences are related to the fact that the authors did not make use of the same notion of necessity while discussing divided modals. If the distinction between divided necessity propositions with restricted and unrestricted subject terms is not nugatory, one should admit that possible beings may have necessary properties without occurring in every possible state of affairs. This condition is fulfilled by a relative *de re* necessity as Buridan and Pseudo-Scotus understood it. That which can be A is of necessity B in this sense iff it could not be actual without being B. This notion of necessity is relative to possible actuality and is weaker than that of simple necessity by which a thing is necessarily something without qualification, i.e., always and in all thinkable states of affairs, and it is stronger than that of temporal necessity by which an actual thing is invariably something in the actual world.[195] Some of Ockham's examples also refer to necessity which is relative to possible actuality, but his syllogistic theory of divided necessity propositions is based on the view

[192] *OSL* II.10, 276-9; III-1.32, 448; III-3.10, 632-4; *BC* II.7.16, 75-6. While compound and divided readings do not differ at this level, they are separated in dealing with standard universal and particular propositions.

[193] For Averroes and Aquinas, see note 84 and Knuuttila 2001, 234.

[194] *BC* II.6.2, 63; *PS* I.26, 312.

[195] For these types of necessity, see Buridan, *In An. pr.* I.25; *BC* V.1, 112; *PS* I.26, 311.

that *de re* necessities pertain to actual beings.[196]

Buridan states that the relations between unrestricted divided modal propositions and the corresponding non-modal propositions are governed by the following principles. There are no valid consequences from divided necessity and possibility propositions to corresponding assertoric propositions, except that a universal negative assertoric proposition follows from an unrestricted universal negative divided necessity proposition. There are no valid consequences from assertoric propositions to corresponding divided necessity propositions. Particular divided possibility propositions follow from universal and particular affirmative assertoric propositions by valid consequences. Buridan adds that a possibility proposition follows from any necessity proposition.[197] Ockham's position with respect to these relationships is different for reasons already mentioned, compound and divided necessity propositions implying corresponding assertoric propositions.[198] As for the relationships between compound and divided modal propositions, Buridan states that particular divided possibility propositions follow from compound universal and particular affirmative possibility propositions, and a universal negative compound necessity proposition follows from an unrestricted universal negative divided necessity proposition.[199] According to Ockham, compound and divided readings of all singular modal propositions are equivalent, provided that the subject terms are demonstrative pronouns or proper names. Divided necessity propositions which are universal, particular, or have as their subject a common term with a demonstrative pronoun do not imply corresponding compound modals, and the same holds *e converso* with the exception that affirmative compound propositions having as a subject of their *dicta* a common term with a demonstrative pronoun imply a corresponding divided necessity proposition. Ockham's rules for universal and particular possibility propositions are the same as those of Buridan.[200]

In discussing divided modal syllogistic in *Tractatus de consequentiis*, Buridan

[196] See, e.g., *OSL* III-1.32, 448.119-122.

[197] *BC* II.6, 64-6. Lagerlund (2001, 150) remarks that Buridan does not want to exclude the consequence from negative assertoric to divided negative possibility propositions which is not mentioned.

[198] *OSL* III-3.11, 637-8.

[199] *BC* II.7, 76-8. In denying that a universal affirmative compound necessity proposition implies a divided necessity proposition, Buridan states that while Aristotle says that 'Every horse is an animal' is necessary, no horse is necessarily an animal, for any horse can fail to exist and consequently fail to be an animal (77). This is a strange example, for it is said earlier that divided affirmative necessity propositions do not imply assertoric propositions, which is in agreement with the relative conception of necessity mentioned above. See also IV.1, 112 where Buridan says that 'A horse is an animal' is not necessarily true by a simple necessity, for it is possible that there will be no horses, although horses are necessarily animals as long as they exist. Perhaps the first example was about simple necessity which presupposed necessary existence. Some problems could be avoided by regarding compound necessity propositions as law-like statements with an ampliated supposition. In this case 'Every horse is an animal' is necessarily true as well as the divided statement that every horse is necessarily an animal, while 'Everyone walking is moving' is necessarily true and the divided statement that all walking are necessarily moving is false. Cf. *In An. pr.* I.25-6. See also the discussion in the introduction to the translation of *Summulae de Dialectica* by G. Klima, xlv-xlvii.

[200] *OSL* III-3.10, 632-4.

first presents the valid moods with necessity or possibility premises. All four first-figure moods corresponding to direct assertoric ones are valid when they are modalized as follows: MMM, LLL, LML, MLM.[201] When assertoric premises are as-of-now statements, LAL and MAM with particular conclusions are valid as well as the universal negative mood in ALA. All ALA moods are valid if the subject of necessity premise is restricted to stand for actual things.[202] If assertoric propositions are assertoric *simpliciter*, equivalent to compound necessity propositions, all four first-figure moods are valid in LAL, ALL, AMM.[203] In outlining the syllogistics for contingency-propositions, Buridan states the validity of all first-figure moods in CMC, CCC, MCM, LCL, CLC and in CAC for particular moods.[204] When the restrictive phrase 'what is' is prefixed to the subject of a divided modal (L', M', C'), all first figure moods are valid in $L'L'L', L'AL', M'L'M', M'AM', C'AC'$.[205]

Valid second and third figure moods are reduced to perfect first figure moods or proved by an expository syllogism or, on few occasions, by *reductio ad impossibile*. First figure divided modal moods without assertoric premises can be delineated as follows: if that which is or can be P is necessarily, possibly or contingently (not) Q and if that which is or can be S is necessarily, possibly or contingently P, then that which is or can be S is necessarily, possibly or contingently (not) Q. It does not matter by which modality the minor term is connected to the middle term — the modality of predication in the conclusion is always that of the first premise. The same principle holds true of the cases where the major premise is modalized and the minor premise is assertoric, but the conclusions are then always particular. Buridan regarded these moods as perfect and clear through the *dici de omni et nullo*.[206]

Buridan made some historical comments on Aristotle's theory claiming, for instance, that Aristotle meant by 'simply assertoric propositions' assertoric propositions which are necessary, that he considered propositions like 'A horse is an animal' as simply necessary because of his view of the eternity and unchangeability of natural species, and that he understood mixed modal syllogisms as divided modal syllogisms.[207] However, he did not make any attempt to reconstruct Aristotle's modal syllogistic as a uniform system; in the light of his modal principles, such a construction was impossible. Some Aristotelian moods are valid only as mixed compound and divided modal syllogisms, some are valid in this way and as divided modal syllogisms with non-ampliated subject terms, and some are valid

[201] *BC* IV.2.4, 115.
[202] *BC* IV.2.10 (118-9), 15-16, 124-5.
[203] *BC* IV.2.11, 120; IV.2.19, 127-8.
[204] *BC* IV.2.21-4, 130-1.
[205] *BC* IV.2.7, 117; IV.2.10, 119; *Summulae* V.7.2-4. Buridan says that if the major necessity premise is restricted and the minor is not, the conclusion is not restricted. He should have added that the conclusions are particular. (The same holds of ALM moods which are mentioned in *Summulae* V.7.3, 355.) If the major necessity premise is not restricted and the minor is, universal conclusions are said to be restricted. Lagerlund (2000, 239) does not pay attention to this qualification.
[206] *BC* IV.2.4, 115; IV.2.10, 119; IV.2.16, 124-5.
[207] *BC* IV.1, 111-112; *In Pr. an.* I. 30.

only with ampliated subject terms. Buridan thought that Aristotle's modal syllogistic was a partial theory of valid modal deductions where the different types of fine structures made use of in dealing with modal premises were not distinguished.[208] Some moods which are accepted in the *Treatise on consequences* are not mentioned in the *Quaestions on the Prior analytics* or in the *Summulae*, and some valid moods which are discussed in the *Summulae* are omitted in the *Treatise on consequences*.[209] Buridan's treatment of modal moods in all works is restricted to those the unmodalized counterparts of which are valid direct moods of three syllogistic figures. He does not treat valid indirect moods or moods with singular terms.

Ockham's divided modal syllogistic also concentrates on the moods which have valid assertoric counterparts, but his discussion is more throughout than that of Buridan. Ockham considers the indirect moods as well, and he treats the combinations in which one of the premise is a compound modal and the other a divided modal. He also mentions some moods with restricted premises which are valid in Buridan's system as well although Buridan does not mention them: e.g., all first figure moods $MM'M', CC'C'$ and the third figure moods $MM'M$ and $M'MM$. As Ockham considered the subject terms of divided necessity propositions as restricted, he did not accept any second figure moods with divided necessity propositions as premises nor any moods with mixed divided modal premises and necessity conclusions.[210] Pseudo-Scotus's discussion of divided modal moods is of the same type as that of Buridan, but it is less detailed.[211]

4 APPLIED MODAL LOGIC

Some twelfth-century thinkers considered that the logic of pure modal terms could be used as a model for studying the logical behaviour of related concepts. It was asked whether the basic rules for modal sentences *de dicto*, viz. (4) and (5), held for other concepts showing *prima facie* similarities with the notions of necessity and possibility. Many logicians in the fourteenth century were interested in this kind of applied modal logic. The extensive lists of terms which were found interesting in this regard included *verum, falsum, per se, scitum, dubium, opinatum, creditum, apparens, notum, volitum, dilectum, obligatum, licitum*, and *illicitum*. [212]

[208] See also *OSL* III-1, 31, 443.
[209] See also Lagerlund 2000, 149-61; Thom 2003, 169-91.
[210] *OSL* III-1, 21 (416-7), 23 (420), 26 (428), 44 (476).
[211] For divided modal syllogistics in Ockham and Pseudo-Scotus, see Lagerlund 2000, 196-29, 171-83; Ockham's modal syllogistic is also discussed in Thom 2003, 141-67. Nicholas Drukken's view of necessity moods was influenced by Ockham's theory; most later medieval authors followed Buridan. See Lagerlund 2000, 134-5, 184-227.
[212] *PS* I.25, 310; I.36, 328-9.

4.1 Epistemic Logic

Knowledge and belief were widely considered as partially analogous to necessity and possibility in late medieval times.[213] Although the inference rules of modal logic *de dicto* were usually not accepted as rules for knowledge and belief, there was a lively discussion of some other important questions pertaining to epistemic logic. The standard Aristotelian reasons for not accepting (4) and (5) with respect to knowledge or belief is stated by John Buridan as follows:

> From compound sentences *de scito, de opinato, de dubitato*, and other similar modals, there is no valid syllogism. It does not follow: 'That every B is A is known by Socrates, and that every C is B is known by Socrates, therefore that every C is A is known by Socrates', because although the two premises are known by Socrates, he nevertheless may perhaps fail to order them in a syllogism, or fail to see that the third follows from them.[214]

Most medieval authors did not operate with the conception of logical omniscience that is included in some modern theories, treating the logic of epistemic notions from the point of view of factual attitudes.[215] However, Peter of Poitiers mentions that according to some twelfth-century thinkers, knowing the antecedent of a sound consequence implies knowing the consequent.[216] Another example is Ralph Strode's (d. 1387) influential treatise on consequences. According to rule 13, if the antecedent of a sound consequence is known, the consequent is also known.[217] The logicians, who commented on Strode's rules, usually considered the reference to a person essential in epistemic contexts, adding the condition that the consequence must not only be valid, but must be known to be valid as well. Strode also stressed the same later in his work.[218]

In the prologue to his *Commentary on the Sentences*, William Ockham states that some acts of the intellect are apprehensive and others are judicative. An apprehensive act can be directed to anything which may occur as the object of a cognitive power. It is an act of entertaining the content of a possible judgment. A judicative act is an act by which the intellect assents to or dissents from the complex objects of apprehension. We assent to what we regard as true and we dissent from what we regard as false.[219] If the assent or dissent which is part of

[213] For medieval epistemic logic, see I. Boh, *Epistemic Logic in the Later Middle Ages* (London and New York: Routledge, 1993).

[214] *Tractatus de consequentiis* IV.1.3, 114.

[215] See, for example, J. Hintikka, *Knowledge and Belief* (Ithaca: Cornell University Press, 1962).

[216] *Sententiae* I.9 (82.139-83.154).

[217] Strode's treatise is edited by W.K. Seaton in *An Edition and Translation of the 'Tractatus de consequentiis' by Ralph Strode, Fourteenth-Century Logician and Frend of Geoffrey Chauser* (Ph. D. diss., University of California, Berkeley, 1973).

[218] For these comments and related discussions, see Boh 1993, 89-125; Boh, 'Four Phases of Medieval Epistemic Logic', *Theoria* 66 (2000), 137-40; Boh 2001, 161-70.

[219] William Ockham, *Scriptum in Librum primum Sententiarum. Ordinatio, prol. and dist.1*, ed. G.Gál and St. Brown, Opera theologica 1 (St. Bonaventure: Franciscan Institute of St.

knowledge in Ockham is considered an act of judicative belief, Ockham's concept of knowledge could be regarded as implying the principle

(21) $K_a p \rightarrow B_a p.$

Robert Holcot explicitly treated assent as a belief, pointing out that the most common sense of believing (*credere*) is to assent to what is stated by a proposition. 'A proposition is believed when it is assented to, and in this way we believe what we know as well as what we formally opine.'[220] Ockham remarks that the term 'knowledge' is sometimes applied to firm cognition of a truth, but he considered this use less proper than those which included a reference to evidence as some kind of justification. Instead of the definition

$$K_a p = B_a p \ \& \ p$$

he prefers

$$K_a p = B_a p \ \& \ p \ \& \ J_a p.$$

($J_a p$ stands for 'the person a is justified in believing that p'.)[221] As shown by Ivan Boh, some later authors were reluctant to accept the first use of knowledge. Their reasons were similar to those which have been presented in the contemporary discussion of the Gettier problem.[222] Some authors were interested in the question of the role of the will in assenting to a proposition. It was usually thought that evident assent was caused naturally and that there were types of non-evident assent which were freely caused, such as the religious assent of faith or the opinative assent based on probabilistic arguments. Robert Holcot argued that beliefs were not freely chosen, but this view was considered over-stated. [223]

The theory of free assent was sometimes associated with the question of making moral decisions in uncertain cases, which anticipated the later controversy over

Bonaventure University, 1967), prol. 1, 16.3-18. For the distinction between apprehensive and judicative acts in late medieval and early modern thought, see K. Tachau, *Vision and Certitude in the Age of Ockcham: Optics, Epistemology and the Foundations of Semantics 1250-1345* (Leiden: Brill, 1988), 117-18; A. Broadie, *Notion and Object: Aspects of Late Medieval Epistemology* (Oxford: Clarendon Press, 1989), 125-48.

[220]Robert Holcot, *In quatuor libros Sententiarum quaestiones* (Lyon, 1518, reprinted Frankfurt am Main: Minerva, 1967), I.1.6.

[221]See William Ockham, *Expositio in libros Physicorum Aristotelis, prol. et libri I-III*, ed. V. Richter and G. Leipold, Opera philosophica 4 (St. Bonaventure: Franciscan Institute of St. Bonaventure University, 1985), prol. 5.29-6.50.

[222]Boh refers to a passage from Peter of Mantua's *Logica* which shows that a person could be said to know (in the first sense of 'scire') that p even when the assent is based on a mistake: 'Let it be posited that Plato is very near you and you know that he is running, but you believe that he is Socrates so that you firmly believe that Socrates is running. But let Socrates in fact be running in Rome, although you do not know this. You thus know that Socrates is running and do not know that Socrates is running, therefore, on the same basis, what is known is doubtful to you' (*Logica* 17rb, translated in Boh, 2000, 136).

[223]See Holcot, *Sent.* I.1.1; Holcot's view is criticized, e.g., in Peter of Ailly, *Questiones super libros Sententiarum* (Paris 1500), princ. III, I D, 1.2 BB. For the discussion of this question in Paris between 1500 and 1530, see Broadie 1989, 149-78.

probabilism and probabiliorism. Thomas Aquinas, Boethius of Dacia, and some other Parisian masters explained Aristotelian dialectical probabilities by stating that what is probable in the sense that most experts accept it is probably true because it is not probable that the majority of well informed experts would be mistaken in the same way.[224] These and other similar examples show that, contrary to what has been sometimes maintained, an intuitive conception of objective frequency probability, different from epistemic probability, was developed in the Middle Ages.[225]

The question of the relationship between epistemic propositions *de dicto* and *de re* belonged to standard fourteenth-century topics of epistemology. It was thought that knowledge statements *de dicto* did not imply knowledge statements *de re* or *vice versa.*[226] Buridan says, however, that when a person knows that some A is B, then of something which is A he or she knows that it is B. The reason for denying this could be that Socrates does not know which A is B. Buridan would agree that in this sense the *de re* reading does not follow from the *de dicto* reading, but there is another kind of *de re* reading (or which might be called so) which does follow from the *de dicto* reading. This is a kind of intermediate reading between pure *de dicto* and *de re* readings. The idea can be formulated as follows. According to Buridan, statements of the type

(22) $\quad K_s(Ex)(Fx)$

imply that there are individuals having property F, although S does not necessarily know which they are. In principle they are identifiable, however, and if we suppose that one of them is z, we can write:

(23) $\quad K_s(Ex)(Fx) \rightarrow (Ex)((x = z)\ \&\ K_s(Fx))$.

From the *de dicto* statement 'Socrates, who is sitting in a cellar, knows that a star is above' it does not follow the *de re* reading understood as 'There is a star which Socrates knows as the star which is above', but the following *de re* reading does follow from it: 'There is a star of which Socrates knows that it is above, although Socrates does not know which star it is.'[227]

Buridan was not the first to employ this reading. In his *De obligationibus*, Walter Burley discussed the following case:

[224] See Thomas Aquinas, *Summa theologiae* II-1.105.2, ad 8; II-2.70.2; Boethius of Dacia, *Quaestiones super librum Topicorum*, ed. N. G. Green-Pedersen and J. Pinborg, Corpus Philosophorum Danicorum Medii Aevi VI.1 (Copenhagen: Gad, 1976), III.14, 187.

[225] For this notion of probability and its role in later medieval thought, see I. Kantola, *Probability and Moral Uncertainty in Late Medieval and Early Modern Times* (Helsinki: Luther-Agricola Society, 1994); for later discussions , see S.K. Knebel, *Wille, Würfel und Wahrscheinlichkeit. Das System der moralischen Notwendigkeit in der Jesuitenscholastik 1550-1700* (Hamburg: Meiner, 2000).

[226] See, for example, the extensive discussion of the question in the second chapter of William of Heytesbury's *Rules for Solving Sophismata* (Venice 1494), translated in Kretzmann and Stump 1988, 436-72.

[227] *Sophismata*, ed. T.K. Scott (Stuttgart-Bad Canstatt: Frommann-Holzboog, 1977) IV,14, trans. by G. Klima in *Summulae de dialectica*, 900-902.

> You know that Marcus is running. But Marcus is Tully. Therefore, you know that Tully is running. And furthermore, 'Therefore, you know who is called Tully.' But it was posited that you don't know this. Solution. This is multiplex: 'You know that Tullius is running.' It may signify that you know the dictum: 'That Tullius is running', and this is false and the consequence is not valid. Or it may signify that of that person who is Tully you know that he is running, and this can be true, although you don't know who is signified by this term 'Tully'.[228]

The same analysis can be found in many fourteenth century writers.[229]

Let us turn to medieval deontic logic. The following equivalences analogous to those between modal concepts were used in the fourteenth-century discussions of the norms:

(24) $-O-p \leftrightarrow Pp$

(25) $-P-p \leftrightarrow Op$

(26) $Op \leftrightarrow P-p$

(27) $-Pp \leftrightarrow O-p$

(28) $Op \leftrightarrow F-p$

(29) $Fp \leftrightarrow O-p$.

O stands here for obligation (*obligatum*), P for permission (*licitum*), and F for prohibition (*illicitum*). One can find some of these equivalences used earlier and there were some twelfth-century writers who anticipated the later habit of treating deontic concepts as a species of modal concepts. According to one definition of modal terms sometimes used by Peter Abelard, necessity is identified with what nature demands, possibility with what nature allows, and impossibility with what nature forbids.[230] At the beginning of his *Ethics*, Abelard asks whether it is possible that that the antecedent is permitted or obligatory while the consequent is forbidden.[231] Later he discusses some problematic cases associated with this question. These remarks form the first known discussion of deontic consequences. Abelard also formulated the question of whether willing the antecedent of a good consequence which is known to be such implies willing the consequent. His view is that if will is understood in the sense of consent and not merely in the sense of wish and if the consequence expresses a relationship between a goal and a

[228]The text is edited in Green 1963, 39.29-40.7.

[229]See, e.g., *The Sophismata of Richard Kilvington*, trans. with introduction and commentary by N. and B. Kretzmann (Cambridge: Cambridge University Press, 1990), 118-20, 314-23; Albert of Saxony, *Sophismata* (Paris, 1502), III, 32-33.

[230]See, e.g., *Dialectica* 385.1-8.

[231]Peter Abelard, *Ethics*, ed. with trans. and notes by D.E. Luscombe (Oxford: Clarendon Press, 1971), 20.1-11. Cf. 38.22-40.5; 40.15.

means, consenting to the antecedent (goal) implies consenting to the consequent (a means).[232]

Some twelfth-century authors realized that even though willing an end effectively implies that a necessary means to it is also willed, a rule (W), analogical to (5) cannot be applied without restriction to efficient will in the form that if Socrates wills that p and he knows that there is a valid consequence from p to q, he also wills that q. The principle holds about the end-means relationship, but the necessary means are not the only consequents of what is willed. Peter of Poitiers formulated a counter-example as follows: if S repents of a sin, S is guilty of a sin, and S wills to repent of a sin, but S does not will to be guilty of a sin.[233] Stephen Langton's counter-example was of the same type: if a man visits his sick father, the father is sick. This man wills to visit his sick father, but it does not follow that he wills the father to be sick.[234]

When it was realized that one cannot apply (W) to effective will without qualification, medieval thinkers took an interest in finding cases in which a rational agent does not will the consequent of what he or she wills. The twelfth-century examples just mentioned specify the case in which to will something in certain circumstances implies that those circumstances prevail, although this is something which the agent does not will. This exception to (W) was thought philosophically interesting and was also used in later medieval deontic logic. Contrary to what Abelard thought, it was realized that (4) and (5) cannot be applied without qualification to the notions of obligation and permission. The reasons were similar to those which demanded the qualification of (W). Another relevant but more problematic idea was that a separately unwanted side-effect of what is willed is willed more indirectly than a separately unwanted means to an end. This line of thought forms the background to Thomas Aquinas's distinctions between directly and indirectly willed things.[235]

One can follow the discussion of the question of how (W) should be qualified with respect to will by tracing the comments on the example of a man who 'wills to be in the mud and have 100 marks' (*in luto esse cum 100 marchis*). This example was considered ambiguous, and it was possible to interpret it (i) as a case of willing reluctantly to become dirty as a means of receiving 100 marks, (ii) as a case of willing to become dirty as a side-effect or (iii) willing something in a situation in which one would become dirty in any case. The third alternative was particularly relevant to later discussions of conditional obligations.

The most extensive fourteenth-century discussion of the logical properties of deontic concepts is to be found in the first question of Roger Roseth's *Lecture on*

[232] *Ethics* 8.21-10.2; 14.14-19; 16.11-18.
[233] *Sententiae* IV.16 in *Patrologia latina*, ed. J.P. Migne, 211, 1199.
[234] *Quaestiones theologiae*, quoted in R. Quinto, 'Die Quaestiones des Stephan Langton über die Gottesfurcht', *Cahiers de l'Institut du Moyen-Âge Grec et Latin, Université de Copenhague* 62 (1992), 77-165, at 129-130.
[235] See, e.g., *Summa theologiae* II.2.64.7-8; *De malo* 2.1. See also Knuuttila 2006, 205-18.

the Sentences, which he finished c. 1335.[236] The opening question of this work is devoted to the problem of whether somebody can be obliged to do something against his or her conscience. In the second article, Roseth first formulates five general principles or rules which define the rationality of a system of norms, and the rest of the article consists of a discussion of various objections to the rules of rationality. In rules 1 and 2, Roseth defines formal rationality: describing behaviour in accordance with the norms must not yield contradictions, i.e., a person should not violate a norm by fulfilling a norm in the same system of norms. New obligations can be added to a system as rational norms without further changes only if they refer to omissions and commissions which one is permitted to will. Roseth pays particular attention to this point because he is interested in the fact that there are omissions or commissions which are permitted, although it is not permitted to will them; as they cannot be legitimately willed, they cannot be prescribed as such in a rational body of norms. Referring to the dictates of right reason, Roseth adds to the first rule that what is prescribed must be in the power of the agent in such a way that fulfilling the obligation does not result in a great disaster. Rules 3 and 4 are related to the Catholic doctrine of the salvation of the soul. Rule 5 refers to the doctrine of God's absolute power, through which an act which cannot be legitimately willed can be changed into an act which can be legitimately willed.[237]

After these general rules, Roseth discusses various possible objections and develops some philosophical and logical ideas pertaining to norms. The second objection runs as follows. It is permitted to will to behave in a way which is not meritorious though the order that one should will to behave in a non-meritorious way is not rational. It is not rational because it could be violated only in a meritorious way. The first part is proved as follows. If you are sleeping, you do not behave in a meritorious way. Furthermore, you know that this is a good consequence: since you are permitted to will the antecedent, you are permitted to will the consequent.

The main point of Roseth's answer to the second objection is that it is mistakenly assumed that if one is permitted to will the antecedent of a good consequence, one is also permitted to will the consequent:

> I accept the following consequence: you are sleeping; therefore you do not benefit from an elicited act. But I deny this consequence: this consequence is good and you know that it is good; therefore if you are permitted to will the antecedent, you are permitted to will the consequent. This is because there are consequences which are good

[236]This work was copied several times during the second half of the fourteenth century and is preserved to us in eight manuscripts, only three containing the complete work. The first article of the book aroused interest as an independent study of natural philosophy and was circulated in several copies. It was printed, together with the second article of the first question, as the first question of *Determinationes magistri Roberti Holcot* at Lyon in 1518. The printed version is abridged and unreliable. The translations below are based on a transcription of manuscripts and quoted from S. Knuuttila and O. Hallamaa, 'Roger Roseth and Medieval Deontic Logic', *Logique & Analyse* 149 (1995), 75-87.

[237]These rules are translated in Knuuttila and Hallamaa 1995, 79-83.

> and which I know to be good the antecedent of which I am permitted to will without being permitted to will the consequent. For example, this consequence is good and known to be good: I repent of my sin; therefore I am in sin. I am permitted to will the antecedent but I am not permitted to will the consequent, because I am permitted to repent of my sins, but I am not permitted to will to be in sin. Therefore the consequence is not valid.

Examples of this kind show that there are obligations which can be rationally fulfilled only in cases in which some norms have already been violated. Rules (4) and (5) could be applied to the notions of obligations and permissions only if there were no conditional norms by which one's conduct is regulated after having violated certain norms. Roseth realized that conditional norms of this kind, called contrary-to-duty imperatives in contemporary literature, prevent one from accepting the rules of inference of modal logic in deontic logic without qualification.

These are not the only problematic cases according to Roseth. There is a second class of conditional obligations and permissions regulating conduct in those situations in which the moral agent has not violated any rules but which he or she cannot will without violating some rule. This group of obligations and permissions could serve as Roseth's answer to the so-called paradox of the Good Samaritan. Roseth's example is that one may accept that one's father will be killed because of a serious transgression. A simpler case would have been a variant of Stephen Langton's example of conditional will; one should will to visit one's sick father, but one is not permitted to will that there be such a situation.

Roseth then discusses the question of how the conditional obligation should be formulated. He thinks that a reasonable *prima facie* formulation is

(30) $\quad p \rightarrow Oq$

where p may stand for something forbidden or permitted such that it is not permitted to will it. Roseth thought, however, that (30) was not a sufficient form. The first addition is formulated in his answer to the following question. If the conditional obligation is formulated so that it is obligatory and allowable to will to repent if one is guilty of a sin, one could ask whether Socrates, who wills to repent without having committed any sin, should repent or not. If he should, he is doing what he ought to do, but then he should not repent. If he should not, he violates a rule and he ought to see that he repents. So if he should not repent, he should repent and *vice versa*.

According to Roseth, the intention of the rule-giver is that Socrates ought to will to repent of his sins only when he is guilty of sin and that the prescribed act of repentance is different from the act of willing to repent when one has not sinned before repenting. Thus Socrates, while willing to repent in this way, violates the intention of the rule-giver, although he seems to fulfil his obligation. In order to avoid difficulties of this kind, one apparently should add to formula (30) the qualification that fulfilling the condition of a conditional obligation does not fulfil the obligation:

(31) $(p \rightarrow Oq)\ \&\ -L(p \rightarrow q)$.

This form is similar to that which von Wright suggests in order to solve difficulties in defining conditional obligation.[238] Rule (31) had certain antecedents in medieval logic. Conditional norms were also discussed in treatises *De obligationibus* which dealt with the logic of disputation and which as such had nothing particular to do with morality or ethics. One example of the interest in normative notions in obligations treatises is the discussion of disputations where the expression 'must be granted' is included in the initial *positum.* It was realized that a *positum* of the type 'That you are in Rome must be granted' can be read prescriptively as a norm or descriptively as a proposition expressing the existence of an obligation. The initial proposition including 'must be granted' is read descriptively in a *positio* disputation, but it is read normatively in the species of disputations called *petitio.* In his *De obligationibus*, Walter Burley discusses some problems of formulating conditional norms in connection with the disputations called *positio dependens*, *positio cadens*, and *positio renascens.* What they have in common is that a proposition begins or ceases to be the *positum* when a condition is fulfilled. The condition is a disputational act. Burley's remarks on how the condition should be formulated show similarities with Roseth's treatment of conditional obligations.[239]

In order to show that (31) is not sufficient, Roseth deals with a variant of a well-known medieval sophism. Let us assume that those and only those who say something true will cross a bridge and that Socrates says: 'I shall not cross the bridge.' It is then asked whether this is true or false. According to Roseth, one should not accept the assumed case. Fulfilling the condition of a conditional obligation must not make it impossible that the obligation is fulfilled. When this restriction is added to (13), the resulting form is as follows:

(32) $(p \rightarrow Oq)\ \&\ -(p \rightarrow q)\ \&\ M(p\ \&\ q)$.

Roseth presents other similar problems associated with the questions of semantic paradoxes in this context.[240]

CONCLUDING REMARKS

While early medieval discussions of modal matters were influenced by ancient modal paradigms such as temporal frequency interpretation of modal notions, the conception of possibility as a power, the ideas of diachronic modalities and essentialist modal assumptions, there were also new insights. Following Peter Abelard,

[238]G.H. von Wright, 'Deontic Logic and the Theory of Conditions' in R. Hilpinen, ed., *Deontic Logic: Introductory and Systematic Readings* (Reidel: Dordrecht, 1971), 169.

[239]S. Knuuttila and M. Yrjönsuuri, 'Norms and Action in Obligational Disputations' in Pluta (ed.), *Die Philosophie im 14. und 15. Jahrhundert*, Bochumer Studien zur Philosophie 10 (Amsterdam: Gruener, 1988), 191-202.

[240]For the original txt, see Ms. Oxford, Oriel College 15, f. 249vb-252ra; see also Knuuttila 1993, 190-5.

Peter Lombard wrote in his influential *Sentences* that 'Things cannot be other than as God foreknows them' is true in the compound sense and false in the divided sense. The truth of the compound sense saves God's infallibility, while the falsity of the divided sense expresses God's freedom and the metaphysical contingency of the future. It is assumed that when something is, it is possible that it is not at that very instant of time at which it is actual. These theological formulations exemplify twelfth-century deviations from the Aristotelian thesis 'What is necessarily is when it is'. This was traditionally understood as implying the principle of the necessity of the present, which was not questioned in ancient modal theories. Since temporally definite propositions about contingent things were regarded as unchangingly true in God's knowledge, the contingency of these propositions also implied the denial of the Aristotelian equation of immutability with necessity. The new modal idea could be characterized as the model of simultaneous alternatives. Its theological foundation was Augustine's idea of God, who as the creator and providence of the world, acts by free choice between alternatives.

In accordance with the new modal conception, possibilities as the objects of divine power were regarded as much more numerous than possibilities associated with natural powers. While exemplification in the actual world was often regarded as a criterion of the genuineness of the types of natural possibility, it was not relevant for divine possibilities. In twelfth-century theology, natural possibilities *secundum inferiorem causam* were said to be possibilities *secundum cursum naturae* and possibilities *secundum superiorem causam* meant divine possibilities. There were similar discussions in medieval Arabic philosophy.

In twelfth-century logical treatises, which were influenced by Boethius's works, one can find ideas not discussed in ancient sources in Abelard's analysis of the types of modal propositions, future contingents and the logic of conditionals and late twefth-century and early thirteenth-century works on the logic of terms, time and modality, some of these being associated with the idea of modality as referential plurality. The increasing reception of Aristotle's philosophy in the thirteenth century gave support to traditional modal paradigms, as is seen in the discussions of modal conversion and Robert Kilwardby's very influential commentary on Aristotle's *Prior Analytics*, in which modal syllogistic is treated as an essentialist theory of the structures of being.

While twelfth-century modal innovations were used to some extent in thirteenth-century theology, they were not extensively discussed in philosophical contexts. Things became different when John Duns Scotus combined the various elements of the conception of modality as alternativeness into a detailed theory. Scotus describes the simultaneous alternatives as the domains with respect to which God chooses the actual world, but the structure of the theory is taken to pertain to logical modalities which are what they are independently of whether anything exists. A possible state of affairs is characterized as expressing something to which to be is not repugnant. What is impossible is a combination of elements which are incompossible.

Even Aristotle said that when a possibility is assumed to be actualized, noth-

ing impossible follows, but he had in mind actualization in the history of the actual world. The denial of an omnitemporally actual state of affairs is said to be impossible. In the thirteenth century, some thinkers qualified Aristotle's characterization of possibility as something which can be assumed as actual without contradiction by explaining that possibilities referring to the same moment of time are not necessarily compossible in the sense that they could be actual at the same time. In Scotus's theory, actualizability as a criterion of possibility does not refer to the already existing actual world, nor is it explained by referring to prospective options of which the unrealized will disappear, as in the theories of diachronic modalities. Temporally definite possibilities are primarily treated as referring to alternatives, as is shown by the definition of a contingent state of affairs: 'I do not call something contingent because it is not always or necessarily the case, but because its opposite could be actual at the very moment when it occurs'.

In the fourteenth century, many authors followed Scotus in explicating the meaning of modal terms by referring to simultaneous alternatives as the basic model. This is also reflected in the reformulation of modal logic with the systematic distinction between modalities *de dicto* and *de re* and the division of *de re* modalities into those with actual subject terms and those with non-actual subject terms. The basic notion was that of logical possibility which was distinguished from the more restricted notion of natural possibility. John Buridan's theory of modal consequences is considered as one of the greatest achievements of medieval logic. Aristotle's modal syllogistics was regarded as a fragmentary theory in which the different types of the fine structure of modal statements were not distinguished. Contrary to thirteenth-century approaches, Ockham, Buridan and their followers did not try to reconstruct Aristotle's theory as such into a uniform system. The new modal paradigms which show some similarities to the background ideas of possible worlds semantics also influenced fourteenth century logic of discourse (obligations logic) and the treatments of applied modal logic, the logic of knowledge and belief and the logic of norms.

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TREATMENTS OF THE PARADOXES OF SELF-REFERENCE

Mikko Yrjönsuuri

In chapter II, 51 of *Don Quixote*, Miguel de Cervantes describes a curious case presented to Sancho Panza as the governor of the island Barataria. There was a river, a bridge, a gallows — and a law. According to the law, everyone crossing the bridge must first tell under oath where he is going. If he is truthful, he may pass freely, but if he lies, he will be hanged on the gallows. Now, a man is brought to Sancho after having taken the oath that he was going to be hanged on the gallows. The judges had been unable to solve the case, for if he was hanged, he had spoken the truth and thus should not have been hanged. And if he was not hanged, he lied, and thus should have been hanged. Even Sancho with his meager intellectual powers is able to see that the case is paradoxical. No solution would accord to the law.

Cervantes did not invent the paradox himself. Instead it was developed in the medieval literature concerning the paradoxes of self-reference, and the formulation is so similar to certain medieval formulations that Cervantes must have got it in some way or other from the logicians. This is, of course, no surprise. *Don Quixote* draws upon many other philosophical discussions of the schools as well.

Aristotle considers in *Sophistical Refutations* 25 (180a27–180b7) whether and in what sense a person who "swears that he will break his oath" can keep his oath, and claims that there are parallel arguments "as regards the problem whether the same man can at the same time say what is both false and true". Although it is very difficult to judge whether Aristotle really had in mind genuine paradoxes of self-reference like the Liar, 'this sentence is false', many medieval authors refer to this passage as the context where Aristotle offers his solution of such paradoxes. There was, thus, a very authoritative source for connecting oaths to the context of self-referential paradoxes.

It is a very distinctive characteristic of the general manner in which medieval authors discuss paradoxes of self reference that they are put into the framework of a supposed situation. In a typical, simple fourteenth-century version of the Liar, there is a person, Socrates, who begins speaking with the sentence 'what Socrates says is false', and says nothing more (*cf., e.g.* [Ockham, 1974, p. 744]). In this way, the paradoxes of self-reference are supposed to arise from something people do, not from linguistic constructions *in abstracto*. In this sense, an oath is a quite natural element to use in constructing a self-referential paradox.

Clearly, oaths were not at the core of medieval discussions of these paradoxes. The core was the truth-predicate applied in such a way as to produce a paradox,

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but even with the truth-predicate analogous paradoxes were constructed in significantly differing ways. The semantic paradoxes of self-reference seem to have been discussed in both Arabic and Latin logic well before the thirteenth century so that the standard example was 'what I utter is false', but various authors developed versions that in many cases were interestingly different in terms of the solutions given for the core paradox. Roughly at the beginning of the fourteenth century treatments of the topic became really a flourishing genre of logic. The most original work seems to have been done in the 1320s and 1330s in Oxford by authors like Richard Kilvington, William Heytesbury and Roger Swyneshed. Especially Thomas Bradwardine's treatment accrued well-deserved fame in the following centuries. These logicians really can be said to have undertaken the task of giving a consistent, theoretically satisfactory solution to the Liar paradox and its associates rather than simply having the interest of studying various logical aspects of the Liar and other congenial paradoxes.

In the following I will go through a selection of medieval texts. The choice of the texts is not straightforwardly based on the historical importance or even quality in logic. The aim is, rather, that the sample gives a picture of the multiplicity and theoretical richness of this particular medieval discussion.

INSOLUBILIA MONACENSIA

The oldest known Latin tract dedicated to the paradoxes of self-reference is an anonymous text written about at the end of the twelfth century, called *Insolubilia Monacensia* according to the current location of the manuscript in Munich. Scholars know about occasional references to the Liar paradox already long before that, like in Adam of Balsham's *Ars disserendi*, which was written in 1132 [Adam of Balsham, 1956, 107]. However, the scarcity of the surviving early treatments of the paradox has made it difficult to give a story how the medieval Latin discussions got started. One alternative is that they originate from Byzantine Greek or medieval Arabic logical texts, which are not known either.[1]

In any case, *Insolubilia Monacensia* presents a theory that is already relatively developed. The word for these paradoxes is *insolubilia*, or "insolubles", and the author explains that this is due to the difficulty in giving a truth value evaluation of the paradoxical sentences. In a manner typical for medieval logicians, he puts the problem in a disputational context. "A solution is," as he says, "an appropriate determination of a thing", which means that the semantic value of a proposition is evaluated in a satisfactory way, typically by saying "yes" (*ita*) or "no" (*non*), or "it is true" or "it is false" [De Rijk, 1966, 104]. That the author is thinking of disputations in fairly technical terms becomes obvious from his comment that

[1] Cf. [De Rijk, 1966, 83–86, 103]; edition of the *Insolubilia Monacensia* on pp. 104–115. The text has been discussed also by, e.g. [Spade, 1973]; see also [Spade, 1987; Martin, 1993; Bottin, 1976]. For a listing of medieval texts, see [Spade, 1975]. For other interesting early treatments, see [Braakhuis, 1967; De Rijk, 1976; Roure, 1970]. Discussions are also to be found in commentaries on Aristotle's *Sophistical Refutations*.

sometimes refraining from an evaluation can be the correct determination. Thus, if you are asked to doubt whatever is put forward to you first in the disputation, answering with doubt is the correct determination for the first proposition, given your obligation. Such an obligation clearly has the taste of a technical duty.[2]

An insoluble is a proposition that cannot be solved in this way: no answer will be satisfactory. Or further, "an insoluble is a circular and necessary deduction to both sides of a contradiction" [De Rijk, 1966, 105]. The author compares two propositions:

> God is one in essence and three in persons
> I utter a falsehood

In his view, the first of these propositions is insoluble absolutely (*simpliciter*). No correct and consistent answer can be given to whether God is one or to whether he is three. But on the other hand, the latter proposition is insoluble only in a certain respect (*secundum quid*) and thus allows for a solution [De Rijk, 1966, 104–105]. This attitude shows strong optimism about finding a solution to the paradoxes of self-reference, and indeed such optimism was to remain dominant throughout the middle ages, even in two senses. First, authors thought that solving the paradoxes is not impossible but only difficult, maybe difficult to the extent of admitting that no solution had yet been found. And second, they did not consider it a problem if no solution is in sight. It seems that no medieval logician saw the threat of the logical systems collapsing in mere inconsistence in the face of the paradoxes. In the eyes of medieval logicians, the Liar paradox was not a threat but a treasure. It gave an especially interesting test case for semantical theories and was indeed used for that work.

The main approach taken by the author of *Insolubilia Monacensia* is to look at how putting a proposition forward relates to asserting it. His solution of the paradox, as far he really presents a solution, is simply cancellation (*cassatio*). It seems that the idea is that the paradoxes somehow fail to be such that they would need to be answered, and thus they ought to be "cancelled". The author divides his treatment to three ways in which the paradox may arise: through a human act (of asserting or such like), through a feature of the (linguistic) instrument, or through the nature of what is asserted [De Rijk, 1966, 105]. The examples are as follows.

The act of uttering is at issue when one considers a paradox like 'I utter a falsehood', 'I lie', or something such. These utterances are such that they allow for inferring a contradiction. Consider whether 'I utter a falsehood' is true or false. If it is true, it can be proved to be false:

Proof.

> That I utter a falsehood is true
> I utter a falsehood and nothing other than this
> This is a falsehood

[2] [De Rijk, 1966, 104].

■

If it is false, it can be proved to be true:

Proof.

> That I utter a falsehood is false
> I do not utter a falsehood and I utter something
> I utter a truth and nothing other than this
> This is a truth

■

The author gives these inferential chains as proofs for the opposites, showing that the utterance "I utter a falsehood" must be cancelled (*est cassandum*) so that one can respond saying "you are not saying anything" (*nil dicis*) [De Rijk, 1966, 105–106].

The problem is, as the author recognizes, that some words undeniably seem to be uttered even if you want to say that there is nothing in need of an evaluation. He distinguishes between asserting (*asserere*) and putting forward (*proferre*) so that both are necessary in order to make a claim: "To claim is nothing but to assert first and to put forward then." [De Rijk, 1966, 106]. However, the author does not really tell what these stages of making of a claim are. He seems to be ready to admit that either or maybe even both of these stages take place when the insoluble sentence is pronounced, although not together. In saying 'I utter a falsehood', the two stages apparently fail to come together. He compares the situation to how the definition of human, 'a rational animal', has two parts both of which refer even to non-human things, but never together [De Rijk, 1966, 106].

As another example of the same kind, the author presents the truth-teller paradox, 'I utter a truth'. He claims that some people dispute its paradoxical character, but difficulties cannot be avoided if one considers the evaluations of two statements by other people, one saying that the claim is true and the other saying that it is false. Thus, the truth-teller's sentence can equally be proved to be true and to be false, but it cannot be both: we have a paradox [De Rijk, 1966, 106–107].

Still staying with examples of the same type, the author considers how the first person pronoun is not necessary to produce the paradox: 'Socrates utters a falsehood' is paradoxical if uttered by Socrates but not if uttered by someone else [De Rijk, 1966, 108–109]. Further, the author notes that direct self-reference is not needed for the paradox. Circular reference will do. Thus, if Socrates says that Plato lies, when Plato says that Cicero lies and Cicero says that Socrates lies, we have essentially the same paradox in our hands even if none of these persons refers to himself [De Rijk, 1966, 109].

Linguistic paradoxes of self-reference are discussed as the second type. For example, if the clause "a proposition not described by its predicate" is used as the predicate, a paradox results. Thus, the following sentence is paradoxical:

> This is a proposition not described by its predicate.

If it is asked whether the predicate of this proposition refers to the proposition itself, no answer can be given. For if the predicate describes the whole, the proposition must be like it says, that is, a proposition not described by its predicate, contrary to what was assumed, and if it does not, then the proposition is described by its predicate, contrary to what was assumed. The author claims that the mentioned clause cannot be a predicate in a proposition, and thus the considered sentence-looking piece of language lacks a predicate and is not a proposition. This of course solves the problem, but leaves one wondering why exactly the clause cannot be a predicate [De Rijk, 1966, 113].

The same arbitrariness of the solution is visible in an even more serious way in another example considered by the author. Suppose that on an otherwise empty page, or embedded within a completely true text it reads

> A proposition written on this page is false.

The author claims: "it must be said that this proposition cannot be written either alone or with truths". But what does this mean? Does the pen stop if one tries to write something like this? The author seems to go back to something like the distinction between asserting and putting forward discussed above. Literally, he distinguishes between writing as "drawing figures" (*figuras protrahere*) and as "representing figures" (*figuras representare*). The latter can, the author claims, be cancelled (*cassari*). Thus, it appears that it is possible to draw on the page something looking exactly like a proposition, but the ink will fail to become letters and words [De Rijk, 1966, 113–114].

The third class of insolubles is discussed so shortly that it is impossible to see what the author exactly means. The example is 'something is not true about itself', and the idea appears to be that such examples are based on what it is to be a proposition [De Rijk, 1966, 115]. However, in the short discussion the example looks like it would belong together with the former example of using 'proposition not referred to by its predicate' as the predicate. That example was classified as a linguistic paradox. In the later treatises on insolubles the distinction between insolubles arising from human acts or from linguistic structure seems to remain a standard classification. The third category seems to disappear, though.

'THE *POSITUM* IS FALSE' AS THE *POSITUM*

As we noted, the author of *Insolubilia Monacensia* discusses the paradoxes of self-reference in a distinctively disputational context, where the respondent evaluates sentences put forward by the opponent. Also, he shows knowledge of the medieval disputational technique of obligations. However, the disputational context plays no substantial role in his ways of setting the paradox or his solutions of it. In particular, he does not present the paradox as a technical obligation in the way medieval obligational disputations were presented. The medieval logical genres of obligations and insolubles thus started as distinct fields. Nevertheless, it seems

that they were closely connected from the very beginning, and many later authors chose to present insolubles as obligational disputations in the technical sense.

As they were known in the thirteenth and fourteenth centuries, the technical obligational disputations consisted of encounters between an opponent and a respondent, with the opponent first assigning the respondent some special duty (*obligatio*), typically that of accepting a sentence known to be false, and then putting forward sentences for the respondent to concede, deny or doubt in a consistent manner. The idea was that the respondent had to pay attention to inferential relationships between different sentences. In the standard form, the disputation would start with the opponent "positing" a sentence, or laying down as the basis of reasoning a false, perhaps impossible sentence, the *positum* as it was called. The respondent should accept the *positum* if he thought that it could be maintained consistently. Then he should answer other sentences so that he concedes consistently everything that logically follows from the presented *positum*. In a way, this technique served for the Latin medieval period as a paradigm of how one should deal with false assumptions in a disputational context.[3]

In the medieval manuscripts from thirteenth century onwards, treatises on insolubles were very often associated with treatises of obligational disputations. Furthermore, paradoxical or at least complicated types of self-reference were often constructed in the obligations treatises. It seems that already the very earliest obligations treatises address issues that are of genuine relevance to the topics discussed in the treatises on insolubles. One such is the anonymous "Emmeran Treatise on False *Positio*", which seems to date from the first half of the thirteenth century.[4]

The author of this text takes considerable effort to discuss exact rules for what the respondent should do if the opponent wishes to lay down as the *positum* something containing 'a falsehood is posited'. As the text tells, the respondent ought not to accept this sentence as the *positum*, since "a contradiction would follow". Interestingly, the author does not say that a contradiction would follow technically in the disputation, but outside it. As he says, the opponent would then say "time is finished", which in these disputations means that one must turn into real discussion of truth values and not care anymore about the duty to grant the *positum* and anything following from it [*Anonymous*, 2001, 199–200]. The idea is simple: when 'a falsehood is posited' is posited, it becomes a sentence that claims itself to be false. The resulting paradox is quite near the standard Liar.

In another interesting move the author considers 'a falsehood is conceded' as the *positum*. It can be accepted, as he points out, since no contradiction follows at this stage. The sentence is not paradoxical as a *positum*, because the response to the *positum* is not called 'conceding' but 'accepting'. However, if it is posited and accepted, and the opponent puts it forward as a proposition, the respondent loses the disputation. The proposition put forward follows directly from the *positum* as a repetition of the same sentence. Therefore, it should be conceded, but it

[3]For deeper discussion of the genre, see, e.g. [Yrjönsuuri, 1994; 2001; Keffer, 2001].
[4]Edition in [De Rijk, 1974], translation [*Anonymous*, 2001].

cannot be, because from conceding only the sentence 'a falsehood is conceded' and nothing else, a paradox would follow. [*Anonymous*, 2001, 203].

In both of these cases, the paradox can be avoiding by the respondent's refusal to go into such situation, since the rules of obligations allow such a move. Without respondent's acceptance nothing can be posited, and without him conceding the crucial sentence nothing dangerous is conceded, and no self-reference arises.

After discussing the case of positing 'a falsehood is posited' (let us call it 'p') alone, the author goes into a discussion of the opponent wishing to posit a molecular sentence containing 'p'. The rules concerning these situations are quite simple, and again the idea is to avoid the paradox. The disjunction $p \vee q$ can be posited if q is true but not if it is false. Similarly, the conjunction $p\&q$ can be posited if q is false but not if it is true. The underlying idea is, of course, that it should be possible to assign truth values without problems. In the case of the disjunction that is true because of q, it can be entertained that the *positum* is true and thus 'a falsehood is posited' is false. Similarly, in the case of positing the conjunction having an unambiguously false conjunct, it can be entertained that 'a falsehood is posited' is true [*Anonymous*, 2001, 200–201]. At this point it may also be pointed out that *Insolubilia Monacensia* too addresses the issue and resolves it similarly. According to that text, 'I utter a falsehood or God exists' can be granted because of the latter disjunct, while 'I utter a falsehood and God exists' is insoluble and must be cancelled because its truth would require the truth of the first conjuct [De Rijk, 1966, 109–111].

According to a further rule in the Emmeran Treatise, if the opponent wants to posit a material equivalence between the *positum* and some other sentence, the respondent must check whether the sentence at issue is true or false: if it is false, the *positum* may not be accepted. If the opponent wants to posit something like "the *positum* and [the first sentence] put forward are similar [in truth value]", the respondent can only accept the *positum* in case the first sentence will be true. Unfortunately the author does not recognize that this ruling allows the opponent to trap the respondent who maybe does not know what the opponent is going to put forward as the first sentence [*Anonymous*, 2001, 201–202].

These rules about molecular sentences containing self-reference show that the intention is not to prohibit reference to the *positum* in the *positum* itself. It is not the self-reference that is taken to be problematic, but the possibly ensuing paradoxes. Thus, the suggestions do not amount to even trying to give a solution to the paradox. Rather, the author sketches ways of seeing how it could arise and how it can be avoided in advance. The picture gets very complicated because of the two-tier structure involving the technical disputational context and the discussion outside the disputation. Somewhat interestingly, the author does not pay attention to how self-referential paradoxes could arise within the technical disputation itself.

In the later treatments of the obligations technique self-referential structures become more rare or merely apparent.[5] Occurrences of the sentence 'the *positum*

[5] As an outstanding exception, *Obligationes Parisiensis* contains material similar to that found in the Emmeran Treatise. Cf. [De Rijk, 1975].

is false' provide an illuminating example. Walter Burley, for example, considers whether one should grant 'the *positum* is false' if it really is. Thus, if the *positum* is 'You are in Rome' in a disputation held in Oxford, one should concede 'the *positum* is false' as the first proposition. According to Burley, this has the implication that then one should deny ''you are in Rome' is the *positum*', which may result in a complicated (and curious) disputation with sentences like '"you are in Rome" is true and irrelevant' being conceded in Oxford [Burley, 1963, 62]. It seems that Burley was thinking about the fact that there is nothing inconsistent in supposing that one is participating an obligational disputation in Rome with a false *positum*, and insofar as this was his idea, he was not dealing with self-referential paradoxes in this context.[6]

THOMAS BRADWARDINE

Despite work done by many authors, including such excellent logicians as Walter Burley and William Ockham, it seems that the medieval discussion concerning the paradoxes of self-reference did not make much visible progress for more than a century. The account given in *Insolubilia Monacensia* appears almost as advanced as some late thirteenth-century ones. This may be partly due to a modern pretension that progress would be marked by the offered solutions becoming better in some respect, while the thirteenth century authors apparently were more interested in forming a clear picture in how self-reference leads to a paradox and in mapping the various versions of the paradox that can be built. Ockham, to mention a different kind of a case, appears to have been simply not very interested in the topic. He ends the relevant section in his *Summa logicae* to the comment that it is there just for the sake of completeness.[7]

In any case, the discussion was brought to a turning point by Thomas Bradwardine in the early 1320s.[8] His discussion of the topic seems to be the first that gives the feeling of a systematic solution cast in a logically careful format. Bradwardine was one of the so-called Oxford Calculators, which was a group of scholars working in Oxford and in effect producing a new mathematical and logical way of doing philosophy that was to have repercussions even until Galileo Galilei and early modern science in general. Bradwardine has in recent discussion been best known for his work in mathematical physics, but it is clear that his solution of the insolubles was a very remarkable achievement. Also other calculators, including Richard Kilvington and William Heytesbury, produced treatments of insolubles that are of a very high logical quality.[9]

Bradwardine's treatise is systematically structured. It starts off with discussions

[6] For an edition of Burley's treatise on the insolubles, see [Roure, 1970].

[7] [Ockham, 1974, 746]. In respect to the substance of what Ockham has to say about insolubles, it seems clear that this part of *Summa logicae* must have been written without knowledge of Bradwardine's work.

[8] For Bradwardine's position in the discussion, see e.g. [Read, 2002, esp. pp. 189–199].

[9] For a general discussion of the Oxford calculators, see e.g. [Sylla, 1982].

and refutations of the earlier solutions of the paradox. This part of the treatise is useful even in a historical sense. After that he presents his own solution in the form of "divisions, definitions, postulates and theses" In other words, he puts forward a kind of axiomatic basis for discussions of insolubles, and derives results that are directly applicable to different kinds of self-referential paradoxes. The rest of the treatise applies the offered solution to a variety of examples, including even "sophisms which seem to be insolubles but are not".[10]

The core of Bradwardine's solution of the paradoxes of self-reference is that a sentence that claims itself to be false signifies not only that it is false but also that it is true. On the basis of such a signification, it is unproblematic to judge that the sentence is false because the falsity can now be associated to the truth-claim. The problematic point in this solution is, as is easy to see, the proof that and the explanation how and why the insoluble sentence signifies itself to be true, since there seems to be nothing such explicitly said in the sentence. Bradwardine deals with the issue in terms of his second thesis, which has been put under close scrutiny by modern scholars.[11] Instead of starting with that, let us however first see how Bradwardine's solution works in a real case, taking up postulates and theses as they are needed in the argumentation.

The first example analyzed by Bradwardine after the discussions presenting the axiomatic basis is cast in the format of an analysis of a very simple obligational disputation. [Bradwardine, internet, 47]. It runs as follows:

> Case: Socrates utters only this: (a) Socrates utters a falsehood
> Proposition: (b) Socrates utters a falsehood

The idea is this. You, the respondent, will have to evaluate the proposition 'Socrates utters a falsehood' on the basis of an imagined case, where Socrates only utters 'Socrates utters a falsehood'. Bradwardine's solution of the paradox is that you ought grant the sentence, because what Socrates says (that is, 'a') would in the imagined case be false. Therefore, the proposition put forward to you (that is, 'b') is true.

The crucial practical edge of the discussion comes from the fact that as said by your opponent, 'Socrates utters a falsehood' is not self-referential, and thus it can be admitted as true, but as Socrates himself utters 'Socrates utters a falsehood', the sentence is self-referential in a problematic way. Bradwardine makes a distinction between these two propositions ('a' and 'b') and points out that in granting 'b' you do not grant the proposition 'a' uttered by Socrates, but "another like it" *(aliam sibi similem)* [Bradwardine, internet, 47]. The discussion thus relies on the standard medieval approach that truth-values are attributed to sentence tokens and not types (in Bradwardine's terms, to sentences as they differ *secundum numerum*, rather than *secundum speciem*; cf. [Bradwardine, internet, 62].

[10] Printed edition of the treatise can be found in [Roure, 1970]. I have used Stephen Read's edition and translation available on the internet [Bradwardine, internet].

[11] See esp. [Read, 2002; Spade, 1981; Spade, internet].

Bradwardine wants to say that the sentence 'a' is false, but the sentence 'b' is true. This becomes even more clear if we look at how he analyses a counter argument, which is as follows [Bradwardine, internet, 47]:

Proof.
'a' and 'b' are equivalent
'b' is true
so 'a' is true ■

To this argument Bradwardine objects that the first premise is false. To prove this, he works out an analysis of what 'a' and 'b' signify, and it is this analysis that really reveals what his solution of the paradox is. Thus, let us take a close look at the argumentation in the text.

The sentence uttered by Socrates, 'a', is the natural point of departure. Bradwardine says:[12]

> (1) 'a' signifies that Socrates utters a falsehood, from which it follows that what is uttered by Socrates is false.

Unfortunately, there are different readings of the consequence mentioned here, depending on the reference of "from which" (*ex quo*). What exactly is the antecedent of the intended consequence? Bradwardine uses the consequence to apply his second postulate (P2), which runs as follows:[13]

> (P2) Every proposition signifies or means as a matter of fact or absolutely <respectively> everything which follows from it as a matter of fact or absolutely.

Here again, the "it" (*ista*) is not too lucid, but the natural reading would be that a sentence means what follows from the sentence itself. In the examined example Bradwardine applies this principle to infer that

> (2) 'a' signifies that what is uttered by Socrates is false.

If he means the principle (P2) in the exact form quoted above, it seems that in (1) he should have said that the consequent at issue follows from 'a' itself. This is however something that the Latin sentence does not directly mean. Either it means that the consequent follows from 'Socrates utters a falsehood' or that it follows from 'a' signifying that.[14]

[12] "*a significat quod Sortes dicit falsum, ex quo sequitur quod dictum a Sorte est falsum.*" Bradwardine, [internet, 46; translation p. 47].

[13] "*Quelibet propositio significat sive denotat ut nunc vel simpliciter omne quod sequitur ad istam ut nunc vel simpliciter.*" Text and translation from Bradwardine, [forthcoming, 38, 39]. Roure [1970] has '*illam*' instead of '*istam*'.

[14] To apply his second postulate correctly, Bradwardine should, it seems, have said that 'a' is (rather than "signifies") 'Socrates utters a falsehood'. However, when medieval logicians used letters as names of sentences they often formulated this by saying that the letter "signifies" the intended sentence. Thus Bradwardine may be intending to say exactly that 'a' names the sentence 'Socrates utters a falsehood' as an explanation why he thinks that 'a' implies that what is uttered by Socrates is false.

The issue makes a difference, as can be seen by considering whether Bradwardine accepts a modified version of (P2), namely,

> (P2)* Every proposition signifies or means as a matter of fact or absolutely <respectively> everything which follows from what it signifies as a matter of fact or absolutely.

There are two ways to attribute (P2)* to Bradwardine. One is to claim that Bradwardine accepted both (P2) and (P2)*, the latter through accepting that every sentence implies what it signifies. However, this attribution has the problem that it forces Bradwardine quite visibly to consequences that he explicitly wanted to avoid.[15] Also, it does not accord very well with the way he puts the example. The other way is to attribute minor textual problems to Bradwardine's formulation and claim that he means to state (P2)* rather than (P2). This would make the example run smoothly.

The question concerns the exact way in which Bradwardine believed that signification is "closed" under the consequence relation. To put it formally (using the colon for the relation of signifying),[16] Bradwardine may mean:

$$\text{(P2)}\ \frac{a \rightarrow p}{\text{`}a\text{'} : p}$$

or he may rather mean:

$$\text{(P2)*}\ \frac{\text{`}a\text{'} : p, p \rightarrow q}{\text{`}a\text{'} : q}$$

The problem with (P2) is that it is somewhat counterintuitive and does not work the way Bradwardine puts his principle to work. The problem with (P2)* is mainly that it is not what he appears to say in the text. To some extent, thus, the question comes down to the historian's methodological choices.

Immediately after Bradwardine's work, medieval scholars started to discuss the nature of propositional significates (*complexa significabilia*). In this discussion, the problem was to determine what exactly is signified by a proposition over and above that which is signified by its terms. Indeed it is possible that Bradwardine's work gave a seminal motivation for such a discussion in a context where standard semantic theories had room only for things and not for states of affairs as signified entities.

Be that problem as it may, we can see what Bradwardine is aiming at in the examined example. Given (2), Bradwardine points out that the subject of the signified proposition 'what is uttered by Socrates is false', or 'what is uttered by Socrates' has in the assumed case only one *suppositum*, or refers to only one thing, which is 'a'. Thus, 'a' signifies the affirmative predication of falsehood to that

[15] Paul Spade has defended this kind of reading of Bradwardine. See e.g. [Spade, 1981]. Stephen Read rejects the reading. See [Read, 2002, 208–214].

[16] I am using the arrow for medieval consequence, which makes it in certain cases ambiguous between conditional and inference.

suppositum [Bradwardine, internet, 47]. This step is taken in virtue of the second part (T1.2) of Bradwardine's first thesis, which is as follows:[17]

(T1) Every proposition whose extreme:

(T1.1) has many *supposita*, signifies or means affirmation or denial for one of them, and
(T1.2) if it has only one, for that.

The idea of this thesis is to formulate for the context the theory of truth conditions that was standard at the time, usually given in the form of *supposita*. Propositions claim something about the things they refer to, which is to say that they affirm or deny something of their *supposita*. Bradwardine aims at a technically satisfactory way of saying that falsehood is affirmed in 'a' of 'a' itself. Thus, the subject term of the latter part of (2), or 'what is uttered by Socrates' can be substituted by 'a' in order to move to:

(3) 'a' signifies that 'a' is false

A reference to (P2) is again needed, and again it is applied in the form (P2)* as allowing one to move from 'a': p to 'a': q when $p \rightarrow q$. Substituting the pronoun 'itself' for 'a', Bradwardine achieves [Bradwardine, internet, 47]

(4) 'a' signifies itself to be false.

Now Bradwardine is almost there, since his second thesis applies to sentences like this. It runs as follows[18]:

(T2) If some proposition signifies itself not to be true or itself to be false, it signifies itself to be true and is false.

Using this thesis, Bradwardine achieves for the analyzed case the crucial result. Thus [Bradwardine, internet, 47]:

(5) 'a' signifies itself to be true

and

(6) 'a' is false.

If we collect the argument together, it runs as follows:

Proof.

(1) 'a' signifies that Socrates utters a falsehood

[17] "*Quelibet propositio cuius extremum multa habet supposita significant sive denotat affirmationem vel negationem pro aliquo istorum, et si unicum, pro isto.*" Text and translation [Bradwardine, internet, 38, 39].

[18] "*Si aliqua propositio significet se non esse veram vel se esse falsam ipsa significant se esse veram et est falsa.*" [Bradwardine, internet, 38; 39].

(2) 'a' signifies that what is uttered by Socrates is false P2
(3) 'a' signifies that 'a' is false T1.2
(4) 'a' signifies itself to be false P2
(5) 'a' signifies itself to be true T2
(6) 'a' is false T2

■

The edge of the argument really comes from the fact that if a similar reasoning is applied to 'b', or the proposition put forward to you in the disputation, it will not go through. The beginning of the reasoning works, of course, in the same way:

(1)* 'b' signifies that Socrates utters a falsehood
(2)* 'b' signifies that what is uttered by Socrates false P2
(3)* 'b' signifies that 'a' is false T1.2

But after this line it is clear that the introduction of the pronoun 'itself' would be fallacious. For 'a' is not 'b', it is a different token of the same type. And because there is no self-reference, Bradwardine's second thesis (T2) cannot be used. The true claim that Socrates utters a falsehood gets separated from the false one.

Bradwardine's solution of the paradox amounts thus to exact logical way of spelling out the fact that whereas 'a' signifies something about itself, there is no such self-reference for 'b'. Thus, the meanings of 'a' and 'b' differ, and they differ so that 'b' means almost everything 'a' does, except that 'a' additionally means that 'a' is true. To be sure, 'b' does not mean that 'a' is true, and it does not mean that 'b' is true.

Even if this is the core of the solution, working it out in an acceptable way was not a simple project. When we look at Bradwardine's successors, we find that in general they did not buy Bradwardine's (T2). There were exceptions, though. At least Albert of Saxony and to some extent John Buridan, as we will see, accepted (T2) in virtue of accepting the stronger claim that all sentences signify their own truth, but this seems not to have been Bradwardine's intention. He appears to say that it is only the paradoxical sentences that signify their own truth.[19]

Bradwardine's proof of the crucial thesis (T2) is difficult to spell out, but it is really the most important part of the solution, since without proof the thesis just is not acceptable. With the proof, Bradwardine can rely only on (P2), which is much easier to accept as it stands. To prove (T2), he first proves that any sentence which signifies that it itself is not true also signifies that it itself is true. Then he extends the result to sentences that signify their own falsehood. As the last part he proves the final bit that because these sentences signify their own truth, they are false. The crucial part of the logic is to found in the first part, which works with the assumption that the sentence either does or does not mean something

[19]The agreements and differences between Bradwardine, Buridan and Albert of Saxony have been spelled out in detail by Stephen Read in [Read, 2002].

else as well. The first part of this proof is relatively understandable, and we can still use 'a' as a name for the sentence at issue (although it must now be taken as a name for an arbitrary sentence satisfying the criterion of self-reference, or any sentence for which the first premise (1) is true).[20]

Proof.

(1)	'a' signifies only that 'a' is not true	Assumed
(2)	'a' is not true	Assumed
(3)	It is not wholly as 'a' signifies	(2), (Def. of truth)
(4)	It is not the case that 'a' is not true	(1), (3), (T1.2)
(5)	'a' is true	(4)
(6)	'a' signifies itself to be true	(1), (2)–(5), (P2)

■

Most steps of this argument seem clear enough. In essence, (3) expresses what it is for 'a' not to be true.[21] Then at step (4), 'the case that 'a' is not true' is substituted for 'wholly as 'a' signifies' since that is all that 'a' signifies according to (1). Then at step (5) the double negation is eliminated. At step (6), Bradwardine applies his principle (P2), which is doing the crucial work.

A reader may wonder about the line (2) and the application of the principle (P2). Is line (2) assumed for the sake of conditional proof? The lines (2)–(5) would then infer "'a' is true' from "'a' is not true'. The consequence would then be used at (6). This interpretation seems a good suggestion, especially in view of applying the principle (P2) on (1) to achieve (6). However, again it seems that it is (P2)* that is needed, since line (2) appears to be what 'a' signifies and not the examined sentence 'a' itself.[22] The reasoning seems to run from 'a' : $\neg T$'a' (cf. (1)), proving (on steps (2)–(5)) the consequence $\neg T$'a' $\rightarrow T$'a' and using it with the principle (P2)* to achieve 'a' : T'a'.

The second part of the proof is more difficult. What if the examined sentence means something else as well? Bradwardine proceeds as follows. Let that other signification be 'B is C', and run the reasoning similarly. Now it is as follows:[23]

[20] In Stephen Read's translation, the text runs as follows: "Let A be a proposition signifying itself not to be true, whether signifying anything else or not. If not, then it follows from the assumption that A is not true, by definition (D1), that it is not wholly as is signified by A, and by A is signified only that A is not true (we supposed), so by thesis (T1.2) it follows that it is not the case that A is not true, that is, that A is true, whence by postulate (P2) A signifies itself to be true." [Bradwardine, internet, 45]. The proof has been analyzed in [Spade, 1981, 120–125; Read, 2002, 211–212], but my analysis differs slightly from both. See also [Read, forthcoming, 8–10].

[21] Bradwardine refers to his definition D1 (see footnote 19).

[22] There seems to be no sufficient reason for assimilating 'a' with "'a' is false', since Bradwardine quite explicitly presents the reasoning as something that applies to any sentence 'a' signifying its own falsity, and "'a' is false' is certainly only one such.

[23] The text continues from footnote 19: "If, on the other hand, A signifies more than that it

Proof.

(1)* 'a' signifies only that 'a' is not true and that B is C Assumed

(2)* 'a' is not true Assumed

(3)* It is not wholly as 'a' signifies (2)*, (Def. of truth)

(4)* It is not the case that 'a' is not true and B is C (1)*, (3)*, (T1.1)

(5)* 'a' is true or B is not C (4)*

(6)* 'a' signifies that 'a' is true or B is not C (1)*, (2)*–(5)*, (P2)

(7)* 'a' signifies that B is C (1)*, (P2)

(8)* 'a' is true or B is not C, and B is C, therefore 'a' is true

(9)* 'a' signifies that 'a' is true. (P2)

■

This reasoning is a bit more confusing, and not only because Bradwardine jumps between first order claims ((4)*, (5)* and (8)*) and claims about the examined paradoxical sentence and its signification ((1)*, (2)*, (3)*, (6)*, (7)* and (9)*) more than in the previous proof. It is, however, possible to see what really happens. After laying the ground with the two assumptions (1)* and (2)*, Bradwardine construes the disjunctive signification (6)* through the conditional proof (2)*–(5)*. The he excludes the latter disjunct at (7)*, and proceeds with the support of the inference (8)* to the main goal (9)*.

At (7)* Bradwardine makes a choice. He could at that step exclude the former disjunct by taking the corresponding proposition "'a' signifies that 'a' is not true' and using it in a similar way, to achieve instead of step (9)* the claim "'a' signifies that B is not C.' That is, he could reason as follows. Assume 'a' : $(\neg T\text{`}a\text{'}\&p)$. Thus $\neg T\text{`}a\text{'} \rightarrow \neg(\neg T\text{`}a\text{'}\&p) \rightarrow (T\text{`}a\text{'} \vee \neg p) \rightarrow \neg p$. By (P2)*, it follows that 'a' : $(\neg T\text{`}a\text{'}\&p\&\neg p)$, and since any sentence follows from the contradiction $(\neg T\text{`}a\text{'}\&p\&\neg p)$, '$a$' signifies everything, including its own truth.

For quite obvious reasons, this is not the way Bradwardine goes. He wants to prove the thesis (T2) in a sensible way, and for this he needs "'a' signifies that 'a' is true' but not much more. Thus, he reasons as follows. Assume 'a' : $(\neg T\text{`}a\text{'}\&p)$. Thus $\neg T\text{`}a\text{'} \rightarrow \neg(\neg T\text{`}a\text{'}\&p) \rightarrow (T\text{`}a\text{'} \vee \neg p) \rightarrow T\text{`}a\text{'}$. By (P2)*, it follows now that '$a$' : $(\neg T\text{`}a\text{'}\&p\&T\text{`}a\text{'})$, which is a much more sensible kind of inconsistence. However, this way of looking at the proof makes it apparent that Bradwardine is in

itself is not true, for example, that B is C, then supposing that A is not true, it follows that it is not wholly as A signifies, that is, that A is not true and B is C, whence it is not the case that A is not true and that B is C, and so, by postulate (P4), A is true or B is not C, and so, by postulate (P2), A signifies that A is true or B is not C, and since it signifies the opposite of the second part of this disjunction (we supposed), namely, that B is C, we have by postulate (P5), from this disjunction with the opposite of its second part that the first part follows, namely, that A is true. Hence by postulate (P2), A signifies A to be true." [Bradwardine, internet, 45]. (P4) and (P5) are ordinary principles of propositional logic; see [Bradwardine, internet, 39].

fact applying the principle (P2)* on the proposition at issue so that it is possible to infer contradictions and thus to make any impossible proposition signify whatever you want. Thus, the idea that the paradoxical sentence signifies its own truth is not very informative, since in the same manner it can be shown to signify whatever.

Let us go back to the example we have analyzed, and the way Bradwardine develops the discussion. He considers an imagined opponent arguing that just as 'a' in the example signifies both "a' is false' and "a' is true', in fact 'b' signifies those two sentences too. As his imagined opponent argues: "'a' being true follows from 'a' being false ... therefore, it follows that from 'b' it follows that 'a' is true, and so 'b' signifies consequentially that 'a' is true, and so 'b' signifies the same as 'a', so 'a' and 'b' are equivalent." The idea seems to be that since Bradwardine shows in his proof of (T2) that $\neg T$'a' $\rightarrow T$'a' under the given conditions, a proposition signifying $\neg T$'a' will signify T'a' as well. Thus if 'b' : $\neg T$'a' it follows by (P2)* that 'b' : T'a', and thus 'b' should be as false as 'a'.

This is a reasoning that Bradwardine cannot accept, and thus he must show that there is something wrong. He needs a way of showing that if you as the respondent say that Socrates utters a falsehood, you do not imply that Socrates utters a truth even if Socrates himself uttering that sentence does commit himself to that implication. The way Bradwardine argues here sheds light on how he understood his crucial principle (P2). He rejects the claim that "that 'a' is true follows from 'a' being false". Bradwardine formulates his point as follows [Bradwardine, internet, 49]:

> "Rather, from 'a' signifying itself to be false it was earlier deduced with certain truths that 'a' signifies itself to be true"

Thus, we should try to understand his proof of the thesis (T2) so that he does not really infer from 'a' or from "a' is false'. The reasoning is concerned with the signification of 'a', not its truth. Also, the reasoning leans on some auxiliary premises. It seems that what Bradwardine has in mind is the way in which moving from (3) to (4) requires (1). For consider the following:

(1)**	'b' signifies only that 'a' is not true	Assumed
(2)**	'a' is not true	Assumed
(3)**	It is not wholly as 'a' signifies	(2)**, (Def. of truth)
(4)**	It is not the case that 'a' is not true	

Here step (4)** is fallacious, since there are no premises about what 'a' signifies. Thus, there is no analogous way of showing that 'b' signifies that 'a' is true. This is what Bradwardine wants. With this kind of an interpretation his solution to the paradox appears to work.

Either Bradwardine did not manage to give a final solution of the Liar, or he did not manage to get understood. He did, however, give so lucid an exposition of the logical structures involved that apparently all important medieval solutions after him are more or less reactions to his work. Further, many of his examples of the different applications are interesting enough to look at in their own right.

RICHARD KILVINGTON

In the early 1320's when Bradwardine composed his treatise on insolubles, Richard Kilvington was his younger colleague who was a bit ahead in his studies. At about the same time, Kilvington's wrote a collection of sophisms.[24] In the medieval tradition of *sophismata*, sophisms were sentences that were for some reason difficult to evaluate. Typically there was a casus assumed at the background, just as in obligational disputations. Most of Kilvington's *Sophismata* deals with limit decision problems of continuous quantities, issues concerning with acceleration and mean speed, and other topics of mathematical physics. The last group of nine sophisms, however, addresses issues of epistemic logic and semantical theory, and the very last sophism is more or less directly dedicated to insolubles. It seems that Kilvington's discussion is to be understood as a reaction to Bradwardine's theories.

One of the most crucial general issues in Bradwardine's solution of the paradoxes of self-reference is the semantic relation between a sentence and its own truth. Kilvington takes this relation under scrutiny, and he proves two crucial results. First, as is proved in sophism S41(l), truth is not a thing, and thus signifying the truth or falsity of a sentence will have to work differently from signifying things. Indeed, Kilvington explains, the truth of a sentence is nothing other than the sentence itself and thus it may appear that in talking about the truth of a sentence you do not talk about anything else than what you talk about when you talk about the sentence itself. And second, as is proved in sophism S47(t)–(y), a sentence and the claim that it is true follow form each other only under very stringent conditions. That is, Kilvington would not have accepted the so called Tarskian biconditionals in any general fashion.

So what is wrong with the equivalence "p' is true, if and only if p'? The main issues Kilvington takes up are existence and signification. Following the standard medieval approach of token-based semantics, Kilvington assumes that truth bearers must be existing sentence tokens, which implies that there is a difference between claiming something about things in themselves and claiming truth for a sentence. A sentence token must exist in order to be true. Kilvington is not satisfied with the existence condition, however. He points out also that claiming truth for a sentence token of a certain type may leave it open what the sentence token means. Thus, "you are in Rome' is true' has nothing to do with your whereabouts if 'you are in Rome' means that man is an animal. Signification is ultimately a contingent matter, and thus the claim "p' is true' does not in general imply p. It seems obvious that Kilvington's real aim is not at such clear cases, but rather at paradoxes of self-reference understood in a Bradwardinian manner.

In the last sophism of his collection S48, Kilvington goes directly to the paradoxes of self-reference. The sophism is based on the following two sentences:

[24]Edited by N. Kretzmann and B. E. Kretzmann in [Kilvington, 1990a]; their translation and commentary in [Kilvington, 1990b]. I refer to the text by sophisma number and paragraph letter. Sylla [1982] gives a good introduction to what sophisms in the medieval sense are.

(p) God exists
(q) Nothing granted by Socrates is known by you

Now suppose a situation where Socrates evaluates and grants one of these sentences and no other sentence, but you do not know which one it is. You are nevertheless asked whether the sentence granted by Socrates is something you know. If the sentence is 'p', no problems of referential circularity arise and you are in the medieval context supposed to know 'p'. However, if the sentence granted by Socrates is 'q', there is a possible self-reference arising. Not for Socrates, of course, but for you.

Can you say that you know 'q'? But 'q' is the sentence that Socrates granted, and if you know it, it is true, and what Socrates granted is not known by you. Thus, if you know 'q', you do not know 'q'. S48(c).

Can you then say that you do not know 'q'? No. For that would mean saying that you do not know the very sentence granted by Socrates, or in other words that nothing granted by Socrates is known by you. But then, by saying that you do not know it, you obviously claim to know that the case is as it signifies, or that it is true, and thus you seem to know it. S48(d).

The core of the sophism is that you do not know which one of the two Socrates grants. The sophism is structured so that the one Socrates grants, is nevertheless named 'a', and then you are asked whether you know 'a'. Thus, the crucial sentence to evaluate in this situation is ''a' is known by you'. As a whole, the sophism shows quite elaborated command of the ways of building a paradox on self-reference. Indeed, the structure is so elaborated that it remains unclear even whether the sophisms should be treated as an insoluble in the medieval sense.

Kilvington starts his solution by saying that many people solve this sophism by saying that it is insoluble, thus referring people back to their solutions of the self-referential paradoxes. But, as Kilvington notes, if the sentence put forward to Socrates is 'p', no self-reference arises, and then one is not facing an insoluble in the technical sense. As Kilvington continues, he first goes into a discussion showing that "this does not follow: 'I know that 'a' is true; therefore, I know 'a'"'(S48, n). In short, he wants to open space for doubting a sentence in contexts where there is a way of referring to the sentence so that under that description you know the sentence to be true. His prime example is one where "'a' is the singular name of the true proposition in the pair 'A king is seated' and 'No king is seated"' (S 48, q). If you know how to use the name 'a', you know that 'a' is true, but this will not help you to know whether any king is seated.

It seems that Kilvington discusses these problems of epistemic logic just in order to open some space between a sentence and the attribution of truth to the sentence. For this is crucial for his solution of the Liar, into which he turns for the rest of the sophism. His solution, namely, depends on the idea that "each insoluble is true in a certain respect and false in a certain respect and neither true nor false absolutely." (S48, cc.) Although Kilvington is not very clear about what he means by "in a certain respect" and "absolutely" in this context, it seems safe to suppose that his approach is a version of Bradwardine's. He seems to think that

when two sentence tokens of the same type have different truth values at the same time, there is a fallacy involved and what really happens is that neither of them is true absolutely but that they we must speak of truth "in a certain respect."

WILLIAM HEYTESBURY

Another Oxford calculator, William Heytesbury wrote in 1335 a work titled "Rules for Solving Sophisms" (*Regulae solvendi sophismata*).[25] The aim of this work, as the introduction tells us, is to advice young students in their everyday duty to deal with sophisms. It seems that such exercises formed at the time the core of the practical side of the study of logic and semantics, and it seems that paradoxes of self-reference held a place of pride there. The first chapter of Heytesbury's *Rules* is dedicated to insolubles.

Right in the beginning Heytesbury tells his readers, "although the insolubles can be solved, nevertheless they have not yet been solved." [Heytesbury, 1979, 15]. Accordingly, his work should not be understood as offering a solution to the paradox. Rather, he aims at telling students how to deal with the paradox in their logic exercises. Essentially, Heytesbury's advice is to avoid problems through not admitting that any examined paradoxical sentence would signify exactly as the common meaning of the terms would make it to signify. Thus, he reverts back to the strategy we saw in the thirteenth century treatises: avoid the paradox rather than face and solve it. But now this strategy has a Bradwardinian twist. Instead of saying that the respondent in a disputation should simply refuse self-referential paradoxes, Heytesbury advices the respondent to avoid admitting that one knows what exactly a sentence like 'I utter a falsehood' means.

Before going into his advice in detail, Heytesbury criticizes earlier solutions. He lists three different views that he takes to be worse than his own approach. The second and third of these are not very interesting. They verge on the question whether insolubles are truth-bearing sentences. The second denies their sentential status. The third allows, in Heytesbury's formulation, that insolubles are sentences and thus truth-bearers, but claims that they just do not have a truth value. Quite righteously Heytesbury wonders what it means to be a truth bearer without a truth value.

The first view has been interpreted as Roger Swyneshed's theory,[26] and Heytesbury's discussion of it is perhaps the most interesting part of the treatise. Since Swyneshed's approach was developed on the basis built by Bradwardine, many of Heytesbury's arguments tackle issues already in Bradwardine, though still not by abandoning the approach. Rather, Heytesbury continues on the same track.

The modification Swyneshed made to Bradwardine's theory was to shift attention from signification to consequences. His idea was that there are sentences

[25] The whole work is edited in [Heytesbury, 1494]. For the chapter on insolubles, see translation in [Heytesbury, 1979].

[26] See Spade's study in [Heytesbury, 1979, esp. p. 72–79]. For a study of Swyneshed's theory, see Spade, 1983; for an edition of the text, see [Spade 1980].

— the paradoxical ones — which have the truth value 'false' not on the basis of their signification but due to their self-falsificatory character. Thus, any sentence implying that it itself is false, falsifies itself and is false because of this falsification regardless of the signification. Thus, it seems that Swyneshed did not buy Bradwardine's principle (P2), or at least he did not think that it would have any significant application in the theory of self-referential paradoxes.

Swyneshed's theory shares the feature of Bradwardine's theory that the paradoxical sentences are deemed false. This has the implication that there at least appear to be contradictory sentences that are both false. Although Bradwardine recognizes the problem, he tries to do away with it. Swyneshed, for his part, accepts the oddity with straight face, buying the strange view that contradictories are sometimes both false. Furthermore, there appear to be cases where two tokens of exactly the same sentence type have different truth values as regards the same time and the same situation ('Socrates utters a falsehood' as uttered by Socrates or by someone else). Both Bradwardine and Swyneshed accept directly this implication of their solution.

One further oddity concerns inferential relationships. Consider the consequence:[27]

> This is false, therefore, this is false,

where both demonstrative pronouns indicate the consequent (let the consequent be 'a' and the antecedent 'b'). According to both Bradwardine and Swyneshed, as a self-referential token 'a' must be deemed false, but 'b' is to be counted as true, because it is not self-referential but instead refers to 'a', which indeed is false. The inference appears valid: the contradictory of the consequent is inconsistent with the antecedent. If the same thing is indicated, 'this is false' and 'this is not false' are contradictories.

Swyneshed accepts the reasoning, confirms that the consequence is valid and allows that in a valid inference there is a true antecedent with a false consequent [Spade, 1979, 189]. This is an interesting choice, since it makes it appear that he did not think that the main point in a consequence would be to account for truth preservation or any of its modal varieties, as has mostly been the approach in modern logic. Rather, this suggests that Swyneshed thought that consequence is a relation between the meanings of the sentences, and it is more or less a coincidence that in most cases it preserves truth.

It seems that Bradwardine would not need to accept the validity of this consequence even if he would accept the same understanding of the consequential relationship, since in his theory the difference between the truth values of 'a' and 'b' is based on a difference in signification. Thus, what is affirmed by the antecedent is only a part of what is affirmed by the consequent. This may of course result in the inference being blocked. Furthermore, Bradwardine tries to show that the contradictory of a self-referential sentence like 'a' is not simply 'this is not false' but more like 'either 'a' is not false or 'a' is not true', which clearly would not

[27] See [Heytesbury, 1979, 29].

be inconsistent with 'this is false' indicating 'a' from the outside [Bradwardine, internet, 51].

For his part, Heytesbury takes it to be odd if consequences of the form

> p, therefore, p

are not formally valid. Since he provides the example as a criticism of the attempted solution, he does not offer any solution of the problem. Instead, he continues with a whole series of examples where logical properties of sentences misbehave if Swyneshed's (and Bradwardine's) solution is accepted. To pick up another example, let 'a' be 'this sentence is not necessary', indicating itself, and let its contradictory 'b' be 'this sentence is necessary,' indicating 'a'. Now, there seems to be no reason to count 'a' as false in the way insolubles are false, and it seems even to signify as is the case, since it does not appear to be necessary. Indeed, if 'a' was necessary, its contradictory 'b' would signify as is the case and thus 'b' would be true. This seems unacceptable. However, it appears that 'a' signifies as is necessarily the case, as Heytesbury puts it, and thus it seems not to be a contingent sentence either. Thus, 'a' is a true sentence that cannot be ascribed a modal status [Heytebury, 1979, 31]. Without modifications, neither Bradwardine's nor Swyneshed's theory will solve the case. And if modifications are made, it seems that they will be necessary for all kinds of logical properties, since corresponding examples arise in many directions.

Heytesbury's own rules about what one should do when faced with an insoluble draw considerably on the disputational context. One really gets the feel that he is giving advice to a student on how to behave in a specific kind of disputation. For example, his third rule concerns the situation where the constructed case has an insoluble sentence and it is assumed together with it that the sentence "signifies as its terms pretend, but not precisely" [Heytesbury, 1979, 49]. In such a case, the insoluble sentence must be granted, but one must deny its truth:

> Case: Socrates is saying only this: 'Socrates is saying what is false', and it so signifies, but not precisely.
> Proposition 1: Socrates is saying what is false. Granted, follows
> Proposition 2: 'Socrates is saying what is false' is true. Denied

Heytesbury seems to follow Bradwardine in judging that the actual sentence uttered by Socrates can be deemed false. This seems to be his reason for denying proposition 2.

The text continues with a discussion of what the meaning difference between proposition 1 and the sentence uttered by Socrates is. In putting this question, Heytesbury seems to be thinking of Bradwardine's theory and the idea that when uttered by Socrates himself 'Socrates is saying what is false' has a different signification than when uttered in a disputation by other people. Heytesbury does not, however, want to specify the difference in meaning. In particular, he does not say anything like Bradwardine's claim that the self-referential sentence signifies its own truth. He advises the respondent to refuse any question concerning precise

signification. Even more strongly, if the respondent is asked to assume that the sentence signifies precisely as terms pretend, the whole assumption should be refused. As a practical advice for a disputation, this may seem satisfactory. But as Heytesbury recognizes, logically the solution is far from perfect.

JOHN BURIDAN

John Buridan's massive textbook of logic *Summulae de Dialectica* ends with a collection of sophisms, seemingly as a section containing exercises. The last set in this collection deals with self-referential propositions and thus also with insolubles. This is not the only location where Buridan discusses paradoxes of self-reference, but it is perhaps the most accessible one. The textbook as a whole was widely used for several centuries, but no other section in it is directly dedicated to self-reference.[28]

Buridan's way of treating these paradoxes becomes obvious through looking at how he has put together the set dedicated to self-reference. Let us start from the second sophism in the set [Buridan, 2001, 956–958]. The sophism is the consequence:

> 'No proposition is negative; therefore, some proposition is negative.'

Buridan's proof for this is straightforward, and for us interesting not so much because of the self-refuting character of the antecedent but because of the connection that the reasoning has to the Tarskian truth-scheme. For Buridan uses the claim "no proposition is negative' is true' as an intermediate step. The crucial claim that he will use in his solution of the insolubles is that all sentences imply their own truth. This is something that he tackles already at this stage of the series. Thus, the following chain appears to be a valid inference:

Proof.

(1) No proposition is negative
(2) 'No proposition is negative' is true
(3) The proposition 'no proposition is negative' exists
(4) A negative proposition exists
(5) Some proposition is negative

■

Buridan's solution of the sophism is that it must be denied. Surprisingly, the step at which he blocks the inference is already (2) [Buridan, 2001, 957]. Thus, he seems to reject a principle that will turn out to be of utmost importance for his solution of the insolubles. However, this is a move where we see a good logician at work.

[28] [Buridan, 2001] has the English translation. See [Buridan, 2004] for a critical edition, and e.g. [Pironet; 1993; 1998] and [Zupko, 2003] for an evaluation of Buridan's different texts on insolubles and more bibliographical information.

The equivalence between a sentence and its truth had been put under scrutiny by Bradwardine and others after him, and thus Buridan could by no means take it for granted and just use for his solution of the paradox. He had to first achieve a firm grip of what exactly is the logical relation between a sentence and its truth.

Consider the consequence

> A man exists, therefore, 'a man exists' is true.

Buridan rejects this with the consideration that it might be the case that there are men but no propositions. He thinks that a proposition needs to exist in order to be true: he too is committed to token-based semantics. In the first sophism of the series, Buridan has accepted 'every proposition is affirmative, therefore, no proposition is negative' as valid inference despite the fact that the consequent is self-refuting and thus cannot be true [Buridan, 2001, 953]. He wants to say that inference is fundamentally a modal relation, and a proposition is not called possible on the basis of whether it can be true, but rather on the basis of whether the case can be as it signifies. Thus, a proposition can be possible without being possibly-true.[29] In the same vein, consequences do not hold on the basis of a connection between truth values, but on the basis of whether things can be as is signified by the antecedent and consequent. In a valid consequence, the case is as is signified by the consequent if it is as is signified by the antecedent. Existence of the propositions will not affect inferential validity in the ordinary way, though it will affect truth. As Buridan notes concerning the example, 'every proposition is affirmative, therefore, no proposition is negative': "something true can entail something false." But as he explains, "when this is concluded, the former is no longer true, but false." [Buridan, 2001, 955].

In the case analyzed in the second sophism, existence of the proposition matters, since the consequent in the consequence 'a man exists, therefore, 'a man exists' is true' is about truth value, not about humans. Buridan notes that the following is a valid inference

> A man exists
> 'A man exists' exists
> Therefore, 'a man exists' is true

Thus the above discussed inference 'a man exists, therefore, 'a man exists' is true' would be a valid enthymeme with reference to a true premise "a man exists' exists', although as it stands it is not valid. A reader familiar with Richard Kilvington's work will at this point wonder why Buridan skips any worry about signification. As we saw above, Kilvington — and Heytesbury — would have here made reservations about what 'a man exists' means. But not Buridan.

However, as Buridan notes, in order to be valid or true, a consequence needs to exist: not only truth but even asserted validity appears to be carried by tokens of

[29] Cf. [Prior, 1969]. The corresponding problem with inferences has been discussed in, e.g., [King, 2001, pp. 129] and [Dutilh Novaes, 2005].

inferences [Buridan, 2001, 957]. But now, does the existence of the consequence not guarantee the existence of its antecedent and thereby also the existence of the proposition 'a man exists'? Yes and no. Whenever the reasoning is evaluated, the antecedent exists, but as Buridan shows, to be valid the consequence should hold also for situations in which it does not exist. This is a distinction Buridan spends some time to clarify, though we can leave it obscure in respect to our limited purposes.[30]

One further sophism, the fourth one in the set, deserves attention before we go to Buridan's treatment of insolubles proper. This sophism still concerns the attribution of truth values. Suppose you say,

I say that a man is a donkey

Do you now utter a truth or a falsehood? Buridan's answer is: a truth, literally speaking. For you indeed say the words 'a man is a donkey', and your claim is that you say those words, and no more. As supporting his claim, Buridan cites the Psalmist saying "the fool hath said in his heart, there is no God." As Buridan notes, this does not commit the Psalmist to the proposition 'there is no God' and indeed this proposition is not in the Psalm as a proposition, but only as a part of a proposition [Buridan, 2001, 960].

But now, if you say,

'Man is an animal' is true,

doesn't this analysis make the sentence false since here 'man is an animal' is not a proposition? It is just the subject term of a proposition, and as such it cannot be ascribed a truth value. Buridan accepts [Buridan, 2001, 960–961]. In his view there is a reading of this sentence that indeed makes it false. For an appropriate evaluation one needs to know what the subject of the proposition, i.e. 'man is an animal' is taken to supposit for. If you are there referring to another linguistic structure, similar to the one you mention but used as a proposition, then your claim may be true. But if you intend your claim to be about the set of words you utter as the subject in this utterance, then the claim is false for the very same reason as 'I say that a man is a donkey' is true in the sophism.

It seems that Buridan's idea in structuring his work is that these sophisms provide exercise and make certain issues clearer before he goes to the genuine paradoxes of self-reference. This takes place as the seventh sophism, where Buridan even gives his general solution to the *insolubilia* [Buridan, 2001, 965–971]. His paradigm version of the paradox assumes the case that all true propositions have been annihilated but false ones remain in existence. Then Socrates utters

'Every proposition is false',

[30]It seems that Buridan distinguishes between validity and truth of a consequence so that an existing consequence (nowadays called 'conditional') is true if and only if the consequence (nowadays called 'inference') from the antecedent to the consequent is valid. Thus, for validity, the conditional need not be formulated, but for asserting the validity one needs to make a claim, a conditional proposition needs to be actually formulated. See [King, 2001].

and the question is whether his utterance is true or false.

After considering briefly some earlier suggestions for a solution, Buridan turns to his own. He takes the view that the paradoxical sentence is false. However, he rejects the view that this is because the sentence signifies or asserts its own truth in the way every sentence does. He admits that this had been his opinion earlier. However, now he argues against it, rejecting the idea that all propositions signify their own truth.

Consider a typical token of the claim made by the defenders of this theory:

> 'Man is an animal' signifies that it is true

There are two ways to read this sentence. The clause 'that it is true' can be taken "materially or significatively", in Buridan's own terms.

Following the former reading, the claim is that the proposition 'man is an animal' signifies the proposition 'this proposition 'man is an animal', is true', which Buridan finds hard to believe, because 'man is an animal' is about men and animals, not about linguistic entities like words and propositions whereas the truth-claim concerns an actually existing linguistic structure.

As for the latter reading, consider an impossible proposition like 'a man is a donkey'. What would it be for it to be true? As Buridan argues, nothing, and it is not true to say that something that is not and cannot be, is signified or understood [Buridan, 2001, 968–969]. Though it is difficult to see what exactly Buridan has in mind here, it seems to be related to his refusal to accept so-called propositional significates (*complexa significabilia*). Maybe the argument is intended to point out that even if you were able to claim that true sentences signify real states of affairs and possible sentences, possible states of affairs, impossible sentences will not signify anything sententially. For what would the impossible states of affairs signified by impossible propositions be?

Claiming that this line is a dead end, Buridan takes another approach. According to him, every proposition implies virtually another proposition such that the predicate 'true' is affirmed for a subject suppositing for the proposition [Buridan, 2001, 969]. This is included in the truth conditions. Thus, no proposition is true unless in this affirmative consequent both the subject and the predicate supposit for the same. To cite first a simple example, we may consider the following consequence

> Man is an animal, therefore, 'a' is true.

Here 'a' refers to the antecedent 'man is an animal'. This appears to be similar to a case discussed above, and the reader ought keep all the appropriate reservations in his mind. For Buridan turns to an example that is more complicated. He looks at the case, where Socrates utters 'no proposition is true'. This proposition is given the proper name 'c'. Now the consequence required by his theory is:

> No proposition is true, therefore, 'c' is true.

As Buridan explains, the antecedent proposition '*c*' cannot be true unless the implied condition is true, that is unless its subject '*c*' and its predicate 'true' supposit for the same [Buridan, 2001, 969].

'Supposite for the same' is Buridan's normal theory of truth, but here he claims carefully and explicitly that this criterion is not quite sufficient. The consequential consideration needs to be added as a condition. It can be argued that by the condition Buridan makes the truth of a sentence dependent on itself. No sentence can be true, unless it fills all the normal conditions and in addition is true. This sounds circular, and thus a problem.[31] Be that as it may, the criterion does help Buridan to evaluate the paradoxical sentences as false. Thus, when it is claimed that the sophism 'every proposition is false' is true because all the propositions that the subject supposits for are indeed supposited for by the predicate as well, Buridan can answer that the sameness of supposition is not enough for the truth of the proposition, since the implied consequent affirming truth to the sentence turns out inconsistent and false. For if one looks at the content of the antecedent, it follows that

> No proposition is true, therefore, '*c*' is not true

And if this consequence is valid together with the one that has the same antecedent but the opposite consequent, it can only be because the antecedent is false and even impossible.

As we noted, Buridan's second sophism in this set shows that consequences of the form

> *p*, therefore, '*p*' is true

are not as such valid. Now we have seen on the contrary that his solution of the insolubles requires or even is based on such consequences being valid. Buridan notes this fact, but points out that consequences of the form

> *p*, and '*p*' exists, therefore, '*p*' is true

are valid. Buridan seems to think that this issue forces him to qualify his theory, but then he also notes that in an insoluble case it is normally assumed that the sentence at issue exists. For this reason, the qualification should not engender problems [Buridan, 2001, 970].

It seems that Buridan thinks that at this point he has given a sufficient explanation of how the core of the paradoxes of self-reference can be solved. He turns to different versions of the paradox and considers further examples.

The series of sophisms contains thirteen more. They develop interestingly different problems resulting from either immediate or mediated self-reference. By the latter Buridan means situations like the one in the ninth sophism, where Plato is supposed to say 'Socrates utters a falsehood' while Socrates says 'Plato utters a

[31] See [Read, 2002, 200–201].

truth'. Here neither of the propositions refers to itself, but together they generate a loop.

Two further sophisms deserve mention here. The eleventh sophism assumes that I utter only the sentence 'I utter a falsehood' ('a'). Buridan applies his theory systematically. Thus, he says that 'a' is false and rejects the proof that it follows that it is true. The crucial arguments are as follows:

> 'a' is false, therefore, 'a' exists
> a, and 'a' exists, therefore, 'a' is true

Buridan accepts both of these two arguments, but rejects that together they would show that "'a' is true' follows from "'a' is false'. As he explains, the conjunctive antecedent of the latter inference is to be rejected because of the first part. Thus, he distinguishes between 'a' is false' and 'a' even in a case where 'a' means that 'a' is false. Indeed, he grants "'a' is false' as true while claiming that 'a' is false [Buridan, 2001, 978].

Then Buridan turns to the issue of contradictories and equivalencies. What should you say to say the same as I say when I say 'I utter a falsehood'? According to Buridan, your sentence should be 'You utter a falsehood, and 'a' is true', 'a' referring to my sentence. Correspondingly, the contradictory of my sentence would be the disjunction: "You do not utter a falsehood, or 'a' is not true." As one can see, in the paradoxical case the contradictory is indeed true [Buridan, 2001, 979–980].

However, by claming that the paradox sentence in fact turns into a conjunction, Buridan shows that he has not really got very far from the theory that the sentence *signifies* its own truth and not only *implies*. For suppose a valid consequence

$$p \rightarrow q$$

If we want to find the contradictory of 'p', we do not in general include 'q'. That is, it seems odd to say that the contradictory of 'p' has the disjunctive form '$(\neg p \vee \neg q)$' whenever something follows from p. This formulation — or the talk about mental language that Buridan goes into in this context — both seem natural only if 'q' is somehow signified by rather than simply follows from 'p'. It seems that Buridan was not able to say what he really meant by his theoretical idea that the insoluble sentences imply rather than signify their own truth.

If compared to Bradwardine's work, Buridan's discussion of the insolubles does not appear very ingenious and original. He wavers and leaves room for doubt, allowing the reader to get the feeling that his solution is sketchy. The work is, however, clearly on a very advanced logical level and the problems obviously result from difficulties in the subject matter. Given the extremely wide circulation of Buridan's *Summulae*, it is very understandable that his solution achieved a very high reputation and a wide influence in the coming centuries.

The seventeenth sophism in Buridan's set is the sentence 'You will throw me in the water'. Buridan writes: "Let us posit the case that Plato is the master of the bridge and that he guards it with such a powerful military support that nobody

can cross it without his permission. And then Socrates arrives, asking Plato with insistence to let him cross. Then Plato, getting angry, vows and takes an oath of the form: 'Certainly, Socrates, if with your first proposition that you will utter you say something true, I will let you pass, but certainly if you say something false, I will throw you in the water'. And then Socrates says to Plato the above mentioned sophism, namely, 'You will throw me in the water'. The problem then is: what should Plato do to keep his promise?"

As a careful, not to say tedious logician, Buridan gives a much fuller analysis of the sophism than Sancho Panza will do later. In Buridan's analysis, Socrates's sentence concerns a future contingent and therefore, its truth value can be determined only later, when it will be seen what happens. Thus, refraining from evaluation does not mean asserting a truth value gap. Further, Plato's claim is a conditional one, and there is no necessary connection between the antecedent and the consequent. In the strict sense, Plato speaks falsely. However, as Buridan notes, one should rather interpret Plato as giving a promise, which is usually said to be true when the person keeps his promise. And Plato does not. As a promise, the utterance is false. Socrates does not fill the condition, because he utters a proposition that is not yet determinately true or false (though it is, in Buridan's view, either true or false). Plato is not able to keep his promise, because through his utterance Socrates makes Plato's promise self-reflexive (*habens reflectionem supra se*). When it is asked, whether Plato needs to keep his promise, Buridan answers negatively, just as Sancho Panza will. Plato cannot keep his promise, and thus he need not. He should not have given a promise vulnerable to paradox.

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DEVELOPMENTS IN THE FIFTEENTH AND SIXTEENTH CENTURIES

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INTRODUCTION

The story of logic in the fifteenth and sixteenth centuries is very much the story of institutional change. Logic remained at the heart of the university curriculum for students in faculties of arts throughout Europe, but there were significant changes both in the curriculum as a whole and in the approach to logic, particularly with respect to the textbooks used and to the style in which they were written. I say 'style' rather than 'language', for while it is true that the first known vernacular logics appeared in Germany in 1534, in Italy in 1547, in France in 1555, and in England in 1551,[1] this development, which was partly a result of printing having made book ownership possible for a wider range of people, had little to do with the universities, where education was and continued to be strictly in Latin at least until the end of the seventeenth century.[2] The textbooks, the written and spoken exercises, the lectures, the disputations were all in Latin, and ability to handle Latin was the entrance requirement. There are two aspects of this use of Latin that I would like to emphasize. First, it meant that university members were part of one European culture, including the European book trade. Just as manuscripts circulated widely in the fifteenth century, so printed texts circulated widely in the late fifteenth and sixteenth centuries. Second, it meant that university members worked in a semi-artificial second language which was no one's mother tongue, and which mainly survived in the Roman Catholic Church and in educational institutions.[3] During the later middle ages, Latin's artificiality took the form of the production of a special technical language for the discussion of logic and logical grammar. With humanism and its emphasis on the study of the language and literature of Classical Greek and Latin, a different kind of artificiality appeared,

[1] For a discussion of various aspects of language during the Renaissance, see Luce Giard, "Du latin médiéval au pluriel des langues, le tournant de la Renaissance", *Histoire Epistémologie Langage* 6 (1984), 35–55. On p. 48 she gives a table of the dates at which grammatical and logical texts in the vernacular first appeared in various European countries, and I have used her material here.

[2] Ong has emphasized the unread look of the surviving vernacular texts as opposed to the dog-eared and annotated Latin texts: see W.J. Ong, *Ramus, Method, and the Decay of Dialogue* (Cambridge, Mass.: Harvard University Press, 1983 [first printed in 1958]), p. 14.

[3] See Giard, "Du latin médiéval", p. 38.

that of imitating the written texts of men who had been dead for fifteen centuries.[4] To understand the significance of these developments for the logician, we have to consider three questions. First, how much of the medieval logic described in the previous chapters survived? Second, insofar as medieval logic survived, were there any interesting new developments in it?[5] Third, does humanist logic offer an interesting alternative to medieval logic?

In Part One of this chapter I shall consider the first two questions in the context of a historical overview in which I trace developments in logic from the later middle ages through to 1606, the year in which the Jesuits of Coimbra published their great commentary on Aristotle's logical works, the *Commentarii Conimbricensis in Dialecticam Aristotelis*.[6] I shall begin by considering the Aristotelian logical corpus, the six books of the *Organon*, and the production of commentaries on this work. I shall then examine the fate of the specifically medieval contributions to logic. Finally, I shall discuss the textbook tradition, and the ways in which textbooks changed and developed during the sixteenth century. I shall argue that the medieval tradition in logic co-existed for some time with the new humanism, that the sixteenth century is dominated by Aristotelianism, and that what emerged at the end of the sixteenth century was not so much a humanist logic as a simplified Aristotelian logic.

In Part Two of this chapter, I shall ask whether the claims made about humanist logic and its novel contributions to probabilistic and informal logic have any foundation. I shall argue that insofar as there is any principled discussion of such matters, it is to be found among writers in the Aristotelian tradition.

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[4]For its deleterious effects, see Giard, "Du latin médiéval", p. 46. For more about the humanist return to the past, see W. Keith Percival, "Grammar and Rhetoric in the Renaissance" in *Renaissance Eloquence. Studies in the Theory and Practice of Renaissance Rhetoric*, ed. James J. Murphy (Berkeley: University of California Press, 1983), pp. 303–330, especially pp. 311–312. See also Brian P. Copenhaver, "Translation, terminology and style in philosophical discourse" in *The Cambridge History of Renaissance Philosophy*, ed. Charles B. Schmitt, Quentin Skinner, and Eckhard Kessler, (Cambridge: Cambridge University Press, 1988), pp. 77–110, especially pp. 104–106. For humanist innovations in logical vocabulary, see G. Nuchelmans, *Late-Scholastic and Humanist Theories of the Proposition* (Amsterdam: North Holland Publishing Company, 1980), part two, passim.

[5]For a full account of these matters, I refer the reader to my own writings, in particular: E.J. Ashworth, *Language and Logic in the Post-Medieval Period* (Dordrecht, Boston: D. Reidel, 1974); E. J. Ashworth, "Traditional logic" in *The Cambridge History of Renaissance Philosophy*, pp. 143–172; E. J. Ashworth, introduction to Robert Sanderson, *Logicae Artis Compendium*, ed. E. J. Ashworth (Bologna: Editrice CLUEB, 1985), pp. XI–LV.

[6]An unauthorized edition appeared earlier, in 1604. The author of the logic commentary was Sebastian de Couto.

Part One: Historical Overview

1 ARISTOTLE: TEXTS AND COMMENTARIES

The *Organon* of Aristotle played a central role throughout the medieval and post-medieval period, at least in principle.[7] During the medieval period, the *Organon* was divided into two parts. The *Logica vetus* or 'Old logic' consisted of Boethius's translations of the *Categories* and *Perihermenias* along with Porphyry's *Isagoge* or introduction to the *Categories*; and it was already known by the beginning of the twelfth century. The *Logica nova* or 'New logic' consisted of the *Prior* and *Posterior Analytics*, the *Topics* and the *Sophistici Elenchi*. Boethius's translations of the *Prior Analytics*, *Topics* and *Sophistici Elenchi* were recovered from about 1120 on, and they were completed by James of Venice's translation of the *Posterior Analytics*. Other translations of all these works were produced during the twelfth and thirteenth centuries, but none of them seem to have shaken the virtual monopoly of Boethius and James of Venice, which endured into the sixteenth century.[8] From the thirteenth century on both the *Logica vetus* and the *Logica nova* were firmly embedded in the university curriculum, though some exception must be made of the *Topics*, at least in the later period. Sometimes it was omitted altogether, and sometimes only certain of its Books were to be read.[9] Even in the sixteenth century, the interest in Aristotle's *Topics* was somewhat modified. The Jesuit *Ratio Studiorum* of 1586 referred to the commitment to follow Aristotle in logic, but noted that the *Topics* (except for Books I and II) and also the *Sophistici Elenchi* should be passed over, though they added that the material of the topics and fallacies should be presented in some more orderly way.[10] These exhortations were shown to be effective by the brief treatment given to both the *Topics* and the *Sophistici Elenchi* in the Coimbra commentary.[11]

From the beginning of the medieval period, Aristotle's text was accompanied by explanation and commentary.[12] In the twelfth and early thirteenth centuries

[7] One has to be cautious because students who were supposed to read Aristotle himself may often have read secondary sources instead.

[8] For details, see B. G. Dod, "Aristoteles latinus" in the *Cambridge History of Later Medieval Philosophy*, edited by N. Kretzmann, A. Kenny and J. Pinborg (Cambridge: Cambridge University Press, 1982), pp. 45–79, especially pp. 74–75; and the introductions to the various volumes in the *Aristoteles latinus* series. The use made of various translations is sometimes difficult to establish: see introduction to Leonine editions of Aquinas's Aristotle commentaries.

[9] For details, see Ashworth, "Traditional logic," p. 143, and N. J. Green-Pedersen, *The Tradition of the Topics in the Middle Ages* (München, Wien: Philosophia Verlag, 1984), p. 90.

[10] K. Kehrbach, *Monumenta Germaniae Paedagogica Band V* (Berlin, 1887), pp. 129–130, p. 138.

[11] *Commentarii Collegii Conimbricensis e Societate Jesu. In universam dialecticam Aristotelis* (Cologne 1607; reprinted Hildesheim, New York, 1976), II, columns 733–766.

[12] For a listing of Aristotle commentaries in the medieval period, see C. H. Lohr, "Medieval Latin Aristotle Commentaries: Authors A-F," *Traditio* 23 (1967), 313–413; "Authors G-I," ibid., 24 (1968), 149–245; "Authors Jacobus-Johannes Juff," ibid., 26 (1970), 135–216; "Authors Johannes de Kanthi-Myngodus," ibid., 27 (1971), 251–351; "Authors Narcissus-Richardus," ibid., 28 (1972), 281–396; "Authors Robertus-Wilgelmus," ibid., 29 (1973), 93–197; "Supplementary

what we find is glossed texts, that is, texts with marginal notes, some of a very simple nature, and some very lengthy and elaborate.[13] By the mid-thirteenth century, the fully-fledged commentary had come to dominate, and during the thirteenth and fourteenth centuries all the great names in philosophy such as Albert the Great, Thomas Aquinas, John Duns Scotus and William of Ockham, wrote commentaries on at least some of the books of the *Organon*. There were two main types of commentary.[14] First, there is the literal commentary, typified by those of Aquinas. The author of a literal commentary divided the work into a series of *lectiones*, and then treated it passage by passage. A paraphrase would be offered, together with an explanation of difficult or important phrases, and points of doctrine would be investigated. Especially in the later middle ages, *dubia* were often added and discussed at some length. Second, there is the question-commentary, whose author did not divide or paraphrase the text, but instead considered a series of questions which arose from points in the text. During the fifteenth century the distinction between the two types of commentary became blurred, and we find question-commentaries which contain paraphrases, and literal commentaries which contain *dubia* whose length and structure allies them to the *quaestio*. Moreover, literal commentaries ceased to be divided into *lectiones*, instead being organized in accordance with the accepted division of Aristotle's text into chapters and tractates.[15] The Coimbra commentary of 1606 is an excellent example of the hybrid question-commentary with its chapter divisions, chapter summaries, textual commentary and lengthy *quaestiones*.

The Coimbra commentary illustrates another trend which became important in the fifteenth century, that towards commentaries on the entire *Organon*.[16] Indeed, the very name *Organon* when used as a title rather than a description seems to be a late fifteenth-century innovation. Commentaries on individual books continued to be written; but the fifteenth-century Thomists at the *Bursa Montis* in Cologne produced a single commentary on the *Logica vetus* and another on the *Logica nova*, as did Johannes Versor in Paris.[17] Also at Paris we find George of Brussels and Pierre Tartaret, whose respective commentaries on the entire *Organon* were first published in 1493, and enjoyed considerable success during the first decades of the sixteenth century. Later sixteenth-century commentaries included that by

Authors," ibid., 30 (1974), 119–144. For a listing of Aristotle commentaries in the period 1500–1650, see C. H. Lohr, *Latin Aristotle Commentaries. II. Renaissance Authors*, Firenze: Leo S. Olschki, 1988. See also C. H. Lohr, "Renaissance Latin Aristotle Commentaries: Authors A-B," *Studies in the Renaissance* 21 (1974), 228–289; "Authors C," *Renaissance Quarterly*, 28 (1975), 689–741; "Authors D-F," ibid., 29 (1976), 714–745; "Authors G-K," ibid., 30 (1977), 681–741; "Authors L-M," ibid., 31 (1978), 532–603; "Authors N-Ph," ibid., 32 (1979), 529–580; "Authors Pi-Sm," ibid., 33 (1980), 623–734; "Authors So-Z," ibid., 35 (1982), 164–256.

[13] Dod, "Aristoteles latinus", pp. 73–74.

[14] Green-Pedersen, *The Tradition of the Topics*, pp. 88–89, distinguishes three.

[15] Green-Pedersen, *The Tradition of the Topics*, p. 91.

[16] For comment, see Lohr, "Renaissance Latin Aristotle Commentaries: Authors A-B", pp. 230–231 or Lohr, *Latin Aristotle Commentaries II*, p. XIV.

[17] For publication details, see W. R. Risse, *Bibliographia Logica Band I. 1472–1800* (Hildesheim: Georg Olms, 1965).

Franciscus Toletus, published in Rome in 1572 and republished many times.

Another feature of the late fifteenth and sixteenth centuries which should be mentioned here is directly due to the invention of printing, and concerns the wide dissemination of the medieval Latin commentators. Walter Burley and Antonius Andreas were particularly important at the beginning of the sixteenth century; and the commentaries of Aquinas reached a peak of popularity in the mid-sixteenth century.[18] The commentaries by, and attributed to, John Duns Scotus were also published throughout the century.[19] This publication history was to have the interesting result that in the Aristotle commentaries of the second half of the sixteenth century we find much more attention paid to Aquinas and Scotus, along with other thirteenth-century figures such as Albert the Great and Giles of Rome, than to the perhaps greater logicians of the fourteenth century such as Ockham and Marsilius of Inghen. This is clearly illustrated by the marginal references throughout the Coimbra commentary. The reprinting and reediting of the great medieval Arab commentator Averroes is also a noteworthy part of sixteenth-century publishing history, which was to have its impact on the style of Aristotle commentaries in the latter part of the sixteenth century.[20] The influence of Averroes on Zabarella (see below) is of particular importance, but his name occurs with some frequency in the already-mentioned marginal references of the Coimbra commentary.

So far I have only mentioned the medieval tradition of Aristotle translation and commentary, but one cannot understand sixteenth-century developments without also considering the impact of humanism. During the fifteenth century Greek manuscripts became increasingly accessible, and proficiency in the Greek language became more and more widespread, first in Italy and then, gradually, in other European countries. The first printed edition of the Greek text of Aristotle was produced in Venice by Aldus Manutius from 1495 to 1498; but before that the Byzantine humanist and philosopher Joannes Argyropulos (ca. 1415–1487) had already produced new Latin translations of the works of the *Organon*. As more and more attention was paid to the actual Greek text, more and more dissatisfaction was felt with the old translations, including those of Argyropulos himself.[21] People came to realize that Aristotle's Greek "was more 'literary' and less technically consistent than early translations had suggested,"[22] and this realization could be used to cast doubt on earlier interpretations of his work. In his preface to a translation of the *Prior Analytics* by Johannes Franciscus Burana, first published in 1524, Hieronymus Bagolinus wrote scathingly of the medieval translators

[18]F. E. Cranz, "The Publishing History of the Aristotle Commentaries of Thomas Aquinas," *Traditio* 34 (1978), esp. p. 163.

[19]The commentaries on the *Prior* and the *Posterior Analytics* are not by Scotus.

[20]See C. B. Schmitt, "Renaissance Averroism Studied Through the Venetian Editions of Aristotle-Averroes (With Particular Reference to the Giunta Edition of 1550–2)," Study VIII in *The Aristotelian Tradition and Renaissance Universities* (London: Variorum Reprints, 1984).

[21]See C. B. Schmitt, *Aristotle and the Renaissance* (published for Oberlin College by Harvard University Press, 1983), p. 72.

[22]L. Jardine, in an earlier version of her chapter "Humanistic logic" in *The Cambridge History of Renaissance Philosophy*, pp. 173–198.

who had "presented all the thoughts of Aristotle as if they were enveloped in a perverted, corrupt, and noisome fog."[23] In the same place he praised Burana for his use of both Greek and Arab commentators, including new translations of the latter from their Hebrew translations. An excellent example of the new Greek-based texts is found in the *Organon* edition of Julius Pacius which was first published in 1584. In it we find the Greek text side-by-side with a new translation designed not only to read well (as was the humanist goal) but also to capture the philosophical significance of Aristotle's words. In the margins we find a commentary dealing with difficult points both of theory and of translation.[24]

However, it must be noted that the humanist approach to Aristotle did cause some problems both for commentators who wished to make use of medieval insights and for publishers of medieval commentaries. The discussion of the *Organon* text had been organized around a series of key terms which appeared in the old translations, and whose disappearance from new translations could make the task of exposition, or of simply locating references, rather difficult.[25] Thus although Johannes Eck in 1516–17 produced a complete *Organon* commentary for the University of Ingolstadt[26] which was based on the translations of Argyropulos, as indeed was the Coimbra commentary of 1606, the Parisian humanist Jacques Lefèvre d'Étaples retained the translations of Boethius for his popular series of paraphrases and annotations on Aristotle's logical works first published in 1503. In the great 1570 edition of the *Opera omnia* of Thomas Aquinas two translations were included for every commentary by Thomas. Thus, for the *Perihermenias* and *Posterior Analytics* commentaries, we find both the *translatio antiqua* and the translation by Argyropulos.[27] It is also significant to note that the popular and widely reprinted translations of Aristotle into Ciceronian Latin by the French Benedictine Joachim Périon seem to have been without much philosophical influence.[28]

The other area in which humanism is particularly important for our purposes is in the edition and translation of the ancient Greek commentators on Aristotle's logic. In his prefatory letter to the first volume of the Greek Aristotle published in 1495, Aldus Manutius had announced his intention of printing the commentaries of Ammonius, Simplicius, Porphyry, Alexander, Philoponus, and Themistius,[29] and although he failed to complete the project, it was continued by his heirs,

[23] J. F. Burana, *Aristotelis Priora Resolutoria* (Paris, 1539), sig. A ii r: "[. . .] quòd anteà non nisi per neotericos quosdam interpretes, qui omnia Aristotelis sensa depravata, corrupta, ac teterrima quadam caligine involuta legebant, exponerentur [. . .]"

[24] For discussion, see Schmitt, *Aristotle and the Renaissance*, pp. 83–85. Pacius also wrote an analytic commentary on the *Organon* which was published separately in 1597.

[25] For discussion, see Cranz, "The Publishing History", pp. 160–162, 169–178.

[26] This work was prescribed by the statutes of 1519/20, but it is not clear how much it was actually used: see Terrence Heath, "Logical Grammar, Grammatical Logic, and Humanism in Three German Universities," *Studies in the Renaissance* 18 (1971), p. 59.

[27] Cranz, "The Publishing History", p. 177.

[28] Cranz, "The Publishing History", p. 178. For discussion of Périon as a translator, see Schmitt, *Aristotle and the Renaissance*, pp. 72–79.

[29] C. B. Schmitt, "Alberto Pio and the Aristotelian Studies of his Time," Study VI in *The Aristotelian Tradition*, p. 55.

particularly in the 1520s and 1530s. New Latin translations followed,[30] and this material had a great impact on the contents and style of new commentaries on Aristotle. This is particularly evident in the works of Agostino Nifo. He used material from medieval commentators, including the thirteenth-century Robert Kilwardby and the fourteenth-century Marsilius of Inghen,[31] but he gave pride of place to the ancient Greek commentators, and at the end of his *Topics* commentary he spoke harshly of those who tried to explain Aristotle while ignorant both of the Greek language and of the Greek expositors.[32] The Jesuits of Coimbra too were to make full use of the Greek commentators.

The most interesting developments in purely Aristotelian logic took place in Italy and concerned the logic of scientific method. Italian universities had always been distinguished from Northern European universities by their strong emphasis on law and medicine, combined with a relatively slight emphasis on theology. The faculties of arts at such places as Padua provided studies leading to a degree "in arts and medicine" and as a result, the main emphasis was placed on logic and natural philosophy, as propaedeutic to medicine.[33] In the fifteenth century works stemming from Oxford and Paris on such topics as the intension and remission of forms were particularly important, but after about 1520 these works fell into sudden oblivion.[34] By the mid-sixteenth century the focus of attention was on the Greek commentators, on Averroes, especially his *Physics* commentary,[35] and on the new Greek Aristotle. The main Aristotelian logic text studied was the *Posterior Analytics*. This work was lectured on every year at Padua,[36] and the Pisa statutes of 1543 prescribed just Porphyry's *Isagoge* and the *Posterior Analytics* for logic.[37] From Gaetano da Thiene (1387–1465) in the fifteenth century through Nifo and Balduino in the sixteenth century we find a gradual refinement of writings on demonstration and scientific method, and this movement culminated in the logical work of Jacopo Zabarella, whose entire academic career was at the University of Padua.[38] Zabarella wrote a long commentary on the *Posterior Analytics*,

[30]Schmitt, "Alberto Pio", p. 56.

[31]E.g., Augustinus Niphus, *Super libros Priorum Aristotelis* (Venice, 1553), f. 14 va. Kilwardby is called 'Culverbinus' and Marsilius is called 'Ingueneus'.

[32]For text and translation, see Jardine, "Humanistic Logic", p. 196.

[33]See C. B. Schmitt, "Aristotelianism in the Veneto and the Origins of Modern Science: Some Considerations on the Problem of Continuity," Study I in *The Aristotelian Tradition*, p. 113; and C. B. Schmitt, "Thomas Linacre and Italy," Study XII in *The Aristotelian Tradition*, p. 49.

[34]C. B. Schmitt, "Hieronymus Picus, Renaissance Platonism and the Calculator," Study V in *The Aristotelian Tradition*, pp. 64–65; "Philosophy and Science in Sixteenth-Century Italian Universities," Study XV in *The Aristotelian Tradition*, p. 316.

[35]N. W. Gilbert, *Renaissance Concepts of Method* (New York: Columbia University Press, 1960), p. 166.

[36]Schmitt, "Thomas Linacre and Italy," p. 50.

[37]C. B. Schmitt, "Philosophy and Science in Sixteenth-Century Universities: Some Preliminary Comments," Study V in *Studies in Renaissance Philosophy and Science* (London: Variorum Reprints, 1981), p. 493.

[38]C. B. Schmitt, "Aristotelianism in the Veneto and the Origins of Modern Science," p. 113. This paper contains some useful corrections of Randall's important work: J. H. Randall, Jr., *The School of Padua and the Emergence of Modern Science* (Padova: Editrice Antenore, 1961).

first published in 1582; but in addition he produced single works on a variety of related themes, from necessary propositions to the species of demonstration.[39] He discussed the two main types of scientific method, the compositive, which is *a priori* and moves from cause to effect, and the resolutive, which is of secondary importance, and moves from effect to cause. In his work *De regressu* he described how the two methods could be united, thus allowing the scientist a method which would enable him to discover new causal relations at the same time as proving his conclusions apodictically. Zabarella's *Opera logica* were read throughout Europe well into the seventeenth century; and were particularly important in Germany. Just as the Coimbra commentary represents the new Aristotelian commentary, so Zabarella represents the new Aristotelian philosopher of logic.

Another development in Aristotelian logic that calls for some comment is the discussion of the fourth figure of the syllogism. To understand the problem, we need to consider the relation of the terms in the premisses to the terms in the conclusion. In standard examples, the subject of the conclusion appears in the second premise, and is called the minor term, and the predicate of the conclusion appears in the first premise, and is called the major term. Whether there can be non-standard examples depends on how one defines the major and minor terms. John Philoponus (ca. 490–ca. 570), in a definition that became popular by the seventeenth century, defined the major term as the predicate of the conclusion. Given this definition, one can easily differentiate syllogisms in which the middle term stands as subject to the major term but stands as predicate to the minor term (first figure), from syllogisms in which the middle term stands as predicate to the major term and stands as subject to the minor term (fourth figure). Moreover, there can be no conclusion which is indirect in the sense that the major term is subject and the minor term is predicate. However, during the medieval period, logicians tended to define the major term as that which appeared in the first premise and the minor term as that which appeared in the second premise. This definition allows indirect conclusions, and many logicians from Theophrastus onward added five indirect modes to the first figure, giving a standard listing of nineteen valid syllogisms (or twenty-four, if one adds the subalternate modes).[40] It also leaves open the possibility of acknowledging the fourth figure, but there are two ways of introducing such a figure. One can take the first figure direct syllogisms, and transpose their premisses, thereby changing the relationship of the middle term to the major and minor terms, and obtaining the indirect modes of the fourth figure. Alternatively, one can take the first figure indirect syllogisms and transpose their

For more information and bibliography, see William A. Wallace, "Circularity and the Paduan *Regressus*: From Pietro d'Abano to Galileo Galilei," *Vivarium* 33 (1995), 76–97.

[39]For Zabarella's works, see *Jacobi Zabarellae Opera Logica* (Cologne, 1597; repr. Hildesheim: Georg Olms, 1966).

[40]We need to remember that whereas modern logicians treat universal propositions about non-existent objects as true and particular propositions as false, so that there are only fifteen valid categorical syllogisms when these are symbolized in classical first-order quantificational logic, late medieval logicians treated affirmative propositions, whether universal or particular, about non-existent objects as false and negative propositions as true.

premisses, thus obtaining the direct modes of the fourth figure.

At this point, a diagram may be of use, with A, B, C as the terms, and with S, P and M to indicate the different structures that result.

Barbara (1^{st} direct)		Bambara (4^{th} *indirect*)	
All A is B	All M is P	All C is A	All P is M
All C is A	All S is M	All A is B	All M is S
$\therefore$ All C is B	$\therefore$ All S is P	$\therefore$ All C is B	$\therefore$All P is S

Baralipton (1^{st} indirect)		Bamalipton (4^{th} *direct*)	
All A is B	All M is P	All C is A	All P is M
All C is A	All S is M	All A is B	All M is S
$\therefore$ Some B is C	$\therefore$ Some P is S	$\therefore$ Some B is C	$\therefore$ Some S is P

Buridan has been credited with the first explicit recognition of an independent fourth figure, but this is a debatable position.[41] All Buridan means by the fourth figure is the four direct modes of the first figure with transposed premisses, which give us only the four indirect modes of the fourth figure. He did not explicitly link the indirect modes of the first figure with the fourth figure.[42] Certainly Johannes Dorp, whose commentary on Buridan's *Summulae* was published at least five times between 1490 and 1504, only seems to accept a fourth figure which is formed from the direct modes of the first figure with transposed premisses.[43] Like others, he referred to Galen's acceptance of a fourth figure, and remarked that Aristotle and Buridan did not discuss it because it was so easily reduced to the first figure. Other logicians, such as Nifo, regarded the fourth figure as constituted by the indirect modes of the first figure with transposed premisses.[44] Yet other logicians, such as Johannes Eck, gave all nine possible modes of the fourth figure, both direct and indirect.[45] Eck was happy to accept the fourth figure, but others, notably Zabarella, who devoted a whole work to the topic, argued at length that it represented an unnatural form of reasoning, and should be rejected.[46] It is only in the second half of the seventeenth century that, owing to the acceptance of Philoponus's definition of major and minor terms, indirect modes were firmly

[41] Hubert Hubien, "John Buridan on the Fourth Figure of the Syllogism", *Revue internationale de philosophie*, 113 (1975), 271-285, especially pp. 284-285.

[42] John Buridan, *Iohannis Buridani Tractatus de Consequentiis*, ed. Hubert Hubien (Philosophes médiévaux 16. Louvain: Publications Universitaires, Paris: Vander-Oyez, S.A., 1976), pp. 82-83. Buridan says that transposition only affects whether the conclusion is direct or indirect, and he notes that the direct modes of the first figure are indirect modes of the fourth "et econuerso". There is no reason to read this phrase as an explicit recognition that there are direct modes of the fourth figure which are indirect modes of the first figure, an assumption that Hubien seems to make (see article cited above). Later in his text (pp. 91-93) Buridan discusses the direct and indirect modes of just three figures.

[43] Johannes Dorp, *Perutile compendium totius logice*, (Venetiis, 1499; repr. Frankfurt/Main: Minerva G.m.b.H., 1965), sig. l 6 va–sig. m 1 ra.

[44] Niphus, *Super libros Priorum*, f. 27 va–vb.

[45] Johannes Eckius, *Aristotelis Stagyrite Dialectica* ([Augsburg, 1516–1517]), f. xii ra–f. xiii rb.

[46] Zabarella, *Liber de Quarta Syllogismorum Figura* in *Opera Omnia*, cols. 101–132.

rejected and the fourth figure accepted.[47]

2 THE FATE OF THE MEDIEVAL CONTRIBUTIONS TO LOGIC

So far I have considered only Aristotle and commentaries on Aristotle. However, this is somewhat misleading so far as university instruction is concerned, since the material in question was aimed at more senior students, including those studying for the M.A. degree. Medieval universities exposed first-year arts students to a wide range of logical writings, mostly contemporary; and during the sixteenth century, although Italian universities continued a tradition of reading Aristotle and his commentators directly,[48] other universities turned more and more to textbooks of one kind and another. What I have considered so far is misleading for another reason as well, since I have totally ignored the development of areas of logic not discussed by Aristotle. It is now time to remedy these defects, and I shall begin by outlining the specifically medieval contributions to logic which have been discussed in previous chapters.

One of the features of medieval logic was the production of independent texts, each focussing on a particular topic. These texts were regarded as complete in themselves, and as not needing any justification in terms of Aristotelian logic. They can be organized into three groups. The core of the first group is formed by the so-called *parva logicalia*, or treatises dealing with the properties of terms, including their reference in various contexts. Here we find tracts on supposition, on relative terms, on ampliation, appellation, restriction and distribution. These core treatises were supplemented in three ways. Logicians wrote about syncategorematic terms, that is, the logical particles such as 'all', 'some', and 'not', which determine the logical structure of a proposition. They wrote about exponible terms, that is, those logical particles such as 'except' and 'only' whose presence requires the analysis of an apparently simple categorical proposition into several conjoined propositions. Finally, they wrote about the proof of terms, or the ways in which the truth-conditions of propositions are affected by the presence of exponible terms, of modal terms such as 'necessary' and 'possible', or of epistemic terms such as 'knows' and 'believes'. Obviously there is a good deal of overlap between writings on syncategorematic terms, on exponibles, and on the proof of terms, since one and the same particle could be discussed in all three types of treatise. The second group contains the 'three tracts of the moderns', that is, the treatises on consequences, obligations and insolubles.[49] The third group is formed by the treatises on sophismata in which problematic or puzzling statements were

[47] For details, see E. J. Ashworth, "Some Notes on Syllogistic in the Sixteenth and Seventeenth Centuries", *Notre Dame Journal of Formal Logic* 11 (1970), 17–33.

[48] Schmitt, "Philosophy and Science in Sixteenth-Century Universities: Some Preliminary Comments," p. 508.

[49] For an interesting discussion of the way in which the phrase '*Logica modernorum*' has been extended in recent times to refer to all the new developments of medieval logic, see N. W. Gilbert, "Ockham, Wyclif and the 'via moderna'," *Miscellanea Mediaevalia* 9 (1974), pp. 111–115.

analyzed and tested against various logical rules. Since these rules were drawn from the areas of investigation already mentioned, including supposition theory and its ramifications, there was considerable overlap between these treatises and those belonging to the first and second groups. Indeed, the latter treatises, as well as commentaries on Aristotle and Peter of Spain, themselves made heavy use of sophismata in order to test the rules they enunciated against possible counter-examples. Thus we get a two-way movement. A treatise on sophismata begins with the sophism and proceeds to the rules; a treatise on, for instance, consequences begins with the rules and proceeds to the sophismata.

As well as treatises on individual topics, medieval logicians wrote *summulae* or general textbooks. The most famous example is the thirteenth-century work by Peter of Spain which gives a fairly complete outline of Aristotelian logic, including categories, syllogisms, topics and fallacies. More than three hundred manuscripts of this work survive, and about two hundred printed editions, mostly with a commentary.[50] However, it would be a great mistake to think that Peter of Spain had no rivals. For those interested in medieval logic, Ockham's *Summa totius logicae* and the somewhat later *Perutilis logica* by Albert of Saxony must not be overlooked. However, so far as the fifteenth and early sixteenth centuries are concerned, perhaps the most important textbook writers are John Buridan and Paul of Venice. Paul of Venice's *Logica parva* is worth considering more closely. Tract 1 presented the material of the *summulae*, and dealt with terms, nouns, verbs, propositions, equipollence, conversion, hypothetical propositions, predicables, categories and syllogisms — everything, that is, that Peter of Spain had covered, except for topics and fallacies, but in much briefer compass. Tract 2 dealt with the material of the *parva logicalia*, tracts 3, 5, and 6 dealt with consequences, obligations and insolubles. Tract 4 was devoted to the proof of terms; and the last two tracts took up objections to the *summulae* and to the consequences. Thus the *Logica parva* was a compendium of mid- to late-fourteenth century logic, as indeed was the very much longer *Logica magna*.[51] The *Logica parva* was very popular. About eighty manuscripts survive and it was printed many times.[52] So far as we know, it was the first medieval logic text to be printed, in 1472, and the last edition was in 1614.

These medieval works dominated the curricula of fifteenth-century universi-

[50]Peter of Spain, *Tractatus called afterwards Summule Logicales*, ed. L.M. de Rijk (Assen: Van Gorcum, 1972), p. C. De Rijk comments that most of the printed editions are adaptations, pp. C–CI.

[51]In his *Logica magna*, Paul of Venice quotes verbatim (though without acknowledgement) from various logicians of the second part of the fourteenth century: see Paul of Venice, *Logica Magna Part II Fascicule 6*, ed. and trans. F. de Punta and M. M. Adams (Printed for the British Academy by the Oxford University Press, 1978) and Paul of Venice, *Logica Magna Part II Fascicule 8*, ed. and trans. E. J. Ashworth (Printed for the British Academy by the Oxford University Press, 1988). (Other volumes in the series tend not to trace Paul's sources.)

[52]For a list of some 81 manuscripts, see Alan R. Perreiah, *Paul of Venice: A Bibliographical Guide* (Bowling Green, Ohio: Philosophy Documentation Center, 1986). See also F. Bottin, "Alcune correzioni ed aggiunte al censimento dei codici di Paolo Veneto," *Quaderni per la Storia dell'Università di Padova* 14 (1981), 59–60.

ties. In Italy, the fifteenth century was characterized by the enthusiastic reception of fourteenth-century Oxford logic, not only as found in the works of Paul of Venice, but also as found in original treatises by such men as Heytesbury, Strode and Billingham.[53] As Charles Schmitt remarked, "Contrary to what is generally thought, the most technical and most highly specialized products of medieval scholasticism continued to remain popular during the Renaissance, precisely at the same time as when humanism was at its height."[54] Indeed, those humanists such as Leonardo Bruni whose fulminations against the '*barbari britanni*' are often quoted, were dead long before the wave of British logic was spent in Italy.[55] Thus the prescribed texts at Padua in 1496 included Paul of Venice's *Logica parva* along with works by Heytesbury, Strode, and the more recent Paul of Pergula (d. 1451), who had taught at Padua.[56] As well as producing his own logic text, Paul of Pergula had commented on Strode's *Consequences*, as had his colleague Gaetano di Thiene.[57] Outside Italy, matters were little different. At Erfurt in 1420, Billingham, John of Holland and Thomas Maulvelt were read;[58] and the Erfurt statutes of 1412 together with the additions of 1449 also mention Heytesbury, Albert of Saxony, Buridan and Marsilius of Inghen.[59] Peter of Spain's *Summulae* were read at Ingolstadt in 1478.[60] Buridan was popular in Poland.[61] At Oxford, a loose collection of largely anonymous texts containing similar material to that found in Paul of Venice's *Logica parva* was read.[62] I will not add more detail to this list; but

[53]See especially W. J. Courtenay, "The Early Stages in the Introduction of Oxford Logic into Italy" in *English Logic in Italy in the 14^{th} and 15^{th} Centuries*, ed. Alfonso Maierù (Napoli: Bibliopolis, 1982), pp. 13–32.

[54]Schmitt, "Alberto Pio and the Aristotelian Studies of His Time," p. 49.

[55]See E. Garin, "La cultura fiorentina nella seconda metà del '300 e i 'barbari britanni'," *La Rassegna della Letteratura Italiana* 64 (1960), 181–195.

[56]Schmitt, "Philosophy and Science in Sixteenth-Century Universities: Some Preliminary Comments," p. 493, p. 520 note 43: "Deputati ad sophistariam teneantur legere *Logicam Pauli Veneti* et *Quaestiones Strodi* cum *Dubiis Pauli Pergulensis* et pro tertia lectione *Regulas* seu *Sophysmata tisberi*."

[57]It is interesting to consider the title of the work published in Venice in 1517: *Consequentie Strodi cum commento Alexandri Sermonete. Declarationes Gaetani in easdem consequentias. Dubia magistri Pauli pergulensis. Obligationes eiusdem Strodi. Consequentie Ricardi de Ferabrich. Expositio Gaetani super easdem. Consequentie subtiles Hentisbari. Questiones in consequentias Strodi perutiles eximii artium doctoris domini Antonii Frachentiani Vicentini.* For Paul of Pergula, see Paul of Pergula, *Logica* and *Tractatus de Sensu Composito et Diviso*, ed. Sister Mary Anthony Brown (St. Bonaventure, N.Y.: The Franciscan Institute; Louvain, Belgium: E. Nauwelaerts; Paderborn, Germany: F. Schöningh, 1961).

[58]L. Thorndike, *University Records and Life in the Middle Ages* (New York, 1944), pp. 296–297.

[59]A. L. Gabriel, "'Via antiqua' and 'via moderna' and the Migration of Paris Students and Masters to the German Universities in the Fifteenth Century," *Miscellanea Mediaevalia* 9 (1974), 467–468.

[60]Heath, "Logical Grammar", p. 49.

[61]M. Markowski, *Burydanism w Polsce w Okresie Przedkopernikanskim* (Wroclaw, Warszawa, Krakow, Gdansk, 1971), passim.

[62]L. M. de Rijk, "*Logica Oxoniensis*: An Attempt to Reconstruct a Fifteenth Century Oxford Manual of Logic," *Medioevo* 3 (1977), 121–164; E. J. Ashworth, "The 'Libelli Sophistarum' and the Use of Medieval Logic Texts at Oxford and Cambridge in the Early Sixteenth Century," *Vivarium* 17 (1979), 134–158.

I should point out that it is a very varied one, and that the emphasis is generally on authors who wrote at least within the previous century, even if they could not be described as recent. The one exception is Peter of Spain, and he was almost inevitably read through the medium of later commentaries.[63] The most popular commentator was Johannes Versor (d. after 1482), who lectured at Paris and was influenced by both Albert the Great and Thomas Aquinas.[64] Risse lists 23 editions of his commentary on Peter of Spain, the last in 1639.[65] Other thirteenth-century figures such as William of Sherwood had not survived at all, and the reemergence of Albert the Great and Aquinas in their role as commentators on Aristotle was to be largely a sixteenth-century phenomenon. It should be pointed out that the reading of the late medieval texts found in the curriculum was not necessarily sympathetic. The Thomists of the *Bursa Montis* in Cologne, whose commentary on Peter of Spain's *parva logicalia* was reprinted a number of times between 1480 and 1507, included the prescribed nominalist texts on obligations, consequences and insolubles in that work, but explicitly disagreed with many of the positions taken.[66]

The big changes in logic teaching were to come in the first half of the sixteenth century. However, at the beginning of the century, particularly in Paris, there were changes internal to the medieval tradition itself. Some medieval topics, including sophismata, syncategorematic terms, and the proof of terms, simply disappeared as the subject of separate treatises, while separate treatises on supposition theory, exponibles, consequences, insolubles and obligations continued to be written. To these were added new treatises on oppositions, or the logical relations between different kinds of categorical propositions, on syllogisms, and on terms. These treatises covered material which had always to some extent been included in *summulae* and commentaries, but which was now the focus of particular attention for such authors as the Scotsman John Mair, and the Spaniards Antonio Coronel, Gaspar Lax and Fernando de Enzinas, all of whom taught at the University of Paris. At the same time new commentaries on Peter of Spain were being published by authors ranging from George of Brussels and Pierre Tartaret in late fifteenth-century Paris, to Johannes Eck in Germany in 1516 and Domingo de Soto in Spain in 1529. The first version of his *Summulae*, which drew heavily on his earlier studies in Paris, included a commentary on the first and fourth books of Peter of Spain's *Tractatus*, together with material on terms, supposition theory,

[63] Cf. Rita Guerlac, *Juan Luis Vives Against the Pseudodialecticians: A Humanist Attack on Medieval Logic* [texts and translation], (Dordrecht, Boston, London: D. Reidel, 1979), p. 19.

[64] On Versor's work as commentator and its reception, see Pepijn Rutte, "'Secundum processum et mentem Versoris': John Versor and His Relation to the Schools of Thought Reconsidered", *Vivarium* 43 (2005), 292–329.

[65] Risse, *Bibliographia Logica I*, p. 282.

[66] I use the following edition: [Lambertus de Monte], *Copulata super omnes tractatus parvorum logicalium Petri hispani. ac super tres tractatus modernorum textui pulcerrime annotata in argumentis et replicis denuo diligentissime correcta iuxta inviolatum processum magistrorum Colonie bursam montis regentium* ([Cologne], 1493). For discussion, see H. A. G. Braakhuis, "School Philosophy and Philosophical Schools: The Semantic-Ontological Views in the Cologne Commentaries on Peter of Spain, and the 'Wegestreit'," *Miscellanea Mediaevalia* 20 (1989), 1–18.

exponibles, insolubles, and obligations.

While much of the material found in the work of the Parisian authors, including Domingo de Soto, stemmed from such predecessors as John Buridan, there were original developments. For instance, in supposition theory logicians began to use the letters of the alphabet as special signs, not only to allow different interpretations of sentences of standard form but also, more importantly, to handle sentences of non-standard form such as "Of every man some donkey is running".[67] In particular, logicians wanted to explain how to provide contradictories for such sentences which preserved the relevant truth conditions, and also how to analyse them using the process called suppositional descent. The letter 'a' was used to indicate that the term following it had merely confused supposition. For instance, if one writes "a. man is not an animal", this sentence turns out to be true because if each of several men is identical to a different animal, it is true to say of each animal that one or more men is not identical to that animal. The letter 'b' was used to indicate that the term following it had determinate supposition. Accordingly, "Every man is b. animal", unlike "Every man is <an> animal", signifies by virtue of the special sign that every man is identical to one and the same animal, and hence is false. Subsequent letters of the alphabet were used to indicate cases of mixed supposition, in which the type of supposition changed during the stages of suppositional descent. The humanist Juan Luis Vives, who had himself studied at the University of Paris, found this procedure particularly repellent. In his diatribe *Against the Pseudodialecticians* he wrote "[. . .] a, b, c, d can make those suppositions confused, determinate, and a mixture of both. Indeed you can add more commixtions than any quack pharmacist ever made —e, f, g, h, i, j, k— so that some of these men already have recourse to letters down as far as the tenth letter of the second alphabet, dreaming up and combining wonderful kinds of suppositions."[68]

Another development concerned the semantics of propositions (where 'proposition' means an occurrent declarative sentence, whether written, spoken or mental), and the problem of what it is they might be said to signify. As Juan Dolz put the question, "Does a proposition signify some thing or some things or in some way [*aliqualiter*]? Is there to be given a *complexum significabile* or not, and if so, is it to be distinguished from the significates of the terms of the proposition?"[69] Dolz and Fernando de Enzinas argued that indicative sentences signify *aliqualiter*, in some way, rather than *aliquid*, some thing.[70] They explained this by arguing that propositions are analogous to syncategorematic rather than to categorematic

[67] See E. J. Ashworth, "Multiple Quantification and the Use of Special Quantifiers in Early Sixteenth Century Logic", *Notre Dame Journal of Formal Logic* 19 (1978), 599–613; reprinted as Study X in E. J. Ashworth, *Studies in Post-Medieval Semantics* (London: Variorum Reprints, 1985).

[68] Guerlac, *Juan Luis Vives*, p. 61.

[69] Johannes Dolz, *Termini* (Parisius [ca. 1511]), f. v ra. Note that the more usual phrase is '*complexe significabile*'.

[70] Dolz, *Termini*; Fernando de Enzinas, *Tractatus de Compositione Propositionis Mentalis* (Lugduni, 1528).

words in their manner of signifying, performing a function other than that of naming.[71] To the question, "What does this proposition signify?" one could reply only by a paraphrase. In particular, questions about the *dictum* of a proposition were answered by replacing the *dictum* with a that-phrase. For instance, "'Man is an animal' signifies man being an animal (*hominem esse animal*)" (in which '*hominem esse animal*' is the *dictum*) was rewritten as "'Man is an animal' signifies that man is an animal (*quod homo est animal*)", thus removing the temptation to think that the *dictum* 'man being an animal' functions as a name. However, they recognized that there were some contexts in which the question "What is this sentence about?" could be answered by naming or pointing to an object. On such occasions a sentence was said to signify categorematically.

Another semantic issue that was discussed by Juan Dolz and other Parisian logicians had to do with the use of proper names.[72] Reflection on John Buridan's account of the problem of identity through time led them to adopt an even more restrictive account of proper names than Buridan himself had countenanced. They argued that a sequence of different concepts of Socrates will be formed through time, even by one acquainted with him, and hence that the name 'Socrates' is no more a proper singular term than is the name 'Seine'. It is radically equivocal, since at every moment it is subordinated to a different concept.[73] Nor is this incompatible with the intentions of the person who originally gave Socrates his name. Strictly speaking, only the baby was called Socrates, but the name was imposed equivalently or as a consequence to signify every successor to the boy by means of different concepts. Domingo de Soto was strongly opposed to this view. He said that to regard the name 'Socrates' as an equivocal term which refers to a succession of individuals is a denial of common speech and of the common sense of the wise men who originally gave terms their signification.[74]

Soto was equally opposed to Buridan's initial account of identity through time which made a division between strict identity which allowed of no change whatsoever, a lesser identity for which the continuity of one part, notably the human soul, was sufficient despite bodily changes, and the weakest identity, based on a continuity of diverse parts succeeding one another. Animals, bodies and rivers have only the weakest kind of identity and so the Seine is the same river now as it was ten years ago because of the continuous succession of water-parts. According

[71] For syncategorematic terms, see E. J. Ashworth, "The Structure of Mental Language: Some Problems Discussed by Early Sixteenth Century Logicians", *Vivarium* 20 (1982), 59–83, reprinted as Study V in eadem, *Studies in Post-Medieval Semantics*. For the significate of propositions, see E. J. Ashworth, "Theories of the Proposition: Some Early Sixteenth Century Discussions", *Franciscan Studies* 38 (1978 [1981]), 81–121, reprinted as Study IV in eadem, *Studies in Post-Medieval Semantics*.

[72] For full discussion and references, see E. J. Ashworth, "Singular Terms and Singular Concepts: From Buridan to the Early Sixteenth Century" in *John Buridan and Beyond: Topics in the Language Sciences 1300–1700*, ed. Russell L. Friedman and Sten Ebbesen (Copenhagen: The Royal Danish Academy of Arts and Sciences, 2004), pp. 121–151.

[73] See, for instance, Dolz, *Termini*, f. xxx ra–rb.

[74] I cite the second edition: Domingo de Soto, *Aeditio Secunda Summularum*, (Salamanca, 1539–1540), f. xviii vb–f. xix rb.

to this account the names 'Socrates' and 'Aristotle' on the one hand and 'Seine' and 'Brownie' (the donkey) on the other have a different status. The former are more properly singular terms than the latter even for those who are acquainted with all of these individuals through time. Domingo de Soto rejected this consequence, arguing that horses, trees and even rivers could enjoy a strong enough identity, by virtue of the continuity of their forms and functions, to give a basis for the application of one singular concept through time.

The topics of special quantifiers and the use of singular terms soon disappeared from the logical literature, and the topic of the *significatum* of propositions ceased to be a central issue, though it was touched on in later Thomistic accounts of the object of judgement.[75] However, another topic to which Domingo de Soto made a very important contribution was longer-lived. This was the classification of signs, both linguistic and non-linguistic. In relation to the speaker, spoken words were said to be instrumental signs, because they were used as instruments of communication, and mental terms were said to be formal signs, because they represented by their very nature. In relation to the things signified, spoken words were said to be conventional signs, and mental terms were said to be natural signs, since their signification did not depend on choice or convention. In the various editions of his *Summulae*, Soto asked whether one should add a category of customary signs, such as napkins on the table as a sign of lunch. In the end he concluded that these signs were natural, albeit founded on a convention.[76] Soto's classifications were elaborated by many later authors, including Sebastian de Couto in the Coimbra commentary and John of St. Thomas (1589–1644) in the seventeenth century.[77]

3 THE NEW TEXTBOOKS

Outside Spain, work in the late medieval tradition came to an abrupt halt around 1530. No new independent treatises in the medieval style or commentaries on Peter of Spain were written; the publication of works written during the first three decades of the sixteenth century ceased, and the publication of such medieval authors as Buridan, Heytesbury and Strode also ceased. Only in Spain did authors

[75] See Nuchelmans, *Late-Scholastic and Humanist Theories of the Proposition*.

[76] For discussion and references, see E. J. Ashworth, "The Doctrine of Signs in Some Early Sixteenth-Century Spanish Logicians" in *Estudios de Historia de la Lógica. Actas del II Simposio de Historia de la Lógica*, ed. Ignacio Angelelli and Angel d'Ors (Pamplona: Ediciones EUNATE, 1990), pp. 13–38; E. J. Ashworth, "Domingo de Soto (1494–1560) and the Doctrine of Signs" in *De Ortu Grammaticae: Studies in medieval grammar and linguistic theory in memory of Jan Pinborg*, ed. B. L. Bursill-Hall, Sten Ebbesen and Konrad Koerner (Amsterdam: John Benjamins, 1990), pp. 35–48.

[77] For Sebastian de Couto, see John P. Doyle, trans., *The Conimbricenses: Some Questions on Signs* (Milwaukee: Wisconsin: Marquette University Press, 2001); for John of St. Thomas [John Poinsot], see John Deely, ed. and trans., *Tractatus de Signis: The Semiotic of John Poinsot* (Berkeley, Los Angeles, London: University of California Press, 1985). For some corrections to Deely's discussion, see E. J. Ashworth, "The Historical Origins of John Poinsot's *Treatise on Signs*", *Semiotica* 69 (1988), 129–147.

continue to produce, if not independent treatises, at least commentaries on Peter of Spain. For instance, the commentary by Alphonso de Veracruz was published in Mexico in 1554, followed by several printings in Salamanca; and in 1571 Thomas de Mercado's commentary made its first appearance in Seville.[78]

In Northern Europe, excluding Spain and Italy for the moment, these changes in publication were accompanied by a change in university curricula. At such universities as Oxford, Cambridge, Ingolstadt and Tübingen, authors such as Agricola and Caesarius were required in place of the medieval texts, and Philip Melanchthon's simplified summaries of Aristotelian logic swept Germany.[79] Later Ramus was to enjoy a runaway success. Yet the most important and influential texts of the second half of the sixteenth century were by no means simplified humanist manuals, and they contained considerably more syllogistic logic than Agricola and Ramus had thought appropriate. Indeed, many of them also contained treatments of such medieval doctrines as supposition theory. In order to understand these developments, we must first consider humanist logic in more detail.

Rudolph Agricola's *De inventione dialectica libri tres*, written ca. 1479 but first published in 1515, is central to the development of humanist logic.[80] Agricola took up the well-known division between invention, that is, the part of logic which finds the subject matter for argumentation by using the Topics, and judgement, that is, the part of logic which organizes and evaluates the resulting arguments, and he focussed almost exclusively on invention, as the title of his work suggests. The work is divided into three Books. In the first, Agricola discusses Topics. In the second, he takes up the nature and role of dialectic, but while he mentions the standard forms of argumentation, his focus is on literary examples and on the use of arguments in actual situations. The last Book considers moving, or the use of emotions, pleasing, and disposition. Much of this material came from rhetoric manuals, and rhetoric itself was restricted by Agricola to questions of style. For our present purposes, the two most important features of Agricola's work are his placing of invention before judgement, and his novel treatment of the Topics. There was nothing new about the discussion of Topics as such, for the three standard works about Topics, namely Aristotle's *Topics*, together with Boethius's two works, *De topicis differentiis* and *In Ciceronis Topica*,[81] had always

[78]For more details, see E. J. Ashworth, "The eclipse of medieval logic" in *The Cambridge History of Later Medieval Philosophy*, ed. Norman Kretzmann, Anthony Kenny and Jan Pinborg (Cambridge: Cambridge University Press, 1982), p. 790.

[79]For the curricula, see Jardine, "Humanistic logic", pp. 800–801; Heath, "Logical Grammar", p. 59, p. 63; Peter Mack, *Renaissance Argument: Valla and Agricola in the Traditions of Rhetoric and Dialectic* (Leiden, New York, Köln: E. J. Brill, 1993), esp. pp. 257–302.

[80]For full discussion of Agricola, see Mack, *Renaissance Argument*.

[81]For Boethius, see *Boethius's De topicis differentiis*, translated, with notes and essays on the text, by Eleonore Stump (Ithaca and London: Cornell University Press, 1978) and *Boethius's In Ciceronis Topica*, translated, with notes and an introduction by Eleonore Stump (Ithaca and London: Cornell University Press, 1988). For discussion, see Eleonore Stump, *Dialectic and Its Place in the Development of Medieval Logic* (Ithaca and London: Cornell University Press, 1989). For a history of Topics commentaries, see N.J. Green-Pedersen, *The Tradition of the*

been firmly part of the logic syllabus. Boethius's first work was more important in the medieval period, though in the fourteenth century it was largely replaced by the account of Topics given by Peter of Spain in his *Tractatus*.[82] In the sixteenth century Boethius's second work became popular again, as did Cicero's original text.[83] Agricola himself was well-acquainted with both Aristotle and Boethius, but he introduced a radical change of approach.

For Boethius, the notion of a Topic had covered two things. First, a Topic was a maxim or maximal proposition, a self-evidently true universal generalization which could either be inserted into an argument as a premiss or which could be appealed to as providing a warrant for an argument. Some of these maxims were turned into formal deductive arguments by medieval logicians, who had a tendency to absorb invention into judgement,[84] but others were not readily formalizable. As such, they provide the perfect nucleus for a developed informal logic. The second sense of Topic for Boethius was the Topic as the Differentia of a maximal proposition, that is, the characteristic that enables us to classify maximal propositions into groups. To list the Topics in this second sense is simply to list the headings under which material can be gathered: 'definition', 'genus', 'cause', 'opposite', 'similar', and so on. What we find in Agricola is a deliberate rejection of the maxims,[85] and of material that might belong to judgement rather than invention. His attack has two parts. With respect to particular Topics, such as the Topics of antecedents and consequents, he claims that their only use is to present valid argument forms, for they have no distinctive material of their own.[86] Hence they must be rejected in their Boethian form. In general, he argues that the purpose of any maxim is simply to present a necessary argument, and that they are quite inappropriate for all the cases in which we are dealing with probabilities. We should note here that Agricola is clearly talking not just of the epistemological status of the premisses of an argument, but of a probabilistic and informal relationship between premisses and conclusion.[87] He also claims that maxims are too restrictive, for they represent an attempt to force a wide range of material into a narrow compass; and that they are of no use to someone who really understands the nature of Topics.[88] What we are left with, then, is the Topics solely as headings under which material can be organised. Their most obvious link with argumentation, both formal and informal, has been broken. Nor, as we have seen, is there any serious discussion of argumentation.

Topics.

[82] Peter of Spain, *Tractatus*, pp. 55–78.

[83] For some details, see E.J. Ashworth, "Boethius on Topics, Conditionals and Argument-forms" in *History and Philosophy of Logic*, 10 (1989), 214–215.

[84] See Stump, introduction to *Boethius's De topicis differentiis*, p. 25.

[85] Vives also rejected maxims: see Guerlac, *Juan Luis Vives*, p. 40.

[86] Rudolph Agricola, *De Inventione Dialectica ... per Alardum Amstelredamum accuratius emendati, et additis annotationibus illustrati* (Köln, 1523: repr. Frankfurt am Main: Minerva G.M.B.H., 1967), pp. 107–108; cf. p. 170. Boethius was aware of the problem: see Stump, *Boethius's In Ciceronis Topica*, pp. 128–131.

[87] Cf. Agricola, *De Inventione Dialectica*, p. 109, pp. 110–113.

[88] Agricola, *De Inventione Dialectica*, p. 176.

Agricola's work was one of the most widely-published dialectic texts of the sixteenth century, receiving more than forty editions.[89] Nonetheless, its fit with the university curriculum in logic was not good, both because it was awkward to place Topical invention before the rest of logic, and because it contained virtually nothing about the rest of logic. The obvious answer to this problem was either to provide supplementary texts, or to write new texts containing appropriate formal material. As Monfasani has pointed out, in the earlier part of the sixteenth century we often find that the *Isagoge dialectica* of George of Trebizond (1395–ca. 1472), which gave a brief outline of Aristotelian logic, was paired with Agricola.[90] A good example of the other approach, that of writing a new text, is provided by Philip Melanchthon, the so-called *Praeceptor Germaniae*. His logic text, first published in 1520, as *Compendiaria dialectices ratio*, but replaced by two later versions, became very popular. In it we can see how the insights of Agricola were transmuted to serve the textbook needs of educational institutions, in much the same way as the Ramist innovations were to be transmuted in the last years of the sixteenth century. On the one hand Melanchthon enjoyed the Agricolan emphasis on clarity of style and the use of literary allusions, which he supplemented by a large number of Biblical allusions; he accepted the importance of the Topics, although he restored discussion of the maxims; and some remarks on order in the first version of his text grew into a full section on logical method as a way of ordering discourse.[91] On the other hand, Melanchthon remained a convinced Aristotelian, who believed that students needed to be taught some formal logic. He placed judgement before invention, and the formal techniques he used were those of syllogistic, while his work included a discussion of the other standard Aristotelian subjects, including the categories and the square of opposition for propositions. At the same time, he purged Aristotle of medieval accretions, approaching him through new readings of the Greek text and of the Greek commentators. Any references to the specifically medieval contributions to logic are most unfavourable, and he relegates supposition theory to grammar.[92]

Such was not the case with the first important English logical text of the sixteenth century, John Seton's *Dialectica*, published in 1545. Seton explained in

[89]Mack, *Renaissance Argument*, p. 257.

[90]See John Monfasani, *George of Trebizond. A Biography and a Study of his Rhetoric and Logic* (Leiden: E. J. Brill, 1976), pp. 333–335. For the text, see Georgius Trapezuntius, *De Re Dialectica Libellus* (Coloniae, 1539; repr. Frankfurt am Main: Minerva G.M.B.H., 1966).

[91]See Philip Melanchthon, *Compendiaria Dialectices Ratio* in *Opera* (Corpus Reformatorum XX, Brunsvigae 1854, repr. New York and Frankfurt am Main, 1963), columns 724–726; and *Erotemata Dialectices* in *Opera* (Corpus Reformatorum XIII, Halis Saxonum, 1846; repr. New York and Frankfurt am Main, 1963), columns 573–578. Sections on method became a standard component of logic textbooks. Particularly after the publication of Zabarella's *Opera logica*, it became customary to add remarks on scientific method as well, in a mixture of subject-matters that was not always happy.

[92]Melanchthon, *Erotemata Dialectices*, cols. 750–752. He writes (col. 750): "Addita est Aristotelis Dialecticae, doctrina verius Grammatica quam Dialectica, quam nominarunt Parvalogicalia, in qua dum praecepta immodice cumularunt, et labyrinthos inextricabiles, sine aliqua utilitate finxerunt, ut: Nullus et nemo mordent se in sacco, etiam illas admonitiones, quarum aliquis est usus, tenebris involverunt."

his introduction that he had written the work because of the absence of a suitable text for the instruction of the young.[93] Aristotle was too difficult, Agricola had deliberately restricted himself to the subject of invention, and Melanchthon's style was not suited to elementary teaching. He added that he hoped through his efforts to make Aristotle more accessible to the young. In the first edition of his work he dealt only with that part of logic called judgement, though, in the editions of 1568 and later, a brief fourth Book on invention, based on Agricola's first Book, was to be added, along with annotations by Peter Carter. The fourth Book was somewhat redundant, since Seton had already given Agricola's classification of the Topics in his second Book. There are frequent references to Agricola throughout the work, as well as to Cicero, Quintilian and Erasmus. However, he did include the bulk of traditional Aristotelian logic from the categories to the syllogism, which he described as the most important part of judgement,[94] and he also retained the strictly medieval doctrine of the supposition of terms, though in a considerably truncated form.[95] It is interesting to compare Seton's text to Robert Sanderson's *Logicae Artis Compendium* which appeared at Oxford in 1615, and was to remain a standard text there well into the eighteenth century.[96] This too contains a full outline of strictly Aristotelian logic, together with brief discussions of the medieval doctrines of supposition, exponibles and consequences, and a full discussion of method as a way of ordering discourse. Both these works offer the student a compendium of simple logical devices for use in formal debate, as well as a "guide to analyzing a Cicero oration or a passage from Ovid."[97]

No account of logic in the sixteenth century would be complete without some reference to Petrus Ramus (Pierre de la Ramée), the most notorious logician of the period. He is known both for his attacks on Aristotle and for the simplified logic presented in his *Dialectique* of 1555, which was published in Latin in 1556 as *Dialecticae libri duo*. The *Dialectique* had two parts. The first, on invention, covered the Topics, and the second, on judgement, presented a deliberately simplified version of the syllogism followed by an account of method as a means of ordering in the arts and sciences. The Topics are presented in an Agricolan way, without maxims, and with a new theoretical foundation. They are not merely useful headings for gathering material, but they represent the mind's natural organization of data.[98] As such, Topics function as categories;[99] and indeed, Ramus emphasized that his Topics are more useful and more natural than Aristotle's ten Categories. If we are asked to discuss war, peace or the state, it is no use to think in terms of

[93] John Seton, *Dialectica* (Londini, 1545), sig. A ii v.
[94] Seton, *Dialectica*, sig. I ii r.
[95] Seton, *Dialectica*, sig. E iv r– v v.
[96] See Ashworth, introduction to Sanderson, *Logicae Artis Compendium*, pp. XIII–XVI.
[97] L. Jardine, "The Place of Dialectic Teaching in Sixteenth-Century Cambridge," *Studies in the Renaissance* 21 (1974), 57. Jardine is referring only to Seton.
[98] See, for instance, Petrus Ramus, *Scholarum dialecticarum seu animadversionum in Organum Aristotelis*, in *Scholae in tres primas liberales artes* (Frankfurt 1581; repr. Frankfurt am Main 1965), p. 54. Agricola suggests the same thought: *De Inventione Dialectica*, p. 9.
[99] Ramus, *Scholarum Dialecticarum*, pp. 118–119.

the category of substance. Instead we must appeal to such Topics as cause, event, and opposites.[100] No reference was made in the *Dialectique* to such standard material as the categories, the square of opposition, conversion, demonstration and fallacies. On the other hand, the work is rich with quotations from the poetry and prose of Classical authors, which must have strengthened the impression among students that logic was both easy and fun.

The publishing history of Ramus's book was remarkable. Ong lists 262 editions, 151 of which appeared in Germany.[101] Despite this fact, it is not clear how much impact Ramus actually had on the university scene. In England, he certainly enjoyed some popularity at Cambridge, but was less well received at Oxford.[102] Such textbooks as that of Sanderson betray little Ramist influence. In Central Europe, Ramus was widely used in the pre-university curriculum, often in conjunction with works by Melanchthon, and the study of Ramus in schools tended to be followed by the study of Aristotle at universities and other higher-level institutions.[103] Generally speaking, university teachers of logic found Ramus's book to display serious deficiencies, and in the 1590s a new school of textbook writers known as the Philippo-Ramists appeared in Germany.[104] Their aim was to combine what was best in Ramus with what was best in the more Aristotelian works of Philip Melanchthon. As a result, they tended to restore all those parts of Aristotelian logic that Ramus had deliberately omitted, and the syllogism was once more presented as central to formal logic.

If France, England and Germany saw the production of slim textbooks exemplifying the marriage between humanism and a simplified Aristotelianism, Spain and Italy were the scene of a much more substantial textbook production. Two important texts, both recommended by the Jesuit *Ratio Studiorum*, were the *Introductio in dialecticam* of Franciscus Toletus and the *Institutionum dialecticarum libri octo* of Petrus Fonseca.[105] Toletus's book was first published in Rome in 1561, and the last of its eighteen editions appeared in Milan in 1621. Fonseca's work was first published in Lisbon in 1564, and the last of its fifty three editions appeared in Lyon in 1625. The works were fairly similar in content, but I shall look more closely at Fonseca's work, which is available in a modern edition.[106] The first thing to notice is that the *Organon* is very much the focus of attention. The material of the *Categories*, *Perihermenias*, *Prior* and *Posterior Analytics*, *Topics* and *Sophistici*

[100]Ramus, *Scholarum Dialecticarum*, p. 147.

[101]W. J. Ong, *Ramus and Talon Inventory* (Cambridge, Mass.: Harvard University Press, 1958). For a full discussion of Ramus, see Ong, *Ramus, Method and the Decay of Dialogue*; N. Bruyère, *Méthode et dialectique dans l'oeuvre de La Ramée* (Paris: Vrin, 1984); Mack, *Renaissance Argument*, pp. 334–355.

[102]See Ashworth, introduction to Sanderson, *Logicae Artis Compendium*, pp. XXVIII–XXIX.

[103]Joseph S. Freedman, "The Diffusion of the Writings of Petrus Ramus in Central Europe, c. 1570–c. 1630", *Renaissance Quarterly* 46 (1993), 98–152.

[104]Ashworth, *Language and Logic*, pp. 16–17.

[105]The *Ratio Studiorum* of 1586 recommended Fonseca: see Kehrbach, p. 131. The *Ratio Studiorum* of 1599 added the name of Toletus: p. 332.

[106]Pedro da Fonseca, *Instituições Dialécticas. Institutionum Dialecticarum libri octo*, ed. J. Ferreira Gomes (Universidade de Coimbra, 1964) 2 volumes.

Elenchi is taken up in turn. However, medieval material is used to supplement Aristotle in various places. Exponibles are discussed, albeit in a simplified form; some non-syllogistic consequences dealing with truth and modality are listed, and the material about supposition and related doctrines is presented as an aid to understanding fallacies. Fonseca warned that these doctrines were "unrefined, uncouth, and remote from use", and that to dwell on them at length was dangerous to good language. However, he remarked, some loss would come from ignoring these doctrines altogether.[107] He had nothing to say about other specifically medieval doctrines covered in the treatises on insolubles and obligations.

The second thing to be noted is the general style and manner of Fonseca's approach. He was frequently concerned to explain the precise intention of Aristotle's remarks, and in his attempt to understand Aristotle, he drew on his knowledge of the Greek language and of Greek authors. He cited not only those who commented on Aristotle, such as Alexander of Aphrodisias and Ammonius, but also Plato and Pindar. Among Latin authors, he drew heavily on Cicero, and there are references to Horace and Virgil. Later authors used include the great Arab commentator Averroes. In all of this Fonseca reflected the renewed Aristotelianism of the sixteenth century. There is a strong philological emphasis, based on the new study of the Greek language, and Aristotle's text is to be read through the eyes not of late medieval logicians but of the Greek and Arab writers. Wider Classical learning is used to illuminate points and to provide apt examples. Insofar as Fonseca did make use of authors from the medieval Latin West, he tended to focus on Thomas Aquinas. Ockham is cited, but the next logician in chronological order to be cited is Rudolph Agricola. The authors popular fifty years earlier have dropped out of sight. In many ways, Fonseca's logic text is far closer to the Coimbra commentary of 1606 than it is to the work of his Spanish predecessors such as Fernando de Enzinas or Domingo de Soto. Nor is this surprising, given that Fonseca taught at the University of Coimbra, and was one of the moving spirits behind the production of the *Conimbricenses*.

Part Two: Claims about Humanist Logic

One of the chief claims about humanist logic is that it rhetoricized logic. For instance, Kristeller spoke of "the attempt to reform logic by subordinating it to rhetoric";[108] Rita Guerlac remarked "Agricola succeeded in rhetoricizing dialectic";[109] and John Monfasani wrote: "in making *inventio* not merely a part of logic, but as Agricola insisted, the greater part, the Northern humanists transformed logic, in all but name, into an expanded version of rhetorical invention".[110] One can see this supposed rhetoricization as the simple result of humanist literary

[107] Fonseca, *Instituições Dialécticas*, pp. 676–678. P. 676: "[...] inculta, horrida, et ab usu remota [...]"

[108] P.O. Kristeller, "Rhetoric in Medieval and Renaissance Culture" in *Renaissance Eloquence*, p. 18.

[109] Guerlac, *Juan Luis Vives*, p. 32.

[110] Monfasani, *George of Trebizond*, p. 303.

studies, but broader and more substantial claims have been made, particularly by Lisa Jardine. She holds that it is a mistake to think that the driving force behind humanist logic was simply disgust at the non-Ciceronian Latin of the Scholastics, and that the philosophical ideals lying behind the work of Lorenzo Valla (d. 1457) and his follower Rudolph Agricola were more important. According to Jardine, both Valla and, to a lesser extent, Agricola were concerned to offer a logic which was linked with Cicero's Academic skepticism rather than with Aristotelian certainties.[111] They wished to offer argumentative strategies for rendering plausible each of the two sides of an undecidable question, or for supporting one of them as, perhaps only marginally, more plausible than the other. They were thus drawn to consider a variety of non-deductive strategies in lieu of the formal techniques which had dominated a large part of medieval logic, especially in the treatises on consequences, and in lieu of Aristotle's own syllogistic. These new strategies included *sorites*, or the classical Heap argument, and classical dilemma, but they centred on the Topics, which, especially as presented by Cicero and Quintilian, seemed to offer a method of classifying non-deductive strategies by their key terms rather than by their form. At the same time, much of Agricola's concern was with the art of discourse as such, that is, with the problem of presenting and organizing complete arguments and narrations, whether written or spoken. It was as a result of this interest both in persuasive techniques and in discourse as such that logic came to embrace much of what had been traditionally regarded as rhetoric, and rhetoric came to be seen as concerned, not with the invention of Topics, but with the ornamentation of discourse.

1 LORENZO VALLA

In order to evaluate these claims, we first need to consider Lorenzo Valla (1405–1457), and his *Dialecticae disputationes*, more accurately called *Repastinatio Dialecticae et Philosophiae*.[112] In this work, Valla certainly attacked a number of Aristotelian positions. For instance, he reduced the number of categories from ten to three, he criticized the square of opposition, and he rejected the third figure of the syllogism as insane. However, he retained a generally Aristotelian framework for logic, and his discussion of Topics was merely a reproduction of Quintilian's material. He classified dialectic as a part of rhetoric, but found few supporters for this move other than Marius Nizolius (1488–1567).[113] He did indeed discuss

[111] Lisa Jardine, "Lorenzo Valla and the Intellectual Origins of Humanist Dialectic," *Journal of the History of Philosophy* 15 (1977), 143–164; "Humanism and the teaching of logic" in *The Cambridge History of Later Medieval Philosophy*, pp. 797–807; "Lorenzo Valla: Academic Skepticism and the New Humanist Dialectic" in *The Skeptical Tradition*, ed. Myles Burnyeat (Berkeley, Los Angeles, London: University of California Press, 1983), pp. 253–286; "Humanistic Logic", pp. 173–198

[112] For all three redactions, see Lorenzo Valla, *Repastinatio Dialecticae et Philosophiae*, ed. G. Zippel (Padua: Antenore, 1982). For a full discussion of Valla, see Mack, *Renaissance Argument*, pp. 22–116.

[113] Mack, *Renaissance Argument*, pp. 115–116.

sorites and dilemma, but he disapproved of them both; and the claim that he introduced a new interest in *sorites* and other classical forms of argument is hardly borne out by the evidence. It is true that some later authors mention *sorites*, but in the hands of Philip Melanchthon and Petrus Fonseca it is identified with the medieval argument 'From First to Last', a linked chain of syllogisms (or conditionals).[114] In his latest logical work, *Erotemata dialectices*, Melanchthon explains that the chain is based on movement from lowest species to highest genus, or from causes to proximate effects.[115] Dilemma does not fare better. Both Jodocus Clichtoveus and Domingo de Soto give lengthy accounts of the lawsuit between Protagoras and Euathlus, as reported by Aulus Gellius, but they assimilate the story to medieval discussions of promises that cannot be kept.[116] Agostino Nifo in his *Dialectica ludicra* has a section entitled *De dilematibus et antistrephontibus: idest de insolubilibus*, but places the classical material in the context of medieval discussions of semantic and pragmatic paradoxes.[117] Two further points casting doubt on Jardine's account of Valla's importance are first, that the evidence is against his having been a sceptic;[118] and second, that the work had only a limited manuscript circulation, and few printed editions.[119]

While Valla seems not to have played the role assigned to him by Jardine, we still need to confront the wider questions of how far logic was rhetoricized, and how far humanist dialectic, or any other type of sixteenth-century logic, was concerned with probabilistic and informal argumentation. In order to do this, we need to make a series of distinctions about the key terms, 'dialectic', 'probability', 'rhetoric', and 'informal', as a lack of such distinctions in the literature has obscured the points at issue.

2 DIALECTIC AND PROBABILITY

In order to understand the issues here we need to consider the background. The Aristotelian syllogism was taken to be the most important form of argumentation,

[114]Melanchthon, *Compendiaria dialectices*, cols. 747–748, has a discussion of *sorites* after the other four forms of argumentation. He also mentions (col. 748) other arguments such as the Crocodile, Asystaton, and Antistrephon, but only to say that they can be omitted. For Fonseca on *sorites*, see *Instituições Dialécticas*, p. 348.

[115]Melanchthon, *Erotemata dialectices*, cols. 624–626.

[116]Jodocus Clichtoveus, *Jacobi Fabri Stapulensis artificiales introductiones per Judocum Clichtoveum in unum diligenter collecte* (Parisiis, 1520), f. 139 v–140r; Domingo de Soto, *Summulae*, f. lv rb–va. For the problem of promises that cannot be kept, see E. J. Ashworth, "Will Socrates Cross the Bridge? A Problem in Medieval Logic", *Franciscan Studies* 36 (1976), 75–84, reprinted as Study XII in Ashworth, *Studies in Post-Medieval Semantics*. For a full discussion of the limited treatment of dilemma and other classical arguments in sixteenth century logic, see Gabriel Nuchelmans, *Dilemmatic Arguments: Towards a history of their logic and rhetoric* (Amsterdam: North-Holland, 1991).

[117]Augustinus Niphus, *Dialectica ludicra* (Venetiis, 1521), f. 156 ra– f. 163 ra.

[118]See John Monfasani, "Lorenzo Valla and Rudolph Agricola," *Journal of the History of Philosophy* 28 (1990), 181–200.

[119]Mack, *Renaissance Argument*, p. 115.

and Aristotle twice defines the syllogism as a discourse (*oratio*) in which, certain things being stated, something other than what is stated follows of necessity from their being so. (*Prior Analytics* 24b18–20; *Topics* 100a 25–26). It is quite possible that the Greek word was intended to be taken in two senses, with the *Prior Analytics* dealing with syllogisms in the technical sense of arguments with two premisses and three terms arranged so arranged that each term appears twice, the middle term in both premisses, and each of the other terms in just one premiss and the conclusion, and the *Topics* dealing with deduction in general. However, the standard medieval and post-medieval assumption was that the technical sense of syllogism is intended in each case.[120] Accordingly, the *Topics* (100a25–101b24) was taken to list four species of syllogism, demonstrative, dialectical, deceitful (the *syllogismus falsigraphus* or *pseudographus*, which argues from falsehoods belonging to the special sciences), and sophistical or contentious (*litigiosus*). We will not be concerned with the last two, which are both fallacious. In the *Topics* (105a10–19) Aristotle also tells us that there are two species of dialectical reasoning (*dialecticarum ratiocinationum*), *syllogismus* and *inductio*. From his *Rhetoric* we learn that rhetoric accompanies dialectic (*Rhetorica assecutiva dialecticae est*), that it is concerned with persuasion, and that it uses two other types of reasoning, enthymeme, which is the rhetorical syllogism, and example, which is the rhetorical induction (*Rhet.* 1355a4–8, 1356a36–b12). These four types of reasoning (*argumentatio*) were neatly summarized by Peter of Spain,[121] and they formed a standard list to be included in logic texts, despite the association of enthymeme and example with rhetoric.

Other standard material was drawn from Boethius's two works on the Topics, *In Ciceronis Topica* and *De topicis differentiis*. He discussed the uses of the words '*logica*' and '*dialectica*'; following Cicero, he described logic as a system of discourse (*ratio disserendi*);[122] and he told his readers that an argument "is a reason that produces belief regarding something that was in doubt".[123] Moreover, he handed on the important distinction between invention (the finding of material for arguments) and judgement.[124]

If we now ask what counts as dialectic and whether it differs from logic, two main answers are possible. One can regard 'logic' and 'dialectic' as merely two names for one discipline, or one can regard dialectic as a sub-part of logic which studies dialectical syllogisms as presented in Aristotle's *Topics*. Both of these answers were generally recognized as acceptable in the medieval and post-medieval period, and one did not exclude the other. A third answer, based on the discussion of dialectical invention by Rudolph Agricola, is that dialectic focusses not on the dialectical syllogism but on Topical discourse and debate, and hence, accord-

[120] Note that the 'technical' sense was normally extended to embrace hypothetical and expository syllogisms.

[121] Peter of Spain, *Tractatus*, p. 56.

[122] Stump, *Boethius's De topicis differentiis*, p. 29.

[123] Stump, *Boethius's In Ciceronis Topica*, p. 29; Stump, *Boethius's De topicis differentiis*, p. 39 ("Argumentum est ratio rei dubiae faciens fidem".)

[124] Stump, *Boethius's In Ciceronis Topica*, p. 25.

ing to Jardine and others, is a part of logic primarily concerned with persuasive techniques and informal argumentation, that is, with non-deductive strategies. In fact, the link between dialectic and disputation goes back to Aristotle, and was generally accepted. Domingo de Soto, for instance, discusses the Greek derivation of the word '*dialectica*', saying that it comes from '*disputo vel dissero*' and so dialectic is a '*disputatoria scientia*'.[125] He applies this assessment to logic as a whole, rather than to a single part. The important question is whether it follows from dialectic's link with the Topics and with disputation that it is concerned with informal argumentation, and if it is, how far this concern extends.

Before this question can be answered, we need to consider the notion of probability. The term '*probabilis*' itself had a fairly general meaning. A probable proposition was likely, plausible, susceptible of proof though not usually of demonstration in the strict Aristotelian sense. It was recognized that there are degrees of belief, and that a probable belief or *opinio* is one that falls somewhat short of certainty, while being more robust than mere *suspicio*, the special province of the rhetorician. It was also recognized that even those propositions which ought to be certain might be matters of opinion for some. As Paul of Pergula remarked, not everyone believes that God exists, even though "God exists" was a standard example of a necessarily-true proposition.[126] No logician that I know of displayed any awareness of statistical or relative frequency interpretations, and the notion of subjective probabilities, according to which premisses support a certain degree of probability attached to the conclusion, comes up only occasionally, particularly in the work of Philip Melanchthon, as we shall see in section 4 below. More usually, however, if an argument was labelled as 'probable' this had to do, not with the replacement of a strict logical relation between premisses and conclusion by some other relation, but with the epistemological status of the premisses. If at least one premiss was about contingent matters, or was wrongly taken to be about contingent matters, the argument, albeit deductively valid, fell into the category of probable or dialectical syllogism. This reading is, of course, based on the belief that Aristotle's reference to 'syllogism' in the *Topics* is to be taken as a reference to a particular kind of formally-valid deductive structure. Thus, demonstrative reasoning involves formal syllogisms with certainly-true premisses, and dialectical reasoning involves the same formal syllogisms with probably-true premisses. Sophistical reasoning may or may not involve formal syllogisms, since different kinds of logical error are possible. On this interpretation, dialectic is clearly a sub-part of logic, or, for those who preferred the term 'dialectic' to the term 'logic', dialectic in the narrow sense is a part of dialectic in the broad sense.

The notion that there are three different kinds of syllogism, differentiated not with respect to their form but with respect to the epistemic status of their premisses, fitted in with the standard ordering of the *Organon*. Terms were discussed

[125]Soto, *Summulae*, f. 3 va. Cf. Franciscus Toletus, *Introductio in dialecticam Aristotelis* in *Opera omnia philosophica I–III* (Cologne 1615–1616; repr. Hildesheim: Georg Olms, 1985), p. 4a.

[126]Paul of Pergula, *Logica*, p. 88.

in the *Categories*, propositions were discussed in the *Perihermenias*, syllogisms in general were discussed in the *Prior Analytics*, and then the three kinds of syllogism were discussed in the *Posterior Analytics*, *Topics*, and *Sophistici Elenchi*, in that order. This arrangement, however, gave rise to some criticism, particularly from Petrus Ramus. He attacked on two fronts. His first target was the division given by Averroes, and championed by Zabarella.[127] Averroes had divided Aristotle's logic into general or universal logic, which was contained in the first three books of the *Organon*, and three special logics, apodictic, probable, and sophistic, which were contained in the last three books. Ramus argued in his *Scholae Dialecticae* that the list of three special logics was unacceptable, partly because the study of sophistical reasoning was no more a part of logic than the study of barbarisms and solecisms was a part of grammar, and partly because the *Posterior Analytics* and the *Topics* were themselves both general in their application.[128] They applied to any epistemological material, and there was only one general doctrine applying to terms, sentences, syllogisms, and method. Of course there is a difference between apodictic, dialectic and sophistic propositions, but affirmation and negation are just as much features of propositions as are truth and falsehood, necessary and probable truth, and the former are not elevated into divisions of logic as such. Nor do we say that the *Perihermenias* is divided into necessary, true-seeming and captious, even though propositions are.[129] Thus there is one logic or dialectic, and it is the art of discourse.[130] Ramus is assuming that indeed formal deductive structures are what counts in argumentation, and that the mistake of his opponents lay only in their supposition that the application of these structures to different kinds of material would produce different kinds of logic.

Ramus's second target had to do with the application of the division between invention and judgement or *dispositio*, as he frequently called it, to the books of the *Organon.* This was not a new issue. For instance, in the thirteenth-century *Topics* commentary ascribed to Robert Kilwardby the author writes that insofar as *inventio* comes before the judgement of what is found, the *Topics* and the *Sophistici elenchi* should come before the *Prior* and *Posterior Analytics*, but insofar as the syllogism as such comes before the demonstrative syllogism, and the demonstrative syllogism before the dialectical, and the dialectical before the sophistical syllogism, the *Prior Analytics* should be followed by the *Posterior Analytics*, *Topics* and *Sophistici elenchi* in that order. He felt that Aristotle approved the second ordering.[131]For both Rudolph Agricola and Petrus Ramus it seemed obvious that invention should precede judgement, and this arrangement became a hallmark of Ramist textbooks, though not of those textbooks that retained traditional Aristotelian material. The issue was discussed by other authors, including the Coimbra

[127]Zabarella, *De natura logicae*, col. 51 ff.
[128]Ramus, *Scholarum Dialecticarum*, pp. 38–42, p. 61.
[129]Ramus, *Scholarum Dialecticarum*, pp. 60–61.
[130]Ramus, *Scholarum Dialecticarum*, pp. 36–37.
[131]Olga Weijers, "Le commentaire sur les "Topiques" d'Aristote attribué à Robert Kilwardby (ms. Florence, B.N.C., Conv. Soppr. B.4.1618)", *Documenti e Studi sulla Tradizione Filosofica Medievale* 6 (1995), p. 126.

commentary in their introduction to the *Prior Analytics*.[132] They write that some more recent thinkers have argued, on the basis of the Stoic division into invention and judgement ("which is not disproved by the Peripatetics"), that the *Prior Analytics* does not precede the *Topics*, on the grounds that dialectic is divided into invention and *iudicium seu dispositio*, and invention is naturally prior to *dispositio*. This view is wrong. First, the two parts spoken of do not deal with the same material. The *Topics* concerns only the invention of probable arguments, but the *Prior Analytics* deals with the disposition of all arguments, with respect to their form. Moreover, given that dialectic is most properly divided into the three *modi sciendi*, defining, dividing and arguing, it is clear that invention and judgement have a role in all three parts. One should start by investigating and finding the parts of a definition, and then aptly disposing them. Certainly we find neither invention alone nor disposition alone in the *Topics* and the *Prior Analytics*.[133] The same is true of the *Posterior Analytics*. And even if in an art invention must occur first, there is no reason why it should be taught first. It seems that one cannot understand invention perfectly unless one has a prior notion of disposition. The Coimbra commentary therefore opted for the standard ordering in which the *Perihermenias* precedes the *Prior Analytics*, which is immediately followed by the *Posterior Analytics* and then by the *Topics*.

The discussion of invention, and the association of judgement and *dispositio* brings us to the question of rhetoric's relation to dialectic.

3 RHETORIC AND ITS RELATION TO DIALECTIC

One can think of rhetoric merely as a means of presenting arguments in a clear and engaging manner, and it is quite clear that, as a result of humanist literary studies and the changes in the educational system, logical writings in general did undergo change. The more tortured formulations of late medieval logic were abandoned, classical examples were used, and classical vocabulary was often substituted for medieval technical terms. However, it is rhetoric as an academic discipline that is more important for our purposes. Again, it is quite clear that it had a more important role in the curriculum than it had enjoyed during the medieval period,[134] but this does not settle the question of its relation to logic. One issue concerns its parts; the other main issue concerns its possible subsumption under logic.

In 1518, a student at Bologna debated the question "Is rhetoric a part of dialectic

[132] *Commentarii Collegii Conimbricensis*, II, cols. 235–236

[133] Cf. Augustinus Niphus, *Commentaria in octo libros Topicorum Aristotelis* (Parisiis, 1542), f. 2 ra–vb. He argued that while there were reasons for attributing both invention and judgement to the individual books of the *Organon*, there were better reasons for following the sharp distinction found in Cicero and Boethius.

[134] For instance, the University of Glasgow's new constitution of 1577 greatly reduced logic teaching while increasing the time given to rhetoric: see J. M. Fletcher, "The Faculty of Arts" in *The History of the University of Oxford III. The Collegiate University*, ed. James McConica (Oxford: Clarendon Press, 1986), p. 158.

or politics?"[135] This was not a new question. Aristotle's *Rhetoric* as well as his *Poetics* had come to be regarded by the late Greek commentators of the School of Alexandria as part of the *Organon*, that is, as part of logic in a wide sense, and the Arabs followed this established tradition.[136] Albert the Great, Aquinas, and Simon of Faversham, among others, followed the Arabs, at least in principle. If we look at Aquinas's analysis of the parts of logic in his *Posterior Analytics* commentary, we find that demonstrative syllogisms produce certainty, while dialectic deals with arguments on the basis of probable premisses that produce opinion or belief, and rhetoric deals with arguments whose premisses produce *suspicio*, an inclination to accept one position rather than its opposite.[137] In the sixteenth century, the most conspicuous defender of this tradition was Zabarella, who devoted a large part of his work *De natura logicae* to arguing that rhetoric and poetics were genuine parts of logic.[138] None of these claims seem to have had any practical impact. One reason for this, at least during the medieval period, was the relative inaccessability of Aristotle's *Rhetoric* and *Poetics*, but during the sixteenth century, when these works were readily available,[139] the consensus seemed to be that while rhetoric should be taught in tandem with dialectic, it should nonetheless be regarded as a separate discipline.

The most important issue is how the five parts of rhetoric, namely *inventio*, *dispositio*, or the structuring and arrangement of material, *elocutio*, *memoria* and *pronuntiatio*, were to be handled. For Agricola and Ramus, *dispositio* was interchangeable with judgement, and both *inventio* and *dispositio* were to be handled by the logician. This arrangement left matters of eloquence to the rhetorician, and anything to do with the Topics, with argumentation, or with the ordering of discourse to the logician. With respect to *inventio*, Agricola insisted that the Topics were to be confined to dialectic, and that rhetorical Topics, which had covered material of a more particular kind, were to be reduced to dialectical Topics.[140] In

[135]Herbert S. Matsen, "Student "Arts" Disputations at Bologna around 1500", *Renaissance Quarterly* 47 (1994), p. 545. Rhetoric came after logic in order of importance as a disputation topic, op.cit., p. 543 and p. 545.

[136]Black, Deborah L. *Logic and Aristotle's 'Rhetoric' and 'Poetics' in Medieval Arabic Philosophy*. Leiden etc.: E.J. Brill, 1990, pp. 17–18; Marmo, Costantino. '*Suspicio*: A Key Word to the Significance of Aristotle's *Rhetoric* in Thirteenth Century Scholasticism.' *Cahiers de l'institut du moyen-âge grec et latin* 60 (1990), 145–198.

[137]Thomas Aquinas, *In Aristotelis libros Peri Hermenias et Posteriorum Analyticorum Expositio* (Turin: Marietti, 1964), Prooemium §6, pp. 147–148. He writes, p. 148: "Per huiusmodi enim processum, *quandoque* quidem, etsi non fiat scientia, fit tamen fides vel opinio propter probabilitatem propositionum, ex quibus proceditur: quia ratio totaliter declinat in unam partem contradictionis, licet cum formidine alterius, et ad hoc ordinatur *Topica* sive *Dialectica*. Nam syllogismus dialecticus ex probabilibus est, de quo agit Aristoteles in libro *Topicorum*. ¶*Quandoque* vero, non fit complete fides vel opinio, sed *suspicio* quaedam, quia non totaliter declinatur ad unam partem contradictionis, licet magis inclinetur in hanc quam in illam. Et ad hoc ordinatur *Rhetorica*."

[138]Zabarella, *De natura logicae* in *Opera omnia*, cols. 71–100.

[139]See Brian Vickers, "Rhetoric and Poetics" in *The Cambridge History of Renaissance Philosophy*, p. 718 for details of the reception of the *Poetics* in the sixteenth century, and p. 721 for the reception of the *Rhetoric*.

[140]Agricola, *De Inventione Dialectica*, pp. 313–319. Boethius had discussed the relationship

the hands of Philip Melanchthon there was no real reduction, but instead a listing of the so-called personal Topics, such as nation, age, education and sex, within dialectic itself.[141] With respect to *dispositio*, Agricola discussed the problem of presenting and organizing complete arguments and narrations, whether written or spoken, at great length. In Melanchthon we find the same interest in *dispositio*. A few remarks on order in the *Compendiaria Dialectices Ratio* grew into a full section on logical method as a way of ordering discourse in the *Erotemata Dialectices*.[142] In turn we find method discussed in the second part of Ramus's *Dialectica*, but as we can see from his references to Agricola and Melanchthon, it is a mistake to think that this was Ramus's innovation. Sections on method became a standard component of logic textbooks, and particularly after the publication of Zabarella's *Opera logica*, it became customary to add remarks on scientific method as well, in a mixture of subject-matters that was not always happy.[143]

To conclude, if we mean by the rhetoricizing of logic that material from the rhetoricians came to be included in logic texts, this is certainly the case. How far the material absorbed included informal persuasive devices is another matter.

4 INFORMAL ARGUMENTATION

Whether or not sixteenth-century logic leaves room for the notion of informal arguments depends on how 'formal' and 'informal' are to be defined. The first thing we need to understand is that logic was never formal in the modern sense. There was no notion of an uninterpreted system with its own set of properties, nor was there any notion of a formal interpretation such that, for instance, any proposition just says T or F. While letters such as 'a' and 'b' could be used to stand for propositions, and rules could be described in neutral language, the basic assumption was always that logic served the purpose of finding and expressing truths. That being said, one might make a distinction between formal and informal arguments in terms of deductive validity, so that an informal argument is one that is acceptable but not deductively valid. This still leaves various possibilities open, with respect to both categories. In the case of valid deductive arguments, we need a further distinction between formal and non-formal arguments. Some arguments are such that their conclusion is true whenever the premisses are true, no matter what non-logical terms are substituted. Other arguments are such that their conclusion is true whenever the premisses are true just because of the

between rhetorical and dialectical Topics: see Stump, *Boethius's De topicis differentiis*, pp. 79–95.

[141] Melanchthon, *Compendiaria Dialectices Ratio*, cols. 751–755; *Erotemata Dialectices*, cols. 659–662.

[142] Melanchthon, *Compendiaria Dialectices Ratio*, cols. 724–726; *Erotemata Dialectices*, cols. 573–578.

[143] For more on discussions of method, see Cesare Vasoli, "The Renaissance concept of philosophy" in *The Cambridge History of Renaissance Philosophy*, pp. 71–72; Jardine, "Humanistic logic", pp. 191–192; Ashworth, introduction to Sanderson, *Logicae Artis Compendium*, pp. XLVIII–L.

semantic link between particular non-logical terms. I dub these 'non-formal'. Such arguments were discussed under the heading of 'materially valid consequences', by those logicians who continued to include sections on consequences in their texts, and who used the substitution criterion of formality.[144] In the case of arguments that are not deductively valid but are nonetheless acceptable, we might be talking about incomplete arguments, notably enthymemes, that need to be re-expressed as formal deductive arguments, or we might be talking about those arguments whose premisses merely support the conclusion, or in some way make it reasonable to accept the conclusion. Only the latter are properly dubbed 'informal'.

At this point, we face another problem. As discussed in Part One, Agricola and Ramus had excluded maxims and rules from the parts of dialectic devoted to invention. This reduced or even eliminated the room for informal argumentation connected with the Topics. Moreover, Agricola had accepted the four standard types of argumentation as the subject of dialectical judgement, while Ramus insisted that the syllogism, whether Aristotelian as in the *Scholae*, or reformed, as in his *Dialectica*, was the general object of discussion under judgement. We also have to take into account the rejection of specifically medieval material by many sixteenth-century logicians. This meant that in a simplified logic manual there was no discussion of types of conditional statement or of types of consequence, any more than there was serious discussion of the standard types of argumentation. Accordingly, it is only in those texts that are more heavily scholastic and Aristotelian in their approach that we find anything more than a hand-waving hint that there are indeed informal argumentative devices. I will consider just a few examples.

I will start with conditionals and consequences. In his commentary on Peter of Spain, Eck divides consequences into those that are merely inferential (*illativa tantum*), those that are both inferential and convincing (*illativa et probativa*), and those that are merely convincing (*probativa tantum*). Merely convincing consequences are those in which the antecedent does not entail the conclusion, but support it with reference to the Topics. They are persuasive, and are used in ethics and rhetoric.[145] That is, they do not really belong to logic.

Petrus Fonseca is less dismissive. Contrary to the view held by Peter of Spain that all conditionals are either necessary or impossible, Fonseca, like Nifo before him,[146] allows for conditional statements that are contingent;[147] and he also argues that that while all formal consequences are necessary, material consequences can

[144]For the issues here, see the previous chapter on medieval theories of consequence. For those who used the containment criterion of formality, the labels attached to what I call non-formal consequences will differ.

[145]Johannes Eckius, *In summulas Petri Hispani* (Augsburg, 1516), f. c vb. "Probativa tantum est in qua antecedens non de necessitate infert consequens, tamen probat probabiliter et topice." He subsequently adds "consequentiam probativam non esse simpliciter bonam consequentiam sed esse persuasivam, et in moralibus ac rhetoricis multum frequentatam, ut Hupertus fuit solus cum Catherina in loco suspecto, ergo stupravit eam".

[146]Niphus, *Dialectica ludicra*, f. 78 vb, f. 79 rb.

[147]Fonseca, *Instituições Dialécticas*, p. 198.

be either necessary or probable.[148] "A man is an animal, therefore every man is an animal" is necessary, given that any man is necessarily an animal, but "A mother loves her child, therefore this mother loves her child" is probable in the sense that there is no necessary link between antecedent and conclusion. The conclusion follows often, but not always. We have here a recognition of genuine informal arguments, but it is not enlarged on. It is presumably based either on Boethius's *De topicis differentiis* where Boethius speaks of arguments which are "readily believable and not necessary", such as "If she is a mother, she loves [her] child",[149] or on Cicero, *De inventione* I.29.46.

Fonseca also has a few relevant remarks in his discussion of the Topic from associated accidents.[150] These are circumstances which are often not necessarily connected, and which give rise to *suspicio*, that epistemic state associated with rhetoric. However, Fonseca explicitly allows that the dialectician may not only teach the invention of such arguments, but use them insofar as they give rise to probabilities. Toletus too had some relevant material in his discussion of the Topic from associated accidents.[151] He noted that there are two modes of argument, necessary and non-necessary. "It is bearing fruit, therefore it has flowered" is necessary, but "People are drinking, therefore there will be fights" (*Bacchanalia sunt, ergo rixae erunt*) is non-necessary. The latter mode is related to other arguments based on natural signs, physiognomy, prodigies, portents, and those once based on auguries but now empty and fallacious. He suggests that they are based on *suspicio*, and they are not, or not all, necessary or probable. In effect, this once more relegates such arguments to rhetoric rather than logic.

Other discussions found in the logical literature relate to the classification of enthymemes. Aristotle had written (*Prior Analytics* 70a10–11) that "An enthymeme is a deduction starting from probabilities or signs [. . .]", where these are two types of proposition, and the same distinction between probabilities or non-necessary signs and necessary signs was found in Quintilian.[152] Commenting on Aristotle's classification, Clichtoveus writes that the first type of proposition is contingent, for instance, "A mother loves her children", and that it gives rise to a dialectical enthymeme, such as: "A mother loves her children, therefore a mother does not hate her children."[153] His example, however, seems more like a non-formal valid deductive argument than it does a genuinely probabilistic informal argument.

In the section on enthymeme found in the *Compendiaria Dialectices Ratio*, Melanchthon emphasizes that rhetoricians and dialecticians use the same *ratio argumentandi* and the same structure of arguments, though dialectic is more cer-

[148] Fonseca, *Instituições Dialécticas*, p. 340.
[149] Stump, *Boethius's De topicis differentiis*, p. 40.
[150] Fonseca, *Instituições Dialécticas*, p. 542–544. He uses the title 'De locis praecurrentium, comitantium et subsequentium', but tells the reader that this is what Cicero called the Topic from adjuncts. In turn, that Topic is the one also known as the Topic from associated accidents (*De communiter accidentibus*).
[151] Toletus, *Introductio in dialecticam*, f. 82 b.
[152] Quintilian, *Institutio oratoria*, V.9.1–8.
[153] Clichtoveus, *Introductiones artificiales* (Parisiis, 1517), f. 100 r–v.

tain and rhetoric, freer.[154] As he had remarked earlier, a good dialectical proof could not be otherwise, but a rhetorical proof can always be put another way.[155] In the section on Topics, however, he takes up the notion of signs, which he divides into necessary and probable.[156] He also uses an explicit probability operator, placing '*probabile est igitur*' in front of the conclusion of an argument from probable signs.He then links the discussion of signs with the personal Topics, saying that they are both less relevant to the dialectician, who needs necessary and evident arguments, than to the rhetorician, but their discussion should not be left to the rhetorician. Arguments using premisses based on signs or the personal Topics need to be put into the form of syllogisms, enthymemes, induction or example. For instance, he gives the argument: "It is unlikely (*verisimile non est*) that students of literature should indulge in pleasures, Coelius is a student of literature, therefore it is unlikely that he indulges in pleasures." The example suggests that he wanted to treat probability operators as part of a deductive system, rather than using them to construct a non-deductive probabilistic logic.

In the *Erotemata Dialectices*, enthymeme is linked with signs, and Melanchthon explains that Aristotle defined enthymeme as an imperfect syllogism whose premisses are probabilities (*ex icotibus*) and signs.[157] However, the point of discussing true-seeming (*verisimilia*) signs is only to warn us against misleading and fallacious arguments. In the section on Topics, signs are now discussed quite separately from the personal Topics, and are divided into those that are necessary and those that are "not necessary but are associated accidents", called '*eikota*'.[158] We have to be very careful using arguments based on the latter, lest we deceive ourselves, as often happens "in suspicionibus et aliis malis persuasionibus." We should remind ourselves here that *suspicio* and persuasion belong to rhetoric. All in all, Melanchthon, despite clear humanist interests, does not write like a logician who is intent on legitimatizing informal, probabilistic argumentation.

Agostino Nifo firmly relegates one kind of enthymeme to rhetoric. In his *Dialectica ludicra*, he quotes Michael Psellus, an eleventh-century Byzantine writer, whose work on rhetoric divided consequences into necessary and contingent.[159] The latter were those that merely persuade without proving or convincing, such as rhetorical enthymeme and example. Nifo went on to claim that there are two sorts of enthymeme, the analytical enthymeme which differs from a syllogism only in that it uses one premiss, and the rhetorical enthymeme, which differs from a syllogism both in its use of just one premiss and in the fact that the suppressed premiss is neither necessary nor probable but merely true-seeming (*verisimilis*).[160] Such a premiss is not dependent on opinion or fact, but on a link with action.[161]

[154]Melanchthon, *Compendiaria Dialectices Ratio*, col. 743.
[155]Melanchthon, *Compendiaria Dialectices Ratio*, col. 733.
[156]Melanchthon, *Compendiaria Dialectices Ratio*, cols. 750–751.
[157]Melanchthon, *Erotemata Dialectices*, cols. 617–619.
[158]Melanchthon, *Erotemata Dialectices*, cols. 704–706.
[159]Niphus, *Dialectica ludicra*, f. 110 va.
[160]Niphus, *Dialectica ludicra*, f. 110 vb–111 ra.
[161]Niphus, *Dialectica ludicra*, f. 111 ra: "assumptio verisimilis non accipitur ab opinionem: vel

By this he seems to mean that it would be plausible for people to act in that way. His example is "Caesar is seeking munitions, therefore he will become a tyrant", where the suppressed premiss is "All those who seek munitions are aiming to become tyrants." He goes on to argue that such consequences obey the regulative principle that the negation of the consequent appears to be inconsistent (*verisimiliter repugnat*) with the antecedent, but while we can call them rhetorical, simply speaking they are just invalid and non-necessary.[162]

I confess that I have not done a complete survey of sixteenth-century logical literature. However, based on the evidence so far, I think it fair to conclude that the view that the sixteenth century made an important contribution to informal logic is at best exaggerated.

CONCLUSION

To conclude this survey of changes in logic during the fifteenth and sixteenth centuries, I shall attempt to isolate the main differences between medieval texts and their post-medieval successors, whether commentaries on Aristotle or introductory textbooks. One obvious difference is the emphasis on following the order and material of Aristotle's *Organon*. Such medieval material as survived was strictly subordinated to this end, and even though Agricola and Ramus had tried to ignore Aristotelian syllogistic and the doctrines propaedeutic to it, such as conversion and opposition, their omissions were rapidly remedied by subsequent textbook writers. Non-Aristotelian logic was on the whole to be excluded, but all of Aristotle himself was to be retained. Fifteenth-century logic is still medieval, but sixteenth-century logic becomes strictly Aristotelian.

However, there is a second difference to which I have so far paid little attention, and which concerns the language used in logical writings. Medieval logicians treated Latin as a technical, almost artificial language. They were deeply concerned with the effects that different word-orders and the addition of extra logical particles had on both meaning and reference, and they frequently tried to express semantic differences through different syntactic structures. The clearest indication of their attitude toward language is the place of sophismata in their logical texts, for sophismata are precisely those tortured fragments of language which best illustrate or raise logical problems;[163] and these logical problems are in turn solved by means of the tools provided by supposition theory and those doctrines allied to it. The language of a post-medieval logician, whether humanist or Aristotelian, commentator or textbook writer, is totally different. Sophismata have completely disappeared, and so too has any attempt to treat Latin as a technical language in which different word-orders represent different logical structures. The propositions used for such operations as syllogistic conversion are presented in an already fully

quia ita est: sed quia verisimile est in actione."

[162]Niphus, *Dialectica ludicra*, f. 111 rb.

[163]For Juan Luis Vives's attack on sophismata, which includes a wild variety of examples (many, I think, invented by Vives) see Guerlac, *Juan Luis Vives*, passim.

standardized form, and they are always relatively simple propositions devoid of repeated logical particles or logical particles found in curious places. Insofar as the language used is complex, it is because the logician is striving to be Classical in style and vocabulary, not because he sees Latin itself as a logical tool. As Luce Giard has put it, "[...] logic no longer bends language as an object to its own uses; rather it itself bends before the uses of language. From being normative and technical, logic becomes descriptive and in a sense pragmatic."[164] Why these changes came about is a delicate question, not easily to be resolved. Humanism coexisted too long with medieval logic for humanism to be the sole explanation, and the return to Averroes and Aquinas shows that mere revolt against anything medieval is not a sufficient explanation either.[165] Changes in grammar teaching;[166] changes in the relation of logic to the study of natural science;[167] and changes in other parts of the university curriculum presumably have a good deal to do with the appearance of a new style of logic.[168] So too do Reformation, Counter-Reformation and the resulting changes in theological studies. The logic needed for the study of Peter Lombard's *Sentences* was not the logic needed for reading the Bible or the Church Fathers. But whether or not reasons can be given for them it, is still quite clear that dramatic changes in logical writing took place in the sixteenth century.

[164]L. Giard, "Du latin médiéval au pluriel des langues, le tournant de la Renaissance," *Histoire Épistémologie Langage* 6.1 (1984), 45. Her words (which I have translated rather freely) are worth quoting in their full context: "Ce qui est en jeu dans ce refus de la logique scolastique, ce n'est pas une discussion de son contenu sur le fond, mais la revendication d'un nouveau statut pour la langue traitée. Dans cette perspective, la saisie de la langue ne peut plus être abandonnée à des instruments théoriques issus ou reçus directement de la logique. En conséquence, le rôle de la grammaire change lui aussi: elle devient un savoir qui doit recenser et expliciter les règles de l'usage de la langue pour les offrir, dans un second temps, à la logique, obligée, pour sa part de s'en accommoder. La logique ainsi entendue ne plie plus la langue-objet à son usage, elle se plie à ses usages; de normative et technicisée, elle devient descriptive et pragmatique en un sens."

[165]Schmitt, "Philosophy and Science in Sixteenth-Century Italian Universities," pp. 316–317.

[166]See Heath.

[167]For sophismata and natural philosophy in the later middle ages, see Edith Sylla, "The Oxford Calculators" in *The Cambridge History of Later Medieval Philosophy*, pp. 540–563.

[168]For some remarks on the curriculum at Oxford, see Ashworth, introduction to Sanderson, *Logicae Artis Compendium*, pp. XXXII–XXXV. For a full discussion of Oxford, see *The History of the University of Oxford. Vol. III. The Collegiate University*, ed. J. McConica (Oxford: Clarendon Press, 1986).

RELATIONAL LOGIC OF JUAN CARAMUEL

Petr Dvořák

1 INTRODUCTION

Juan Caramuel y Lobkowitz, born in Madrid in 1606, was educated at Alcalá (philosophy) and, as a member of the Cistercian religious order, in Salamanca (theology). After spending some time at various universities in Spain and Portugal, he settled in Louvain for several years where he received a doctorate in theology (1638). There he sided with the Jesuits in a famous controversy over Jansenism. In 1647 he became an abbot of a Benedictine abbey in Prague, Bohemia, and, somewhat later, the vicar general of the Archbishop of Prague (not the archbishop, as is sometimes mistakenly supposed). Being active in the political struggles of his time and carrying out the project of re-Catholicisation perhaps too vigorously, he made himself many enemies even within the Catholic camp. Consequently, he left Central Europe for Italy in 1657. There he was appointed a bishop, first of a rather poor diocese in Southern Italy, in an exile of sorts far away from the major centers of power, later in Vigevano, near Milan, where he died in 1682.

Being an extremely prolific writer, Caramuel produced well over 50 different works of varying length (and quality), from short *opuscula* to thousand-page volumes, some of them appearing in several editions. These works range from treatises in the fields of mathematics and logic, through works on linguistics, music, art and architecture, to books on political theory, philosophy and moral theology. He is chiefly known as a defender of probabilism, a doctrine of moral theology which fell into disfavor in the latter half of the 17^{th} century (cf. [Fleming, 2006]). This might be one of the reasons why his work in other fields, namely logic, did not exert the influence it deserved in his own time and in later centuries. A second reason for neglect is undoubtedly the fact that the *locus classicus* of formal logic in Caramuel, his *Theologia Rationalis* (1654), appeared under the title of "Rational Theology" rather than "Rational Philosophy" (see below), thus perhaps remaining largely undiscovered by the logician.

There are two areas of particular interest in Caramuel's logic: his "moral logic", i.e. a thorough analysis of arguments in legal discourse, but, above all, his relational logic, the first systematic attempt at non-reductive formal logic of relational statements and arguments built from them aimed at extending the powers of syllogistics. The following contribution will introduce Caramuel's logical work in general (Chapter 2) and its most important aspect, the so-called oblique or discrete logic, in particular (Chapter 3).

2 CARAMUEL'S LOGIC

"Rational Theology", an anthology of sorts, brings together more or less independent logical treatises, some of which were published separately either prior to 1654 or later.[1] Designating a set of works in the fields of formal and applied logic, the title appears to be misleading for two reasons: first, the relation of the content to Aquinas and his principal *Summa* is rather unclear and vague at best. Second, the title intended by the author, *Philosophia Rationalis*, was changed at will by the printer, as Caramuel reports in the printed list of his works composed in Vigevano.[2]

Theologia Rationalis consists of two main parts, *Praecursor Logicus*[3] and *Metalogica*.[4] Let us introduce briefly the former part first. From the standpoint of formal logic, not all works included in the first part of "Rational Theology" are of equal interest. *Grammatica Audax* ("The Bold Grammar")[5] deals with grammar and contains only a brief summary of the standard logic of the time.[6] On the other hand, *Logica Vocalis, Scripta et Mentalis* ("Spoken, Written and Mental Logic") is a fully-fledged logical work presenting a scholastic Aristotelian-style "upright" logic (*logica recta*).[7] Logic treats of language and the emphasis is put on its spoken form. Hence, "Spoken Logic", presenting the material in the classic order of term-proposition-argument, is the most extensive. By contrast, "Written and Mental Logics" add issues peculiar to their specific modes of language, either written or mental. The view that logic primarily deals with language rather than concepts clearly betrays a nominalist approach. The formal organization of the material into the levels of language — spoken, written and mental — appears to be novel.

It is only *Logica Obliqua*, however, (abbreviated as *LO* below) which marks a truly original contribution to the field in its content. It amounts to a systematic

[1] *Theologia Rationalis* was published under two full titles: "Juan Caramuel's Rational Theology or Liberal, Philosophical and Scholastic Meditations, Notes and Observations on the Golden Summa of the Angelic Doctor in Two Volumes", and "Rational Theology Explaining The Bold Grammar, Spoken, Written, Mental, Upright, Oblique and Herculean Dialectics as well as Metalogic through Examples Human and Divine and Making Clear the Entire First Part of the Angelic Doctor['s Summa] in the Same Course and Work".

[2] J. Caramuel, *Ioannis Caramuelis [...] opera omnia, quae prodierunt in lucem: interseruntur etiam libri aliqui, qui ultimam manum subierunt, et lucem opportunam exspectant*, vol. XVI, *Miscellanea in-folio*, in Biblioteca Universitaria di Pavia.

[3] *Praecursor Logicus* proper consists only of *Grammatica Audax*: *Caramuelis Praecursor Logicus Complectens Grammaticam Audacem, cuius Partes Sunt Tres...* Caramuel speaks, however, of *Herculis Logici Labores Tres* as *Praecursoris Logici Pars Altera*, so one can assume that *Grammatica Audax* and the various "Logicae" form *Praecursoris Logici Pars Prima*. Hence the entire four-volume structure could be rightly denominated *Praecursor Logicus*.

[4] Re-issued independently in 1682 as *Metalogica Scholastica* with book X omitted from the volume.

[5] Published independently in 1651.

[6] *Grammatica Audax*, Pars III Critica, med. I Logica, pp. 65–74. One should notice the quantification of the predicate term in the part devoted to contemporary logic, for this is an important presupposition of Caramuel's own relational logic.

[7] Re-issued separately under the title *Logica vocalis, scripta, mentalis, obliqua*, Vigevano 1680.

presentation of the logic of binary relational statements on the levels of proposition (Part I) and argument (Part II). We shall take up the matter below in rather more detail.

Herculis Logici Labores Tres ("Three Labors of Hercules, the Logician") appears to be an interesting collection of three studies devoted to three logical problems.[8] The first Labor called *Nil-negans* ("Negating nothing") reduces the system of valid modes of the syllogism based on the reduction of the negative copula to the negation of the predicate, i.e. term negation. Caramuel appears to have defended this so-called infinite logic (negated terms are called "infinite") as part of his bachelor exams in philosophy at Alcalá, early on in his career.[9] The second Labor entitled *Non-omnis* ("Not every") deals with the issue that in natural language — Latin — the compound expression *non omnis* (not every) is equivalent to "some... not... and some...," for instance, "not all students smoke" is equivalent to "some students do not smoke and some do", rather than to "some...not..." only, as is the purely logical meaning. It seems that the meaning of the natural language quantifier includes something over and above the logical one. Caramuel explores this idea and shows the logical properties of statements including Latin *non omnis*: equivalences and oppositions of these statements as well as valid modes of syllogism. Finally, the third Labor under the name *Contingens* ("Contingent") is a systematic treatment of modal logic. The basic idea is that *contingens* (contingent) is not equivalent solely to "possible", but "possible and possible not".[10]

The second part of Rational Theology, *Metalogica*, or — as the full title reads — *Metalogica: Disputationes de Logicae Essentia, Proprietatibus et Operationibus Continens* ("Metalogic Containing Disputations on the Essence of Logic, its Properties and Operations"), rather than being a work in formal logic, is a set of treatises, formally books, in the fields of the philosophy of logic (Book I), philosophical logic (Books VII, IX), metaphysics (Books II, III, IV, V, VI, VIII, X) and informal logic (Book X). The topics covered range from the nature of logic, the theory of proposition and semantic paradoxes to issues of a metaphysical kind (beings of reason, universals, individuation, self-evident principles etc.). The last book, tenth in number, called *De Severa Argumentandi Methodo* ("On a Rigorous Method of Argumentation"), is the most extensive of all.[11] The content ranges from informal logic to logic applied to specific problems in philosophy and theology.

Why is Metalogic needed, according to Caramuel? He draws a parallel between logic and music, both directing sound, but for a different purpose. As the knowledge of the production of musical sound, music, needs musical theory ("metamusic"),[12] so does the knowledge of the production of language sound — logic — need metalogic. One might object that the knowledge of the production of language sound is grammar, rather than logic. According to Caramuel, the lat-

[8] Sometimes listed as published independently in 1651, which is doubtful, however.

[9] Belazzi [1982], p. 17.

[10] This was by no means obvious at the time; cf. [Roncaglia, 2003]. For a detailed treatment in Caramuel cf. [Dvořák, 2005].

[11] It appeared as an independent work in 1644.

[12] Caramuel refers to A. Kircher's *Musurgia universalis* and calls it "metamusic".

ter characteristic applies to both disciplines; they differ as to their goal, however. While the goal of grammar is to speak correctly, the goal of logic is to discover truth.[13] Hence, grammar also needs its "metagrammar".[14] The goals Caramuel assigns to metalogic are partly those dealt with today in the philosophy of logic and partly in philosophical logic and metaphysics. This has to do with an alternative understanding of metalogic in Caramuel as dealing with topics belonging to both logic and metaphysics. Hence, metalogic is metaphysical logic, rather than the philosophy of logic (meta-logic).

What is the nature of logic according to Caramuel? Logic is the ability (faculty) to coordinate phonemes and characters in language and the corresponding ideas in the mind. By this, a natural order of things is saved.[15] Hence, for Caramuel, the order of language corresponds to the order of things. One cannot coordinate words or infer statements from other statements at will. Caramuel is thus a naturalist rather than a constructionist in his philosophical outlook on logic. Also, logic is both the faculty (*facultas*) as well as a set of rules (*methodus*) given to a specific purpose. Logic gives the rules of coordination on various levels (apprehension, judgment and argumentation), directing linguistic and mental operations with the goal of manifesting or inferring truths hitherto unknown. As is commonly the case in scholastic logic, Caramuel does not differentiate semantics from formal logic proper, for the rules concern both semantic content rules of compossible combinations of expressions as well as the rules accounting for logical form. The subject of logic is also dual: linguistic expressions both spoken and written as well as the mental operations or acts behind them. Even though Caramuel devotes the greatest amount of space in his logical writings to analysis on the language level (comparing the relative sizes of his "Spoken" and "Mental Logics"), a psychologist, mentalist streak in Caramuel's conception of logic clearly cannot be brushed aside.[16]

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[13] And the goal of rhetoric, also having to do with the production of language sound, is to speak ornately.

[14] This is what Caramuel sets out to provide in his *Grammatica Audax*. To speak correctly means, among other things, to avoid ambiguity. Caramuel, recognizing at least five senses of "being" and consequently regarding the verb "to be" (*esse*) as vague and ambiguous, coins a host of new expressions (*sare, sere, syre, sore, sure*...) to capture the various subtle meanings projected onto the term by the Scholastics, thus making the language of theology and philosophy more precise, in the hope of resolving scholastic dilemmas in these disciplines. He does this systematically in his later work *Leptotatos latine subtilissimus* (Vigevano 1681). For an explanatory account of this endeavor cf. Sousedík [1991]. This seems to be a key part of Caramuel's universal grammar.

[15] *Metalogica*, Lib. I, disp. I, pp. 3-4.

[16] *Metalogica,* Lib. I., disp. VI., § 2, p. 30: *Pono secundo nomine Logicae a nobis hanc intelligi scientiam, quae si Mentalis sit, tres dirigat mentis operationes: si Vocalis, tres lingua operationes illis correspondentes: et tandem si Scripta, tres correspondentes linguae et menti.* („Secondly, I claim that under the name of "logic" we will understand such a science, which — if mental — directs three operations of the mind, if spoken, [directs] three language operations corresponding to the mental ones, if written, [directs three operations] corresponding to [the operations] of language and the mind").

The talk of compossible combinations of expressions reminds one of possible worlds discourse. Indeed, Caramuel speaks of possible worlds (*mundos possibiles*) in *Metalogica* in relation to divine knowledge.[17] As has been stated, book X of "Metalogic" applies logic to various philosophical and theological problems. His aim is to clarify the various logical stages in God's knowledge and thus draw a parallel between human knowledge related to decision-making and that of God. As one knows (i) possible characters, say *a, e, i, o,* and *u*, (ii) the production of this or that character conditional on the knowledge of movements of the pen and (iii) the actual character to be written based on the decision which character to write, so God knows (i)' possible worlds *a, e, i, o, u*, (iii)' that *e* is or will be realized while *a, i, o, u* will not, based on the previous decree of his will. Strangely enough, the theological parallel of (ii), i.e. (ii)', God's conditional (or middle) knowledge, is not mentioned (probably because Caramuel regards it explanatorily superfluous, as is clear from his adherence to the Thomist theory of physical pre-determination).

We see Caramuel use possible worlds in theological models and theories.[18] Possible worlds also serve in philosophical explanations, e.g. in explaining the ontological status and foundation of *possibilia*. And once again, a parallel between grammar and metaphysics is used. Characters now represent neither possible worlds, nor *possibilia*, but perfections: as characters combine into words, so do perfections into possible creatures. As words combine into statements and those into books, so creatures combine into possible worlds (from today's perspective we could regard statements as corresponding to states of affairs). The books actually written represent the actual world (*hunc mundum*), and those capable of being written all possible worlds (*omnes possibiles*).[19] For Caramuel, the rules governing the combinations of perfections and creatures (determining what is possible) are probably ultimately grounded in the divine essence, as his example shows, for he sees God as identical with all possible (simple) perfections, thinking all possible strings of perfections (*possibilia* and possible worlds), while creating some of them

[17] *Metalogica,*, Lib. X, Theological Theses (on God's omniscience and omnipotence), Thesis I, p. 190ff. For Caramuel on possible worlds cf. Dvořák [2000].

[18] Earlier on in *Metalogica* (Lib. X, Pars II, Positiones Metaphysicae, Thesis VIII, p. 74) one can find a neat theological speculation employing *possibilia*; this time to clarify and understand matters of dogmatic nature. Consider the following application: when God wanted to create Adam, A, he first came to know, by the first type of knowledge – the knowledge of simple apprehension — a great number of people- whom he could have created had he so wished: B, C, D... etc. (note that here individual characters stand for *possibilia*, i.e. individual occupants of possible worlds). By the same act of knowing by which God knows A, he also knows the whole of his posterity. Since effects become individual through their causes, no descendant of Adam could have been created by any descendant of B and vice versa. By choosing A God relegated B, C, D...etc. to the ranks of the purely possible. Yet Adam sinned, and the Original Sin afflicts all of his descendants, future ones as well as merely possible ones. This does not afflict the descendants of B, so God chose the Virgin Mary, the mother of God, who was exempt from the Original Sin, from among B's posterity, implanting her miraculously among the descendants of Adam. By doing so he suspended the validity of the principle of the essentiality of origin (effects being individuated by their causes), to put it in Kripkean terms. Thus in Caramuel, the principle is only physically necessary, not logically necessary.

[19] *Grammatica Audax*, Pars III, med. IV, art. VI, p. 119.

ad extra as someone pronouncing various combinations of characters (speaking) out loud. This is consistent with Caramuel's naturalism in logic mentioned above.

Yet Caramuel also employs the idea of possible worlds and *possibilia* in order to capture the meaning of modal expressions in "Three Labors of Hercules, the Logician". The term used here is somewhat different, i.e. "another world" (*mundus alter*).[20] While exploring the various meanings of "possible" he is making use of the then familiar distinctions within *possibilia*, yet he uses them in order to clarify both the ontological statuses of both this, i.e. the actual, world, and the other world. Caramuel says that when "possible" is used in the pure sense, then this world is not possible, while the other is. The other world does not exist in fact (*de facto non est*), yet could be created (*sed condi potest*). When "possible" is used in the abstract sense, then both worlds — the actual as well as the other — are possible in this sense.[21]

It comes as no surprise to learn that Caramuel, famed for his moral theology, also dealt with logic in moral and legal argument. This is to be found especially in his *Pandoxion physico-ethicum* ("Physico-ethical summary of belief") (1668) and somewhat extended in *Moralis seu Politica Logica* ("Moral or Political Logic") (1680). One would expect to find some predecessor of deontic logic in Caramuel's treatment, but the nature of legal argument, as Caramuel sees it, rather gives rise to (i) the logical analysis of the scale or gradation of natural language quantifiers and (ii) some considerations in epistemic logic. For in moral or legal argument, what one finds is very often an inference from a statement where fewer than all members of a certain class are subjects of predication to a statement whose truth is consequently only more or less probable. *Syllogismus iuridicus seu moralis* consists of a general major premise, which states some moral or legal norm (*ius*), while a singular minor states some fact (whose occurrence is to be proved at the trial by the accuser). The conclusion, also singular, is then the sentence.[22] For instance:

1. Whoever injured some person with the intention of causing death is to be guilty of murder and sentenced to death
2. Peter injured John with the intention of causing death

Therefore, Peter is to be guilty of murder and sentenced to death

The minor consists of two parts, the first (Peter injured John) is established by the testimony of the witnesses, while the second, intention, is to be proved by another moral or legal argument:

[20] *Herculis Logici Labores Tres*, *op. cit.,* p. 49ff.

[21] According to Caramuel, the difference could also be captured terminologically: while in the pure sense a world (another world) is said to be *in* potency in relation to existence, in the abstract sense a world (this world) is said to *have* potency to exist.

[22] *Moralis seu Politica Logica*, Lib. I, Logicae Moralis Synopsis, sect. XVI, pp. 41-45.

RA 1. Rarely anyone (almost noone) boasts of a death crime which he or she did not commit.

PI 2. But Peter boasted of inflicting on John a serious injury in a particular way

DI. Therefore, it would be reckless to believe that he did not accomplish the crime

The abbreviations "RA", "PI", "DI" stand for syllables indicating the quantity of the particular statement and hence, taken together, the mood of the syllogism (all being of the first figure). Caramuel gives the following table:[23]

Abbreviation	Quantifier	Mood	The mode of the conclusion
CA	absolutely all (*cuncti omnes*)	Camilli	really certain
FA	almost all (*fere omnes*)	Fallitis	morally certain
PL	most/majority (*plures*)	Placidi	more probable
MA	half (*media pars*)	Magistri	equally probable
PA	a minority/not half (*pauciores*)	Paridis	less probable
MU	many (*multi*)	Mugavit	scarcely probable
PU	few (*pauci*)	Pudici	reckless
RA	almost no (*rari fere nulli*)	Rapidi	reckless
NO	no (*nulli*)	Nobilis	against manifest truth

A great amount of *Moralis seu Politica Logica* concerns what we would today call the philosophy of law. However, in the second book, Caramuel analyses the so-called moral modes and the logical properties of propositions containing them: "it is certain that" (*certum esse*), "it is probable that" (*probabile esse*) and "it is doubtful that" (*dubium esse*).[24]

There does not seem to be any treatment of deontic logic operators in Caramuel's "Moral or Political Logic". The closest one gets might be the operators "must" (*debeo*) and "can" (*possum*), which Caramuel seems to be using in their non-deontic meaning, however. He suggests, for the sake of simplicity and ease, to reduce "it is necessary that" (N), "it is impossible that" (I) and "it is contingent that" (C) to "must" (D) and "can" (P). Caramuel presents the following

[23] *Ibid.*, p. 44. Caramuel does not seem to distinguish corresponding positive and negative scales. For instance, one can say very many, i.e. almost half (positive), or a minority, i.e. less than a half (negative), in relation to the same number.

[24] *Ibid.*, Lib. II, cap. 2, sect. IX, p. 149.

equivalences (where "Pos" means "it is possible that"):[25]

$$N \leftrightarrow D$$
$$N \leftrightarrow \neg P \neg$$
$$I \leftrightarrow D \neg$$
$$\text{Pos} \leftrightarrow P \leftrightarrow \neg D \neg$$
$$\text{Pos} \neg \leftrightarrow P \neg \leftrightarrow \neg D$$
$$C \leftrightarrow P \wedge P \neg \leftrightarrow \neg D \wedge \neg D \neg$$

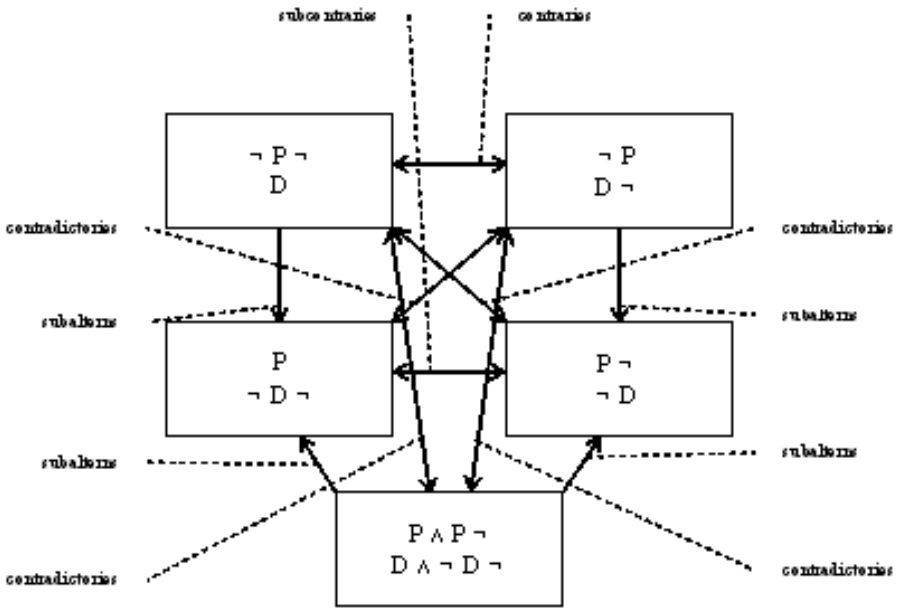

Figure 1.

The latter modals could easily be assigned their deontic meaning with their logical behavior essentially remaining the same. This is precisely the case in the third Labor of the "Three Labors of Hercules, the Logician" entitled "Contingent", containing a somewhat more extensive discussion of the various modes treated explicitly as deontic.[26] Hence *debeo* is to be rendered as "should" or "ought

[25] *Ibid*, Lib. I, Logicae Moralis Synopsis, sect. X Doctrina de Modalibus ad Summam Reducitur Facilitatem, pp. 36-37. The abbreviations are mine.

[26] *Herculis Logici Labores Tres*, Labor III *Contingens*, Pars Ulterior, art. II De Modis Debiti et Obligationis, p. 56-58.

to" (obligation), rather than "must" (necessity), with the corresponding *possum*, "can", taken as permission. Caramuel also seems to be using "must" in the same context: "it is necessary that I do", but he does not differentiate its usage from obligation ("should"). This could be understood as non-deontic necessity or compulsion opposed to ability (see below). So Caramuel discusses "should (do)", "can (do)" and "must (do)" (*debeo facere*; *possum facere*; *necesse est, ut faciam*), focusing on "should" whose analysis is the same as above. "*Facere*" ("to do") makes explicit that the meaning has to do with human action in contrast to "*esse*" ("to be") used in the examples in "Moral or Political Logic", where the meaning was "real", i.e. metaphysical and physical. The "can" of ability clearly re-surfaces in Caramuel's discussion of the relationship between "can" and "should", or, more precisely, the inference from "can" to "should". Caramuel admits this inference on analogy with the inference from possibility to actuality in necessary matter (i.e. some necessary connection between the subject and the predicate is possibly true, therefore, it is true). He admits the argument from the existence of a certain norm obliging a particular person in conjunction with the ability of this person to obey the norm to the conclusion that this person should obey the norm. It is clear that Caramuel understands "can" in the sense of the absence of some hindering conditions, in the non-deontic sense of ability, not in the sense of permission that could stand for *non debeo non facere*. Hence, the inference he has in mind is an altogether different inference from the immediate illegitimate inference from permission to obligation. To put it in a nutshell, Caramuel explicitly distinguishes between real modes and modes having to do with human action, arguably not discriminating within the latter group, at least not explicitly, between the deontic modes of permission and obligation on one hand and ability and compulsion on the other. Consequently, he does not discuss the interrelations among these modes.

In the same work Caramuel analyzes the epistemic mode of knowledge (*scio*) and the mode of willing (*volo*). Both are treated on a par with the universal affirmative, and again the opposing forms of modals within the logical square are formulated using negation in the same way as with *debeo* ("should" or "must") above.[27]

And once again, there follows the analysis of modes having to do with evidence and probability.[28] In relation to probability, the oppositions examined hold between different degrees of probability, and thus they really concern the respective quantifiers used: *mere, magis, minus, aeque*. It is clear that the latter three quantifiers are relational in the sense of essentially involving comparison between probable propositions. The examination of comparative degrees of probability, and probability in general, amounts to the logical background of Caramuel's moral-theological theory, namely that of probabilism, opposing probabiliorism. According to the latter, out of two competing moral norms in a particular situation of moral choice, in the absence of proof of either of the two, one is to opt for the more probable. Caramuel disagrees: one is morally justified to opt for either of

[27] *Ibid.*, p. 58.
[28] *Ibid.*, art. III, p. 59-61.

the two provided that it possesses at least a minimal degree of probability.

Finally, a mixture of epistemic and theological modes is dealt with: self-evident proposition, demonstrative, evident, morally certain, *de fide*. The middle part of this particular Herculean task ends with the discussion of theological modes having to do with censure: different degrees and types of erroneous doctrinal propositions.[29]

3 RELATIONAL LOGIC

By far the most interesting contribution of Caramuel to formal logic is his oblique logic. A key part of this is what we would call relational logic dealing with the formal structure of relational statements and their properties (Pars I of *Logica Obliqua*, pp. 407-429) and the theory of argument consisting of such propositions (Pars II, pp. 429-503).[30]

Propositio obliqua seu discreta or relational statement is a statement including a complex predicate which could be further analyzed and whose analysis is important from the point of view of securing and determining validity within an argument. Apart from a verb,[31] this complex predicate consists of the so-called *terminus obliquus*, oblique term, grammatically a noun in a case other than the nominative (*casus obliquus*). For instance, in the statement "every man commits some sin" the term "sin" is oblique, for the Latin equivalent *peccatum* would be in the accusative form. In contrast, the subject term "man" is in the nominative (*terminus rectus*). Thus oblique logic contrasts with "upright" logic dealing with statements in which both terms, the so-called extremes, are *recti* or upright.[32] The oblique predicate term or extreme, called *connotatus* by Caramuel, is what we would call the second member of a binary relation, while the upright subject term or extreme, called *connotans* by Caramuel, would be the first member.

As for the form of oblique statements or propositions, Caramuel recognizes three types or genera: The first type, called preposition by Caramuel, places the oblique

[29] *Ibid.*, art. IV and V, p. 61-67.

[30] Properly speaking, *logica obliqua* covers more than relational logic, for Caramuel treats also propositional logic under the same heading (the so-called "hypothetical" propositions). The meaning of the term "discrete" will become clear below.

Caramuel's relational logic was presented for the first time in the studies of S. Sousedík (and K. Berka) in the 1960s and early 70s, cf. Sousedík [1969] and Berka–Sousedík [1972]. Since then only very few studies have appeared, for instance, Hernández [1992]. So far the most extensive treatment can be found in Dvořák, [2006]. Besides relational logic, the latter monograph also explains the problem of *non omnis* in Labor II of the Herculean Labors and selected issues from *Metalogica*, e.g. logical paradoxes (*insolubilia*), change of propositional truth value, future contingents, etc. It also covers an interesting polemic of Leibniz against Caramuel on *antistrephus*. For the truth-value of propositions about future contingents in particular cf. Dvořák [2002].

[31] The verb is called "adjective" in contrast with the copula "to be" which is traditionally called "substantive", for it was viewed as a substance of which the predicate is an accident. Hence Caramuel calls statements of this type "adjective".

[32] It is quite interesting to note that in Caramuel's work an analogical contrast exists within his theory of architecture, i.e. that between *architectura recta* and *obliqua*. Cf. J. Caramuel, *Architectura civil recta y oblicua*, Vigevano 1678.

term or *connotatus* into subject position, the second type, called postposition, into predicate position, the third type, called composition, into both. Caramuel presents the following examples (the oblique term is in the brackets):

(i) The soul which committed a sin will die (sin)

(ii) The soul aspires to glory (glory)

(iii) The soul which committed a sin will be sentenced to eternal damnation (sin, eternal damnation)

One can see that the division into subject and predicate concerns the principal predication. In the first and third examples, the oblique term is a part of an embedded predication attached to the principal subject. The first example is thus an upright statement with an oblique predication embedded within it. In contrast, the third example is an oblique statement with an oblique predication embedded within it.

In his treatment of oblique statements, Caramuel concentrates on the second type, i.e. postpositions, with some treatment of type one, prepositions, leaving compositions out.

Caramuel in *Logica Obliqua* extends syllogistics to include relational logic, i.e. to handle syllogisms consisting of relational statements, either some or all:

> Every man commits some sin
> *Every sin is a moral fault*
> Therefore, every man commits some moral fault
>
> Every ant is greater than every atom
> Every elephant is greater than every ant
> Every elephant is greater than every atom

While the former is a specimen of an argument Caramuel calls mixed relational syllogism (*syllogismus obliquus mixtus*), for, apart from relational statements in a premise and the conclusion, it contains an upright premise (*propositio recta*), the latter is of a pure relational form (*syllogismus pure obliquus*). Prior to Caramuel these oblique syllogisms were typically reduced to non-oblique ones (Ockham, Jungius) and thus accommodated within the standard Aristotelian system of figures and moods. The reduction was based on the reduction of oblique statements into non-oblique or upright ones. The oblique form of a proposition, seen as somehow deficient, was thus from the point of view of formal logic not recognized as an independent form, different from a non-oblique one. Thus the resulting logic could be called the logic of relations but not relational logic proper. Only when the oblique form is recognized as a logical form in its own right can one speak of relational logic or relational syllogistics. This is what happens in Caramuel for the first time, though it is true that his more famous predecessor, Joachim Jungius (1587–1657), recognized a non-reducible form of relational argument or inference,

that of *a rectis ad obliqua* (from terms in the nominative to oblique terms), but regarded it as a type of non-syllogistic immediate inference.[33] A step further would be to say that the standard upright form is a special case of the oblique form; hence, really, the oblique form is paradigmatic for any statement. We shall see below that this is arguably the path Caramuel will eventually take.

There seem to be two possible ways of analyzing a relational statement, for instance "every man commits some sin":

every man is [some sin is committed by]

The first way regards the complex predicate consisting of a verb and an oblique term to be an embedded predication; hence the oblique term is regarded to be a (subordinate) logical subject. Hence, there are two subject-predicate structures present within a relational statement and, consequently, two *copulae*. This rather conservative view of the logical form of a relational statement accommodates the latter statements within a quantifier-subject-copula-predicate upright scheme, without reducing them to this form altogether. (The relational form is clearly seen as different from the non-relational one, but not substantially, as one can see). Caramuel takes this approach in his initial formal symbolism introduced to grasp the logical form of the relational statement. Apart from the signs for quantifiers "a", "i", "e" (all, some, no), the asterisk symbol * divides the subject part from the predicate part. For instance, "a*i" means (every... is some..., e.g. "every man commits some sin"). The introduction, however, of two symbols for negation, copula negation "ñ" and quantifier negation "n", and their possible positions in Caramuel show that Caramuel assumes there are two *copulae* in relational statements. The following are the possible places of negation:

(i) n-*-

(ii) -ñ*-

(iii) -*n-

(iv) -*ñ-

[33] For example: A circle is a plane figure. Therefore, whoever draws a circle draws a plane figure.

J. Jungius (Junge), *Logica Hamburgensis, hoc est, Institutiones Logicae In usum Schol. Hamburgensis conscriptae, et sex libris comprehensae*, ed. R. W. Meyer — J. J. Augustin, Hamburg 1957 (1^{st} edition 1638), Lib. II, c. 4, § 6; Lib. III, c. 1. § 5. For the logic of relations in Jungius cf. e.g. Ashworth [1967] and Dvořák [2006].

Caramuel also suggests the reduction of oblique forms into upright ones, but more for the sake of showing the usefulness of his new system, rather than as a necessary procedure to exhibit the true form disguised or clouded by the oblique form, as in Jungius. Cf. *Ibid.*, Lib. II, c. 16, § 1: *Crypsis est affectio Syllogismi oratione externa propositi, qua ejus forma ita occultatur, ut cum sit legitimus, talis tamen non esse videatur* ("Disguise is a property of a syllogism put forth in an external linguistic formulation, in which its form is so hidden that in spite of being valid, it does not appear so").

It is clear that (ii) and (iv) are both negations of the copula, but in each case a different one. The first stands outside the predicate structure, the second within it. "Every man does not commit any sin", añ*i, and "every man non-commits some sin", a*ñi, are different, for the first is equivalent to "every man non-commits every sin". Caramuel makes the former logical form with outside negation even more explicit: "añ,q*i", where "q" stands for the relative pronoun (*qui*): "every man is not the one, who commits some sin".[34]

In his relational syllogistics, however, Caramuel settles for another, less conservative and more innovative approach. The symbolism is simplified. The asterisk and relative pronoun signs are dropped. Thus "ai" means "every... some..." as in "every man commits some sin". The key idea in this analysis is the generalization of the copula from the semantically rather empty verb "to be" to any meaningful verb, e.g. "commits", amounting to a major step in the direction towards contemporary predicate logic. Hence, there is only one copula present in the relational statement. As for negation, only one symbol for negation is introduced, "n", either for quantifier negation (n- - and -n-), or for the negation of the (inner) copula (--n). There is no place for the negation of the outer, non-predicate copula, for none is needed. More precisely, since the form (ii) -ñ*- is equivalent to (iii) -*n-, one does not need (ii) and can do with (iii), second quantifier negation. This allows for the simplification, for two different negation symbols are no longer necessary. The position before or after the quantifier makes all the difference now.

If the verb "to be" is only one copula among many possible ones, the standard quantifier-subject-copula-predicate upright form is but a special case of relational statement quantifier-subject term-copula-quantifier-predicate term form. Thus, grammatically, even though there is no predicate quantifier in "every man is an animal", it is just not made explicit, but logically, it is present ("every man is some animal").[35] It is precisely this move of generalizing the copula and the consequent

[34] "a*ñi" could be rendered as "every man is the one, who does not commit some sin". Cf. *LO*, Pars I, disp. I De Propositione Obliqua in Genere, pp. 407-415.

[35] Caramuel's survey of contemporary logic in *Grammatica Audax* Pars III, art. 1 shows that the quantification of the predicate term is not an uncommon feature. Caramuel himself in *LO* Pars II quite naturally occasionally rewrites "a" with "ai". This would mean that the second from the two squares of opposition presented below is the traditional square of A, E, I and O statements. Caramuel could thus be credited with discovering another one.

Caramuel also speaks, however, of the extended number of moods in relational logic in comparison with the classical Aristotelian syllogistics owing to discrimination of two forms within the universal affirmative statement (A): either ai or aa: J. Caramuel, *LO,* Pars II, disp. IX, p. 432: *Quatuor isti modi constant ex universalibus et affirmativis, et tamen differunt inter se; ut ... intelligatur, quanto sit ditior obliqua Dialectica caeteris; siquidem quatuor aut pluribus modis praemissas suas disponit, quas omnes rejiceret antiqua ad* Bammada*, et si non cognosceret hunc modum, (non enim Aristotelicus sed Platonicus est) ad* Barbara. ("These four moods arise from universal affirmatives; however, they differ among themselves. In order ... to see to what extent the oblique logic is richer than others, as it disposes its premises in four or more ways, which ancient logic would all classify as the mood Bammada, or if it did not recognize this mood [for it is not Aristotelian, but Platonic], as the mood Barbara."). This would mean that the traditional square of opposition of A, E, I and O statements contains within itself both squares, as it were. The first alternative, however, appears to be more likely the case.

The duality pointed out might be related to the two ways of analyzing relational statements, for

view of relational form as paradigmatic which guides A. de Morgan in his *Formal Logic*, 1847 and later works in developing his relational logic, far more powerful than Caramuel's, because more general.

As A. de Morgan does, Caramuel recognizes eight basic logical forms of relational statements. These are gained from possible logical forms of relational statements based on equivalences. One can find two equivalence rules used in Caramuel's writing:[36]

1. nx ↔ y where x and y stand for contradictories (e.g. e and i)
2. xn ↔ y where x is any statement (e.g. e) and y is its contrary or subcontrary (e.g. a)

What we get are two equivalent eight-member sets of forms:[37]

ai	ee
aa	ein
ii	ine
ia	inin
ea	ain
ei	ae
ina	iin
ini	ie

Based on their logical relationships of oppositions, the forms in both of these sets could be arranged into two squares of oppositions (with the traditional relationships):

the second way is more akin to the strategy of making the predicate more precise by analyzing its embedded predicational structure, as in Fred Sommers (*The Logic of Natural Language*, Clarendon, Oxford 1982). In a marginal note Caramuel recognizes relational syllogism to have four terms; cf. *LO*, Pars II, disp. IX, p. 432.

[36] *LO*, Pars I, disp. I, De Aequipollentia Propositionum, pp. 411-412.

[37] *LO,* Pars I, disp. I, De oppositione obliquarum, pp. 412-413.

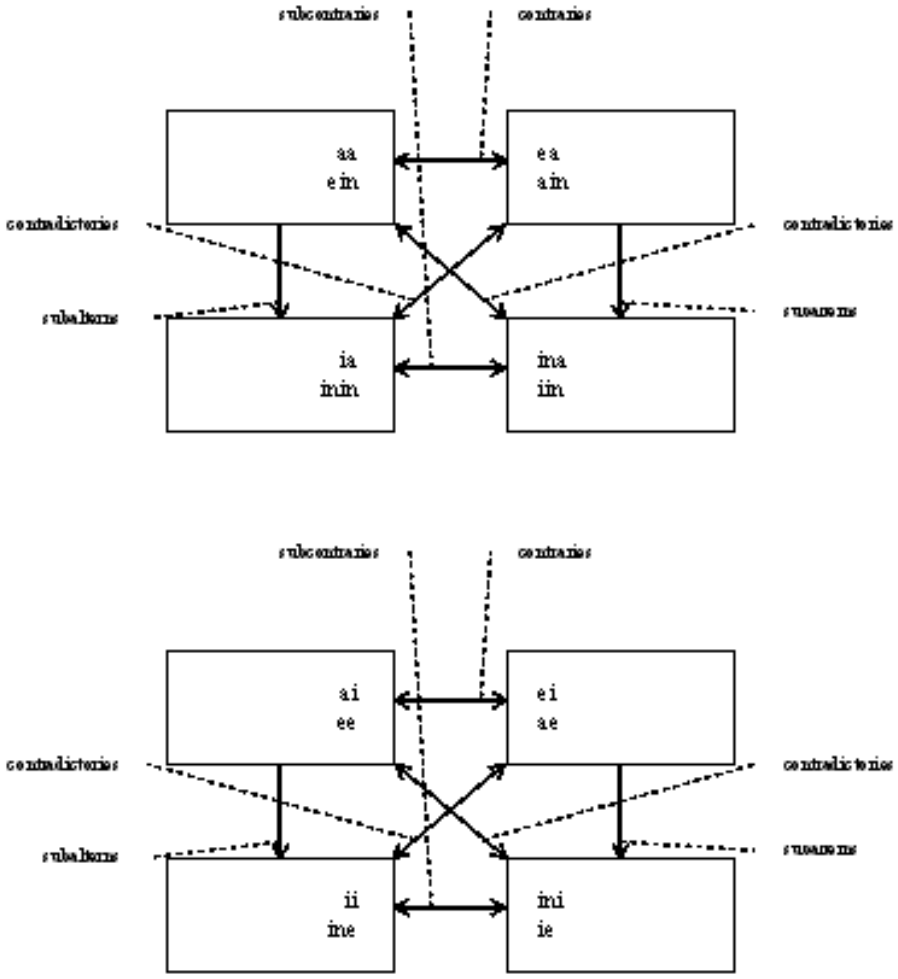
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Figure 2.

In dealing with relational syllogisms, Caramuel distinguishes the following figures of the pure relational syllogism:[38]

A is related to M M is related to B A is related to B	**Platonic figure**
M is related to A B is related to M B is related to A	**Figure I**
A is related to M B is related to M B is related to A/ A is related to B (indirect)	**Figure II**
M is related to A M is related to B B is related to A/ A is related to B (indirect)	**Figure III**

The letters A, B, M stand for terms, where M is the middle term and A, B are the extremes. All the terms are quantified and the quantification is given by the mood (see below). The expression "is related to" stands for a transitive verb, the extended copula. We can see that the logical form of the syllogism

> Every ant is greater than every atom
> Every elephant is greater than every ant
> Every elephant is greater than every atom

is that of Figure I.

Caramuel gives valid moods for each of the figures above. By way of an example, let us present the valid moods under Figure I:

1. **aa.aa.aa.**
2. aa.ai.ai.
3. aa.ia.ia.
4. aa.ii.ii.
5. ai.aa.ai.
6. ai.ai.ai.
7. ai.ia.ii.
8. ai.ii.ii.

[38] Caramuel deals with the pure relational syllogism in *LO*, Pars II, disp. IX Syllogismorum Pure Obliquorum Formas et Figuras Expendens, pp. 432-436. Only this type of syllogism is presented in Sousedík [1969].

We see that our specimen of a pure relational syllogism is of mood 1 (called *Parat astra famam* by Caramuel).

Our second specimen syllogism

> Every man commits some sin
> *Every sin is a moral fault*
> Therefore, every man commits some moral fault

is of the mixed type, as already known. The four figures could be distinguished also for the mixed relational syllogism.[39] Our example is of the most common Platonic figure:

> A is related to M
> M is B
> A is related to B

As we can see, the mood is ai.a.ai, and it is one of the valid moods Caramuel states for this figure. (Caramuel gives names to the moods; the one at issue is dubbed *Altis ab astris*):

	aa.a.ai.		ein.a.ai.
	aa.a.ii.	25.	ein.a.ein.
	ae.a.ain.		ein.a.ii.
	ae.a.ea.		ia.a.ii.
5.	ae.a.iin.		ian.a.iin.
	ae.a.ina.		ian.a.ina.
	ai.a.ai.	30.	ina.a.iin.
	ai.a.ein.		ie.a.iin.
	ai.a.ii.		ie.a.ina.
10.	ai.a.iin.		ine.a.ine.
	ain.a.ain.		ine.a.ii.
	ain.a.ea.	35.	ien.a.ii.
	ain.a.ina.		ii.a.ii.
	ea.a.ain.		iin.a.iin.
15.	ea.a.ea.		ina.a.iin.
	ea.a.iin.		ina.a.ina.
	ea.a.ini.	40.	ine.a.ii.
	ee.a.ai.		ine.a.ine.
	ee.a.ee.		ini.a.iin.
20.	ee.a.ein.		ini.a.ina.
	ei.a.ain.		
	ei.a.ea.		
	ei.a.iin.		

[39] Caramuel treats the mixed relational syllogism in *LO*, Pars II, disp. XI De Syllogismo Obliquo Mixto, pp. 442-457.

As has been said, apart from pure and mixed relational syllogisms in which the oblique statements were of the type called postposition, Caramuel now gives valid moods in all four figures of mixed relational syllogisms where the oblique statements are prepositions.[40] Recall that while postposition places the oblique term, *connotatus*, into the predicate position (of the principal predication, one might say, although postpositions do not contain any other predication, "principal" thus being redundant), preposition, containing embedded predication attached to the subject, places *connotatus* into the predicate position of the embedded predication. The numbering (and naming) of the moods is taken from postpositional mixed syllogisms. Caramuel points out the moods in which the change from postposition to preposition results in the mood being invalid. Some prepositional moods do not have correlates in postpositions. Consequently, they are listed without any number. By way of an example, let us present the prepositional moods only in the Platonic figure, since the postpositional moods in the same figure are given above:

M . . . invalid (*malus*)
B . . . valid (*bonus*)
P . . . dangerous (*periculosus*)

1.	aa*a.ai* M	24.	ein*a.ai* M
2.	aa*a.ii*	25.	ein*a.ein* M
3.	ae*a.ain* M	-	ein*a.ei*n M
4.	ae*a.ea*	26.	ein*a.ii*
-	ae*a.ae*	27.	ia*a.ii*
5.	ae*a.iin*	28.	ian*a.iin* B
6.	ae*a.ina*	-	ian*a.ii*n M
7.	**ai*a.ai*** M	29.	ian*a.ina*
8.	ai*a.ein*	30.	ina*a.iin* B
9.	ai*a.ii*	-	ina*a.ii*n M
10.	ai*a.iin* P	31.	ie*a.iin*
11.	ain*a.ain*	-	ie*a.ii*n M
12.	ain*a.ea*	32.	ie*a.ina* M
13.	ain*a.ina*	33.	ine*a.ine*
14.	ea*a.ain* M	34.	ine*a.ii*
15.	ea*a.ea*	35.	ien*a.ii*
16.	ea*a.iin* M	36.	ii*a.ii*
-	ea*a.ii*n B	37.	iin*a.iin* B
17.	ea*a.ini*	-	iin*a.ii*n M
18.	ee*a.ai* M	38.	ina*a.iin* B
19.	ee*a.ee*	-	ina*a.ii*n M

[40] Caramuel treats the prepositional relational syllogism in *LO*, Pars II, disp. XII De Praepositivo Obliquorum Genere, pp. 458-464.

20.	ee*a.ein* M	39.	ina*a.ina*
21.	ei*a.ain* M	40.	ine*a.ii*
-	ei*a.ai*a B	41.	ine*a.ine*
22.	ei*a.ea* B	42.	ini*a.iin* B
-	ei*a.ei* B	-	ini*a.ii*n M
23.	ei*a.iin* M	43.	ini*a.ina* M
-	ei*a.ii*n B		

If the upright statement is universal negative:

44. aa*e.aius* M
45. aa*e.iius*
46. ai*e.aius* M
47. ai*e.iius*
48. ia*e.iius*
49. ii*e.iius*

Incidentally, if we change our postpositional specimen syllogism

> Every man commits some sin
> *Every sin is a moral fault*
> Therefore, every man commits some moral fault

into prepositional

> Every man, who commits some sin, is evil
> *Every sin is a fault*
> Therefore, every man, who commits some fault, is evil

we see that it is one of those syllogisms where the change from postposition to preposition results in its invalidity (see mood 7 in Caramuel's list of postpositional moods in the Platonic figure above, which he names *Astris a validis*).

What has been said concerning relational syllogisms here suffices as an illustration of Caramuel's approach. Caramuel's is a logic which is not only an extension, but also a fundamental revision of the traditional syllogistics as such. Not only does Caramuel in his opinion present logic of types of discourse previously not treated (*Sum traditurus Novam Logicam nempe Obliquam, de qua Antiqui Dialectici nihil aut parum.* "I am about to introduce new logic, truly oblique, about which the ancient dialecticians [said] nothing or very little"), but he thinks syllogistics is based on a false principle if this is to be regarded as the most general logical principle:[41]

[41] J. Caramuel, *LO,* Lectori, p. 406. The text continues: *Sum, inquam, traditurus LOGICAM a veteri toto distinctam, et oppositis fundamentis inhaerens... ...majorem enim, quam Oraculi loco habent Peripatetici, falsam assero: et novam viam ingressus Terminis realiter distinctis utor, illisque novas propositiones et novos syllogismos fabricor.* ("I say I am about to introduce logic wholly distinct from the old one and based on opposite foundations ... I claim that the major premise — which Aristotelians take as a dogma — is false [Caramuel means the syllogistic principle of identity]. Having set out on a new way, I use really distinct terms from which I

Things, which are identical with some third, are identical themselves.[42]

In relational statements the terms are really distinct (discrete), not identical. Thus Caramuel also dubs his system "discrete logic". The most general principle of this logic could be hypothesized as the following:

* Things, which are *related* to some third, are *related* themselves.

It is only to be regretted that Caramuel's system remained little known and did not exert a greater impact on the logic of his time. One could only speculate how the discipline would unfold and what it would be like today had it been otherwise. It is our hope that this historical injustice is remedied as time goes on and the subtle genius of Caramuel achieves proper recognition at last.

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create new propositions and new syllogisms.") *Ibid.*, p. 406: *Non est animus diruere Antiquam LOGICAM ab aliis aut etiam a nobis summo studio firmatam: sed illi Novam hanc accensere, et numerum augere Musarum.* („I have no intention to destroy ancient logic established by others and also by us through greatest zeal, but to add to it this new logic and to extend the number of Muses")

[42] Cf. also J. Caramuel, *Theologia Moralis Fundamentalis...* Frankfurt am M. 1652, Lib. I, fund. XLV, § 1177.

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PORT ROYAL: THE STIRRINGS OF MODERNITY

Russell Wahl

1 INTRODUCTION

Logic or the Art of Thinking, popularly known as the Port-Royal *Logic*, was probably the most important logic text book from the time after the mediaeval period until the middle of the nineteenth century. Published anonymously, it was authored by two associates of the abbey of Port Royal des Champs, Antoine Arnauld (1612-1694) and Pierre Nicole (1625-1695). It is unclear how much each contributed to the work, although there are indications that Nicole wrote the first and second discourses and also the additions, and that the rest was very much a collaborative effort.[1] According to the forward, the initial draft was written in just a few days with the aim of teaching a student to master "everything useful about logic" in four or five days. This draft was expanded into a published version in 1662 and the book went through five editions during the lives of the authors and over fifty editions in French, thirteen in Latin, and nine in English before the twentieth century. The sixth English edition, entitled *The Port-Royal Art of Thinking*, was, according to its subtitle, used in the first class of the course of education pursued at the Universities of Cambridge & Oxford.[2] Arnauld and another associate of Port Royal, Claude Lancelot published (also anonymously) the *Grammaire générale et raisonnée* in 1660. This work is also known as the *Port-Royal Grammar*. Several points from the first part are taken from the *Grammar* and the first two chapters of the second part, added in the fifth edition, were taken straight from Part II, Chapter 2 of the *Grammar*.

While including discussions of many topics found in previous logic works, the Port-Royal *Logic* is primarily a statement of the new logic of the seventeenth

[1] This is from a remark by Racine, who also said that Arnauld wrote the entire fourth part. See the first note in [Arnauld and Nicole, 1662, 365], for the discussion of the composition of the *Logic*. There is also some further discussion in [McKenna, 1986] which reveals just how difficult it is to determine what was written by Nicole and what by Arnauld. All the page numbers for quotations from the Port-Royal *Logic* cited here will be from the critical edition by Clair and Girbal. These pages are the only pages cited where the work is not specified. This edition is based on the fifth (1683) edition with reference to variations in other editions, and has become the standard edition of the Port-Royal *Logic*. There is now a good translation by Jill Buroker, as *Logic or the Art of Thinking* [Buroker, 1996b], which includes the page references from Clair and Girbal.

[2] This is the subtitle on the sixth English edition listed in [Arnauld and Nicole, 1662, 9].

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century, the logic of ideas. Like other authors of the seventeenth century, Arnauld and Nicole were very critical of the old logic, which for them included not only the "scholastic" work on syllogisms, but also the humanistic logic of Ramus. They saw their logic as new and they were particularly influenced by Descartes, who had also been critical of syllogistic logic, and they incorporated many of his doctrines into their work. This new seventeenth-century logic, with its rejection of much traditional logic and its concern with clarifying ideas and determining the truth of simple propositions, has often been criticized by more recent logicians of mixing psychology and epistemology with logic. Often it has been compared unfavorably with mediaeval logic.[3] Despite the fact that most of its topics are closer to the mediaeval period, the Port-Royal *Logic* has a very modern feel to it, and covers several topics now included in more informal introductory logic courses, such as discussions of clarification of concepts, informal fallacies, causal reasoning, and probability, as well as more traditional accounts of propositions and syllogisms.

2 THE NATURE OF LOGIC

Descartes had not written a specific treatise on logic but had developed his method in the unpublished *Rules for the Direction of the Mind* and his later *Discourse on Method*. A manuscript of the *Rules* had been made available to Arnauld and Nicole, and they even incorporated a portion of this work in their own.[4] Descartes' focus was on clarifying simple ideas and developing a method which would lead to new truths. His view was that the mind had no trouble making inferences,[5] but that it did have trouble discerning what was before it clearly and distinctly. His method was developed to help the mind solve problems and more clearly discern truths. Descartes recognized two operations of the intellect which help it reach this goal, intuition and deduction. Intuition, he said, was not "the fluctuating testimony of the sense or the deceptive judgement of the imagination ... but the conception of a clear and attentive mind, which is so easy and distinct that there can be no room for doubt about what we are understanding" [Adam and Tannery, 1964-76, X, 368; Cottingham *et al.*, 1985-1991, I, 14]. He was less clear on what he

[3] [Kneale and Kneale, 1962, 315–320], is relatively more sympathetic toward the Port-Royal logicians, but [Bochesnski, 1961, 258], does not mince words when summing up classical seventeenth century logic: "Poor in content, devoid of all deep problems, permeated with a whole lot of non-logical philosophical ideas, psychologist in the worst sense." Norman Kretzmann's long article on the History of Semantics [Kretzmann, 1967, 378–379], is also quite critical of the Port-Royal Logic. [Ashworth, 1974] has some opening remarks which are also quite critical of logic during this period.

[4] Adam and Tannery state that Clerselier gave Arnauld and Nicole the manuscript of the *Rules* after the first edition of the *Logic* appeared in 1662. The portion from Rule 13 included in Part IV, Chapter 2, was included in the second (1664) edition of the *Logic* [Adam and Tannery, 1964-76, X, 352].

[5] Descartes is notorious for stating that "while our experiences of things are often deceptive, the deduction or pure inference of one thing to another can never be performed wrongly by an intellect which is in the least degree rational" [Cottingham *et al.*, 1985-1991, I, 12; Adam and Tannery, 1964-76, X, 365]. This was an opinion shared by Arnauld and Nicole.

means by deduction, but suggested that deduction is necessary, since "very many facts which are not self-evident are known with certainty, provided they are inferred from true and known principles through a continuous and uninterrupted movement of thought in which each individual proposition is clearly intuited" [Adam and Tannery, 1964-76, X, 369; Cottingham *et al.*, 1985-1991, I, 15]. Deduction for Descartes appears to involve the intuition of a necessary connection between two distinct ideas which are clearly intuited. The items intuited are said to be 'objects' [*objectum*] [Adam and Tannery, 1964-76, X, 365; Cottingham *et al.*, 1985-1991, I, 12] but also 'propositions' [*propositionis*] [Adam and Tannery, 1964-76, X, 379; Cottingham *et al.*, 1985-1991, I, 20]. In the *Discourse*, too, the method focused on attaining knowledge by breaking problems down to their simplest objects and then, moving from "the simplest and most easily known objects" to "ascend little by little, step by step, to knowledge of the most complex . . . " [Adam and Tannery, 1964-76, VII, 18; Cottingham *et al.*, 1985-1991, I, 120]. The examples he has in mind appear to be simple proportions that hold between line lengths.[6]

While Descartes was concerned about apprehending simple propositions and the relation between them, he was also concerned with clarifying simple ideas, and seeing relations between simple ideas. The apprehension of relations of different line lengths is taken as the model for clear and distinct perception, which is then used to explain immediate inferences from one proposition to the next.

Descartes' view of logic as a tool for the discovery of truths and his criticism of the old logic is present in the Port-Royal *Logic* as well. Throughout the *Logic* there is a criticism of formal logic as being useless for discovering truths. Arnauld and Nicole, however, did include discussions of such formal topics as conversion of propositions and validity of syllogisms. But even when discussing these topics, they were quick to remark that they view these matters of little importance. A striking indication of their anti-formalism is the complete lack of variables in the exposition. The authors give concrete examples of the various syllogisms and it is clear that what is at issue are questions of form, but they never present the syllogisms or categorical propositions with variables.

Arnauld and Nicole saw logic as "the art of thinking", which, with Descartes, they understood as the art of discerning the true from the false (15). Others of the time classified logic as the art of reasoning well,[7] but in the second discourse Arnauld and Nicole specifically rejected this characterization as too narrow: ". . . the purpose of logic is to give rules for all actions of the mind, and for simple ideas as well as for judgments and inferences . . . " (27). The rules in question are rules to help discern the true from the false (15). The rules for valid arguments will form part of this discipline, but only a small part. Arnauld and Nicole also included rules for the determination of the truth of simple judgements or ideas, and in fact, following Descartes, thought of this issue as the more pressing. However, unlike Descartes, Arnauld and Nicole did think that the rules for figures of syllogisms

[6]See [Adam and Tannery, 1964-76, VII, 20; Cottingham *et al.*, 1985-1991, I, 120-1]. For more on Descartes's theory of inference, see, [Gaukroker,1989] and [Grosholz, 1991].

[7]See the discussion in [Ashworth, 1974, 29].

have some use (22), but that more formal topics such as the demonstration of the validity of those rules, and the rules of conversion were less useful and were only included as exercises of the mind. In the first edition there was an inclusion of the reduction of the syllogisms to syllogisms of the first figure, but they omitted this in the later editions, judging it to be worthless.

Standard logic works were organized with sections on terms, propositions, and arguments and often a final section on method. This organization can be read into Aristotle's *Organon*, with the *Categories* dealing with terms, the *De Interpretatione* dealing with propositions, the *Prior Analytics* dealing with arguments and the *Posterior Analytics* and the *Topics* with method. This manner of dividing the topic was quite common in the medieval period. The Port Royal authors followed this pattern, but their choice of titles suggest that their concern was specifically with operations of the mind, rather than with language. They organize the *Logic* in the following way:

> Part I Containing reflections on ideas, or the first action of the mind, which is called conceiving.
>
> Part II Containing reflections people have made about their judgments.
>
> Part III On reasoning
>
> Part IV On method.

From this table of contents, the charge of psychologism appears to be justified, since it seems to suggest an investigation in the workings of the mind rather than in logic proper. However, while some irrelevant psychological considerations have crept into the discussion of logic, by and large they are kept to a minimum. It is true that, following Descartes, Arnauld and Nicole thought that our minds were structured so as to yield correct concepts and judgments when we use our reasoning correctly, and so they saw logic, or the art of thinking, as an art to help improve our natural cognitive faculties. Thus the section headings suggest that the cognitive faculties rather than the objects of those faculties are the topic of inquiry. However, the analysis Arnauld and Nicole give is of the concepts and propositions judged rather than the operations of the mind which lead to them. For the most part, considerations from empirical psychology do not infect their discussion of logic. However, as is often the case with works on informal logic even today, their discussions of errors in concept formation and in inference do make reference to psychological tendencies and even to theological concerns.[8]

While the contribution to anything we would now recognize as formal logic is small, the *Logic* does contain a more sophisticated semantics than it might first seem, and uses that semantics, especially the distinction between comprehension and extension made in the first book, to later use in their analysis of propositions and arguments.

[8]See the discussion of why people have certain obscure ideas in Part I, Chapters 9 and 10, and the discussion of fallacies in Part III, Chapter 20.

3 IDEAS OR TERMS

The first section of traditional logic books was generally a discussion of terms, or even words. The Port-Royal *Logic* begins instead with ideas. This is not as big a shift as it may seem, as certain medieval writers, for example Ockham, thought there were spoken, written and mental terms, and the latter were primary.[9] Arnauld and Nicole are not always consistent in their terminology, switching every now and then from "idea" to "term". The Port Royal use of "idea" indicates their view of the priority of thought over language. In the *Grammaire*, Arnauld and Lancelot defined words as "distinct and articulate sound which men have made as signs to signify their thoughts" [Arnauld and Lancelot, 1660, II, 1, 27]. This definition was used to give a reason for studying the words and was included in the *Logic* (103-4) when parts of the *Grammaire* were added to the section on judgments with the aim of clarifying the role of the verb.

The *Logic* begins with the claim that "the word *idea* is one of those that are so clear that they cannot be explained by others, because there are none more clear and simple" (39). Despite this pronouncement, Arnauld and Nicole immediately launch into confusions about ideas which they intend to clear up. They first remark that they do not mean images "painted in the fantasy", but rather "all that is in the mind when we can say truthfully we are conceiving something in whatever manner we conceive it" (41). Following Descartes, they rejected the view of Hobbes and Gassendi that ideas are simply derived from sense images. Like Descartes, again, Arnauld and Nicole give examples of such ideas as that of a chiliagon or God, which are not associated with an image. In a later work, *On True and False Ideas*, Arnauld made it clear that he thought of an idea of something as the mental perception of that thing. Like Descartes, he thought our minds, when used correctly, would correctly grasp the natures of things. The properties of those things would be contained, in Descartes' language, *objectively* in the ideas. So, for example, *omniscience* is contained objectively in the idea of God, and *having angles which add up to two right angles* is contained objectively in the idea of a triangle. While these remarks suggest ideas have a propositional content, Arnauld and Nicole maintained that ideas are prior to judgement and they identified propositions with judgements.

An idea can be clear and distinct or obscure and confused. Arnauld and Nicole again followed Descartes here, although they in fact blur his distinction between clarity and distinctness in their explanation. Descartes had said a perception was *clear* "when it is present and accessible to the attentive mind ... and stimulates it with a sufficient degree of strength and accessibility", and *distinct* when "as well as being clear, it is so sharply separated from all other perceptions that it contains within itself only what is clear" [Adam and Tannery, 1964-76, VIII -1, 22; Cottingham *et al.*, 1985-1991, I, 208]. The example Descartes gives of a clear idea which is not distinct is of a pain, for the pain is quite strongly present to the mind, but the idea is obscure because "people commonly confuse this perception with

[9] See [Loux, 1974, 49–51].

an obscure judgement they make concerning the nature of something which they think exists in the painful spot and which they suppose to resemble the sensation of pain; but in fact it is the sensation alone which they perceive clearly" [Adam and Tannery, 1964-76, VIII-1, 22; Cottingham *et al.*, 1985-1991, I, 208]. Descartes is known for saying that extension and motion are contained clearly in the idea of material substance, but that heat, color, etc. are not so contained. An idea (or perception) of material things which included extension and motion, but didn't clearly exclude these other things would be clear but not distinct. One which only included extension and motion would be clear and distinct. After giving a close approximation to Descartes' own account of clarity and distinctness, Arnauld and Nicole added that the obscurity of ideas arises from their confusion, so in the end, they take clarity and distinctness to be the same thing (70). With respect to Descartes' example of an idea of pain being clear but not distinct, they say that the sensation itself is both clear and distinct. "That the sensation is in the hand", which was what was not distinct, is also, they said, not clear (70). The obscurity is a result of the confusion. So what we are concerned with in clarifying ideas really involves making them distinct. In retrospect this was also Descartes' concern.

Given Arnauld's view of an idea as the mental awareness or perception of a thing, it is the perceiving that is clear or obscure, not what is perceived. This point is sometimes overlooked, as Arnauld and Nicole occasionally classify ideas as clear and distinct simply with respect to their objects. Thus they say that "the idea each person has of himself as a thing that thinks is very clear, and similarly the ideas of everything dependent on our thought, such as judging, reasoning, doubting ..." (70-1) Presumably the suggestion is that when someone is thinking of these things, these ideas are clear because what is contained in them will strike the person having them forcefully. Similarly they say that "confused and obscure ideas are those we have of sensible qualities such as colors, sounds, odors, tastes, cold, hot, weight ..." (71). Properly speaking whether these ideas are confused or clear would be relative to the conceiver. Given that they said the idea of pain confined to the sensation is clear and distinct, it seems possible to have a clear idea of sensible qualities, provided any bodily location is excluded from them. Similarly, ideas of external objects can become less obscure by reflection. Descartes himself speaks of a confused idea of the sun and also a clearer one.

Arnauld and Nicole make a distinction between the *comprehension* of an idea and its *extension*. This distinction, roughly corresponding to the present distinction between intension and extension, is important for the analyses of propositions and syllogisms given in Parts II and III. The comprehension of an idea consists of "the attributes that it contains in itself, and that cannot be removed from it without destroying it ..." (59). Those features that Descartes would have said are contained objectively in the idea will be included in this comprehension. For example, the comprehension of the idea of a triangle will contain extension, shape, three lines, three angles, and the equality of these three angles to two right angles. The extension of the idea are "the subjects to which the idea applies" (59). These include the lower species as well as the individuals. While some have seen this

inclusion as a serious confusion, it does not appear to make much difference in the application in the *Logic*.[10] It is a mistake, I believe, to read into the *Logic* a prelude to set theory. Like Descartes, Arnauld and Nicole were nominalists with respect to universals, and they did not have a concept of a collection as itself an abstract object. There were universal ideas, and these were said to extend to species, which were themselves ideas, but the only non-ideas to which all these apply are particulars.[11] Thus it is fairly harmless when they say that the extension of the idea of a triangle includes all the species of triangles (59).

It is less harmless that they lump together two methods of narrowing the extension of an idea. The first method is to form a complex idea by adding an attribute which cuts down the extension, for example adding the attribute of having a ninety degree angle to the general idea of a triangle and obtaining the complex idea of a right triangle. The second method of narrowing the extension of an idea is to join it with the idea of an indeterminate part, which is their analysis of "some triangle". This way of understanding the particular quantifier results in some confusion in the account of propositions.

The distinction between the extension and comprehension of an idea is first used in clarifying the distinction in complex ideas between restrictive and non-restrictive additions. Arnauld and Nicole point out that when we add to the comprehension of an idea by including a further property, we can express this by using an adjective or a relative clause, e.g. "A transparent body" or "a body which is transparent." These they call "complex terms". Those additions to the comprehension that do not narrow the extension are called "explications" (i.e. non-restrictive) and those that do narrow the extension are "determinations". For example, adding the idea of the attribute *mortal* to the idea of *man* will not narrow the extension of the idea, while adding the idea of *transparent* to the idea of *body* will. In the case of singular terms, all such additional complexity will simply be explicative, as in "Alexander, the son of Philip," or "Aristotle, the prince of philosophers". In general it appears that the relation of the comprehension to the extension is reciprocal in that as attributes are added to the comprehension, the extension is narrowed. The principle is not universal, though, for in the case of explications the addition to the comprehension does not further narrow the extension.

One might hold that explications are not really additions, for on the Cartesian view accepted herem even if I don't think of it, *having angles adding up to two right angles* is included in my idea of a triangle. While this appears to hold of what Descartes said were ideas of true and immutable natures, it isn't at all clear that it should apply in the case of singular terms. For example it isn't clear whether, *being the teacher of the son of Philip* is included in the comprehension of the idea of Aristotle. Arnauld rejected this view in his correspondence with Leibniz, although he did agree that God's knowledge of an individual included all truths about him.[12] In any case, there isn't any reason to attribute this view to the Port-

[10] See, for example Norman Kretzman's discussion in [Kretzman, 1967, 379].
[11] This is made explicit in the discussion of universals. (I, 7, 60)
[12] Letter to Leibniz, 13 May 1686, in [Mason, 1967, 25–27].

Royal *Logic*, although it is fairly clear that singular terms do have a comprehension as well as an extension, and this is certainly true of definite descriptions. It is clear that the Port-Royal *Logic* holds the principle that if the comprehension of an idea A includes the comprehension of an idea B then the extension of B includes the extension of A, but there isn't any reason to think Arnauld and Nicole hold the converse, that if the extension of the idea A is included in the extension of B, then the comprehension of B is included in the comprehension of A. Such a principle would make explications simply a making explicit the comprehension already contained in an idea. Unfortunately there is one spot where Arnauld and Nicole come close to expressing this view. In Part III, chapter 10 they discuss a general method of testing arguments and seeking premises, and are in particular discussing a syllogism in Disamis,

> Some saint is poor.
>
> All saints are friends of God.
>
> Therefore some friend of God is poor.

The general method holds that the conclusion is contained in the major premise and that the minor premise makes this explicit. They say that for the major premise to contain the conclusion, "it is necessary and sufficient that the term *some saint* contain the term *some friend of God*." For this to be the case, they continue, "it is necessary that *friend of God* be contained in the comprehension of the idea *saint*" (213). What is at issue here is the truth of the minor premise. Certainly if *friend of God* is contained within the comprehension of *saint*, we could know this, but I don't think we need to read into this remark the claim that this must be true for any such A proposition to be true.

Complex terms, and the distinction between explicative and determinative additions play an important role in the analysis of propositions, particularly the discussion of incidental (or subordinate) propositions. This discussion will be important to the discussion of inference. One tension which arises immediately is the relation of complex ideas to such incidental propositions. Arnauld and Nicole follow tradition by holding that ideas are prior to propositions, yet they hold that "it is the same thing to say 'a transparent body' or a 'body that is transparent"' (66). Yet the first appears to be a complex idea while the second contains, they hold in Part II, an implicit assertion. As we shall see, the line they wish to draw between conception, which is of ideas, and judgment, which is of propositions is not as easy to make as it seems. But to explore this further, as well as other issues involving terms, it will be useful to look at their account of propositions.

4 PROPOSITIONS

The *Logic* has a traditional view of propositions as containing two terms and a copula. One term is the subject term, the other, the predicate; the subject is that of which one affirms or denies something, and what is affirmed is the predicate

(called the *attribut* in the *Logic*). Arnauld and Nicole go on to say that these ingredients are not enough to make a proposition, but that the "mind must join or separate them" (113). This activity of the mind is signified by the copula (*est* in their example) or by the copula along with negation particles (*n'est pas* in their example). As Jill Buroker has pointed out, this treatment of the proposition appears to leave Arnauld and Nicole no room for the mind's simply conceiving of a proposition without affirming or denying it, because on their view it is the act of the will in either affirming or denying that creates the proposition in the first place. It also leaves them open to Frege's charge that this kind of analysis of negation is incoherent.[13] Let me deal briefly with both these difficulties.

Buroker contrasts the Port-Royal account of propositions with that of Descartes. On Descartes' view, there appears to be no sharp line between conceiving ideas which are non-propositional and thinking a proposition. The understanding can have before it an idea with a propositional content, and then the faculty of the will comes in when a person makes a judgement. Affirmation or denial is then made on a propositional content, and the will can also be suspended, in which case the proposition will just be considered. Now as Buroker points out, this last step does not seem to be open to Arnauld and Nicole, for they bring the act of affirmation or denial into the very building of the proposition; on their view it appears that without the judgement involved in the affirmation or denial, there is no proposition to consider.

As Arnauld and Nicole claim that negation involves the mind separating ideas apart, it may appear that Frege's argument in "Negation" applies to them. In that essay[14] Frege argued that false propositions, or thoughts, had just as much being and unity as true ones and that "our act of judgment can in no way alter the make-up of a thought" [Geach and Black, 1970, 122]. He was especially concerned with the view that negation was an act of the person making the judgment which dissolved the proposition or separated it into parts [Geach and Black,1970, 123–124]. This is just the view that Arnauld and Nicole appear to put forth.

Buroker is right about the Port-Royal characterization of propositions. The unity of the proposition as a proposition does appear to be the function of the verb. Starting with the fifth edition of 1683, Arnauld and Nicole included as the first two chapters of the second part one chapter "of words as related to propositions" (II, 1) and one "of verbs," (II, 2) taken pretty much verbatim from the *Grammaire Générale et Raisonée*. The second chapter, taken from Part II, Chapter 13 of the *Grammaire*, asserts that the chief function of the verb is its role in affirming, "to indicate the connection we make in our minds between the two terms of a proposition" (109). In the *Grammaire*, Arnauld and Lancelot argued that participles were not genuine verbs because they contained no assertion, giving as examples "Peter living" and "Peter is living", the first being merely a complex idea containing the two ideas, and the second being a genuine assertion. The inclusion of this section in Part II of the *Logic* was to emphasize the role of the

[13]See [Buroker, 1996a, 9–10].

[14]The essay was first published in 1919 and appears in [Geach and Black, 1970, 117–135].

asserting verb in propositions. All propositions are either affirmations or denials. Subordinate clauses which are not affirmed are not propositions on this view, and so it does not appear to be possible to consider a proposition without affirming it, since it appears that it is the ingredient of affirmation or denial which creates the proposition.

It should be noted, though, that despite this view, Arnauld and Nicole do appear to include in ideas what we would call propositional content. For them, as much as for Descartes, *having angles which add up to two right angles* is contained within the idea *triangle*. Descartes' view was that the clear and distinct perception of this containment would determine the will to make a judgement. The same is the case for Arnauld and Nicole, although they would deny that prior to assertion we are considering a proposition. Instead they would say we are contemplating a complex idea. With respect to relative clauses, Arnauld and Nicole actually get into a difficulty. In Part I, chapter 8 they hold that "a transparent body" and "a body which is transparent" are the same thing, and are complex terms, not propositions (65). But in Part II, Chapter 6 they refer to relative clauses as incidental propositions, but none the less propositions, and thus want to include an assertion in them (122). They come to the somewhat unsatisfactory position that in the case of explicative relative clauses the clause as well is asserted, but in the case of determinative clauses there is a "tacit or virtual" assertion that combination is possible. Thus if I say that "Alexander, who was the son of Philip, defeated the Persians" I am asserting that Alexander was the son of Philip (125), and if I say "A transparent body has weight" on this view I am not asserting that there are transparent bodies, but that it is possible that there be.

In the *Grammaire*, the focus is on explicative clauses in the discussion of relative pronouns. There, Arnauld and Lancelot held that when someone says,

> Invisible God created the visible world,

there are "three judgments in my mind contained in this proposition, for I judge first that God is invisible, second that he created the world, third that the world is visible" [Arnauld and Lancelot, 1660, II, 9, 68]. So while there is only one verb occurring here, what seem to be complex ideas, *invisible God* and *visible world,* contain judgments.

Probably the best way to understand the Port-Royal view is to see that in one sense they were more sensitive than Descartes to the difference between a proposition and a complex idea. They emphasized the occurrence of what Bertrand Russell would later call the verb as a verb, as opposed to the verb as term as the crux of this difference.[15] However, since they thought of assertion and negation as acts of judging, they ended up with a view which makes the very unity of the proposition dependent on the act of the judging mind. In fact, they did not have

[15]See [Russell, 1903, Chapter IV, especially sections 48 to 53]. There are actually many similarities between the analyses given in the Port-Royal *Logic* and the *Principles of Mathematics* on this point, although Russell is very clear, as was Frege, that the propositions are not formed by the act of judging.

to do this, since they seem to recognize some unity in complex ideas and this unity would serve well as what Russell later called propositional concepts or "unasserted propositions".[16] By their own criteria, there is no assertion when I say simply "the visible world," since there is no act of affirming or denying anything. I could, for example, be contemplating denying its existence. They could then have said that the act of asserting was the act of judging that the parts of the complex idea go together, and the act of negating involved judging they did not. The propositional concepts would then have a unity, i.e. the unity of a complex term, though not one created by the asserting will. Unfortunately, they did not address this difficulty in this manner.

The Port-Royalists were taken by the fact that the sentence,

> Invisible God created the visible world

was equivalent to the sentence,

> God, who is invisible, created the world, which is visible.

This last sentence contains verbs occurring as verbs and it seems that someone who states this is committed to the truth of the claims "God is invisible" and "the world is visible". With respect to the proposition "Invisible God created the visible world," these other propositions are probably better understood as presupposed rather than asserted. There is a place in the *Logic* where Arnauld and Nicole allow that a proposition can still be true even though the incidental propositions are false:

> . . . the falsity of the incidental proposition does not ordinarily preclude the truth of the main proposition. For example, "Alexander who was the son of Philip defeated the Persian." This proposition should be considered true even if Alexander was not the son of Philip, because the affirmation of the principal proposition affects only Alexander, and what is joined to it incidentally, although false, does not prevent it from being true that Alexander defeated the Persians. (125)

However, they hold that when the incidental proposition is related as in "Alexander the son of Philip was the grandson of Amintas" the falsity of the incidental proposition would make the principal proposition false. Unfortunately, they did not elaborate this example, for it is not clear why, for example, if Amintas had had two sons, Philip and Hector, and unbeknown to everyone Alexander was the son of Hector, we would not gloss the example the same as the one above. In the first section, Arnauld and Nicole point out what they call the ambiguity of such expressions as "the true religion". They say,

[16]The example Russell uses is "The death of Caesar" as opposed to "Caesar died". It is clear from the Port-Royal discussion in Part II Chapter 2 that the first of these would simply be a complex idea and not a proposition. See especially p.111.

> . . . the words "true religion" signify but a single and unique religion, which is in truth the Catholic religion, since that is the only true one. But because each nation and each sect believes that its religion is the true one, these words are highly equivocal in people's mouths, although by error. And if we read in a historian that a prince was zealous about the true religion, we could not know what was meant unless we knew this historian's religion. For if he were a Protestant, it would mean the Protestant religion; if he were an Arab Moslem who spoke thus about his prince, it would mean the Moslem religion; and we could judge that it was the Catholic religion only if we knew that the historian was Catholic. (67)

Jean-Claude Pariente has noted that Arnauld used this distinction in his controversy over the refusal of the Jansenists to sign the formulary, and says it anticipates Keith Donnellan's distinction between referential and attributive uses of descriptions.[17] His description of Arnauld's own account of the distinction, though, suggests a closer connection to the distinction made between speaker's reference and semantic reference, although the two are related.[18]

Given the account of a proposition as either an affirmation or a denial of the relation of two terms, it is clear that negation will be seen as part of the copula. However, whether a proposition is universal or particular on this account appears to be a function of the subject term. Each idea or term, whether general or singular, has an extension as well as a comprehension. One way of limiting or further determining that extension was to form a complex idea which has a greater comprehension and cuts back the extension. The other way was to join to it "only an indistinct and indeterminate idea of a part, as when I say, some triangle . . . " (59). The view here is that the quantifiers 'all' and 'some' are not part of the propositional form, but attach to a term as a way of determining its extension. So that 'all men' is a complex term in the proposition "all men are mortal", as is "some men" in "some men are over six feet tall". While this is the official view of the Port-Royal *Logic*, in practice they treat propositions, and not merely subject terms as universal or particular.

While each idea has an extension, Arnauld and Nicole mark the distinction of universal and particular propositions as a distinction of whether the terms are "taken" throughout their entire extensions or only through an "indeterminate part" of that extension.[19] In fact, the Port-Royal *Logic* defines the distinction

[17] [Pariente, 1985, 206]. The reference is to [Donnellan, 1966].

[18] Quoting from texts collated by Father Quesnel and reprinted in Volume 22 of [Arnauld, 1775-1783], Pariente marks the distinction between which object is actually determined by the property and which object is thought of by the person who utters the description. This way of making the distinction appears closer to the distinction between speaker's reference and semantic reference. See [Kripke,1977] and [Donnellan, 1978].

[19] ". . . les termes universels peuvent être pris ou selon toute leur étendue, en les joignant aux signes universels exprimés ou sous-entendus, comme *omnis, tout*, pour l'affirmation; *nullus*, *nul*, pour la negation, *tout homme*, *nul homme* . . . Ou selon une partie indéterminée de leur étendue, qui est lorsqu'on y joint le mot *aliqui*, quelque, comme *quelque homme*, *quelque hommes*, ou

between universal and particular propositions on the basis of whether a common term is taken in its entire extension or whether it is taken as an indeterminate part (115). Singular propositions are then classified with universal, even though their subject is not a common term, because they are always taken through their entire extensions (115). Despite the fact that the quantifier expressions are seen grammatically as attaching to the subject terms, there is a sense in which *propositions* are to be understood as universal or particular and not simply their subject terms. The real interest in the Port-Royal *Logic* is in whether a term in a proposition is taken throughout its entire extension or not. This way of phrasing things replaces the account of "distributed terms" familiar to students of what is now taught as syllogistic logic. As we shall see, the Port-Royal analysis of propositions and inferences relies very heavily on this concept.

Jean-Claude Pariente has suggested that the Port-Royal *Logic* is best understood as holding an identity theory of the copula. E. A. Moody [Moody, 1953, 36] characterized two understandings of the copula from mediaeval semantics. The *identity* theory holds that in some sense the copula in an affirmative proposition asserts some kind of identity. Clearly with such cases as "All men are mortal" the view has to be understood not as identifying the subject term with the predicate term, but as saying that the proposition asserts an identity of the extension of the subject term with part of the extension of the predicate. Ockham has been associated with the identity theory,[20] but he made it clear that he does not require that the subject and predicate be identical, but rather, "that the subject and predicate supposit for the same thing".[21] The other theory, the *inherence* theory, did not take the copula as a sign of identity, but as a sign of the inherence of the property signified by the predicate term into those objects which were supposited by the subject term. The identity theory was very much motivated by nominalism, since the earlier theory took the predicate term to refer to a universal or common nature. Pariente takes some remarks from the Port-Royal *Logic* concerning conversion, and a further remark from Arnauld's *La perpétuité de la foi* as evidence that the copula is taken as a symbol of identity.[22]

It seems on the face of it that the nominalism of Port Royal would not have them adopt the inherence theory, and the language as Pariente points out, does suggest the identity theory. However, the situation is a bit more complex, as the discussion of the function of the copula in affirmative propositions takes into consideration both the comprehensions and extensions of the ideas. This discussion is most explicit in the account of conversion of propositions. There Arnauld and Nicole give four "indubitable axioms" concerning affirmative propositions:

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d'autres selon l'usage des langues. (114)

[20] See [Moody, 1953, 36–37].

[21] Ockham, *Summa Logicae*, II, Ch. 2. See [Freddoso and Shuurman, 1980, 86].

[22] [Pariente, 1985, 263]. The remark from *The Perpetuation of the Faith* was "the nature of all affirmative propositions is to indicate that the subject is the same thing as the attribute [predicate]" [Arnauld, 1775-1783, III, 167]. In the chapter introducing conversion in the *Logic*, Arnauld and Nicole say "... it is clear that the nature of affirmation is to unite and to identify ... the subject with the attribute, since this is what is signified by the word *is*." (168)

1. The attribute is put in the subject by an affirmative proposition according to the entire extension the subject has in the proposition.

2. The attribute of an affirmative proposition is affirmed according to its entire comprehension.

3. The attribute of an affirmative proposition is not affirmed according to its entire extension if it is in itself greater than that of the subject.

4. The extension of the attribute is restricted by that of the subject, such that it does not signify more than the part of its extension which applies to the subject. (170)[23]

Thus in the proposition "All F are G", by (1) and (2) the comprehension of G is affirmed of all of the extension of F, and by (3) and (4) the extension of G is not affirmed according to its entire extension, but is restricted to the part of its extension which is common to F. It is somewhat difficult to interpret the last two axioms, but it seems that in some sense the claim is that in affirmative universal propositions, the whole of the extension of the subject is signified, but not the whole of the extension of the predicate. These claims replace the doctrine of distribution, where the term is distributed when it signifies its entire extension, and is undistributed when it doesn't. Pariente takes the second two axioms to indicate that in the case of an affirmative proposition "All F are G" what is being asserted is an identity between the extension of F and a part of the extension of G (namely those G's that are F, given Axiom 4). The proposition "Some F are G" then would assert an identity between an indeterminate part of the extension of F and an indeterminate part of the extension of G. Now Pariente, though, also claimed that in the case of "All F are G", the Port Royal view is that the comprehension of G is included as a subset of the comprehension of F [Parient, 1985, 266]. As Jill Buroker has pointed out, this claim does not appear to follow from these axioms, nor is it plausible in the case of non-necessary universal affirmatives and even less plausible in the case of particular affirmatives. When I affirm "All swans are white", I am claiming that the extension of *swan* is a subset of the extension of *white*. Am I also claiming that the comprehension of *white* is contained in the comprehension of *swan*? This would only be plausible if we thought of the comprehension of an idea as including all the ideas which apply to the entire extension. Given the view that the comprehension was essential to the idea and the extension was not, this position is implausible. Again it seems very implausible, to use Buroker's example, that when we say "some bears are white" we are including the comprehension of *white* in the comprehension of *bear*, or in the comprehension of the idea of some particular bear [Buroker, 1994, 10–11].

While there are places where Arnauld and Nicole seem to hold a theory of the copula as asserting an identity between the extension of the subject and the

[23] An important feature of axiom 1 is that the subject of a universal proposition has the entire extension of the idea, while that of a particular has only an indeterminate part of that extension.

extension of the predicate and this seems most supported by the third and fourth axioms above, the easiest reading of the first two axioms simply is a predicative view of the copula, where the entire comprehension of the predicate is predicated of the extension of the subject term — either the entire extension of the idea in the case of the universal proposition, or an indeterminate part in the case of the particular. As Buroker has pointed out, these first two axioms suggest an asymmetry between subject and predicate which doesn't fit well with the identity theory. However, some remarks at the end of the fourth axiom ("... when we say that humans are animals, the word 'animal' no longer signifies all animals, but only those animals which are human." (170)) suggest something like the identity theory and the unfortunate view that the significance of the word 'animal' when in predicate position is different in different universal affirmative statements.

These four axioms were given as a prelude to the account of conversion, the only immediate inference discussed in the Port-Royal *Logic*. The rejection of the conversion of the A proposition is taken as a consequence of these axioms, for in a proposition such as "all men are animals", all ideas contained in the comprehension of "animals" are asserted of the entire extension of "men," but by the third axiom, nothing is claimed about the entire extension of "animals". Thus the proposition, "all animals are men" is not contained in "all men are animals". Following this analysis, Arnauld and Nicole give two rules for the conversion of affirmative propositions, the first that universal affirmative propositions can be converted by adding a mark of particularity to the attribute which becomes the subject, and the second that particular affirmative propositions can be converted without any change. Given that in "all men are mortal" the extension of the term "mortal" that is in question is only a part of the total extension of the idea, from the fourth axiom, that part of the extension is said to be identical with the extension of the term "men", so "some mortals are men" will be true.

As Jean-Claude Pariente has pointed out, the Port-Royal *Logic* gives a separate account of negation. The negation of a categorical proposition is not defined in terms of the truth values of propositions negated, but separately. It then will follow from this account that the E proposition will take the opposite truth value of the I and the O will take the opposite truth value of the A. Again with the account of negative propositions, there is a part of an identity theory, as the account opens by saying that the nature of a negative proposition is "to conceive that one thing is not another" (173). Three further axioms are given specifically concerning negative propositions:

1. A negative proposition does not separate all the parts contained in the comprehension of the attribute from the subject: but it separates only the total and complete idea composed of these attributes together.

2. The attribute of a negative proposition is always taken generally [i.e. throughout the entire extension].

3. Every attribute denied of a subject is denied of everything contained in the extension of the subject in the proposition [i.e. the entire extension of the

> idea in the case of a universal, and an indeterminate part in the case of a particular]. (173-4)

The fifth axiom states that if I assert the proposition "No F are G", and the idea G contains in its comprehension the ideas H and K, I am not denying that there are F's that are H or F's that are K, but only that there are no F's which have both attributes. The sixth axiom, which corresponds to the claim that the predicate term of negative propositions is distributed, is argued for somewhat awkwardly by the statement that "if triangle is denied of squares, everything that is a triangle will be denied of squares". "Everything that is a triangle" clearly refers to the extension of "triangle" here. Presumably in "some figures are not triangles" the indeterminate part of the extension of "figures" would not be any of the species of triangles, and no member of the indeterminate part would be a member of the extension of "triangle". From these axioms, Arnauld and Nicole derive a third rule, that universal negative propositions can be converted. While they don't give a further rule about particular negatives, they argue that given these axioms, they cannot be converted. The argument for the third rule is that since, given the sixth and seventh axioms, and the fact that in the universal propositions the entire extension of the subject is in question, a proposition such as "No F are G" separates the entire extension of F from the entire extension of G, so this separation will be "mutual and reciprocal." This is not the case with the particular negatives, since in those cases the entire extension of the subject term is not in question, but only an indeterminate part. So "some person is not a physician" makes an assertion concerning only an indeterminate part of the extension of "person" while "some physician is not a person" makes a claim concerning its entire extension.

In a brief section on the square of opposition the Port-Royal *Logic* gives the traditional relations of contradictories, subcontraries, contraries and subaltern (117). Unfortunately, there is no discussion of existential presupposition. It is clear that Arnauld and Nicole hold that the affirmative propositions carry existential presupposition, but they are not explicit with respect to negative propositions, which were held by many mediaeval logicians not to have any existential presupposition, thus preserving all the relations of the square.[24] On this view the immediate inference of obversion doesn't hold, and there is no mention of this inference in the Port-Royal *Logic*. Jean-Claude Pariente has argued that since ideas which are species of the general ideas are counted in the extension of terms, there is a sense in which all terms, even those which do not have any existing particulars in their extensions, do have things in their extension, and he uses this to justify a claim of existential presupposition for all terms.[25] As far as I can tell, the text of the *Logic* does not really lend support to this interpretation. There is, though, a

[24] See [Moody, 1953, 50–51], for a discussion of Buriden on this point, and Freddoso, "Ockham's Theory of Truth Conditions", [Freddoso and Schurrman, 1980, 18], for a discussion of Ockham.

[25] See [Pariente, 1985, 246]: "These observations explain at least in part the indifference of Port-Royal to problems posed for logic by empty terms. When the logic is presented as a calculus applying to ideas, and more precisely to relations between extensions of ideas, the possibility or impossibility of applying these ideas to concrete realities has no effect on the calculus itself."

discussion in the section on incidental propositions and compound subjects where the *Logic* suggests one can affirm an A proposition without asserting the existence of anything which falls under its subject term.

In any case, the section on the square of opposition is brief, and the section on modal propositions (130) is also extremely brief. There is no discussion of modal syllogisms.

5 COMPOUND PROPOSITIONS AND INCIDENTAL OR SUBORDINATE PROPOSITIONS

The Port-Royal *Logic* includes a discussion of a variety of propositions which do not fit easily into the pattern of simple categorical propositions. Arnauld and Nicole characterize compound propositions as those that have either complex subjects or complex predicates. In fact, most of their examples are propositions in which just one or the other terms is complex, such as propositions of the form "either a or b is F", or "b is F and G." However, they allow both terms to be complex as well, and in their discussion of conditionals include such conditionals as "If the earth is immovable, the sun revolves". As they see all conditionals as statements of inference, these conditionals containing different terms and predicates in their subordinate propositions are said to be statements of "mediate inference". All the other cases of compound propositions they discuss involve complex subjects or predicates, although it is possible to extend their account generally.

Compound propositions are divided into two classes, those in which the composition is explicit, and those in which it is not, these latter being called "exponibles". The explicit ones include copulatives ("P and Q"), disjunctives ("Either P or Q"), conditionals ("If P then Q"), causals ("P because of Q"), relatives (other than causal, e.g. "As a person lives, so he dies") and discretives ("P but Q"). The original (first edition) list of exponibles included exclusives ("Only F *is* G"), exceptives ("All F, except G, are H"), comparatives ("A is more F than B"), inceptives ("Beginning at t, P"), desitives ("P ended at t"), and reduplicatives ("A, insofar as it is B, is C"). In the later editions, the inceptives and desitives were treated together, and the reduplicatives simply mentioned under the causals.

These propositions were for the most part not treated as truth-functions, with the exception of basic conjunctions, or copulatives. With respect to these, Arnauld and Nicole say "the truth of these propositions depends on the truth of both parts" (132).[26] However, under each of these classifications there is a discussion

[26]It is interesting to note that for the example of a copulative with both a complex subject and a complex predicate Arnauld and Nicole take a negative proposition which in English is naturally seen as a negation of a disjunction, but which in Latin and French are easily seen as a conjunction of negations. The line is from Horace:

> Non domus & fundus, non aeris acervus et auri, Aegroto Domini deduxit corpore febres, non animo curas.

Which they translate as *Ni les maisons, ni les terres, ni les plus grandes amas d'or & d'argent ne*

of what propositions constitute the contradictory of the given propositions, and in some of these discussions Arnauld and Nicole appear to be closer to a truth-functional analysis of the propositions. Disjunctions are not given a simple truth-functional analysis, as the authors hold that the truth of the disjunction depends on "the necessary opposition between the parts, which must not permit a middle." The contradiction of a disjunction is said simply to be a proposition denying the disjunction, without further explication. The truth of discretives, such as "Happiness depends not on wealth but on knowledge", is said to depend "on the truth of both parts and the separation between them". This suggests a connection stronger than a mere conjunction. Nevertheless, Arnauld and Nicole do not say that a discretive proposition without a contrast between the terms is false, but rather that it is ridiculous [*ridicule*]. When it comes to stating what contradicts discretive propositions, they say that such propositions can be contradicted in three ways:

> Happiness depends on wealth and not knowledge
>
> Happiness depends neither on wealth nor knowledge
>
> Happiness depends on both wealth and knowledge.

As they started with a proposition of the sort "Not P but Q," the propositions denying the discretive are the three possibilities, "P and not Q," "Not P and not Q," "P and Q." This gives a nice modern truth table for "Not P and Q." Such a list was not given for the denial of copulatives (i.e. conjunctions). Instead of recognizing that the contradiction of the discretive is equivalent to a disjunction, they remark that the last two of these alternatives are copulatives and so say, "Thus we see that copulatives are contradictories of discretives" (137).

With respect to conditionals, Arnauld and Nicole say that to determine their truth "we consider only the truth of the inference," stating that even if both propositions are false, "if the inference from the one to the other is valid, the proposition insofar as it is conditional is true". This way of putting things clearly rules out a simple truth-functional analysis. In their analysis of the denial of "if you eat the forbidden fruit, you will die" they give "although you eat the forbidden fruit, you will not die", which is then seen as equivalent to "It is not true that if you eat the forbidden fruit, you will die" (135). This may suggest an analysis closer to modern truth functions, but we should understand that "although" is not treated simply as a truth function. Unfortunately it isn't clear whether they accept their own analysis as a complete account of the denial of a conditional, since by their account for this proposition to be true the *inference* has to be valid. They could conceivably hold that in the case that someone dies after eating the fruit, the proposition may still be false because the inference from "you eat the fruit" to "you die" is not valid.

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peuvent ni chasser la fiévre du corps de celui qui les possede, ni déelivrer son esprit d'inquietude et de chagrin. (132)

The exponibles were thought of as implicit compounds because while they appear to be categorical propositions, on analysis they are seen to contain at least two separate assertions. I will just mention two of them, the exclusives and the exceptives. Exclusive propositions (propositions such as "Only F and G") tend to be treated by modern logic books as a variant on the categorical proposition "All G are F". The Port-Royal *Logic* treats exclusives as a conjunction of two propositions, but it discusses only such propositions which have a singular subject, such as "Virtue alone is admirable", or "Only God is worthy of being loved for his own sake" (138). These propositions are seen as a conjunction of two propositions, e.g. "Virtue is admirable" and "Nothing else is admirable". This restriction to singular subjects is also found in their treatment of exceptives. The example they give, "No ancient philosophers, except the Platonists, recognized God's incorporeality" is treated as the compound of "the Platonists recognized God's incorporeality", and "No other ancient philosophers did". A similar treatment is given to the proposition, "Except for the sage, all people are truly mad (*fous*)" (141). We are not given an explicit treatment of exclusives which do not have a singular subject, although the suggestion is that these too would be considered as having compound subjects. "Only the brave deserve the fair", would presumably be seen by Arnauld and Nicole as asserting that the brave deserve the fair and no others do. With respect to the exceptives, the rule given is that in these propositions we "affirm a thing of an entire subject with the exception of some of the inferiors of this subject, which we show by an exceptive particle that this thing does not apply to them ..." (140). This account includes propositions such as "None but the brave deserve the fair," which would be treated as asserting the same two claims as above.

6 CATEGORICAL SYLLOGISMS

The Port-Royal discussion of syllogisms is a mixed bag. On the one hand, Arnauld and Nicole maintain their skepticism toward formal logic and the importance of the study of reasoning or inference, saying that "there is reason to doubt whether it is as useful as is imagined", and "it happens rarely that we let ourselves be misled by arguments that are false merely because the conclusion is badly drawn" (177-178). Yet despite this dismissive attitude, the treatment of syllogisms is fairly careful and quite clear.

The need for reasoning, they hold, is a consequence of our limited minds. The suggestion seems to be that a mind which had sufficiently clear and distinct ideas would not need any reasoning at all. Reasoning only comes in when we cannot decide on whether a proposition is true or false by considering the ideas which it contains. To help determine whether a proposition is true we bring in a third idea, the middle term, which helps us see the relation between the subject and predicate of the proposition in question. An important feature of this way of viewing things is that the inquirer always starts with the conclusion. The premises are sought as an aid in determining the truth of the conclusion, but arguments do not have

premises as their intellectual starting points. One consequence of this view is that in the proper analysis of arguments the conclusion is to remain unchanged. Any reductions which involved tampering with the conclusion were rejected, and claims that fourth-figure syllogisms could be eliminated by reversing the premisses were also rejected, since the major premise was defined as the premise containing the predicate term of the conclusion.

Despite their opening discussion of the process of reasoning, which appears to apply only to categorical syllogisms, the Port-Royal authors also recognize conjunctive syllogisms, which include conditional syllogisms and disjunctive syllogisms. The simple syllogisms are, paradoxically, divided into those that are complex and noncomplex. The complex syllogisms are not sorities, but syllogisms which contain complex terms in subject or predicate position.

Aristotle is often seen as giving a justification of all the syllogisms by first displaying syllogisms which were self evident to the mind (those of the first figure) and then giving arguments which show that the valid moods of the other figures can be justified based on arguments using conversion, contradictory relations and the first figure valid syllogisms. This system of transformations can be seen as a formal deductive system. While Arnauld and Nicole were aware of this method of reducing syllogisms to the first figure, and included in the first edition a chapter (Chapter 9) on how to figure out what the reduction would be based on the letters of the "artificial words," they generally heaped scorn on this method, beginning the chapter with the sentence: "This chapter is extremely useless" (203).

Their explanation of the validity of the correct syllogisms is instead a semantic one. Based on their analyses of affirmative and negative propositions, they give arguments for rules of syllogisms which capture all the valid syllogisms and exclude the invalid ones. Given their view that the mind easily grasps the valid syllogisms, one may wonder why they even bother with this justification. In fact, with respect to simple syllogisms they think that the rules only serve to exercise the mind (178). However, they also think that such an analysis will lead to general considerations which will also apply to complex syllogisms (205).

The four axioms from which the rules derive are,

1. Particular propositions are contained in general propositions of the same nature (quality) and not the general ones in the particular.

2. The subject of a proposition being taken universally or particularly is what makes the proposition universal or particular.

3. The attribute of an affirmative proposition never having a greater extension than the subject, is always considered as taken particularly, because it is only accidental if it is sometimes taken generally.

4. The attribute of a negative proposition is always taken generally. (183)

On the basis of these axioms they argue for the rules of syllogisms: (1) that the middle term must be taken generally at least once; (2) that the terms of

the conclusion cannot be taken more universally in the conclusion than in the premisses; (3) that no conclusion can be drawn from two negative premises, (4) that a negative conclusion cannot be proved from two affirmative premises (5) that the conclusion always follows the weaker part, and (6) that nothing follows from two particular premises. While they say they derive these rules from these axioms, rules (3), (4) and the part of (5) involving negative propositions (namely, that an affirmative proposition cannot be drawn from a negative premise) actually are based on the account of negative propositions mentioned above, namely that they assert a separation of the subject from the predicate. There is no claim that this set of rules is in any way minimal. The part of (5) involving particular and universal propositions (namely that a universal conclusion cannot result if there is one particular premise), and (6) are derived from previous rules.

With these rules, Arnauld and Nicole come up with a list of 10 "conclusive" [*concluan*s] moods. Curiously, they arrive at this list prior to the discussion of the figures. The four conclusive affirmative moods are AAA, AII, AAI, IAI, the six negative are EAE, AEE, EAO, AOO, OAO, EIO. (AEO is excluded since that would only hold if the stronger AEE held, and so is not seen as a separate syllogism; EAO, though holds in the third and fourth figure where EAE does not.) The conclusive moods appear to be those which may be valid in one or the other figures. In the actual discussion of figures, the authors refer to the valid moods as simply the moods which are possible within that figure. The list above serves as an upper bound on the possible valid moods within a figure, and the authors then argue for the smaller lists within the figures by cutting the list further by means of rules they develop for each figure. They also give "foundations" for the first three figures. These foundations appeal to the axioms given above and the general accounts of affirmative and negative propositions. The foundations given for the first figure, for example, are,

> Principle for affirmative moods: *Whatever applies to an idea taken universally applies also to everything of which this idea is affirmed, or which is the subject of this idea, or what is included in the extension of the idea*, for these expressions are synonymous.
>
> Principle for negative moods: *Whatever is denied of an idea taken universally is denied of everything of which this idea is affirmed.* (193)

These principles are then used to justify the validity of the syllogisms, with Barbara and Darii seen as illustrating the first principle and Celarent and Ferio illustrating the second.

The principles become more complicated with the other moods. For the second figure one foundation is given for Cesare and Festino, and another for Camestres and Baroco. Cesare and Festino are seen as grounded in the same principle for negative moods found in figure 1. Here Arnauld and Nicole point out that these are variations from the first mood with the major premise (E in both cases) converted. The foundation for Camestres and Baroco is the following principle: *Everything*

which is included in the extension of a universal idea applies to none of the subjects of which the idea is denied, the attribute of a negative proposition being taken throughout its entire extension ... (197). Illustrating this with an instance of Camestres, they point out that if "All P are M" is true, then P is included in the extension of M. If "No S are M" is true, then M is denied of each S, and thus P also will be denied, since the denial, "No S are M," denies the entire extension of M to S. While the concern with reduction was dismissed as useless, it is interesting that a vestige of reduction is included in these sections, for Cesare and Festino are justified by pointing out, as Aristotle argued in the *Prior Analytics* (27a), that legitimate conversions yield Celarent and Ferio. While the arguments concerning Camestres and Baroco are not indirect in the manner of the *Prior Analytics*, the reasoning is still quite close to Aristotle's.

With respect to the third figure, these principles are given for the moods:

> Affirmative moods: When two terms can be affirmed of the same thing, they can also be affirmed of each other, taken particularly.
>
> Negative Moods: When of two terms one can be denied and the other affirmed of the same thing, they can be denied particularly one of the other (*l'un de l'autre*)(199)

While no further explication is given, it is easy to see how the principle grounds the affirmative moods, for in Darapti, "All M is P" and "All M is S", both P and S are affirmed of the entire extension of M, so while there may be further instances of P and of S, there will at least be instances in common, namely the M's. Both conclusions, "Some S is P" and "Some P is S", are able to be drawn from these premises, given the rule.

In the case of the negative moods, it is easy to read the principle as suggesting that S can be denied of P and P can be denied of S. This, of course, is not the case. Jean-Claude Pariente pointed out that Arnauld and Nicole's language inadvertently suggests that they have forgotten that the O propositions are not convertible [Pariente, 1985, 343]. However, if we understand the line "*l'un de l'autre*" not as the usual "of each other" but instead as "the one of the other" we can read the principle as licensing only that the term denied of the middle term can be denied of the term affirmed of the middle term, and not the other way around. In this way the principle would justify Bocardo, that is that from "Some M is not P" and "All M is S", I can infer "Some S is not P," but would not license the inference from these premises to "Some P is not S".

Pariente points out that behind the accounts of the third figure syllogisms is a method similar to that of the *ekthesis* (setting out) proofs given by Aristotle in *Prior Analytics* for the third figure syllogisms. The *ekthesis* proofs of Bocardo involves choosing the M which is not P as an S which is not P (*Prior Analytics* 28b 21-22). While the discussion appears to incorporate the reduction so much scorned by the *Logic*, it is intended as giving simple rules to help the mind make the correct inferences.

With the fourth figure, Arnauld and Nicole give a set of rules, but do not bother to give any justifying principle, since they hold the fourth figure to be "unnatural".

7 A GENERAL METHOD FOR EVALUATING SYLLOGISMS

In their account of complex syllogisms, Arnauld and Nicole do two things. First they give an account, at least for some specific examples, of how to rewrite complex syllogisms into non-complex ones. The examples generally have the middle term as only part of a complex term. For example in the argument,

> Divine law commands us to honor kings
>
> Louis XIV is [a or the] king.
>
> Therefore, divine law commands us to honor Louis XIV. (206)

"kings" is the middle term, but occurs in a part of the predicate of the first premiss. The syllogism is rewritten ultimately as the non-complex syllogism:

> Kings ought to be honored
>
> Louis XIV is king.
>
> Therefore, Louis XIV ought to be honored. (207)

This rewriting, though, is governed by the observation that the term *king* is taken generally (that is, through its entire extension) in the first premiss, along with the observation that it is not the entire predicate of the first proposition.

The second thing they do is to form a general principle from these considerations. Since a valid (*bon*) argument is one where the conclusion is contained in the premises, they say that in any syllogism, there will be a proposition, called the *containing* proposition, which contains the conclusion, but only implicitly, and another proposition, called the *applicative* proposition which shows this. Thus in the above example, they would hold that the first premise, "Divine law commands us to honor kings," contains the conclusion, and the second premise shows that it does. With syllogisms such as *Barbara* they say that either premise can be taken to be the containing premise though it is customary to take the major premise as that one, since it is "more general" (212). In negative syllogisms, though, they hold that the negative premise is always the containing one. They then argue that all the rules that governed the syllogisms can be derived from this general observation. What they argue for, in effect, is that what we call the rules of distribution follow from this general claim about valid arguments. They argue that if a syllogism is valid, in order for the conclusion to be contained in the premises it cannot have a term taken more generally than in the premise, and for the applicative premise to show the subject of the conclusion can be substituted in the containing premise, the middle term will have to be taken generally at least once. It is in the demonstration of this last claim by an example where Arnauld and

Nicole come close to the claim that in such propositions as "All F are G", what is being affirmed is that the extension of F is contained in the extension of G and the comprehension of G is contained in the comprehension of F. This claim suggests that there can be no accidental generalizations.

The discussion begins with the task of finding an additional premiss which will lead to the conclusion of an I proposition, in this case, the proposition "Some friend of God is poor", along with the initial premise that "Some saint is poor". (This is given as an example of a proposition where the middle term is not distributed. Given that the conclusion is affirmative, the only other possible choice would be "All who are poor are saints" from which the same reasoning as given for this one would apply, though they would clearly think this premise is less plausible.) What they argue is that any proposition from which this follows must take the middle term universally. Their argument is,

> For it is obvious that in order for this proposition *some saint is poor* to contain the conclusion *some friend of God is poor*, it is necessary and sufficient that the term *some saint*, contain the term *some friend of God*, since the other term [i.e. *poor*] they have in common. Now a particular term doesn't have any determinate extension and contains certainly only what is included in its comprehension and its idea ... And so in order that the term *some saint* contain the term *some friend of God*, it is necessary that *some friend of God* be contained in the comprehension of the idea of *saint*. (213)

What they are arguing is that the premiss containing the middle term must be universal with the middle term as the subject, which is correct. However, in order to get to this point, they seem to make a stronger claim, namely that the *only* way we could come to such a proposition is if the comprehension of the predicate be contained in the comprehension of the subject. They then argue that whenever this is the case, the extension of the subject will be included in the extension of the predicate, which is the case. Unfortunately this suggestion, that the only way to obtain the universal proposition "All saints are friends of God" is if the comprehension of *friends of God* is included in the comprehension of *saints*, has the apparent consequence that in all A propositions, the comprehension of the predicate is contained in the comprehension of the subject. In other works, they appear to hold not just the principle,

> *Whenever the comprehension of B is contained in the comprehension of A, the extension of A is contained in the extension of B*,

but the stronger biconditional. As I pointed out in section IV, this stronger biconditional would seem to make all universal propositions necessary. A charitable reading here might suggest that in this case, the only way we might be able to come to the conclusion "Some friend of God is poor" was to recognize as a premiss "All saints are friends of God". In this case the only way we might do that is by

reflecting on the comprehensions of the subject and predicate. Arnauld and Nicole appear to push for the stronger claim to buttress the general principle as the sole principle for validity.

Even without this example, the general principle needs supplementing. There is still an explanatory gap between the general claim about containment, and the other principles. Arnauld and Nicole attempt an account of the two "distribution" principles, but do not attempt to give an account of the negation rules from this general claim. The negation rules follow from the account of negative propositions, not from the general containment principle.

After giving this general account, Arnauld and Nicole go on to give illustrations of the use of their containment principle both in the case of valid arguments, and in the case of invalid arguments. Some of these examples work very well, but when tackling a famous sophism, it isn't clear their method works. The sophism in question,

> *Whoever says you are an animal speaks the truth.*
>
> *Whoever says you are a goose says you are an animal.*
>
> *Thus whoever says you are a goose speaks the truth.* (216)

they claim can be solved by pointing out that *animal* in the first premise is not "taken generally" and so does not contain *goose*. As Pariente has pointed out, this doesn't quite address the problem of the sophism, for they have not yet told us why we can't take the middle term as "person who says you are an animal". Unlike the other examples, where the actual middle term is embedded in another term in one of the premises, this case appears to be a straightforward case of Barbara requiring another analysis [Pariente, 1985, 371]. On the other hand, Arnauld and Nicole do recognize that this middle term is not ordinary, in that it contains a subordinate proposition, "You are an animal".

8 SYLLOGISMS OF PROPOSITIONAL LOGIC

There is a brief discussion of other kinds of syllogisms, including syllogisms in propositional logic, which the authors call *conjunctive* syllogisms, and also a discussion of arguments with just one premise, which have conditional conclusions. With respect to the conjunctive syllogisms, the Port-Royal authors recognize three kinds: conditional syllogisms, disjunctive syllogisms and copulative syllogisms. The conditional syllogisms discussed include modus ponens and modus tollens, though not with those titles. In their account of conditional syllogisms there is also a discussion of the fallacies of affirming the consequent and denying the antecedent. The disjunctive syllogism is presented in two forms,

$P \vee Q$	and	$P \vee Q$
not $\sim P$		P
$\therefore Q$		$\therefore \sim Q$

This is no surprise, given that the disjunction is taken as exclusive. What they call the "copulative syllogism" actually is a parallel to the disjunctive syllogism, since it involves a negated conjunction. Here only the one form of argument is allowed,

$\sim (P \cdot Q)$
P
$\therefore \sim Q$

The other form,

$\sim (P \cdot Q)$
$\sim P$
$\therefore Q$ is explicitly excluded. Unfortunately there is no further development of these syllogisms nor attempts to see the connection between disjunction and conjunction. That insight would have to wait.

Arnauld and Nicole see these syllogisms as syllogisms which contain the entire conclusion in one premiss, and thus see their general method as covering these kinds of syllogisms as well.

9 DILEMMAS AND FALLACIES

The remainder of the third part of the *Logic* focuses on mistakes in reasoning. This concern is consistent with their account of logic as a tool for distinguishing truth from falsehood. The authors see not just accounts of correct reasoning but also various considerations which might lead someone to draw a false conclusion as part of the subject of logic. Many of these considerations have to do with factors quite separate from what would now be considered logical considerations. In fact, a treatment of informal fallacies fits nicely with the overall view of logic as propounded by Arnauld and Nicole, and does not fit well with a more formal account of logic. The brief chapter on dilemmas (Part III, 16) nicely characterizes dilemma arguments as ones where a whole topic or field is divided up, and then negative or positive conclusions are drawn from each part. The concern is not with the formal properties of such arguments but with the dangers of dilemma arguments and the easy fallacies which may occur in these arguments. The first fallacy is what we would call a "false dilemma," where the alternatives do not exhaust the field. The second is simply when one of the arguments from one of the parts does not hold. While they do not use these terms, a response to a dilemma citing the first charge would be "going between the horns" and a response to one citing the second charge would be "grasping a horn". The authors mention a third caution with respect to dilemma arguments, namely that dilemmas which draw contrasting extremes from the different parts can often be turned around into counter dilemmas.

The account of dilemmas does not involve the form of the argument, but rather the truth of the premisses, or the difficulty with a subordinate argument which

can be seen as a premiss. Given the account of logic advocated in this work, especially the second discourse where logic was seen as giving rules for all actions of the mind, it is no surprise that this concern is included in logic. But it is also the case that dilemma arguments which have faulty premisses can mask that fact by having the same form as sound arguments. It is important to remember that Arnauld and Nicole see the whole purpose of an argument as a process of showing the truth of the conclusion, and so in some sense starting with the conclusion as a problem. Right reasoning involves a clear progression from clearer premises to the problematic conclusion. In the case of the fallacious dilemmas and the other fallacies discussed in chapters 19 and 20 of the third part, there is a pattern of reasoning that gives the picture of a correct argument, but does not succeed.

The section on fallacies reviews the standard fallacies mostly taken from Aristotle's *Sophistical Refutations*: the *ignoratio elenchi* or proving something other than what was in question; *petition de principe*, or supposing for true what is in question; *non causa pro causa* or taking for a cause what isn't a cause; imperfect enumeration, which here is not so much a hasty generalization, but a failure to consider options; the fallacy of accident; composition and division; the fallacy of taking something affirmed conditionally to something affirmed unconditionally; and fallacies of ambiguity. A brief remark is then made about incomplete or defective inductions, postponing that discussion to the section on method. The discussion of these fallacies is clear and quite lively, but not especially innovative.[27] In many of their examples, Arnauld and Nicole use the discussion to accuse Aristotle of committing several of the fallacies he himself talks about.

The discussion of these traditional fallacies is supplemented by a section on fallacies committed in ordinary life and discourse. These are divided into sophisms of self-love, and sophism arising from the objects themselves. The section on sophisms of self-love, which contains some of the more entertaining examples in the *Logic*, focuses on reasons other than the truth for which one may hold something. The opening remarks of this section is similar to much later discussions of what is sometimes called the fallacy of subjectivism.[28] Arnauld and Nicole have in mind such arguments as the following:

> I am from such and such a country, therefore I ought to believe that a certain saint preached the Gospel there. (262)
>
> I love him, therefore he is the most able man in the world: I hate him, therefore he is worthless. (263)

[27]For a further discussion of these fallacies, see [Woods, 2000, 36–38]. Woods calls this first group "scientific sophisms" because of a remark Arnauld and Nicole make at the beginning of the next chapter contrasting sophisms of ordinary life and discourse to those involving matters of science (260).

[28]See for example, the discussion in Kelley, *The Art of Reasoning*, which treats subjectivism as a fallacy [Kelley, 1998, 126–127]. Kelley's analysis of the pattern of reasoning as I believe/want p to be true therefore p is true. Compare this with the passage in the *Logic* (263), where discussing the "sophisms and illusions of the heart", Arnauld and Nicole say these consist in transporting our passions to the objects of our passions and judging them as we want or desire them to be.

> Someone else said it, it is therefore false: it wasn't I who wrote this book, therefore it is bad. (266)

These fallacies are attributed to *amour propre*, or love of self, a sin of great concern to the Jansenists of Port Royal. It might be thought natural to include in this section an account of *ad hominem* arguments or arguments from authority, as the difficulty with these forms of reasoning seem related. However, while some of these examples border on *ad hominem* arguments, the concern here is strictly with forms of thinking which will lead to accepting false conclusions and which come from an inflated sense of self and one's own situation. *Ad hominem* arguments are treated instead in the second section, although not explicitly. That section, concerning fallacies arising from "the objects themselves", is where the Port-Royal authors treat the argument from authority and cases which would best be classified as *ad hominem*. This section also deals with, among other things, weak inductions.

It may seem curious to include examples of *ad hominem* and arguments from authority in a section on fallacies arising from "the objects themselves", but the connection is their view that such fallacies have as their root a reliance on appearance. While the Cartesians were always on guard against taking features of appearance (such as color, heat, etc.) to be features of genuine objects, this is not the concern of this section, as this problem was addressed in the first section, on ideas. The "object" of concern in most of this section appears to be the very words or circumstances of an argument or statement. Thus the manner of presentation of something and who is presenting it, will influence a mind into thinking the statement true. These are the "objects" whose appearances are in question. Authority and *ad hominem* are thus treated in a like manner, and not as a way of countering an argument but as a way people are lead astray by appearance:

> It isn't that anyone expressly reasons this way: He has a hundred thousand pounds income, therefore he is right: he is of noble birth, therefore what he states must be believed as true: this is a man with no wealth, therefore he is wrong: nevertheless something like this occurs in the mind of most people and influences their judgment without them being aware of it. (284)

We can perhaps characterize the concern here not with simply taking someone who is not an authority as an authority, but the concern is with what we might call a circumstantial *ad verecundiam* to complement the circumstantial *ad hominem*.[29]

10 METHOD

The discussion of method in the *Logic* is a curious amalgamation of original material and material taken from various works by Descartes and Pascal. To logicians

[29] There is also a very brief discussion of misplaced authority. The passage quoted here actually is classified as a fallacy of manner. [Woods, 2000, 40–41] points out that Locke characterized the reasoning here as *ad verecundiam*, although he did not also classify it as a fallacy.

of the twenty-first century the concern with ordering ideas and judgments so as to obtain systematic knowledge will appear to be a subject better suited to epistemology than logic. But if we remember that for Arnauld and Nicole logic is to give rules for all the operations of the mind with the goal of discerning the true from the false, the inclusion of this material makes sense. The part on method includes a criticism of skepticism, a discussion of the Cartesian methods of analysis and synthesis, and toward the end a discussion of reasoning from less than perfect information, which includes one of the very early discussions of probability.

With respect to the method of analysis, or resolution, Arnauld and Nicole borrow heavily from Descartes, importing directly Rule 13 from Descartes' *Rules for the Direction of the Mind*, which gave examples of the method such as solving the problem of how a clever statue of Tantalus could have been constructed so that the water from the cup spilt as soon as it reached the lips of the figure. Analysis is best seen as a method of discovery, starting from a problem and proceeding from the particular problem to the more general principles. This is the method Descartes himself advocated and used in the *Meditations*.[30] The four rules given in the Port-Royal *Logic* for proper conduct of this method are taken from Descartes' *Discourse on Method* (306). The method of synthesis or composition Descartes thought was ill suited to metaphysics, since he thought the first principles were the hardest things to come by epistemologically. Synthesis does not yield first principles, but demonstrates conclusions from them. Descartes saw analysis, though, as a method of discovery of these principles. He did state that synthesis may be "suitable to deploy in geometry as a follow-up to analysis", [Adam and Tannery, 1964-76, VII, 156; Cottingham *et al.*, 1985-1991, II, 111] but in his own *Geometry*, he did not give axioms and definitions and then proceed to prove theorems, but approached particular problems using the method of analysis.

In the Port-Royal *Logic*, though, synthesis is considered the "most important method" as it is used "to explain all the sciences" (306). The rules given for proper reasoning and ordering apply to this method, rather than analysis. There is also a discussion of the dangers of synthesis, a discussion which mirrors earlier discussions of problems occurring with respect to the activities of clarifying ideas, forming judgements, and reasoning. Arnauld and Nicole begin by giving five rules, two for clarifying terms, involving definitions, one for axioms, and two for demonstrations (308). These three groups correspond to the three first parts of the *Logic*. After a further discussion of the rules and of the dangers of the method, they conclude with a list of eight rules, 2 for ideas, two for axioms, two for arguments, and two for method. Here is the list:

For definitions:

1. Leave no term even slightly obscure or equivocal undefined.

2. In definition use only those terms perfectly known or already explained.

[30] Analysis is praised by Descartes as being "the best and truest method of instruction" [Cottingham *et al.*, 1985-1991, II, 111; Adam and Tannery, 1964-76, VII, 156].

For axioms:

3. Require in axioms only those things which are perfectly evident.

4. Accept as evident that which needs only a little attention to be recognized as true.

For demonstrations:

5. Prove all propositions which are even a little obscure, employing in the proofs only prior definitionsm axioms agreed upon, or propositions which have already been demonstrated.

6. Never exploit the equivocation of terms by failing to substitute mentally the definitions which restrict them and explicate them.

For method:

7. Treat things, as much as possible, in their natural order, beginning with the most general and simplest and explaining everything that belongs to the nature of the genus before going on to the particular species.

8. Divide, as much as possible, each genus into all its species, each whole into all its parts and each difficulty into all its cases. (334)

The major difference between this set of rules and that given for the method of analysis simply involves the rules for demonstrations, requiring the explicit use of definitions and axioms. The rules for method closely follow remarks Descartes himself made with respect to analysis, and the rules for definitions and axioms can be seen as a further explication of Descartes' first rule in the *Discourse on Method*.

Given the general claim that we need no rules for thinking, these rules are cautionary rules about not paying enough care. This final statement of the rules is formulated after a discussion of the defects, or flaws, of the geometers' method. These defects are not said to turn the arguments away from their purpose, but instead make arguments circuitous and therefore less likely to lead to understanding. The list includes, among other things, proving things that do not need to be proven, focusing on conviction rather than understanding, and proving things by *reductio ad absurdum* (*démonstrations par l'impossible*). This last is criticized for showing that something is the case without showing why it is so. Arnauld and Nicole criticize Euclid for using this method of demonstration when things could be proved positively, but do leave the suggestion that there are things that can be proved only by this means (329).

11 PROBABILITY AND CONTINGENT REASONING

At the end of the fourth part of the *Logic*, there is an interesting discussion of reasoning about contingent things in a setting of imperfect knowledge. This section

contains one of the first discussions of probability involving explicit measuring of odds.[31] There is an awareness that the methods of analysis and synthesis as set out are designed for inquiry into necessary truths, those concerning geometry and other true and immutable essences. These are contrasted with those which "concern existing things, and above all human and contingent events" (339). These include future contingencies and things which may or may not have happened in the past. It is clear that these things cannot be decided by a mere contemplation of a clear and distinct perception, but require weighing of probabilities with respect to the accompanying circumstances, including human testimony. The circumstances which belong to the fact itself are called "internal" (340) and those that concern "the people whose testimony has lead us to believe it" are called "external".

Arnauld and Nicole apply their account of reasoning in less than ideal circumstances to such questions as the belief in particular miracles and the question of whether one should doubt the correctness of the date of a contract signed by two notaries. In this discussion they give as background information that out of 1000 contracts so signed, 999 are not antedated. Given this background information, it could be taken as "very certain" that a contract so signed would not be antedated. But the authors go on to point out that this certainty would be undercut by further information, about the two notaries, for example (350). There is, then, a sensitivity to the issues concerning the probability of something on the basis of evidence.

This discussion ultimately leads the authors to questions of probability, using games of chance as a model. Here they talk about the rationality of participating in certain kinds of games of chance. The rudiments of a foundation for a probability calculus are given very briefly here:

> There are games where ten people each put an ecu, there is only one who wins the whole and the others lose. Thus each has the risk of losing one ecu and can win nine. If one considers only the gain and loss itself it would seem that each has an advantage. But it is necessary to consider as well that if each could win nine ecus and only lose one, it is nine times more probable with respect to each that he will lose his ecu and not win the nine. Thus each, himself, has nine ecus to hope for and one to lose, nine degrees of probability to lose an ecu and one to win nine. This puts things in a perfect equality. (353)

The authors conclude that it is not reasonable to participate in a lottery where the manager of the lottery takes a cut (353). The conclusions from this discussion are then applied to probabilities determined not mathematically, but from past frequencies; as one example, Arnauld and Nicole argue for the irrationality of fearing thunderstorms. People are rightly impressed by the severity of being struck by lightning, but need also consider the likelihood of such an occurrence. A rational

[31] Hacking suggests that the final chapter of the Port-Royal *Logic* is the first occasion in print where probability is measured [Hacking, 1975, 77].

decision will then be to avoid small harms, which are more likely. The book concludes with an early statement of Pascal's wager argument for the rationality in believing in God.[32] For even though the above reasoning works with respect to finite things like thunderstorms, infinite goods like eternity and salvation can never be balanced against any temporal advantage (355).

This discussion of probability is indeed brief, but is important, as Ian Hacking has argued, since it contains the first clear statement of the issues and the first use of quantities of probability as a guide to decision making under imperfect knowledge.

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[32] Pascal's *Pensées* was published (posthumously) eight years after the first edition of the Port-Royal *Logic*. Pascal died in 1662. [McKenna, 1986, 204–206], suggests that the argument here was presented in a lecture given by Pascal at Port-Royal in 1658, attended by Nicole. A 1660 manuscript of the *Logic* contains the chapter on uncertain reasoning (Chapter 13), but not the final chapters which mention the games of chance and the hints of the wager argument.

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Index

Abbo of Fleury, 39, 41, 42
Abelard, 34, 45, 56, 521, 526, 528, 529, 534–538, 567, 568
Abelard, P., 157, 158, 172, 217, 350, 469, 476, 483
About Inivocation, 193
absolute concepts, 411
absolute terms, 191, 410
abstract common terms, 429
abstract intention, 378
abstract intentions, 381
abstraction, 393, 398
absurd syllogism, 117
Abvelard, 563
accusative-infinitive, 215
accusative-infinitives, 218
accusitive case, 177
actus intellectus, 102
Adam of Balsham, 142
Adams, M., 55, 265
adjectives, 174, 175
adverb, 94
affirmative, 159, 160, 164, 178, 179
affirmative proposition, 188
  particular, 188
  universal, 188
affirmative propositions, 679
Agricola, R., 625–627, 630–631, 633, 635, 637–639
Akbert the Great, 527
Al-Ghâî, 506
Alan of Lille, 519
Alassio, F., 512
Alberic of Paris, 142
Albert of Saxony, 195, 225, 261, 263, 434, 436, 445, 446, 450–452, 455, 457, 470, 485
Albert the Great, 20, 512, 517, 531, 540, 545, 546
Alcuin, 25, 29, 48
  Dialectica, 36
  On the Faith of the Holy Trinity, 24
Alexander of Aphrodisias, 9, 13, 521, 529
Alfarbi, 541
Alucin, 22, 23
ambiguity, 207
Ammonius, 9, 508, 511, 524, 525
ampliate, 419
ampliation, 417
ampliation and restriction, 202
ampliative, 416
ampliative context, 416
analogy, 366, 368, 369
analysis and synthesis, 695, 696
Andrews, R., 550
Anselm, 157
Anselm of Canterbury, 1, 27, 47, 515, 516
  De Grammatico, 47, 51–56
  De incarnatione Verbi, 56
  Monologion, 47–48, 55–56
  On Truth, 51
  Philosophical Fragments, 47–56
  Proslogion, 47–48, 56
antiqua, 404
Antolic, P., 506

appellata, 416
appellation, 188, 192
appellative terms, 416
applied modal logic, 559, 569
appropriation, 546
Apuleius, 21, 24, 28, 505
    Peri Hermeneias, 2–4, 39, 41
Aquinas, 20, 375, 507, 508, 513, 528, 531, 556, 562, 564
Aquinas, T., 351
Archytas, 9
argumentation
    types of, 633, 639
argumenti sedes, 123
Aristotle, 1, 6, 38, 83, 143, 150, 157–159, 161, 165, 166, 171, 172, 179, 181, 192, 198, 214, 218, 224, 349, 505–510, 514, 516, 523–525, 529–536, 538–545, 547, 552, 553, 558, 559, 568, 569, 670, 686, 688
    Categories, 2, 5–7, 9, 20–22, 28–30, 34, 36, 38–40, 44, 45, 55, 56
    Categories, glosses to, 37
    On Interpretation, 2, 4–6, 8, 9, 11, 14, 21, 22, 28–31, 34, 36, 38–40, 50, 56
    On Interpretation, glosses to, 37
    Organon, 610–615, 635
    Posterior Analytics, 19, 615
    Prior Analytics, 4, 20, 21
    Rhetoric, 3
    Sophistical Refutations, 20
    Topics, 3, 609, 633, 636
    commenatary on, 610
    commentary on, 615
    transations of, 614
    translations of, 609, 613
Arnauld, A., 165, 667–698
Ars Burana, 178
Ars Emmerana, 178
Ars Meliduna, 142
ascent and descent, 241–248
Ashworth, E. J., 158, 221
assent, 560–561
assertibility, 424
Augustine, 2, 5, 29, 31, 48, 359, 506, 517, 518, 520, 521, 550, 568
    On the Quantity of the Soul, 26
    On the Trinity, 24
    Ten Categories, 2, 3, 5
Averroe, 512, 515
Averroes, 506, 513, 527, 530, 540, 541, 544–546, 556, 613, 615
Avicenna, 371, 506, 513, 536, 537, 540, 541, 544

Bacon, R., 351, 353, 358, 385, 531, 532, 538, 540, 541
Balić, C., 549
Barbara, 117
Bardout, J.-C., 551
Barney, S., 233, 235
Baroco, 117
Baur, L., 523
Bazzi, P., 515
BBuridan, J., 478
Becker, O., 509
Becker-Freysing, A., 531
begins, 185, 186
belief, 561–562
belief contexts, 215
Berengar of Tours, 43–45
Biard, J., 221, 521
Bieler, L., 519
Bilingham, R., 485
Billingham, R., 439, 440, 442, 466, 471, 473, 476
bivalence, 523, 525, 528
Blackwell, R. J., 531
Blaise of Parme, 436, 447, 448
Blank, D., 508
Bobzien, S., 523, 533
Boccardo, 117
Boehner, Ph., 552
Boethius, 1, 2, 31, 34, 36, 38, 39, 48, 83, 158, 171, 192, 349, 366,

408, 505–512, 514, 516, 517, 519, 524–532, 536, 562, 568, 609, 625–626, 633, 640
Consolation of Philosophy, 7, 39, 46
Introduction to Categoriecal Syllogisms, 14–20
On Division, 14
On Hypothetical Syllogisms, 15–20
On Topical differentiae, 18–22
On the Categorical Syllogism, 18, 14−−20
Theological Treatises, 7, 30, 48
commenatary on Categories, 36
commenatry on Cicero's Topics, 40
commentaries on Isagoge and On Interpretation, 41
commentary and first commentary to On Interpretation, 30
commentary of Cicero's Topics, 20
commentary on Cicero's Topics, 18–49
commentary on the Topics, 42
commentary on Categories, 37, 51
commentry on Categories, 10, 25
first commentary on Isagoge, 12
first commentary on On Interpretation, 11, 23–25
first commentry on Porphyry's Isagoge, 7
first commentry on On Interpretation, 25
logical monograph, 38–40, 42, 56
second commentary on Isagoge, 12
second commentary on On Interpretation, 38, 45
Boethius of Dacia, 353
Boh, I., 552, 560, 561
Bok, N. W. den, 549
Boradie, A., 561
Borgnet, A., 531
Bos, E. P., 249, 534
Braakhuis, H. A. G., 506, 535, 545, 554
Bradwardine, T., 433, 439
Brandt, S., 505
Brinkley, R., 442, 467, 485
Brower, J. E., 529
Brown, S., 552, 554
Brown, St., 560
Buchheim, T., 506, 530, 538
Buchner, H., 518
Bullough, D., 25
Buridan, J., 171, 173–176, 178–181, 188, 191, 193–195, 198, 204, 206, 207, 209, 211, 215, 218, 219, 221, 225, 241–244, 247, 249–255, 257, 259, 261, 263, 266, 269, 270, 358, 389, 403, 433, 434, 444, 445, 448–452, 458–463, 469, 470, 472, 473, 475, 476, 481, 484, 528, 553–560, 562, 569, 617, 623
Burley, W., 195, 202, 220, 225, 241–244, 250, 258, 261, 263, 399, 403, 407, 408, 436–438, 440, 443, 450, 458, 467, 473, 479, 480, 482, 484–490, 492–494, 496, 498, 552, 562, 567
Burnett, Ch., 505, 506
Buroker, J., 667, 675, 680
Burridan, J., 436
Buser, W., 446, 485
Busse, A., 508

Campsall, R., 542–544, 546–548, 553, 554
Candidus Wizo, 25
Cantin, A., 518
Caramello, P., 512, 515
Carrier, J., 545
case, 190

Cassiodorus, 1, 2, 5, 22, 25
Instiutions, 21
Catach-Rosier, I., 521
categorematic, 159
categorematic concept, 414
categorematic terms, 409
categorical proposition, 177
categorical propositions, 159–170
categorical syllogism, 21, 86
rules for, 686–689
categorical syllogisms, 84, 166, 685–689
categoricals, 185
causals, 171
cause
contingent, 515
higher, 518–519
lower, 518–519
necessary, 514–515
causes of distribution and confusion, 233
causes of the modes, 250–253, 269–270, 272–273
ceases, 185, 186
Charlemagne, 22, 23, 27
Charles the Bald, 25
Chartres, 38
Chatton, W., 384
Cicero, 1, 6, 40, 158, 505
Topics, 2–5, 17, 21, 22, 39, 42
Claudianus Mamertus, 25
Clichtoveus, J., 632, 640
Coimbra comentators, 635
Coimbra commentators, 610–614, 624, 630, 636
common concept, 393
common nature, 396, 397, 409
common term, 160
comparative, 201
complete phrase, 126
complete significabilia, 403
complex propositions
causals, 683
conditionals, 683, 684
conjunction, 683
discretives, 683, 684
disjunction, 683, 684
exceptives, 685
exclusives, 685
exponibles, 683, 685
reduplicatives, 683
complex terms, 180
complexe significabile, 438, 444
complexe signficabilia, 404
complexe significabile, 427
composite sense, 426
compositional semantics, 404
compossibility, 551, 554, 569
compound conditional, 138
compounded and divided, 181
comprehension and extension, 672–673, 680, 681, 690
conceivable things, 204
concept, 187, 199
formal or subjective, 396, 397
objective, 396–398
concepts, 186, 190, 407
concrete intention, 378
concrete intnetions, 381
conditional, 116
conditional statements, 639–640
conditionals, 171
confessions
Augustine, 1
confused, 223–276
confusion in imaging, 90
congruity, 355
Conimbricenses, see Cimbra commentators624
conjunction, 134
conjunctions, 171
conjunctive conjunction, 134
connexivism, 136
connotation, 366, 409
connotative concepts, 411
connotative terms, 190–192, 416
consecutio, 127
conseqeunce

modal, 557
consequence, 173, 434, 467
absolute, 472, 473, 477, 478
accidental, 473, 529
ad necessarium rule, 482, 483
as-of-now, 472, 473, 477, 478, 480
definition of, 468, 471–473, 483
deontic, 563–564
different from conditionals, 471, 472, 478, 479
epistemic, 560
ex impossibili rule, 482, 483, 493
formal, 471, 473–478
material, 473–475, 477, 478
modal, 522, 552, 569
natural, 473, 529
rule of, 468
rules of, 478–482
consequences, 172, 639–642
consignification, 368
constantia, 139
Constantinople, 39
construction, 355
containment, 128
context, 149
contigency
natural, 540
contingency, 114
ad utrumlibet, 514–515, 532
alternative, 550–551
contingent matter, 507–509
indefinite, 540
indfefinite, 532
natural, 532, 546
ad utrumlibet, 555
contradictories, 162
contraposition, 178
contraries, 132, 162
convention, 359
conventional sign, 360
conversion, 173, 178, 179, 681, 682
simple, 178
conversion by contraposition, 178
conversion per accidens, 164
conversions, 164
copula, 98, 134, 159, 175, 177, 190, 401, 404, 535, 554, 678–681
correspondence condition, 424
correspondence conditions, 423, 425
Corsi, G., 525, 552
counterfactuals, 521, 530, 536, 556
Courtnay, W., 522
Craig, W. L., 520
Cum sit nostra, 199, 235
curriculum
university, 609, 619–621, 625, 627, 629, 636

Dal Pra, M., 525
Damian, P., 45, 50
On Divine Omnipotence, 45–47, 50
De Interpretatione, 179
de dicto and de re (compound and divided), 546–548, 555, 552 – –558, 567
de re, 112
de sensu, 112
De Rijk, L. M., 531
de Rijk, L. M., 506, 508, 512, 520, 532, 535, 536, 541, 549
declensions, 190
definition of truth, 422, 423, 425
Dekker, E., 549
demonstrative logic, 372
denoting phrases, 224
deontic logic, 563–567, 569
conditional obligation, 556–557
norm, 556
obligation, 565, 567
permission, 565, 567
Descartes, R., 668, 669, 672, 673, 675, 676, 695, 696
descent, 231
conjunctive, 453, 455–457
disjunctive, 453, 455, 456
nominal, 453, 455–457
propositional, 453

determinate modality, 114
determinations and explications, 673, 674, 676
determinism, 514–516, 525
diachronic, 113
diachronic (prospective) alternatives, 517, 525, 528
dialectic, 158, 633, 634
Dialectic, 23–24
Dialectica Monacensis, 225, 235
Diaectica Monacensis, 178
dialectics, 372
Dici de omni et nullo, 546–547, 558
dicta, 402
dictio, 355
dictum, 212
dictum, 104
Diels, H., 529
differentia, 123
Diodorus, 512, 513
Dionysius the Areopagite, 30
disjunction, 135
disjunctions, 171
disjunctive conjunction, 134
disjunctive syllogism, 686, 691
divided sense, 426, 427
Dod, B. G., 505, 506
Dolz, J., 622–623
domain of interpretation, 428
Dombart, B., 518
Domingo Soto, 390
Dondaine, H.-F., 532
Döring, K., 533
Dorp, J., 265, 448, 464, 465
Dreyer, M., 550, 551
Drukken, N., 559
Duin, J., 515
Dulong, M., 522
Dunne, M., 527
Dunphy, W., 515
Duns Scotus, 369, 370, 380, 385, 517, 548, 550, 552
Dutilh Novaes, C., 552

Eadmer, 51
Ebbesen, S., 506, 522, 523, 531
Ebert, T., 533
Eck, J., 614, 617, 621, 639
eliminative definition, 410
empty, 164
empty terms, 162–164, 248
entailment, 123
enthymeme, 633, 640–642
enuntiabile, 402
enuntiabile, 147
enuntiabilia, 403, 404
Enzinas, F. de, 621–623
epistemic logic, 560–563, 569
  de dicto vs. de re, 562–563
equipollence, 178
equipollences, 168–170
equivocation, 192, 193, 197, 366, 368
essentialism, 513, 521, 529–531, 545, 548, 551, 567–569
evidence, 561
exceptives, 185
Excerpta Norimbergenses, 178
Excerpta Isagogarum, 40
exigere, 127
existence, 402, 405, 409
existentia rei, 104
existential import, 178, 417, 418, 428
existential presupposition, 138, 682, 683
exponibles, 185, 186, 259
expository syllogism, 545, 558
extinctive negation, 108

fact, 105, 126
Fallacie Parvipontane, 150
Fallacie Parvipontane, 193
fallacies, 157, 238–274, 692–694
  ad hominem, 694
  ad verecundiam, 694
  false dilemma, 692
  ignoratio elenchi, 693
  non causa pro causa, 693
  subjectivism, 693

fallacy of form of expression, 224
fallacy of univocation, 150
fallacy theory, 150
falsity, 104
Ferrer, V., 447, 456, 457
Ferrybridge, R., 441, 483
fictum-theory, 408
Fidora, A., 506
figure, 126
finite, 179
first intention, 373, 376
first intentions, 376, 381, 382
Fland, R., 442, 471, 476, 485
flatus vocis, 91
Fleury, 38, 41
Fonseca, P., 629–630, 632, 639–640
foreknowledge, 515, 520
formal distinction, 406
forms, 106, 186
Frede, D., 524
free will, 515, 517–518
Frege, G., 675
Friedman, R. L., 528
future contingents, 510–511, 523, 519––528, 568
future tense, 204

Gál, G., 552, 560
Gabbay, D. M., 506
Gambatese, A., 554
Garland the Computist, 508, 529
Gaskin, R., 524
Gaunilo, 49
Geach, P., 397, 399
Gelber, H., 549
general names, 85
genitive, 182–185, 255
genitives, 209
Gerbert of Aurillac, 39, 40
    De rationale et rtione uti, 41
Gersonides, 512, 545
Gettier problem, 561
Geyer, B., 520
Gilbert of Poitiers, 34, 521–523
Gilbert of Potiers, 142
Giraldus Odonis, 443
global quantificational import, 270–272
Goris, H., 520, 522
Goris, J. M. G., 515
Gottschalk, 28
Grabmann, M., 512
Grammaire générale et raisonnée, 667, 671, 676
grammatical case, 176
Green, R., 536, 563
Green-Pedersen, N. G., 562
Green-Pedersen, N. J., 506
Gregory of Nyssa, 30
Gregory of Rimini, 403, 438, 444
Guilfoy, K., 529
Gunzo of Novara, 38

Häring, N. M., 519
Hacking, I., 697, 698
Hagen, C., 529
Hallamaa, O., 565
Helias, P., 353
Henry of Ghent, 520, 549
Henry, D. P., 55
Hervaeus Natalis, 370, 382
Heytesbury, W., 434, 439, 440
Hilpinen, R., 567
Hincmar of Rheims, 28
Hintikka, J., 509, 516, 560
Hissestte, R., 515
Holcot, R., 561, 565
Holopainen, T., 49
Honnefelder, L., 550, 551
Hubien, H., 552
Hughes, G. E., 552
Hughes, T. E., 555
humanism, 609–610
hypothetical, 170
hypothetical syllogism, 121, 469
hypothetical syllogisms, 21

Iamblichus, 9, 10
ideas, 671–674

clear and distinct, 671
identiry theory, 100
Identity theory of predication, 422
identity theory of predication, 413
immediate significata, 398, 416
immobile, 231
impersonal construction, 111
imposition, 347, 357, 360
impossibility
accidental, 512, 537
impossible hypothesis, 529–530
impossible matter, 507–509, 553–554
natural, 518, 530
impossible things, 204
Inati, S. C., 537
incomplete phrase, 102
indefinite, 160
indefinites, 94
indirect proof, 529
individualizing factors, 398
inference rules, 533
inferior to a superior, 258, 273
infinite, 179
infinitizing conversion, 177
infinitizing negation, 177–179, 254, 420
inherence theory, 100, 413
inherence theory of predication, 366
inseparable accident, 530, 529−−530
insolubilia, 434, 439, 440, 447
instantiae technique, 143
intellected intention, 373
intelligible being, 380
intelligible species, 352, 363, 380, 381
intensional transitives, 218
intensions, 404
intentio, 371
intention, 352, 370, 373, 382
intentionality, 383
intentions, 373
interjection, 94
intransitive verbs, 174, 175
intrinsic/extrinsic middles, 469, 470, 474
Introductiones Parisienses, 178
intuition and deduction, 669
invention and judgement, 625, 628, 635–637, 639
Isidore, 22, 42
Etymologies, 23
Isidore of Seville, 1, 2, 22, 36, 42
Etymologies, 22
Israel, S., 36
Iwakuma, Y., 509, 522, 523

Jackson, B. D., 505
Jacobi, K., 534–536
James of Venice, 505
Jerome, 45
John Buridan, 552, see Bridan, J.623
John Duns Scotus, 549, 568
John of Holland, 442, 447, 471, 485
John Philoponus, 521
John Scottus Eriugena, 1, 25–34, 48
Periphyseon, 28
Martianus glosses, 27, 32
Periphyseon, 35–36
Jolivet, J., 522
judgement, see invention
Jullien, V., 551
Jungius, J. (Junge), 645, 655, 656

Kalb, A., 518
Kaluza, Z., 522
Kantola, I., 562
Karger, E., 185, 192, 240, 265, 554
Kenny, A., 526
Kilvington, R., 434, 439, 440, 485
Kilwardby, R., 362, 375, 507, 529, 531, 532, 539–541, 545–547, 568
Kilwardy, R., 353
King Charles's Work against the Synod, 24
King, P., 552
Klima, G., 221, 552, 557, 562
Kluxen, W., 550
Kneale, M., 533
Kneale, W., 533

Knebel, S. K., 551, 562
Kneepkens, C. H., 506, 554
knowledge, 561–562
Knuuttila, S., 509, 516, 517, 522, 530, 551, 552, 556, 564, 565, 567
Knuuttila,S., 550
Kretzman, N., 167
Kretzmann, B., 563
Kretzmann, N., 232, 234, 258, 508, 510, 512, 514, 524, 526, 531, 535, 536, 538, 562, 563
Kukkonen, T., 506, 518

Lafleur, C., 545
Lagerlund, H., 532
Lagerlund, S., 543, 544, 547, 548, 552, 553, 555, 557–559
Lambert of Auxerre, 198, 199, 225, 227, 229–232, 512, 531, 532, 538, 539, 541
Langranc, 44
Langton, S., 564
language of logic texts, 609, 642–643
Lavenham, R., 441, 471, 476, 485
Leff, G., 347
Leftow, B., 516
Leibniz, G., 673
Leiden Voss. lat. F70, 37
Leidrad, 30
Leipold, G., 561
Lewis, N., 523, 526
li, 195
Libera, A. de, 522, 531, 532
liberal arts, 353, 372
locality, 232
logic
  art vs. science, 372
  general and special, 635
  informal, 631, 634, 638–639
  intentionalistic vs. terminist, 371
  matter vs.subject, 373
  subject, 370
logic in Eastern Europe, 435, 436, 444, 446
logic in Italy, 435, 436, 439, 441, 443, 447
Logica: Cum sit nostra, 178
Logica: Ut dicit, 178
logica nova, 609
logica vetus, 609
logical monograph, 38–42
Lohr, Ch. H., 542
Lombard, P., 518, 520
Looman-Graaskamp, A. H., 549
Lorenz, K., 506
Loux, M. J., 191, 195
Lovejoy, A. O., 509
Luscombe, D. E., 563
Lutz-Bachmann, M., 506

Macarius, 26, 27
Macrobius
  commentary on Somnium Scipionis, 36
Magee, J., 505
Maggiòlo, M., 515
Maimonides, M., 512, 513
major premise, 165
Mandonnet, P., 515
Manekin, Ch. H., 512
Mangione, C., 525, 552
Marc, P., 515
Marenbon, J., 505, 519, 520, 522, 525, 538
Marius Victorinus, 2, 4–7, 15, 17, 21, 22, 41, 531
  On Definition, 3, 5, 22, 38
  Topics, 6
Marmo, C., 530
Marrone, S. P., 551
Marsilius of Inghen, 222, 225, 249, 251, 436, 446, 447, 450, 459, 461, 462, 470, 485
Martianus Capella, 2, 3, 6, 17, 22, 26, 28, 39, 42
  De Nuptiis IV, 37
Martin of Alnwick, 466, 485
Martin, C. J., 17, 521, 529, 530, 538

material essence realism, 86
Maulevelt, T., 444, 454, 455
Maulfelt, T., 266
maximul proposition, 122
Maximus the Confessor, 30, 32
meaning
grammatical vs. semantic, 354
mecessotu
what nature demands, 521
mediate conditional, 141
medieval curriculum, 433
Meiser, C., 505
Melanchthon, P., 627, 632, 638, 640–641
Melidunenses or Robertini, 142
mental concept, 193, 219
mental imaging, 89
mental language, 189, 190, 192, 415
mental propositions, 190, 216
mental word, 198, 352
Merton College, 435, 438, 439
meta-language, 418, 424, 429
metalogic, 645, 647–649
metaphysics, 517, 521–522, 540, 542, 550
method, 638
scientific, 615–616, 638
middle term, 119, 165
Migne, J. P., 564
Migne, J.-P., 505
Mignucci, M., 508, 525, 526
Minio-Paluello, L., 38, 505, 520
minor premise, 165
mobile, 231, 232, 234, 262, 267
mobility, 232, 233, 237, 241, 249–250
modal propoistion
equipollences and oppositions, 552
structure, 553
modal proposition
de dicto and de re (compound and divided), 532–539
de dicto and de re, 520
de dicto and de re (compound and diided), 542–544
de dicto and de re (compound and divided), 512
per se vs. per accidens, 537–539
conversion, 538–544, 554, 554 − −556
determinate vs. simple, 537
equipollences and oppoistions, 555
equipollences and oppositions, 531–532, 567
modal matter, 553, 507 − −554
restricted vs. unrestricted, 554–559
singular, 555
structure, 535, 540, 558
temporal determination, 536–538
modal propositions, 211–214, 434
modal syllogism, 119
modal syllogistics, 537, 544–548, 551
modal terms
adverbial, 533–535
ampliation, 506, 536, 540–541, 546
categorematic, 535
impersonal, 524
nominal, 533–535
syncategorematic, 535
modal words, 204
modality, 231
as alterativeness, 568–569
as alternativeness, 517–518, 523, 521− −536
as alternatives, 507
diachronic, 516–517, 569
divine, 506, 518, 517 − −521, 568
extensional interpretation, 507, 507− −513
frequencey(statistical) interpretation, 543
frequency (statistical) interpretation, 509–510, 512–513, 517, 567
impersonal, 534
potency model, 513–516, 567
temporal, 516–517, 536–538

mode of signifiying, 354
mode of signifying, 353
modes of being, 355, 376
modes of equivocation, 367
modes of personal supposition, 401
modes of signifying, 368
modes of understanding, 355
modesof signifying (modi significandi), 355
Modists, 353, 376
modus essendi, 354
modus intelligendi, 354
modus significandi, 354
Mojsisch, B., 538
molecular proposition, 172
molecular propositions, 170
Mondadori, F., 528
Montani or Albricani, 142
mood, 127
Moody, E. A., 679
Moore, P. S., 522
moral logic, 645, 650, 651, 653
MS Cologne Dombibliothek 191, 37
MS Milan Ambrosiana B71, 35
MS Orleans Bib, 38
MS Paris BN 12949, 35, 36
MS Paris BN 2788, 37
MS Paris BNF n. acq. lat, 38
MS Saint Gall 274, 36
Mugnai, M., 525, 552

name, 120
natural sign, 359, 360
natural signs, 359
nature, 127
necessity, 104
  per se vs. per accidens, 512, 529, 538–540, 543–546
  conditional, 511, 528, 537
  determinate vs. simple, 537
  natural, 518
  necessary (natural) matter, 509, 507−−545
  relative, 556–557
  simple, 511, 523, 528, 537, 556–558
  temporal, 511, 556
necessity of the past, 522, 525, 527
necessity of the present, 511, 516–517, 523, 525–528, 543, 548–549
negating negation, 177, 178
negation, 414, 461–464, 675
negative, 159, 160, 164, 178, 179
negative proposition, 188
  particular, 188
  universal, 188
negative propositions, 681, 682
New Logic, 158
Nicole, P., 165, 667–698
Nielsen, L. O., 528
Nifo, A., 615, 632, 641–642
Nifo, A¿, 617
nominal definition, 191
nominal definitions, 192
nominal mode, 111
Nominales, 142
nominales, 522–523
nominalist ontology, 390
nominative, 177, 184
non-existence, 405, 409
non-extensional contexts, 197, 215–221
non-relational nouns, 183
non-thing, 217
Normore, C., 526, 551, 552
Notker Labeo, 39–40
Notker of St Gall, 39
Nuchelmans, G., 508, 522
number, 190

O'Donnell, J. ., 536
Obertello, L., 505
object language, 424, 429
object-language, 418
objective being, 380, 382
objective contents of thought, 384
obligations, 484

antiqua responsio, 440
nova responsio, 440
antiqua responsio, 485, 486, 495
casus, 487
development, 484–486
different kinds, 486
game interpretation, 486–489
game intrepretation, 496
is dynamic, 490, 491, 495, 496
nova responsio, 485, 486, 492, 495, 497
positum, 486–489, 492–496
response, 487
responses, 493, 496, 497
rules of, 486–489, 493–495
obligations logic, 506, 530, 536, 549, 562, 567, 569
oblique lgoic, 645
oblique logic, 654, 657, 663
obversion, 178, 179
Ockham, 679
Ockham, W., 645
Ockham, William of, 351
Oesterle, J. T., 507, 508, 513, 528
Old Logic, 158
omnipotence, 517–519, 521
omniscience, 519–522, 526, 550–551, 560
On Interpretation, Categories, 158
On Interpretation, 192
only, 185
ontological reduction, 392, 409
oppositeness, 104
oppositions, 161
oratio, 117

Panaccio, C., 410, 411
paradoxes of implication, 529
parasitic, 183, 184, 255, 273
parasitic terms, 176
Pariente, J.-C., 678–682, 688, 691
Paris, 41
part of speech, 355
participation, 86
participle, 97, 98, 175, 183
participles, 176, 177
particular, 160
particular negative, 178
particular negatives, 163, 222
Parva logicalia, 618
Parvipontani or Adamitae, 142
Pascal's wager, 698
Pascal, B., 694, 698
past and future tenses, 203
past tense, 204
Paul of Venice, 171, 175, 195, 208, 225, 247, 250, 251, 258, 261–263, 267, 436, 441, 442, 452, 455, 461, 466, 467, 473, 476, 477, 485, 620
Paul of Venice, 619
Pera, C., 515
Periphyseon, 28, 29
Perler, D., 512
personal constructions, 108
pertaining to the soul, 204
Pession, P. M., 514
Peter Abelard, 520, 523, 531–533
Peter Aureol, 370, 383, 385
Peter Auriol, 528
Peter Lombard, 523
Peter of Ailly, 190, 439, 447, 561
Peter of Ireland, 527
Peter of Mantua, 561
Peter of Poitiers, 519, 522, 560, 564
Peter of Spain, 159, 165, 167, 171, 175, 179, 185, 188, 198, 205, 222, 224–227, 229, 232, 235, 236, 254, 368, 506, 512, 531, 532, 541, 545, 619, 621, 639
Philoponus, 530
phrase, 98
Pinborg, J., 505, 526, 562
Pines, Sh., 512
Pinzani, R., 537
Plato
Cratylus, 9
Sophist, 14

pleonotetic, 400
Plotinus, 8, 518
Pluta, O., 538
Porphyry, 1, 5, 8, 11, 83, 505, 529, 530
  Isagoge, 2, 5–7, 12, 14, 15, 20–22, 25, 28–30, 34, 37, 38, 40, 44, 56
  Isagoge, glosses to, 36
  commentry on Categories, 10
Porretani or Gilebertini, 142
Port-Royal Logic, 667–698
possibilia, 418
possibility, 104
  possibile vs. contingens, 531–532
  abstract, 530, 556
  alternative, 509, 517–518, 530, 548, 551
  determinate vs. simple, 537
  implies actuality, 537, 543
  logical, 550–551, 569
  natural, 519, 568
  Philonian, 521
  potency, 513–516
  prospective, 517
  supranatural, 530
  synchronic alternatives, 517
  what nature permits, 521
possible things, 203, 213
possible worlds, 404, 551, 555–556
potency
  active, 514, 517
  full, 516
  improper, 516
  passive, 514
  proper, 516
  supernatural, 514
predicate, 85, 159
predicate term, 100
predication, 112
  as of now, 509
  essential (per se), 507–508, 539–541
  identity theory, 553
  impossible, 508
  natural (necessary), 508
  simple, 509
predicative force, 97
prenex forms, 271
preposition, 95
present tense, 204
Priest, G., 264
primary and secondary signification, 357
primary significate, 363
Prior Analytics, 158, 166, 172
Priscian, 199
probabilism, 562
probability, 562, 634, 641, 697, 698
promise, 220
proof of propositions, 434, 440, 442, 466
proper names, 623–624
Properties of Discourse, 235
Properties of Discourse, 225
properties of terms, 391
propoistion
  tensed, 522–523
proportion, 367
proposition, 97, 159
  as of now (simpliciter) assertoric, 509
  as of now (ut nunc) assertoric, 509, 546–548, 558
  simply (simpliciter) assertoric, 545–548, 558
  temporally assertoric, 545
  temporally definite, 522
  temporally indefinite, 509, 512, 522
  tensed, 512, 553
  truth conditions, 553
propositional modal logic, 533, 554
propositional signification, 421
propositions
  semantics of, 622–623
Prudentius of Troyes, 28
Pseudo-Scotus, 406, 477, 478, 481,

484, 552–556, 559
pseudo-Scotus, 470, 473, 476
psychologism, 670

quantification over non-existent, 417
quantification theory, 400
quantifier, 120, 413
quantifiers
special, 622
quasi res, 107
quasi-entities, 397
Quine, W. V. O., 405, 418
Quinto, R., 564
quotation, 196
quotation marks, 194, 195, 197

Radulphus Brito, 370, 377, 382
Ramée, Pierre de la, see Rmus, P.628
Ramus, P., 628, 629, 635, 637, 639
Ratio studiorum, 609, 629
ratio consignificandi, 354
Ratramnus of Corbie, 30
Read, S., 221, 264–266, 506, 530, 552
real definition, 191
real propositions, 403
realist, 186, 193
reductio ad impossibile, 558
reduction per impossibile, 167
reference, 365
Reichenau, 38
relational nouns, 183
relative clause, 182
relative clauses, 180, 209
relative pronoun, 181, 182
relevance implication, 128
Remigius of Auxerre, 28
Rescher, N., 537
restricted variables, 399, 427
restrictions, 180
restrictive relative clauses, 180, 181
Rheims, 40, 41
rhetoric, 630–631, 633, 636–638, 640–642
Ribaillier, J., 519
Richer, 40
Richter, V., 561
Rijk, L. M. De, 192, 193, 225, 261
Robert Grosseteste, 523
Robert of Melun, 142
Roscelin, 83
Roseth, R., 564–567
Rudavsky, T., 514, 520
Rudolph, U., 512
rules of inference, 256, 238−−259, 274
Russell, B., 676

Saint Anselm, 91
Sanderson, R., 628
Schabel, C., 528
Schmutz, J., 551
scientia sermocinalis, 92
scope, 108, 157, 158
Scott, T. K., 562
Scotus, 551
Scotus, J. D., 363
Seaton, W. K., 560
second adjacent, 175
second intention, 374
second intentions, 370, 375, 376, 381
secondary imposition, 362
secondary meaning, 361
secondary significate, 363
sedond intentions, 383
semantic complexity, 414
semantic definition of validity, 425
semantic signification, 357
semantic triangle, 356, 392, 407, 414
semantic values, 404
semantically complex, 415
semantics of propositions, 467
separative negation, 109
sequence of tense, 210
Serene, E., 516
sermo, 92
Seton, J., 627–628
Sharples, R. W., 517
Shiel, J., 9
Siger of Brabant, 515
sign, 359

signfication, 93
signgication, 192
significata of propositions, 401
signification, 186, 187, 349, 351, 360, 392, 397, 398
significatum, 398
signifies, 187
signify, 204, 215
signs, 624, 641
signum quantitatis, 413
Simon of Faversham, 370, 375
Simon of Tournai, 519
simple conversion, 164, 174
simple modality, 114
Simplicius, 529
simultaneous (synchronic) alternatives, 511, 516–519, 522, 527, 543, 548, 550, 568
singular, 160
Sophistical Refutations, 192
Sophismata, 619
sophismata, 498, 618–642
Sophismata, 485
sophismata, 434, 439, 448
Sophistical Refutations, 172
Sorabji, R., 508, 523
sorites, 631, 632
Soto, D. de, 621–624, 632, 634
sound, 355
Southern, R., 51
Spade, P. V., 52, 262, 266, 411, 506
Spath, R. J., 531
Speca, A., 17
species, 360
speculative grammar, 353
Spiazzi, R., 507
Spiazzi, R. M., 509, 528
Spiazzi, R. S., 515
square of opposition, 85, 110, 161, 401, 418, 427, 508, 533–535
St Gall, 35, 38, 40
state of affairs, 105, 427
status, 89, 106
stoic logic, 3, 4, 6, 15–18
Stoics, 157, 158, 171, 513
stoics, 514, 516, 523–524, 533
Street, T., 506, 512, 537, 540, 541
Strode, R., 441, 442, 471, 473, 476, 485, 560, 620
Stump, E., 167, 258, 505, 515, 535, 562
subalternations, 162
subcontraries, 162
subject, 85, 159
subject oflgoic, 382
subject term, 100
substantivated, 180
substantivated adjective, 175
substantivated adjectives, 174
superior to an inferior, 257, 273
superlative, 201
suposition
  simple, 197
suppoistion
  personal, 399
  personal determinate, 461
supposaition
  personal confused and distributive, 453
supposit, 204
supposita, 395, 402, 428
supposition, 157–276, 399, 434, 448, 449, 621–622, 625, 628, 630
  ascent and descent, 452
  common, 222
  confused distributive material, 461
  determinate, 222–276
  determinate material, 460
  discrete, 222
  discrete material, 460
  distributive, 222–276
  material, 187, 193–198, 399, 416, 449–451, 457–461
  merely confused, 222–276
  merely confused material, 460
  modes of: personal, 222–276
  modes of: the early view, 222–241

narrow distributive, 267–276
personal, 187, 193, 194, 197, 412, 449–451, 458, 459, 461
personal collective, 446, 455–457
personal confused and distributive, 456, 457, 461–465
personal determinate, 453, 455, 457, 462–465
personal merely confused, 453, 455, 457, 461–465
simple, 187, 193, 194, 202, 222, 399, 416, 450–452, 458
wide distributive, 267–276
supposition function, 400
supposition theory, 157–276, 506, 533–534, 544
suppositional
descent, 401
supposits, 189
suppositum, 398
Swyneshed, Roger, 439, 440, 485, 486, 492–498
syllables, 93
syllogism, 173, 382
forth figure of, 616
fourth figure of, 617
syllogisms, 165
types of, 632–635
syllogisms with variation in tense, 120
syllogistic, 401
syllogistic rule, 127
Synan, W. A., 542
syncategoremata, 185, 404, 415
syncategorematic, 159, 177
syncategorematic concept, 414
syncategorematic signs, 185
syncategorematic terms, 411, 461, 462, 469, 475
synchronic, 113
synonyms, 190

Tachau, K., 561
Tarski, A., 423, 424
temporal conditional, 139
temporal propositions, 171
Ten Categories, 29, 34–38, 40
tenses, 206–211
term, 90
abstract, 365
concrete, 365
terms, 190
accidental, 540–541, 544
substantial (essential, necessary), 544, 540 − −545
the matter of modal propoistions, 509
Themistius, 2, 5, 6, 18
Theodulf of Orléans, 22, 24, 25
third adjacent, 175
Thirlkel, W. E., 531
Thom, P., 537, 539–541, 544–546, 552, 559
Thomas Aquinas, 368, 370, 372, 373, 512, 515, 517, 520, 524, 527, 530, 532
Thomas of Cleves, 436, 446, 455–457
Thomas of Erfurt, 353
time, 414
timelessness, 519, 523
token-based, 425
token-based semantics, 413
Toletus, F., 613, 629, 640
topic from an equal, 131
topic from an integral whole, 130
topic from genus, 130
topic from immediates, 132
topic from opposites, 132
topical theory, 123
Topics, 469
topics, 468, 625–626, 628–629, 631, 637–638, 640
personal, 638, 641
topics from substance, 129
topid from antecedent or consequent, 131
Tours, 23
Tractatus de Univocatione Monacensis, 192
Tractatus Anagnimi, 178

transitive verbs, 176, 177, 255
translatio, 149
Treatise on Univocation, 192, 193
triangle
    semantic, 349
truth, 104, 425
truth conditions, 188
truth-value
    changing, 510, 512, 522–523, 548
    determinate (definite) vs. indeterminate (indefinite), 524–528
    lacking, 528
    unchanging, 510, 512, 522

uc nunc, 171
ultimate significata, 402
ultimate significate, 397, 416
understanding, 90
universal, 160
universal affirmative, 178
universal affirmatives, 163
universal names, 91
universal things, 396
universals, 186, 441, 449, 450, 452, 458
University
    nations, 435, 445, 446
    Oxford, 433, 435–437
    Paris, 433, 435–437, 443, 444, 448
univocation, 192

validity, 425
Valla, L., 631–632
value assignment function, 400
van der Lecq, R., 528, 552
variant interpretations of the modes, 260
Veldhuis, H., 549
Verbeke, G., 508
verbs, 177
Verification
    summary, 422
via antiqua, 389, 399, 406, 407, 412, 421, 428
via antiqua semantics, 392
via moderna, 389, 428, 429
via moderna semantics, 392
Vio Cajetan, T. de, 398, 403, 404
virtiual implication condition, 424
virtual implication, 427
vis inferentiae, 123
Vives, J. L., 208, 622
voice, 96
Von Perger, M., 31
Vos, A., 549
Vuillemin, J., 533

Wadding, L., 552
Wallies, M., 521
Warichez, J., 519
Waterlow, S., 514, 516
Wcliff, 220
whiz deletion, 181
will, 564, 567
William of Auxerre, 519
William of Champeaux, 56, 57, 83, 508, 520
William of Heytesbury, 562
William of Moerbeke, 508
William of Ockham, 20, 171, 184, 187, 188, 190–192, 194, 195, 198, 200–202, 207, 216, 221, 225, 241–244, 248, 251–253, 258–261, 263–266, 269, 270, 358, 384, 389, 408, 436, 438, 443–445, 449, 451, 458, 461, 463, 465, 467, 469, 470, 472–475, 481, 551–557, 559–561, 569
    Mental Language, 411
William of Sherwood, 169–171, 178, 181, 185, 188, 195, 197–199, 201, 225, 227, 229, 231–233, 235–237, 274, 531, 532, 538
Wippel, J. F., 520
Wodeham, A., 217, 403
Wood, R., 550, 551
Woods, J., 506, 693, 694
Wright, G. H. von, 516, 567

Wyclif, J., 441, 442, 450, 452, 467

York, Cathedral School of, 23
Yrjönsuuri, ., 549
Yrjönsuuri, M., 506, 552, 567

Zabarella, J., 613, 637
Zycha, I., 518